Aspects of Diatonic Sets
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ASPECTS OF DIATONIC SETS

John Clough

Since Forte's seminal work of 1964,[1] the literature dealing with what may be loosely described as the theory of pitch class (pc) sets has grown steadily. As was natural, given the direction of Forte's work, most of this literature has been concerned with 12-pc set theory and its applications in the analysis of "atonal" works. Although the possibility of an appropriately shaped pc set theory for the diatonic case was noticed early on, the subject received no significant attention in the literature until Regener's brief treatment in his review of Forte's later work.[2] Space is also devoted to the topic of diatonic sets in at least one recent textbook,[3] a fact which may indicate (or bring about) increasing awareness of it on the part of the theoretical community.

The present article develops diatonic pc set theory more fully, offering the necessary apparatus of abstractions, followed by two applications in musical analysis.[4]

Theory of Diatonic pc Sets

Diatonic Chord Types

As Regener has pointed out, the seven diatonic note classes A, B, C, . . . , G, irrespective of inflection by sharps and flats, may be taken as the universal set and subjected to the same formalism as the twelve chromatic pc's.[5] Thus [A, C, E], [A, C-sharp, E], and [A, C-flat, E], are all equivalent collections in such a diatonic world, in the same way that [A, C, E], [A, B-sharp, E], and [A, D-double-flat, E], are equivalent in the 12-pc world.

To regard C, C-sharp, and C-flat, as members of the same note class may seem to be counterintuitive. It implies that we recognize, for example, infinitely many "triads" which are different in structure with respect to the 12-pc universe, as equivalent structures within the diatonic universe. We can circumvent this problem for the moment by observing that the diatonic universe of seven pc's may be constructed in various other ways which permit application of the same formalism. For example, we may simply stipulate that the universe consists of the seven pc's of any chosen major scale. In this case we still admit triads of "different" structure (major, minor, diminished), but their structural equivalence is arguable with reference to a familiar musical object. The important point is that the formalism applies indifferently to any universe of seven elements which can be cyclically ordered. The manner in which actual sounded or notated pitches may be legitimately reduced to seven elements of cyclic structure (and indeed whether they may be so reduced) is an analytical question which must be answered, for any particular object of study, before the theory of sets here developed can be applied.

In keeping with the above, I make no constraints at this juncture on the rules for formation of the diatonic universe, beyond the condition that it be isomorphic to the set of integers 0, 1, 2, . . . , 6, modulo 7. For the sake of convenience and simplicity in constructing illustrations, however, I shall assume in the following discussion that the integers represent the notes of the C major scale, with C = 0.

Following Lewin, I use the term *interval* here to mean the "directed distance from one pc to another," measured in an upward direction.[6] *Interval* thus pertains to an ordered pair of pc's: the interval from C to C is zero; that from C to D is 1; from C to E is 2; . . . ; from C to B is 6. Or–numerically–from 0 to 0 is 0, from 0 to 1 is 1, etc. And, of course, from 6 to 0 is 1, from 6 to 1 is 2, etc.

Following Regener, I use the term *chord* to mean the interval structure of a set of pc's or any of its transpositions. The structure is indicated by a sequence of numbers, always adding to seven in the diatonic case, listing the intervals between successive pc's, given all pc's arranged in ascending order within an octave, with the first pc repeated at the end. Thus the set [C, E, G], or [0, 2, 4], forms the chord (223). (The chord notation is always enclosed in parentheses.) Since *chord*, by definition, refers to an unordered set, the rotations of (223), namely (232) and (322), designate the same chord as (223). (A rotation is constructed by transposing the first interval of the notation to the end of the sequence. Additional rotations are constructed by repeating this operation. Thus the number of distinct rotations of a chord of n pc's is less than or equal to n.)[7]

The normal form of a chord (or, more precisely, of a chord notation), for which Regener suggests (and I adopt) the term *interval normal form* (INF), is evidently still an unsettled matter. Departing here from Regener, I choose as the INF that rotation having the largest last interval, and in cases where that criterion does not yield a unique INF, the rotation having also the smallest first interval (and if a unique INF is still not found, the smallest second interval, and third interval, etc., as necessary to determine a unique INF).[8] In all that follows, the INF notation for chords will be used, except where explicitly stated otherwise.

By *interval class* I shall mean a complementary pair of intervals i and j such that $i = 7 - j$. There are thus three interval classes in the diatonic universe (excluding zero):

*ic*1 (1≡6, mod 7) — traditional 2nds and 7ths
*ic*2 (2≡5, mod 7) — traditional 3rds and 6ths
*ic*3 (3≡4, mod 7) — traditional 4ths and 5ths

The interval vector is thus a three-digit register. It is enclosed in square brackets, without internal commas (to distinguish it from a pc set). For example, (223), the triad, has the interval vector [0 2 1].

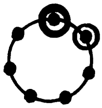
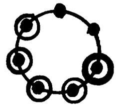
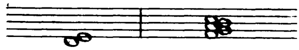
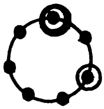
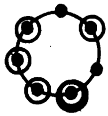
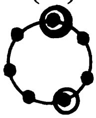
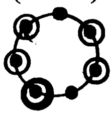
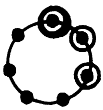
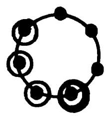
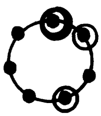
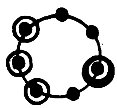
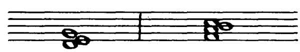
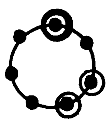
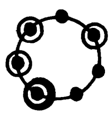
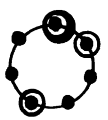
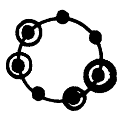
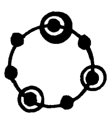
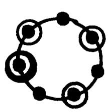
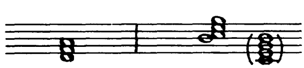
Like the question of which kinds of pitch equivalence are appropriate in the construction of a diatonic universe, the question of inversional equivalence is essentially music-analytical; the formalism is readily applicable whether or not inversion is a recognized operation. For the diatonic case, there is no great difference between the chord repertoire generated with inversional equivalence and that generated without it: there are just four diatonic chords whose inversions differ from the original chord, namely (124) and its complement (2113), and their inversions (214) and (1123). All of these are treated as distinct chords in the enumeration to follow.

(To notate the inversion of a chord, simply reverse the sequence of intervals. For example, (124) inverted is (421), the INF of which is (214), a different chord; (223) inverted is (322), the INF of which is again (223), the same chord.)

With few exceptions, diatonic chords can be generated by superimposition of a single interval. The exceptional cases are the aforementioned (124) and its inversion (214), which are all-interval trichords of content [1 1 1], and their complements, of content [2 2 2].

Table 1 shows the complete roster of diatonic chords of two through five pc's, with interval vectors and generators. Complementary chord pairs are placed opposite one another on the same line. Here and in the

Table 1. Diatonic Chords of 2-5 pc's

							generator
	(16)		(11114)		[100]	[433]	1
	(25)		(11122)		[010]	[343]	2
	(34)		(11212)		[001]	[334]	3
	(115)		(1114)		[210]	[321]	1
	(124)		(2113)		[111]	[222]	—
	(214)		(1123)		[111]	[222]	—
	(133)		(1213)		[102]	[213]	3
	(223)		(1222)		[021]	[132]	2

following tables, an abstract example of each pc set is shown on the staff—abstract in the sense that, since no clef is given, the notes display the normal form of the set without reference to specific pc's. In Table 1, sets are also shown in "clock" notation: dots on the circumference of a larger circle represent pc's (clockwise is "up"). Circled dots indicate, by their relative locations, the structure of a chord. A heavy circle around a dot indicates a pc from which the INF can be counted in a clockwise direction.

Ordered Trichords

As shown in Table 2, each of the five unordered diatonic trichords corresponds to six distinct ordered paris of intervals. These can be generated simply by writing down an ordered set of three pc's corresponding to each of the five trichords, permuting the order of the pc's, and computing the resulting pairs of intervals.[9] An alternate way of generating the 30 ordered trichords is to note the 6 X 6 = 36 ordered pairs of intervals (the members of an ordered pair may be identical). Six of these pairs will produce a redundant third pc. Therefore we have again (6 X 6) - 6 = 30 ordered trichords by this reckoning.

Ordered Source Trichords

Applying the classical operators T, I, R, RI, over the 30 ordered trichords, reduces this collection to nine ordered source trichords. Labelled *a* through *i*, these source trichords correspond to the five unordered trichords and to the 30 ordered trichords as shown in Table 2 and summarized in Table 3.

Trichord Chains

Consider a succession of pc's in which all of the ordered trichords beginning in odd order positions are equivalent (it follows that all of the ordered trichords beginning in even order positions are also equivalent). Such a succession will be known here as a trichord *chain*. As shown in Table 4, each of the nine ordered source trichords can generate two distinct chains (one for the P or RI form, and one for the I or R form). Alternatively, we can think of each chain as characterized by an unordered pair of (possibly identical) intervals (excluding zero) and compute 6 + 5 + 4 + 3 + 2 + 1 = 21. However, three pairs produce a redundant third pc. There are therefore 21 - 3 = 18 distinct trichord chains by this reckoning also.

In the case of Table 4, it is particularly necessary to remember that the given staff notation models only one of many possible realizations which represent each chain of pc's. Furthermore, Table 4 gives no indication of the complete "cycle" of each chain (the number of pc's before the entire chain begins to repeat). For those chains having two

Table 2. Ordered Diatonic Trichords

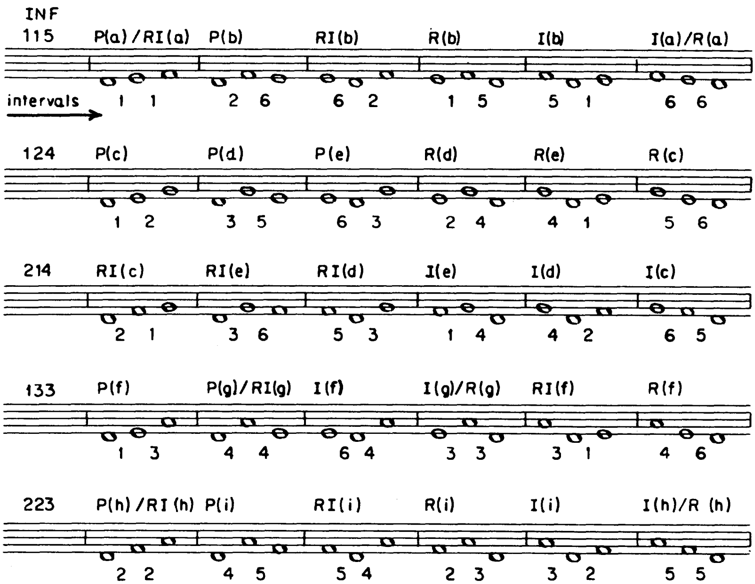


Table 3

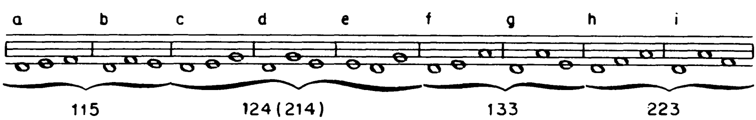
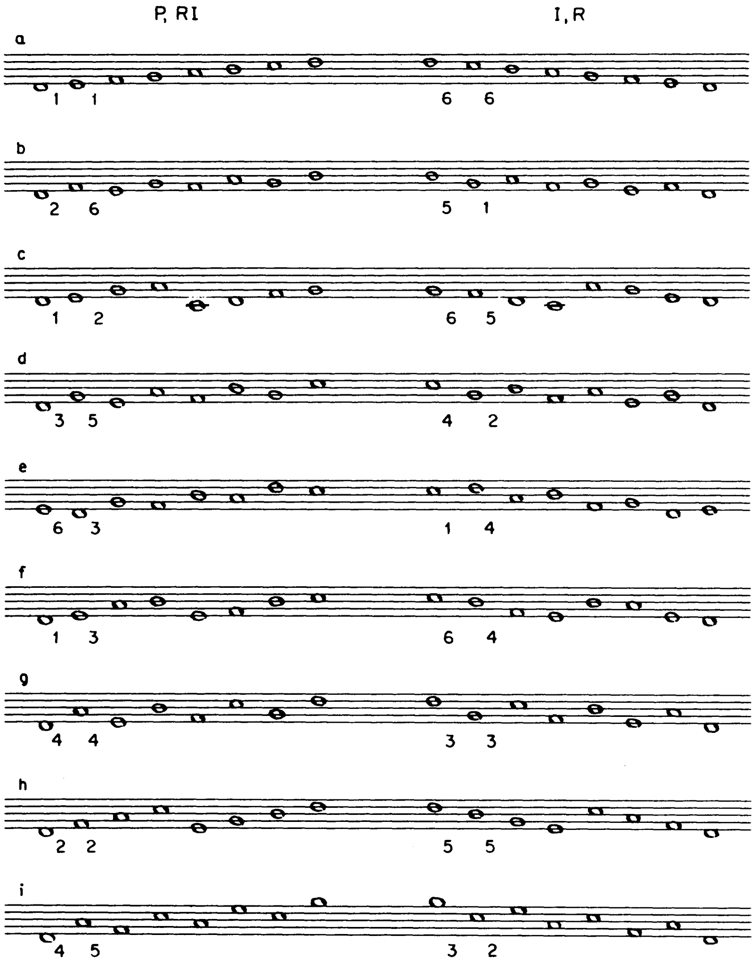


Table 4. Diatonic Trichord Chains



equal intervals in the generating pair, the complete cycle is seven pc's; for other chains it is 14 pc's.

Obviously, trichord chains may also be conceived as successions of equivalent ordered dichords, articulated by equal intervals, or (as suggested above) as repetitions of a particular interval pair. Certain registral deployments of pc's may bring about strong tetrachordal groupings, as reflected in the notation of several chains in Table 4. Consider, for example, chain *h* as an incipient model of the opening measures of Brahms's Fourth Symphony. (The reader will have no difficulty finding other such models in Table 4.)

Analytical Applications

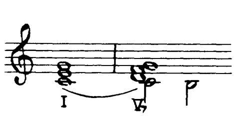
A striking feature of the repertoire of diatonic chords is its smallness. If transpositional but not inversional equivalence is recognized, it consists of just 20 chords (16 as represented in Table 1, plus one chord each of zero, one, six, and seven pc's). By comparison, the 12-pc repertoire is bewilderingly large–352 chords![10]

This compactness of vocabulary carries with it the potential for relatively simple cognitive processing. To the Western musician, an aural or visual scanning of the diatonic chord repertoire is a perusal of *familiar* objects. We have heard and seen them all countless times; we *know* them; we are therefore prepared to attribute a variety of significations to them. Curiously, however, there is in traditional analysis an unyielding disparity between two classes of diatonic trichords and tetrachords: tertian chords and scale segments may have structural significance, while others may not.

Diatonic chords which are neither tertian nor scale segments are, of course, thoroughly explored in classical theory. But they are understood only in their local significance as they relate to tertian chords through a repertoire of linear types, i.e., passing tone, appoggiatura, etc. While not disavowing the contrapuntal and decorative functions of such chords, I suggest that they also serve structural functions at various levels, and offer two studies below in support of this claim.

Mozart's *Jupiter* Symphony, first movement

When $\hat{1}$ is substituted for $\hat{7}$ in V_7, the tetrachord (1213) results, as shown in Example 1. This ubiquitous sonority, familiar to all tonal musicians in its manifold contrapuntal contexts, is so powerful in its suggestion of V -I polarity that, even when presented completely out of context, it tends to be felt as a tonal phenomenon.[11] The opening measures of Mozart's Jupiter Symphony display horizontal projections of this tetrachord, set against a temporal hierarchy of I and V in unusual ways.



(1213) (1222)

Example 1

Example 2. Mozart's *Jupiter* Symphony, first movement

Example 3

In the analysis given in Example 2, the familiar symmetrical pattern I V/V I appears at level 2 in mm. 1-8. However, textural, dynamic, and linear factors articulate the beginnings of mm. 1, 5, and 9, so that at level 1 the onset of V occurs not in m. 3, but in m. 5, equispaced between m. 1 and m. 9. In mm. 3-4 and 7-8, there are elaborations of V and I, respectively (level 3 in Example 1); it is here that the horizontal projections of (1213) are found, to which we come presently.

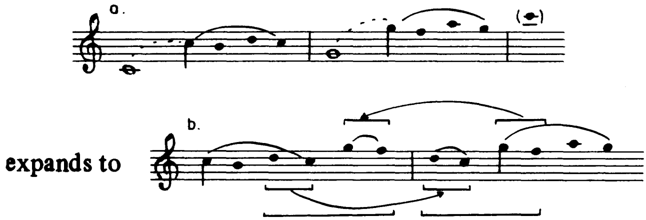
It is tempting to read the melody of mm. 7-8 as a transposition of mm. 3-4 with G–F substituted for E–D (Example 3). There is a better explanation, however: in Example 4 a generative scheme for the melody is proposed. The initial tonic and dominant of mm. 1-2 and 5-6, announced in octaves, are each continued by a harmonized encirclement figure (Example 4a). The encirclement figures are then linked as shown by the small brackets in Example 4b, the tail dyad of the first figure being interpolated before the second figure, and the head dyad of the second figure being interpolated after the first figure. This generates all of the melodic pitches of mm. 3-4 and 7-8 in the correct order. As shown by the large brackets of Example 4b, pitches 3 through 6 are identical to pitches 7 through 10.

As a result of this linking process, the original encirclement figures are located at relatively different temporal positions within their respective two-measure time spans. Interestingly, however, both occur within the V-I segments of the level 3 elaborations over the same bass pitches, and, collectively, they exhaust the traditional repertoire of possible two against one, strong-to-weak contrapuntal configurations with upper voice in stepwise descent against stationary lower voice (Example 5).

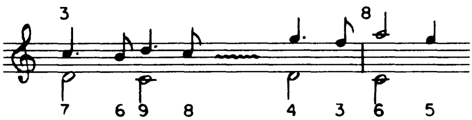
We are now ready to observe the instances of the (1213) tetrachord. In mm. 3-4, the tetrachord [C, D, F, G] is the initial verticality. A tetrachord of identical pc content is also projected by the attacks on each beat of the melody in those measures (Example 6).

As the slurs in Example 4b suggest, the 12 pitches of mm. 3-4 and 7-8 exhibit 4 + 2/2 + 4 symmetry. This is shown more clearly in Example 7. Deleting the rhythmically weak notes of the encirclement figures, we obtain the series of eight pitches in Example 8, which now contains the strong notes of the encirclement figures (tetrachord A of Example 8) together with the "linking" dyads (tetrachord B). Tetrachords C and D are wholly within mm. 3-4 and 7-8, respectively. All of these four tetrachords are of structure (1213) and, because of the generation process, we have the identities of pc content (though not of order) $A = D$ and $B = C$. (Note that in this context, tetrachords C and D are isolated by a different set of criteria than those exercised for the isolation of the melodic [C, D, F, G] in the case of Example 6.)

(If only the initial pitch of each weak-strong dyad is considered,



Example 4

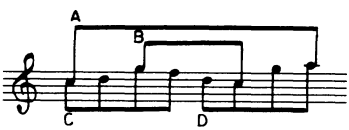


Example 5

[C, D, F, G] = (1213)

Example 6

Example 7



Example 8

(133) (133)

Example 9

then mm. 3-4 and 7-8 reduce to two trichords of structure (133) [Example 9]. This result is less rich in relationships, but, significantly, it still contains chords which are non-tertian and non-scale-segment.)

My claim here is that, while an explanation of horizontal (1213) tetrachords in mm. 1-8 partly in terms of tertian structures is pertinent (indeed quite apparent in the [C, D, F, G] of mm. 3-4, as explained above), the various manifestations of (1213) argue for the conception of that chord's role as truly structural within the context of this time span.

Beethoven's Bagatelle, Opus 119/1

The design of this Bagatelle (the complete score is given in Example 10) may be understood as the initialization, interruption, resumption, development, and completion of a double trichord chain based on the interval pair 4 1. That pair is associated with the non-tertian, non-scale-segment (124) trichord and its inversion (214) (see Table 4, item e).[12] Mm. 1-4 initialize the pattern of parallel thirds descending through the interval of a fourth, followed by a transposition of that material at the lower third. This much is shown in Example 11a and, with passing tones deleted, in Example 11b. As indicated by the arrow in Example 11a, the upper voice of the second of these two descents imitates the lower voice of the first descent at the upper octave. This suggests an ordering of the fourth descents as in Example 11c, with the middle one as a common cell between the two voices, all of which projects a short trichord chain with the interval series 4 1 4 1 4.

This short chain already contains a repetition of the initial pc (D). Since, however, the chain has a complete cycle of 14 pc's (Example 12), repetition of the pc series is not threatened at this point.

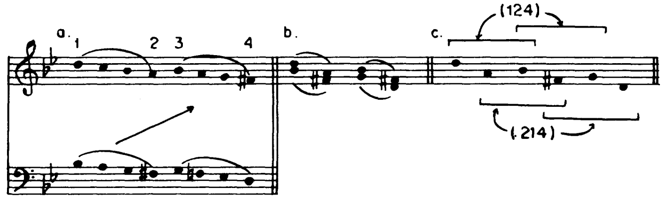
The remainder of the first section (mm. 5-16) produces restatements of the same chain segment but no continuation. Example 13 presents mm. 5-8 stripped of several embellishing notes and shows melodic relationships of those measures in correspondence to mm. 1-4. Here a different fourth descent (D to A, labelled cell *a* in Example 13) is treated imitatively, and the accompanying material (cell *b*) completes the inverted counterpoint of mm. 5-6. The upper voice of mm. 6-8 constitutes a compressed restatement of mm. 1-4, upper voice, set against a bass configured similarly to cell *b*. Mm. 9-16 repeat mm. 1-8, except for the tonic cadence in m. 16.

The impressive E-flat of m. 17 continues the trichord chain. It is the next member of the series established in mm. 1-4 (Example 11c), and by supplying the lower third of the isolated G of m. 16, continues the pattern of parallel thirds (Example 14, mm. 1-17). Mm. 17-32 may be regarded as a prolongation of [E-flat, G].

The retransition of mm. 33-36, the varied repetition of the first

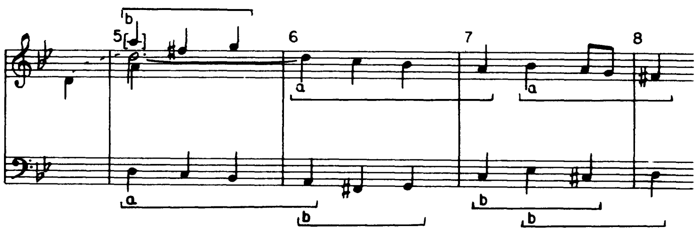
Example 10. Beethoven, Opus 119, no. 1

Example 10 (continued)

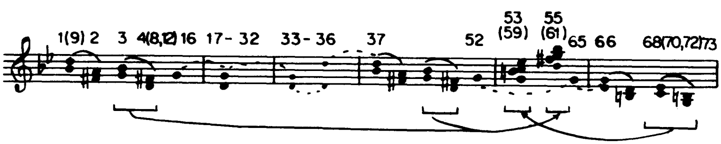


Example 11

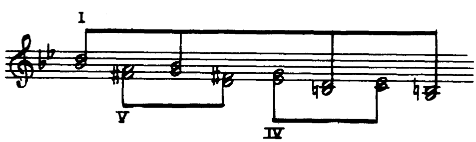
Example 12



Example 13



Example 14



Example 15

section in mm. 37-52, and the first coda in mm. 53-65, to which we return presently, all forego continuation of the trichord chain. That function is reserved for the second coda, mm. 66-74. These measures contain a version of mm. 1-4, transposed as though to C minor, but perceived as a plagal cadence. Continuing the trichord chain, they recall the long-abandoned [E-flat, G] of mm. 17-32, and add three more thirds in parallel motion, concluding on [G, B] (Example 14, mm. 66-74).

The figural material of the first coda (mm. 53-65) may now be understood as a development of the thirds of mm. 3-4 and 68-69, as indicated in Example 14. In re-asserting the [G, B-flat] and [D, F-sharp] of mm. 3-4, and isolating the G as in m. 16, mm. 61-65 prepare the return of [E-flat, G] in m. 66, in similar fashion to the preparation of its original appearance in m. 17.

Example 15, a reduction of Example 14, presents a summary of the work as a double trichord chain. Note the phase relationship between the two chains: the lower voice "leads" and the upper voice "follows" by a "time" interval of two notes. This characteristic is manifest contrapuntally in the imitation in mm. 1-4, and harmonically in the functional scheme shown in Example 15.

NOTES

1. "A Theory of Set-Complexes for Music," *Journal of Music Theory*, VIII (1964), 136-83.
2. Eric Regener, "On Allen Forte's Theory of Chords," *Perspectives of New Music*, XIII/1 (1974), 191-212. A discussion of the diatonic case appears on pp. 199-201.
3. Robert Cogan and Pozzi Escot, *Sonic Design* (New York: Prentice-Hall, 1976), pp. 140-41.
4. The essential ideas of this paper have been shared with colleagues and students for roughly the past ten years, a process which has contributed greatly to their development. The material was first presented in a public forum in January, 1977, as part of the lecture series, "Current Trends in Music Theory," sponsored by the Music Department at the University of Chicago.
5. "On Allen Forte's Theory of Chords," pp. 199-200.
6. David Lewin, "Forte's Interval Vector, My Interval Function, and Regener's Common-Note Function," *Journal of Music Theory*, XXI (1977), 194-237. The definition of interval appears on p. 196.
7. Interval notation of a chord contains a built-in redundancy: given all but one of the intervals listed in the notation, the missing interval can be computed by adding the given intervals and subtracting the result from seven. Thus, it would be possible to adopt the convention of dropping the last interval from the notation without loss of information. Since each interval of the notation is an intrinsic part of the chord structure, however, it is reasonable to

retain them all in the notation. This practice has the added advantage of facilitating visualization of rotations.

8. The algorithm with which Regener computes the INF ("On Allen Forte's Theory of Chords," pp. 196–197) is simpler than that for computing the INF which corresponds to pc normal form. Regener's INF also permits a more meaningful lexical ordering of INF's. Despite these advantages, I prefer the INF given here, because of its correspondence to what would seem to be the intuitive normal form of a chord in pc notation.
9. Where the size N of the universe is non-prime, the permutations will not necessarily represent distinct ordered trichords (or tetrachords, etc.). For example, if N = 6, the six permutations of the trichord [0, 2, 4] represent only two distinct ordered trichords, corresponding to the ordered interval pairs 2 2 and 4 4.
10. A tabulation is given in George Perle, *Serial Composition and Atonality* (Berkeley, Calif.: Univ. of California Press, 1977), 109, 148-55 (4th ed.)
11. Here and in the following discussion we take the C major scale as the diatonic universe. Note that the diatonic tetrachord (1213) may correspond to any one of the following 12-pc chords: (1416), (1425), (2325), and (2415), all of which are strong in tonal suggestion. (We are concerned only with (2325) in the present context.)
12. As several analysts have pointed out, the melodic style of Beethoven is characterized by the prevalence of these trichords, and sequences based upon them. A recent exploration along these lines is Walter Schenkman's "The Tyranny of the Formula in Beethoven," *College Music Symposium,* XVIII (1978), 158-74.