RELATING MUSICAL CONTOURS:

EXTENSIONS OF A THEORY FOR CONTOUR

Elizabeth West Marvin and

Paul A. Laprade

Cognitive psychologists and music theorists have, for many years, understood that human perception of pitch cannot simply be modelled along a single continuum from low to high.[1] Thus representational models for pitch perception have been developed by psychologists to reflect a number of related dimensions,[2] among them the tendency of listeners familiar with Western tonal music to group octave-related pitches into equivalence classes. Nevertheless, in spite of this tendency, listeners are for the most part unable to recognize familiar melodies which have been distorted by octave displacement unless the melodic contour remains invariant. So important is the role of contour in the retention and recognition of well known melodies that even the size of the interval between successive pitches may be altered, and subjects will usually recognize the tune if the contour remains unaltered.[3] Further, experimentation has shown that listeners frequently confuse a fugue subject with its tonal answer—that is, they identify the two as identical on the basis of their equivalent contours and diatonic scale types, despite the fact that their pitch contents differ.[4]

By extention to a non-tonal context, we may predict that listeners will be more likely to assume that non-equivalent sets belong to the same set class if their contours are the same. In fact, W. J. Dowling and D. S. Fugitani have offered experimental justification for the premise that listeners retain brief non-tonal melodies solely in terms of their contours.[5] Thus we may surmise that given the same or similar rhythmic pattern, listeners are generally able to perceive equivalence or similarity among musical contours more easily than among pitch-class sets in melodic settings.[6] Figure 1, for example, illustrates two instances drawn from the music of Alban Berg in which melodic patterns share contour identity but not set-class identity. The melodies of Figure 1A appear about six bars apart in the second movement of the *Lyric Suite*. Surely the listener will associate these two on the basis of their identical contours and rhythmic similarity, in spite of the fact that their intervallic and pitch contents differ. The first melody is a member of the set class 10-4, while the second belongs to set class 10-3. The melody of Figure 1B, drawn from the second movement of Berg's Violin Concerto, may be divided into two parts as marked. The second unit is an intervallic expansion of the first, but may be heard as a same-contour imitation of the first. As in the previous example, each unit belongs to a different set class—the first to 4-27 and the second to 4-20.

For purposes of musical analysis and description, music theorists have also found it useful to divide musical space into a number of interrelated spaces,[7] most commonly into pitch space (a linear space of pitches which extends from the lowest audible range to the highest) and pitch-class space (a cyclical space of twelve pitch classes that assumes octave equivalence and, because of its closed group structure under transposition (addition mod-12) enables equivalence classes not possible in pitch space).[8] Recently a number of theorists have focused their attention upon the examination of another type of musical space, which has been called contour space.[9] In formulating this concept, music theorists recognize the fact that listeners may perceive similarity or equivalence among the contours of two phrases quite apart from accurately recognizing pitch or pitch-class relationships between them, as noted above. In order to reflect this aspect of musical perception in analysis, new theories for comparing contours are necessary. Criteria by which contours may be judged equivalent have already appeared in the literature in publications by Robert Morris and Michael Friedmann. This article takes Morris's contour-space equivalence relations as its point of departure, develops a prime form algorithm and table of c-space segment classes, posits similarity measurements for c-space segments and segment-classes of the same or differing cardinalities, and applies these tools in musical analysis.

Contour Equivalence. Robert Morris defines contour space (c-space) as a type of musical space "consisting of elements arranged from low to high disregarding the exact intervals between the elements."[10] These elements are termed "c-pitches" ("cps") and are "numbered in order from low to high, beginning with 0 up to n-1," where n equals the cardinality of the segment, and where the "intervallic distance between the cps is ignored and left

a. Berg: *Lyric Suite,* (mvt. II), vln. I, mm. 66–67 and 72–73

b. Berg: Violin Concerto: (mvt. II), bassoon, mm. 35–36

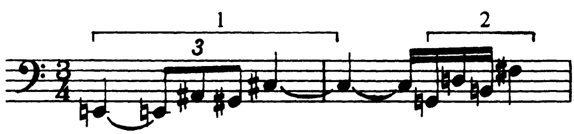


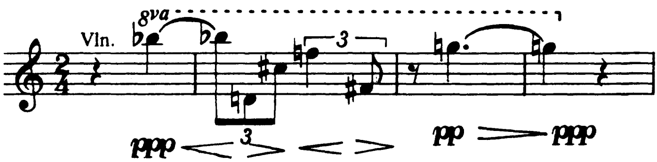
Figure 1. Same-Contour Melodies

undefined."[11] (See Glossary for definitions of technical terms.) The decision not to define the intervallic distance between c-pitches reflects a listener's ability to determine that one c-pitch is higher than, lower than, or the same as another, but not to quantify exactly how much higher or lower. In this respect, Morris's theory differs from that of Michael Friedmann. Many of the issues addressed in the latter part of Friedmann's article hinge upon the concept of contour intervals that measure the distance between c-pitches.[12] In our formulation, however, as in Morris, the intervallic distance between cps will remain undefined.

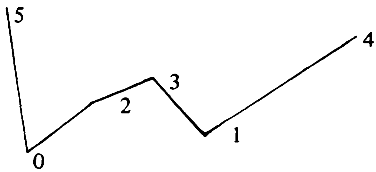
Musical contours are by definition ordered; thus, we will define a c-segment (cseg) as an ordered set of c-pitches in c-space.[13] Csegs will be labelled throughout this paper by capital letters; the cps which make up csegs will be denoted by lower-case letters. Further, we define any ordered sub-grouping of a given cseg as a c-subsegment (or csubseg). A csubseg may be comprised of either contiguous or non-contiguous c-pitches from the original cseg, as shown in Figure 2. The contour diagrams used in this figure appear throughout our discussion as graphic representations of contour shape. Such diagrams make relationships among contours fairly easy to spot visually; thus, we see that csubsegs B and C are inversionally related, while A and D appear to be equivalent contours. More formal definitions of contour equivalence, the operation of inversion, and other relations among contours follow.

We propose a "normal form" for csegs and an operation by which csegs that are not in normal form may be reduced to this form. The elements of a cseg of n distinct c-pitches are listed in normal form when the cseg's c-pitches are numbered from 0 to (n − 1) and are listed in temporal order. A csubseg's elements may retain the same numbers assigned to these cps in the original cseg, or may be renumbered through "translation." Translation is an operation through which a csubseg of n distinct c-pitches, not numbered in register from 0 to (n − 1), is renumbered from 0 for the lowest c-pitch to (n − 1) for the highest c-pitch in the csubseg, as illustrated by the asterisks in Figure 2.[14]

Morris's comparison matrix (COM-matrix) will be used to compare contours in c-space, to define equivalence relations, and to develop our similarity measurements for musical contours. The comparison matrix is a two-dimensional array which displays the results of the comparison function, COM(a,b), for any two c-pitches in c-space. If b is higher than a, the function returns "+1"; if b is the same as a, the function returns "0"; and if b is lower than a, COM(a,b) returns "−1."[15] The repeated instances of the integer "1" are omitted in the COM-matrix, as shown in Figure 3 below. Each of the matrices throughout this article, has symmetrical properties in which the diagonal of zeros from the upper left-hand to lower right-hand corner (the "main" diagonal) forms an axis of symmetry. Each value in the upper right-hand triangle is mirrored on the other side of the main diagonal



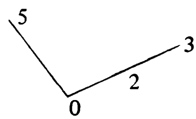
Webern, op. 10/1, mm. 7–10



= < 5 0 2 3 1 4 >

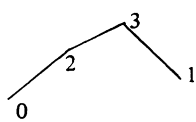
Selected csubsegs of cardinality 4:

A:



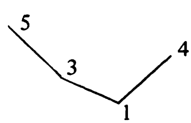
= < 5 0 2 3 > = < 3 0 1 2 >*

B:



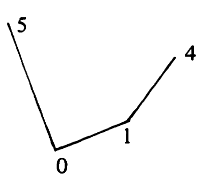
= < 0 2 3 1 >

C:



= < 5 3 1 4 > = < 3 1 0 2 >*

D:

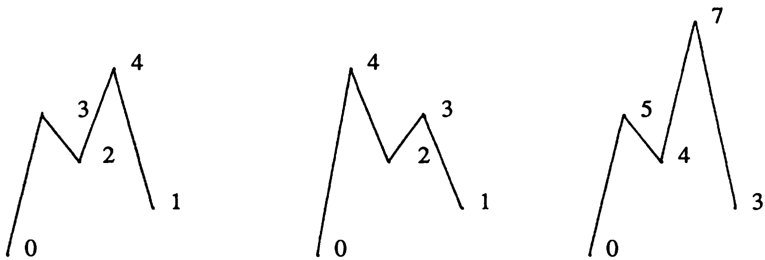


= < 5 0 1 4 > = < 3 0 1 2 >*

***Normal order by translation.**

A and B are contiguous; C and D are non-contiguous c-subsegments.

Figure 2. C-Subsegments



A = < 0 3 2 4 1 >

	0	3	2	4	1
0	0	+	+	+	+
3	-	0	-	+	-
2	-	+	0	+	-
4	-	-	-	0	-
1	-	+	+	+	0

B = < 0 4 2 3 1 >

	0	4	2	3	1
0	0	+	+	+	+
4	-	0	-	-	-
2	-	+	0	+	-
3	-	+	-	0	-
1	-	+	+	+	0

C ≐ < 0 5 4 7 3 > *

	0	5	4	7	3
0	0	+	+	+	+
5	-	0	-	+	-
4	-	+	0	+	-
7	-	-	-	0	-
3	-	+	+	+	0

Csegs A and C are equivalent because they generate identical COM-matrices.

* Normal form of < 0 5 4 7 3 > = < 0 3 2 4 1 > by translation.

Figure 3. Comparison Matrices

by its inverse. This symmetrical structure is a natural consequence of the fact that contour-pitch COM-matrices only compare a cseg with itself.

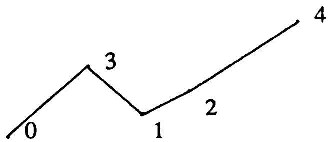
The comparison matrix provides a concise profile of a cseg's structure in much the same way as Friedmann's Contour Adjacency Series (CAS),[16] except that the COM-matrix furnishes a much more complete picture since it is not limited simply to relationships between adjacent contour pitches. Indeed, the CAS appears as a subset of the COM-matrix, as the first diagonal above and to the right of the main diagonal, as shown in Figure 4A. Each of the diagonals to the right of the main diagonal is termed INT_n,[17] where n stands for the difference between order position numbers of the two cps compared; that is, INT_4 compares cps that are four positions apart. INT_1 displays the results of the comparison function for each pair of adjacent cps as Figure 4B shows: $< + - + + >$ for the comparisons 0 to 3, 3 to 1, 1 to 2, and 2 to 4. INT_2 shows each comparison between a given c-pitch and a second cp twice removed from the first: $< + - + >$ for 0 to 1, 3 to 2, and 1 to 4. Likewise, INT_3 displays each comparison between two cps three positions apart: $< + + >$ for 0 to 2, and 3 to 4. Finally, INT_4 shows the comparison between two cps four positions apart. In this case, the predominance of pluses over minuses in each of the INTs illustrates the generally upward motion of this contour.

The information provided by the COM-matrix gives a much more accurate profile of cseg structure than INT_1 alone, since c-pitches may be compared not only consecutively, but also non-consecutively with respect to relative height. By way of example, Figure 5 contrasts several csegs which share an identical INT_1 but which differ a great deal with respect to their overall musical contours, a fact which is reflected in their respective comparison matrices.

Two contour equivalence classes based upon the comparison matrix have been proposed by Morris. The first of these is made up of all c-segments which share the same comparison matrix; thus, the first and third contours of Figure 3 preceding were equivalent csegs since they produced identical COM-matrices. Further, equivalent csegs may be reduced to the same normal order by our translation operation, as shown in Figure 3. The second contour equivalence relation, the c-space segment class (csegclass), is an equivalence class made up of all csegs related by identity, translation, retrograde, inversion, and retrograde-inversion. The inversion of a cseg P comprised of n distinct cps is written IP, and may be found by subtracting each c-pitch from $(n - 1)$.[18] The retrograde of a cseg P (written RP) or its inversion (written RIP) consists of the c-pitches in cseg P or IP in reverse order. Two csegs belonging to the same c-space segment class may be reduced to the same prime form according to the prime form algorithm we introduce below. Csegclasses, as distinct from csegs, will be labelled with underscored capital letters.

Figure 6 shows representatives of csegclass $\underline{P}$, consisting of its prime

A:



	0	3	1	2	4	
0	0	+	+	+	+	-- INT_4
3	-	0	-	-	+	-- INT_3
1	-	+	0	+	+	-- INT_2
2	-	+	-	0	+	-- INT_1
4	-	-	-	-	0	-- main diagonal

INT_1 = < + - + + > (= CAS) INT_2 = < + - + >
INT_3 = < + + > INT_4 = < + >

B:

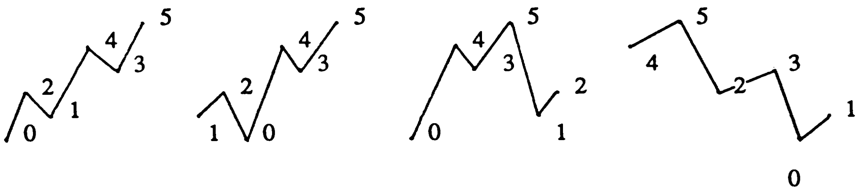
INT_1 -- <0 3 1 2 4>
+ - + +

-
INT_2 -- <0 3 1 2 4>
+ +

+
INT_3 -- <0 3 1 2 4>
+

INT_4 -- <0 3 1 2 4>
+

Figure 4. Structure of the Com-Matrix



A = < 0 2 1 4 3 5 > B = < 1 2 0 4 3 5 > C = < 0 4 3 5 1 2 > D = < 4 5 2 3 0 1 >

	0	2	1	4	3	5
0	0	+	+	+	+	+
2	-	0	-	+	+	+
1	-	+	0	+	+	+
4	-	-	-	0	-	+
3	-	-	-	+	0	+
5	-	-	-	-	-	0

	1	2	0	4	3	5
1	0	+	-	+	+	+
2	-	0	-	+	+	+
0	+	+	0	+	+	+
4	-	-	-	0	-	+
3	-	-	-	+	0	+
5	-	-	-	-	-	0

	0	4	3	5	1	2
0	0	+	+	+	+	+
4	-	0	-	+	-	-
3	-	+	0	+	-	-
5	-	-	-	0	-	-
1	-	+	+	+	0	+
2	-	+	+	+	-	0

	4	5	2	3	0	1
4	0	+	-	-	-	-
5	-	0	-	-	-	-
2	+	+	0	+	-	-
3	+	+	-	0	-	-
0	+	+	+	+	0	+
1	+	+	+	+	-	0

Each contour has INT_1 = < + - + - + >.

As shown by contour graphs, contours A and B are most similar; A and D most dissimilar.

Figure 5. Comparisons Among Selected Csegs Where INT_1 = < + − + − + >

form, < 0 1 3 2 >, together with its inversion, retrograde, and retrograde-inversion, and the COM-matrix for each. The inversion, retrograde and retrograde-inversion of a contour P are also defined by Morris in terms of specific transformations of the COM-matrix for P,[19] as illustrated in Figure 6. The COM-matrix for IP, for example, merely exchanges each "+" from the P matrix for "−" in the IP matrix and likewise exchanges "−" for "+." The matrix for RIP is related in a somewhat more abstract manner, as though the P matrix had been "flipped" around the secondary diagonal (the diagonal proceeding from the lower left-hand corner to the upper right-hand corner). Finally, the COM-matrix for RP combines both the flip and the exchange features.

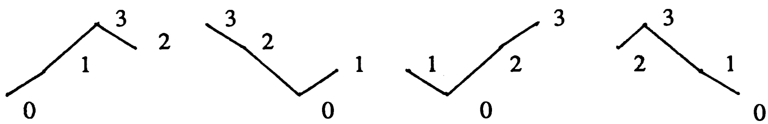
Two csegs belonging to the same c-space segment class may be reduced to the same prime form. Simply expressed, our prime form algorithm consists of three steps:

1) If necessary, translate the cseg so its content consists of integers from 0 to $(n - 1)$,
2) If $(n - 1)$ minus the last c-pitch is less than the first c-pitch, invert the cseg,
3) If the last c-pitch is less than the first c-pitch, retrograde the cseg.[20]

If for steps 2 and 3 the first and last cps are the same, compare the second and the second-to-last cps, and so on until the "tie" is broken. Figure 7 illustrates the use of this algorithm for several csegs and shows that each is a member of the same csegclass. A listing of all c-space segment classes of cardinalities 2 through 6 may be found in the Appendix to this article. We exclude larger csegs because of limitations of space.

Similarity Relations. The similarity of two csegs or csegclasses may be measured in two ways: either by comparing their structural profiles as summarized in the COM-matrix, or by examining their common csubseg structure. The first of these we will call the contour similarity function (CSIM) and the second, the contour embedding function (CEMB).[21] Both are designed to return a decimal number which approaches "1" as csegs become more similar. A function which returns the value "1" compares two equivalent csegs.[22]

The contour similarity function, CSIM(A,B), measures the degree of similarity between two csegs of the same cardinality. It compares specific positions in the upper right-hand triangle of the COM-matrix for cseg A with the corresponding positions in the matrix of cseg B in order to total the number of similarities between them.[23] For each compared position of identical content, this total is incremented by 1. Such a similarity measure, if it were simply to total the number of identical matrix positions, would not yet yield a uniform model of similarity among csegs of various



P: <0 1 3 2>   I: <3 2 0 1>   RI: <1 0 2 3>   R: <2 3 1 0>

	0	1	3	2
0	0	+	+	+
1	-	0	+	+
3	-	-	0	-
2	-	-	+	0

	3	2	0	1
3	0	-	-	-
2	+	0	-	-
0	+	+	0	+
1	+	+	-	0

	1	0	2	3
1	0	-	+	+
0	+	0	+	+
2	-	-	0	+
3	-	-	-	0

	2	3	1	0
2	0	+	-	-
3	-	0	-	-
1	+	+	0	-
0	+	+	+	0

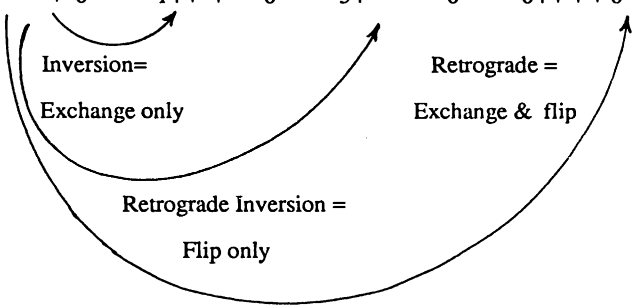


Figure 6. C-Space Segment Class < 0132 >

Csegs:	< 0 3 1 2 >	< 3 0 2 1 >	< 1 2 0 4 >	< 3 2 4 1 >
1. **TRANSLATE**, if not consecutive integers 0 to (n - 1):	OK	OK	Translated = < 1 2 0 3 >	Translated = < 2 1 3 0 >
2. **INVERT**, if (n - 1) minus last cp < first cp	OK	Inverted = < 0 3 1 2 >	Inverted = < 2 1 3 0 >	OK
3. **RETROGRADE**, if last cp < first cp	OK	OK	Retrograde = < 0 3 1 2 >	Retrograde = < 0 3 1 2 >
PRIME FORM:	< 0 3 1 2 >	< 0 3 1 2 >	< 0 3 1 2 >	< 0 3 1 2 >

All four csegs belong to the same c-space segment class.

Operations:

To translate, renumber the cseg with consecutive integers from 0 to (n - 1), where n equals the cardinality of the cseg.

To invert, subtract each cp from (n - 1).

To retrograde, place the cps in reverse order.

Figure 7. Prime Form Algorithm

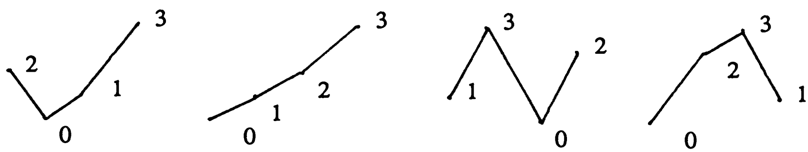
cardinalities. That is, a similarity measurement of 3 between two three-note csegs would signify a much higher degree of similarity than would a similarity measurement of 3 between two seven-note csegs.[24] In order to create a more uniform measurement, the number of identical positions will be divided by the total number of positions compared;[25] thus CSIM(A,B) will return a decimal number which signifies greater similarity between csegs as this number approaches 1. Figure 8 illustrates CSIM(A,B) for various csegs of cardinality 4. As the contour diagrams of Figure 8 show, contours A and D have an inversional relationship. They are, in fact, RI-related and are members of the same csegclass, c4-4. Our measurement CSIM(A,B) as yet accounts only for similarity between csegs, not csegclasses; thus, an extension of the similarity measurement is needed.

We define the similarity function CSIM($\underline{A}$,$\underline{B}$) to compare the similarity between two csegclasses. CSIM($\underline{A}$,$\underline{B}$) returns the largest decimal number, or 1, obtained by comparing the COM-matrix of one cseg representative of csegclass $\underline{A}$ with four cseg representatives (P, I, R and RI) of csegclass $\underline{B}$. Therefore, CSIM($\underline{A}$,$\underline{B}$) indicates the degree of highest possible similarity between two csegclasses. If the two csegs are members of the same c-space segment class, CSIM($\underline{A}$,$\underline{B}$) will return a value of "1". Figure 9 offers two examples: if we compare the csegs A: < 0 2 3 1 > and B: < 3 1 0 2 > for similarity, CSIM(A,B) accurately reflects their total dissimilarity and inversional relationship with respect to contour (CSIM(A,B) = 0), but not the fact that these csegs belong to the same c-space segment class. CSIM($\underline{A}$, $\underline{B}$), however, returns the value "1" since the two csegs are members of csegclass c4-4. In the second example of Figure 9, csegs C and D are not members of the same csegclass; CSIM($\underline{C}$,$\underline{D}$) returns the value .80.

One of the most intuitively satisfying ways of judging similarity in csegs of differing cardinalities is to count the number of times the smaller cseg is embedded in the larger.[26] We can do this in one of two ways: either by examining the two COM-matrices to determine the number of times the smaller cseg's COM-matrix is embedded in the COM-matrix of the larger cseg, or by looking at all possible csubsegs within the larger cseg and determining by translation how many are equivalent to the smaller cseg. We propose a contour embedding function (CEMB(A,B)) in which the number of times cseg A is embedded in cseg B is divided by the total number of csubsegs of the same cardinality as A possible, in order to return a value which approaches 1 for csegs of greater similarity. The formula for determining the number of m-sized subsets of an n-sized set is:[27]

$$\frac{n!}{m!\,(n-m)!}.$$

Figure 10 illustrates two rather dissimilar csegs of unequal cardinality: CEMB(A,B) = 2/20 = .10. Cseg c3-1 < 0 1 2 > is embedded only twice in cseg c6-96 < 4 5 2 3 6 1 >, as the contiguous csubset < 2 3 6 > and



A = <2 0 1 3> B = <0 1 2 3> C = <1 3 0 2> D = <0 2 3 1>

	2	0	1	3
2	0	-	-	+
0	+	0	+	+
1	+	-	0	+
3	-	-	-	0

	0	1	2	3
0	0	+	+	+
1	-	0	+	+
2	-	-	0	+
3	-	-	-	0

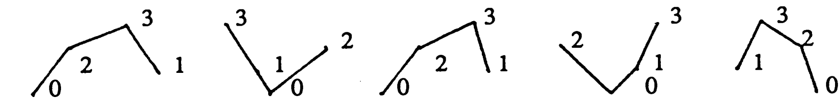
	1	3	0	2
1	0	+	-	+
3	-	0	-	-
0	+	+	0	+
2	-	+	-	0

	0	2	3	1
0	0	+	+	+
2	-	0	+	-
3	-	-	0	-
1	-	+	+	0

CSIM(A,B) = 4/6 = .67
CSIM(A,C) = 3/6 = .50
CSIM(A,D) = 2/6 = .33

Figure 8. CSIM as Similarity Measurement for Csegs of the Same Cardinality

CSIM(A, B): A = < 0 2 3 1 > B = < 3 1 0 2 >



A

	0	2	3	1
0	0	+	+	+
2	-	0	+	-
3	-	-	0	-
1	-	+	+	0

PB

	3	1	0	2
3	0	-	-	-
1	+	0	-	+
0	+	+	0	+
2	+	-	-	0

IB

	0	2	3	1
0	0	+	+	+
2	-	0	+	-
3	-	-	0	-
1	-	+	+	0

RB

	2	0	1	3
2	0	-	-	+
0	+	0	+	+
1	+	-	0	+
3	-	-	-	0

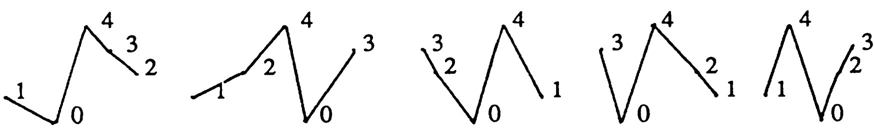
RIB

	1	3	2	0
1	0	+	+	-
3	-	0	-	-
2	-	+	0	-
0	+	+	+	0

CSIM(A, PB) = 0/6 = 0
CSIM(A, IB) = 6/6 = 1
CSIM(A, RB) = 2/6 = .33
CSIM(A,RIB) = 4/6 = .67

CSIM(A, B) = 1

CSIM(C, D): C = < 1 0 4 3 2 > D = < 1 2 4 0 3 >



C

	1	0	4	3	2
1	0	-	+	+	+
0	+	0	+	+	+
4	-	-	0	-	-
3	-	-	+	0	-
2	-	-	+	+	0

PD

	1	2	4	0	3
1	0	+	+	-	+
2	-	0	+	-	+
4	-	-	0	-	-
0	+	+	+	0	+
3	-	-	+	-	0

ID

	3	2	0	4	1
3	0	-	-	+	-
2	+	0	-	+	-
0	+	+	0	+	+
4	-	-	-	0	-
1	+	+	-	+	0

RD

	3	0	4	2	1
3	0	-	+	-	-
0	+	0	+	+	+
4	-	-	0	-	-
2	+	-	+	0	-
1	+	-	+	+	0

RID

	1	4	0	2	3
1	0	+	-	+	+
4	-	0	-	-	-
0	+	+	0	+	+
2	-	+	-	0	+
3	-	+	-	-	0

CSIM(C, PD) = 6/10 = .60
CSIM(C, ID) = 4/10 = .40
CSIM(C, RD) = 8/10 = .80
CSIM(C, RID) = 2/10 = .20

CSIM(C, D) = .80

Figure 9. CSIM for C-Space Segment Classes

the noncontiguous < 4 5 6 >. In Figure 10B, the complete matrix of cseg < 0 1 2 > is found as a contiguous subset of the large cseg's matrix, while Figure 10C shows the matrix of < 0 1 2 > embedded as a noncontiguous subset. The c-pitches associated with each position of the embedded matrix are members of the csubseg < 0 1 2 >. Note that in the noncontiguous instance, the entire structure of each embedded row and column must remain intact in order to reflect the csubseg relation accurately. It is for this reason that CEMB(A,B) must consider the structure of the embedded COM-matrix as a whole rather than the upper right-hand triangle alone. In figure 10D, the pluses of the upper right-hand triangle of the smaller cseg's matrix have been circled in non-adjacent positions of the larger cseg's matrix. If the rows and columns are not violated, the corresponding matrix entries for the main diagonal and lower left-hand triangle (indicated in the figure by squares) are incorrect for the embedded subset. Thus, the information given in the upper right-hand triangle is not alone sufficient to identify c-subsegments.

Since the embedding function checks for non-contiguous subsets as well as contiguous ones, it accounts for such instances as a contour which we perceive as generally rising, even though it also includes some descents. In Figure 11A, for example, the embedded csubseg < 0 1 2 > appears repeatedly, both as a non-contiguous and a contiguous subset of < 0 2 1 3 4 >, and its role in our perceiving this contour as an ascending line is clearly audible. As the comparison matrix and corresponding contour diagrams show, < 0 1 2 > is embedded seven times in the larger cseg. CEMB(A,B) can also be found by extracting all three-note csubsegs from the larger cseg, translating each to normal form, and counting the number of times < 0 1 2 > is found, as shown in Figure 11B.

Although the CSIM and CEMB functions provide an adequate measure of similarity between most csegs (of equal or unequal cardinality), they are not alone sufficient to describe relationships between any two csegs. For example, our embedding function only describes relationships between csegs of differing cardinalities. What of the situation in which two csegs of equal cardinality share one or more common csegs? Following Rahn's generalization of David Lewin's embedding function,[28] we propose two additional functions which count the csubsegs mutually embedded in csegs A and B. Csegs A and B may be of equal or unequal cardinality. CMEMB_n(X,A,B) counts the number of times the csegs, X (of cardinality n), are embedded in both csegs A and B. The variable "X" may successively represent more than one cseg-type during the course of the function, as shown in Figure 12. Each cseg X must be embedded at least once in both A and B; then, all instances of X are counted in both A and B. The total number of mutually-embedded csegs of cardinality n is divided by the number of n-cardinality csubsegs possible in both csegs to return a decimal number approaching 1 as the csegs A and B are more similar. Generally, CMEMB_n(X,A,B) returns

CEMB(A,B)

A = < 0 1 2 > = c3-1
B = < 4 5 2 3 6 1 > = c6-96

A - MATRIX OF c3-1:

	0	1	2
0	0	+	+
1	-	0	+
2	-	-	0

<0 1 2>

B - MATRIX OF c3-1 EMBEDDED AS CONTIGUOUS SUBSET OF c6-96:

	4	5	2	3	6	1
4	0	+	-	-	+	-
5	-	0	-	-	+	-
2	+	+	0	+	+	-
3	+	+	-	0	+	-
6	-	-	-	-	0	-
1	+	+	+	+	+	0

<2 3 6> = <0 1 2>

C - MATRIX OF c3-1 EMBEDDED AS NON-CONTIGUOUS SUBSET OF c6-96:

	4	5	2	3	6	1
4	0	+	-	-	+	-
5	-	0	-	-	+	-
2	+	+	0	+	+	-
3	+	+	-	0	+	-
6	-	-	-	-	0	-
1	+	+	+	+	+	0

<4 5 6> = <0 1 2>

D - UPPER RIGHT-HAND TRIANGLE:

	0	1	2
0	0	+	+
1	-	0	+
2	-	-	0

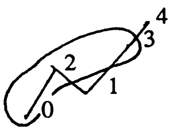
	4	5	2	3	6	1
4	$\boxed{0}$	+	-	-	+	-
5	-	0	-	-	+	-
2	$\boxed{+}$	$\boxed{+}$	0	+	+	-
3	+	+	-	0	+	-
6	$\boxed{-}$	$\boxed{-}$	-	-	$\boxed{0}$	-
1	+	+	+	+	+	0

	4	5	2	3	6	1
4	$\boxed{0}$	+	-	-	+	-
5	-	0	-	-	+	-
2	+	+	0	+	+	-
3	$\boxed{+}$	$\boxed{+}$	-	0	+	-
6	-	-	-	-	0	-
1	$\boxed{+}$	$\boxed{+}$	+	+	$\boxed{+}$	0

Figure 10. CEMB(A,B)

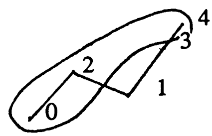
Matrix embedding: A = < 0 1 2 >   B = < 0 2 1 3 4 >

	0	1	2
0	0	+	+
1	-	0	+
2	-	-	0



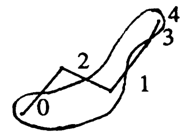
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

csubsegs: < 0 2 3 >



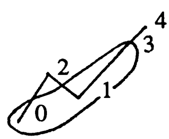
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 0 2 4 >



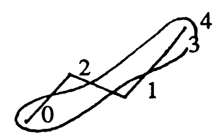
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 0 1 4 >



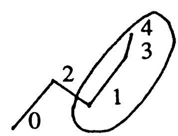
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

csubsegs: < 0 1 3 >



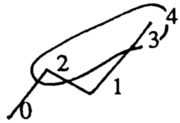
	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 0 3 4 >



	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

< 1 3 4 >



	0	2	1	3	4
0	0	+	+	+	+
2	-	0	-	+	+
1	-	+	0	+	+
3	-	-	-	0	+
4	-	-	-	-	0

csubseg: < 2 3 4 >

CEMB(A,B) - 7/10 = .70

Figure 11 A. CEMB(A,B): Additional Examples

A = < 0 1 2 >   B = < 0 2 1 3 4 >

Possible csubsegs are:

< 0 2 1 > = < 0 2 1 >
< 0 2 3 > = < 0 1 2 >*
< 0 2 4 > = < 0 1 2 >*
< 0 1 3 > = < 0 1 2 >*
< 0 1 4 > = < 0 1 2 >*
< 0 3 4 > = < 0 1 2 >*
< 2 1 3 > = < 1 0 2 >
< 2 1 4 > = < 1 0 2 >
< 2 3 4 > = < 0 1 2 >*
< 1 3 4 > = < 0 1 2 >*

* Embedded < 0 1 2 > identified by translation.

CEMB(A, B) = 7/10 = .70.

Figure 11 B. Embedded Csubsegs by Translation

Cseg A = c5-26: $\langle 1\ 0\ 4\ 3\ 2 \rangle$	**Cseg B = c5-24: $\langle 2\ 0\ 1\ 4\ 3 \rangle$**
Csubsegs of A:	**Csubsegs of B:**
$\langle 1\,0\,4\,3\,2 \rangle = \langle 1\,0\,4\,3\,2 \rangle$	$\langle 2\,0\,1\,4\,3 \rangle = \langle 2\,0\,1\,4\,3 \rangle$
$\langle 1\,0\,4\,3 \rangle = \underline{\langle 1\,0\,3\,2 \rangle}$	$\langle 2\,0\,1\,4 \rangle = \langle 2\,0\,1\,3 \rangle$
$\langle 1\,4\,3\,2 \rangle = \langle 0\,3\,2\,1 \rangle$	$\langle 2\,0\,1\,3 \rangle = \langle 2\,0\,1\,3 \rangle$
$\langle 1\,0\,3\,2 \rangle = \underline{\langle 1\,0\,3\,2 \rangle}$	$\langle 2\,0\,4\,3 \rangle = \underline{\langle 1\,0\,3\,2 \rangle}$
$\langle 1\,0\,4\,2 \rangle = \underline{\langle 1\,0\,3\,2 \rangle}$	$\langle 0\,1\,4\,3 \rangle = \langle 0\,1\,3\,2 \rangle$
$\langle 0\,4\,3\,2 \rangle = \langle 0\,3\,2\,1 \rangle$	$\langle 2\,1\,4\,3 \rangle = \underline{\langle 1\,0\,3\,2 \rangle}$

CMEMB_4(X, A, B) = 5/10= .50

$\langle 1\,0\,4 \rangle = \underline{\langle 1\,0\,2 \rangle}$	$\langle 2\,0\,1 \rangle = \langle 2\,0\,1 \rangle$
$\langle 1\,0\,3 \rangle = \underline{\langle 1\,0\,2 \rangle}$	$\langle 2\,0\,4 \rangle = \underline{\langle 1\,0\,2 \rangle}$
$\langle 1\,0\,2 \rangle = \underline{\langle 1\,0\,2 \rangle}$	$\langle 2\,0\,3 \rangle = \underline{\langle 1\,0\,2 \rangle}$
$\langle 1\,4\,3 \rangle = \underline{\langle 0\,2\,1 \rangle}$	$\langle 2\,1\,4 \rangle = \underline{\langle 1\,0\,2 \rangle}$
$\langle 1\,4\,2 \rangle = \underline{\langle 0\,2\,1 \rangle}$	$\langle 2\,1\,3 \rangle = \underline{\langle 1\,0\,2 \rangle}$
$\langle 1\,3\,2 \rangle = \underline{\langle 0\,2\,1 \rangle}$	$\langle 2\,4\,3 \rangle = \underline{\langle 0\,2\,1 \rangle}$
$\langle 0\,4\,3 \rangle = \underline{\langle 0\,2\,1 \rangle}$	$\langle 0\,1\,4 \rangle = \langle 0\,1\,2 \rangle$
$\langle 0\,4\,2 \rangle = \underline{\langle 0\,2\,1 \rangle}$	$\langle 0\,1\,3 \rangle = \langle 0\,1\,2 \rangle$
$\langle 4\,3\,2 \rangle = \langle 2\,1\,0 \rangle$	$\langle 0\,4\,3 \rangle = \underline{\langle 0\,2\,1 \rangle}$
$\langle 0\,3\,2 \rangle = \underline{\langle 0\,2\,1 \rangle}$	$\langle 1\,4\,3 \rangle = \underline{\langle 0\,2\,1 \rangle}$

CMEMB_3(X, A, B) = 16/20 = .80

Common csubsegs are underlined.

Figure 12. CMEMB_N (X, A, B)

a higher decimal number for embedded csegs of smaller cardinality since there are fewer cseg types, and therefore a higher probability of inclusion in both csegs A and B. Thus a refinement of the function is necessary.

ACMEMB(A,B) counts the total number of significant mutually-embedded csegs of cardinality 2 through the cardinality of the smaller cseg, and adjusts this to a decimal value by dividing by the total number of possible subsets of A and B (excluding the null sets for each and the one-note csubsegs).[29] Figure 13A shows the adjusted mutual embedding function for two csegs of the same cardinality, and 13B for csegs of differing cardinalities.

Finally, we generalize our embedding functions for csegclasses in much the same manner as the CSIM function. That is, CEMB($\underline{A}$,$\underline{B}$), CMEMB_n (X,$\underline{A}$,$\underline{B}$) and ACMEMB($\underline{A}$,$\underline{B}$) will compare the csubseg content of cseg A with each of the four transforms of cseg B (PB, IB, RB and RIB) and return the highest of these values. Thus, if A and B are members of the same csegclass, each of these functions will return a value of "1."

Extensions of the Theory for Context-Dependent Analysis. Up to this point, we have considered relations among contours without extensive reference to the musical contexts in which these contours appear. The application of contour theory to context-dependent analysis poses a number of problems, not the least of which is the segmentation of the music into meaningful units. Friedmann has discussed segmentation in some detail; his examples provide considerable insight into this difficult problem.[30] A second context-dependent issue with considerable theoretical ramifications is the common occurrence of repeated notes within a musical contour.[31] Consecutive repeated notes pose no problem, since they may be treated as single contour pitches, as shown in Figure 14A. We propose that csegs containing nonconsecutive repeated c-pitches be numbered in order from low to high with 0 representing the lowest pitch and $(n - 1 - r)$ the highest; repetitions of a c-pitch are represented by the same integer. Here the variable "n" stands for the cardinality of the cseg, while "r" equals the number of times any c-pitch is repeated. Thus, the contour of the melody in Figure 14B is $< 1\ 2\ 3\ 0\ 3\ 1 >$. The cardinality of the cseg is 6, cp 1 is repeated once and cp 3 is repeated once; thus the cps are numbered from 0 to 3, since $(n - 1 - r)$ equals $(6 - 1 - 2)$ or 3. Translation of a cseg including repeated notes is defined as the renumbering of the cseg with integers ranging from 0 to $(n - 1 - r)$. The inversion of a repeated-note cseg is calculated by subtracting each cp from $(n - 1 - r)$. Previously stated definitions of R and RI still hold. Our prime form algorithm also holds, although "ties" may occur more frequently (if for steps 2 and 3 the first and last cps are the same, the second and the second-to-last cps are compared, and so on until the "tie" is broken). The COM-matrices of repeated-note csegs differ from previous COM-matrices only in the fact that the repeated notes generate zeros in positions other than along the main diagonal.

A: CSEGS OF EQUAL CARDINALITY

A = <0 1 2 3> B = <0 2 1 3>

Csubsegs of A:	<0 1> = <0 1>	Csubsegs of B:	<0 2> = <0 1>
	<0 2> = <0 1>		<0 1> = <0 1>
	<0 3> = <0 1>		<0 3> = <0 1>
	<1 2> = <0 1>		<2 3> = <0 1>
	<2 3> = <0 1>		<1 3> = <0 1>
	<1 3> = <0 1>		<2 1> = <1 0>
	<0 1 2> = <0 1 2>		<0 2 1> = <0 2 1>
	<0 1 3> = <0 1 2>		<0 2 3> = <0 1 2>
	<0 2 3> = <0 1 2>		<0 1 3> = <0 1 2>
	<1 2 3> = <0 1 2>		<2 1 3> = <1 0 2>
	<0 1 2 3> = <0 1 2 3>		<0 2 1 3> = <0 2 1 3>

17 csegs mutually embedded in both csegs; **ACMEMB(A, B) = 17/22 = .77**

B: CSEGS OF UNEQUAL CARDINALITY

C = <0 2 1 3 4>

Csubsegs of C:	<0 2 1 4> = <0 2 1 3>	<0 2 1> = <0 2 1>	<0 2> = <0 1>
	<0 2 3 4> = <0 1 2 3>	<0 2 3> = <0 1 2>	<0 1> = <0 1>
	<0 1 3 4> = <0 1 2 3>	<0 2 4> = <0 1 2>	<0 3> = <0 1>
	<0 2 1 3> = <0 2 1 3>	<0 1 3> = <0 1 2>	<0 4> = <0 1>
	<2 1 3 4> = <1 0 2 3>	<0 1 4> = <0 1 2>	<2 3> = <0 1>
		<2 1 3> = <1 0 2>	<2 4> = <0 1>
		<2 1 4> = <1 0 2>	<1 3> = <0 1>
	<0 2 1 3 4> = <0 2 1 3 4>	<2 3 4> = <0 1 2>	<1 4> = <0 1>
		<1 3 4> = <0 1 2>	<3 4> = <0 1>
		<0 3 4> = <0 1 2>	<2 1> = <1 0>

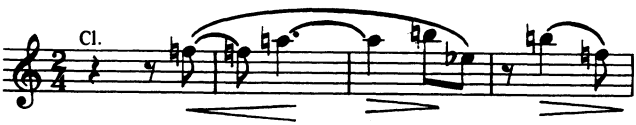
29 csegs mutually embedded in csegs A and C; **ACMEMB(A, C) = 29/37 = .78**
33 csegs mutually embedded in csegs B and C; **ACMEMB(B, C) = 33/37 = .89**

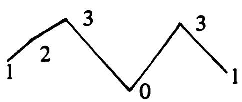
Figure 13. ACMEMB(A,B) for Sets of Equal Cardinality

A. Repeateu-Note Segments Consecutive Repeated Notes:
Webern, op. 10/1, mm. 10–11

<0 5 3 2 4 1> NOT <0 5 3 3 2 4 1>

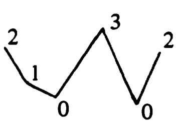
B. Repeated-Note Segments, Non-Consecutive Repeated Notes:
Webern, op. 10/1, mm. 3–6



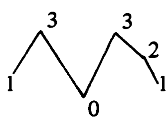


P = <1 2 3 0 3 1>

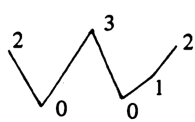
	1	2	3	0	3	1
1	0	+	+	-	+	0
2	-	0	+	-	+	-
3	-	-	0	-	0	-
0	+	+	+	0	+	+
3	-	-	0	-	0	-
1	0	+	+	-	+	0



IP = <2 1 0 3 0 2>*



RP = <1 3 0 3 2 1>



RIP = <2 0 3 0 1 2>

***To invert, each cp is subtracted from (n–l–r), where n represents the cardinality of the cseg and r is the number of times a particular cp is repeated. In this instance, r=2, since cp 1 is repeated once and cp 3 is repeated once.**

Figure 14

The csegclass name of a repeated-note cseg is a hyphenated composite label, based on the cseg's similarity to nonrepeated-note csegclasses. The cardinality of the cseg appears to the left of the hyphen. To the right of the hyphen, separated by slashes, are the ordinal numbers of two related csegclasses. The first ordinal number represents the csegclass label of some cseg whose COM-matrix is identical to that of the repeated-tone cseg except that it contains a plus in the place of each 0 in the upper right-hand triangle. The second ordinal number represents the cseg which contains a minus in each of those positions. In Figure 15A, csegclasses c5-2 and c5-4 differ from the repeated note cseg in only one position each; the composite label is rc5-2/4 ("rc" stands for "repeated-note csegclass").[32] Two repeated notes will result in two zeros in the upper right-hand triangle, as shown in Figure 15B, and so on. The CSIM function will return the same value when measured between a repeated-note cseg and the csegclasses represented in its composite label (or between those two csegclasses), since each of these csegclasses differs precisely in the positions of the COM-mtarix where a "0" appears for the repeated-note set. Therefore, the name of the repeated-note cseg allows us to generate the COM-matrix of the repeated-note cseg (and therefore the normal form of the cseg itself) merely by comparing the csegclasses in its name. Finally, our similarity and embedding functions[33] still hold for repeated-note csegs, as for nonrepeated-note csegs.

Analytical Applications. We have chosen to illustrate some analytical applications of the preceding contour theories in the first of Anton Webern's *Fünf Stücke für Orchester,* Opus 10. The movement divides into four two- and four-bar phrases—A (mm. 1–2), B (mm. 3–6), C (mm. 7–10), and D (mm. 10–11)—plus a concluding 1-bar "codetta" of a single reiterated pitch. The two central phrases are joined in an antecedent-consequent relationship. Both consist of a broad solo line played in the upper register over a sustained celesta trill. Both melodies have substantial accompaniments: a series of chords beneath the antecedent phrase, and a thicker, more contrapuntal accompaniment to the consequent. Flanking this central portion on either side are opening and closing sections of sparser texture, consisting of solo lines without accompaniment. The first and last bars of the movement feature striking instances of *Klangfarbenmelodie,* while the second and penultimate bars consist of unaccompanied solo lines on distinctive, coloristic instruments. Thus the opening and closing sections frame the central portion in a symmetrical arrangement, as shown in Figure 16.

Each of the four principal melodies forms a melodic contour of cardinality six. Yet in each case the six cps are partitioned differently in terms of rhythm, register, and/or timbre: the first as 3 | 3, the second as 4 | 2, and the third as 5 | 1. The final melody is interrupted by rests and has no change in instrumentation; thus it forms a 6 | 0 partition. Comparison of set-class membership reveals that no pair of melodies belongs to the same set class.

A: CSEG WITH ONE REPETITION

A = < 0 1 2 3 2 >

$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 2 \\ \hline 0 & 0 & + & + & + & + \\ 1 & - & 0 & + & + & + \\ 2 & - & - & 0 & + & \boxed{0} \\ 3 & - & - & - & 0 & - \\ 2 & - & - & 0 & + & 0 \end{array}$$

Csegclass label = 5-?

Related Matrices:

B = c5-2: < 0 1 2 4 3 >

$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 4 & 3 \\ \hline 0 & 0 & + & + & + & + \\ 1 & - & 0 & + & + & + \\ 2 & - & - & 0 & + & \boxed{+} \\ 4 & - & - & - & 0 & - \\ 3 & - & - & - & + & 0 \end{array}$$

C = c5-4: < 0 1 3 4 2 >

$$\begin{array}{c|ccccc} & 0 & 1 & 3 & 4 & 2 \\ \hline 0 & 0 & + & + & + & + \\ 1 & - & 0 & + & + & + \\ 3 & - & - & 0 & + & \boxed{-} \\ 4 & - & - & - & 0 & - \\ 2 & - & - & + & + & 0 \end{array}$$

Therefore: **A = rc5-2/4.**

CSIM(A,B) = CSIM(A,C) = CSIM(B,C).

B: CSEG WITH TWO REPETITIONS

D = < 1 2 3 0 3 1 >

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 0 & 3 & 1 \\ \hline 1 & 0 & + & + & - & + & \boxed{0} \\ 2 & - & 0 & + & - & + & - \\ 3 & - & - & 0 & - & \boxed{0} & - \\ 0 & + & + & + & 0 & + & + \\ 3 & - & - & 0 & - & 0 & - \\ 1 & 0 & + & + & - & + & 0 \end{array}$$

Csegclass label = 6-?

Related matrices:

E = c6-145: < 1 3 4 0 5 2 >

$$\begin{array}{c|cccccc} & 1 & 3 & 4 & 0 & 5 & 2 \\ \hline 1 & 0 & + & + & - & + & \boxed{+} \\ 3 & - & 0 & + & - & + & - \\ 4 & - & - & 0 & - & \boxed{+} & - \\ 0 & + & + & + & 0 & + & + \\ 5 & - & - & - & - & 0 & - \\ 2 & - & + & + & - & + & 0 \end{array}$$

F = c6-154: < 2 3 5 0 4 1 >

$$\begin{array}{c|cccccc} & 2 & 3 & 5 & 0 & 4 & 1 \\ \hline 2 & 0 & + & + & - & + & \boxed{-} \\ 3 & - & 0 & + & - & + & - \\ 5 & - & - & 0 & - & \boxed{-} & - \\ 0 & + & + & + & 0 & + & + \\ 4 & - & - & + & - & 0 & - \\ 1 & + & + & + & - & + & 0 \end{array}$$

Therefore: **D = rc6-145/154.**

CSIM(D,E) = CSIM(D,F) = CSIM(E,F).

Figure 15. Csegclass Labels for Repeated-Note Csegs

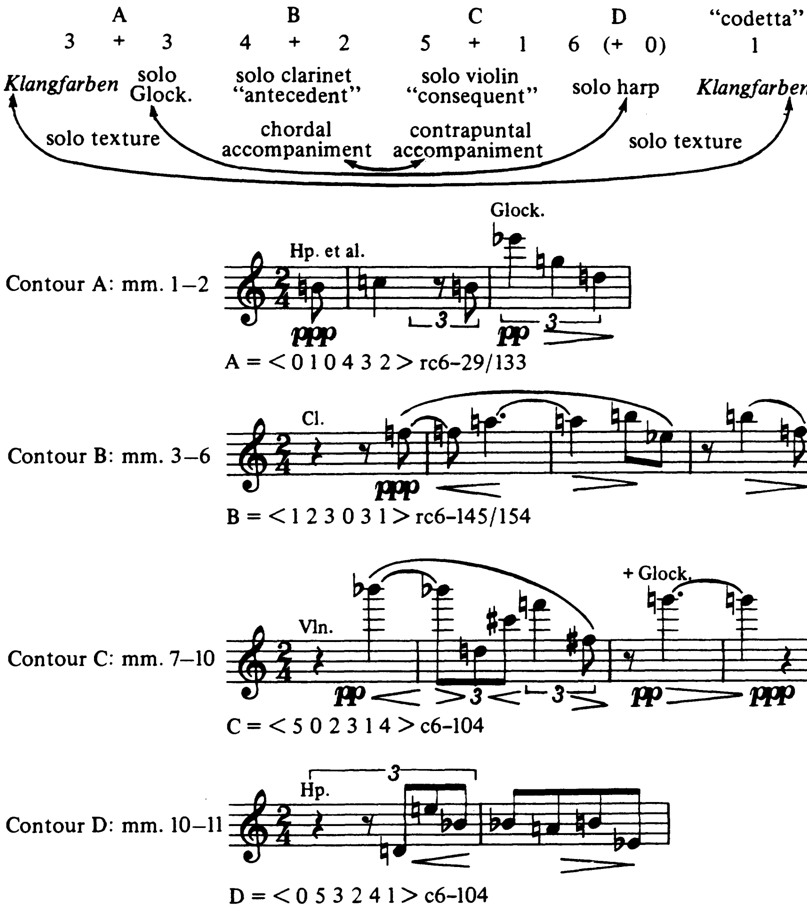


Figure 16. Primary Melodic Contours in Webern, op. 10/1

In fact, since two are repeated-note csegs, the cardinalities of the pitch-class sets differ; the first is a pentachord, the second a tetrachord, and the last two, hexachords. Although the two hexachords do not belong to the same set class (C = 6-Z44, D = 6-Z6), they are members of the same c-space segment class, c6-104. Contour D immediately follows C musically, and is its contour inversion. This, of course, is a much more precise relationship than simply reversing the pattern of ups and downs between adjacent c-pitches (a reversal of signs in the INT_1); that is, changing < + − − + − > to < − + + − + >. In this case, such a reversal of signs is instead reflected throughout the entire COM-matrix. Further, the ordering of cps in c6-104 produces a successive pattern of preserved adjacencies between the inversionally-related contours:[34]

C = < 5 0 2 3 1 4 >
D = < 0 5 3 2 4 1 >.

The relationship between successive contours is, for the most part, one of high dissimilarity: CSIM(A,B) and CSIM(B,C) equal .27 and CSIM(C, D) equals 0. On the other hand, connections between the opening melodies and the concluding one are much stronger (CSIM(A,D) = .53 and CSIM (B,D) = .60). Thus the third melody, at the highpoint of the movement, has the contour most dissimilar from those which precede and follow it, a contour which sets it apart from the others (CSIM(A,C) = .40, CSIM(B,C) = .27 and CSIM(C,D) = 0).

All four of the primary melodies are related by their csubseg structure. Each has c4-6 embedded at least once as four successive cps, often prominently positioned. Yet in no case do these successive pitches belong to the same set class, despite their membership in the same csegclass. For example, contour A ends with < 0 4 3 2 > (or, by translation, < 0 3 2 1 >), and is immediately followed by its retrograde in the first four cps of contour B, < 1 2 3 0 >. This segmentation into fours is aurally suggested by the isolation of these tetrachords by rests on either side. Like contour B, contours C and D begin with c4-6 as the first four cps. Contour C begins with < 5 0 2 3 >, which is the inversion of the original csubseg as stated in A (by translation < 5 0 2 3 > becomes < 3 0 1 2 >, and by inversion, < 0 3 2 1 >). Contour D's initial tetrachord is a return to < 0 3 2 1 > as initially appeared. Finally, csegclass c4-6 appears embedded as noncontiguous csubsegs in contours A, C, and D as well. It occurs a total of three times in A and five times in D, and is in fact the only four-note csubseg these two contours share ($CMEMB_4$(X, A, D) = 8/30 = .27). Contour C also contains five embedded statements of c4-6, but in the inverted form.

Secondary melodic material (of cardinality four or greater) is shown in Figure 17 as contours E through H. In contour F, c4-6 appears again in exactly the same form as in contour C, the melody which it accompanies. Thus the contours of the violin and cello lines (mm. 7–8) form a heter-

Contour E: mm. 4–6

E = < 2 0 1 3 > c4–4

Contour F: mm. 7–8

F = < 3 0 1 2 > c4–6

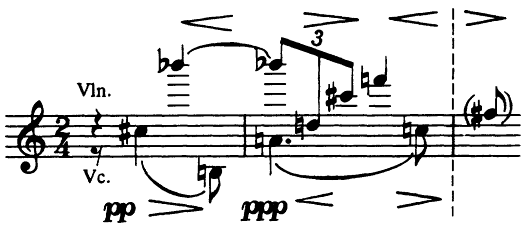
Contour G: mm. 6–7

G = < 0 2 1 3 > c4–3

Contour H: mm. 8–9

H = < 2 0 1 0 3 > c5–14/20

Figure 17. Secondary Melodic Material: Webern, op. 10/1



Both contours are c4–6, < 3 0 1 2 >.

Figure 18. Contour Heterophony: Webern, op. 10/1, mm. 7–8

ophonic texture of overlapping statements of c4–6 in close temporal proximity, as shown in Figure 18. Contour heterophony occurs only at this high-point of the piece, where the contrapuntal texture is most complex. In every other case, the csegclass of the accompanying line is not an embedded csubseg of the melody it accompanies; thus the distinction between melody and accompaniment is clear.

Finally, only two possible csegclasses exist for c-segments of cardinality three. Therefore, occasional instances of recurring three-note csubsegs may be of relatively trivial analytical importance. The distinctive repeated-note csubseg rc3-2/2, $<0\ 1\ 0>$, occurs with enough frequency throughout the movement to warrant discussion, however. This "neighbor note" motive opens the movement with its vivid *Klangfarben* scoring. Its inverted form is embedded repeatedly in contour B which follows, as the contiguous cps $<3\ 0\ 3>$ and as the noncontiguous csubsegs $<1\ 2\ 1>$, $<1\ 3\ 1>$ (twice), and $<1\ 0\ 1>$. Further, it occurs as the central three consecutive cps of contour H. Most striking, however, is its prolonged statement over the course of measures 3 through 10—first in the extended trill (which in itself contains repeated instances of $<0\ 1\ 0>$) and then in the continuation of this line in the trumpet/harp of m. 9 and celesta/cello of m. 10. This extended $<0\ 1\ 0>$ clearly refers back to the opening gesture, even with respect to its instrumentation.

* * * *

If music theorists model analytical theories to reflect aural perceptions, then a theory which describes relationships among musical contours is certainly overdue. The theory detailed above defines equivalence and similarity relations for contours in contour space. The analysis that follows briefly illustrates how specific contour relationships may be used to shape a formal scheme, to differentiate melody from accompaniment, to associate musical ideas that belong to different set classes, and to create unity through varied repetition.

GLOSSARY

COM-matrix (comparison matrix) - a two-dimensional array that displays the results of the comparison function, COM(a,b) for any two c-pitches in c-space. If b is higher than a, the function returns "+"; if b is the same as a, the function returns "0"; and if b is lower than a, COM(a,b) returns "−."

C-pitches (cps) - elements in c-space, numbered in order from low to high, beginning with 0 up to $(n - 1)$, where n equals the number of elements.

C-segment (cseg) - an ordered set of c-pitches in c-space.

C-space (contour space) - a type of musical space consisting of elements arranged from low to high disregarding the exact intervals between elements.

C-space segment class (csegclass) - an equivalence class made up of all csegs related by identity, translation, retrograde, inversion, and retrograde-inversion.

C-subsegment (csubseg) - any ordered subgrouping of a given cseg. May be comprised of either contiguous or non-contiguous c-pitches from the original cseg.

INT_n - any of the diagonals to the right of the main diagonal (upper left-hand to lower right-hand corner) of the COM-matrix, in which n stands for the difference between order position numbers of the two cps compared; that is, INT_3 compares cps which are 3 positions apart.

Inversion - the inversion of a cseg S comprised of n distinct cps is written IS, and may be found by subtracting each c-pitch from $(n - 1)$.

Normal form - an ordered array in which elements in a cseg of n distinct c-pitches are numbered from 0 to $(n - 1)$ and listed in temporal order.

Prime form - a representative form for identification of cseg classes, derived by the following algorithm: (1) if necessary, translate the cseg so its content consists of integers from 0 to $(n - 1)$; (2) if $(n - 1)$ minus the last c-pitch is less than the first c-pitch invert the cseg; (3) if the last c-pitch is less than the first c-pitch, retrograde the cseg. Appendix One lists the csegclasses and their corresponding labels as used in this paper. The first number of the label represents the cardinality of the csegclass and the second number represents its ordinal position on the list: thus c5-12 represents the twelfth contour on the list of five-note csegclasses.

Translation - an operation through which a csubseg is renumbered from 0 for the lowest c-pitch to $(n - 1)$ for the highest.

SIMILARITY MEASUREMENTS:

ACMEMB(A,B) - counts the total number of mutually-embedded csegs of cardinality 2 through the cardinality of the smaller cseg and adjusts this to a decimal value by dividing by the total number of possible subsegs of A and B (excluding the null set for each and the one-note csubsegs).

CEMB(A,B) - counts the number of times cseg A is embedded in cseg B, then divides this sum by the total number of csubsegs of the same cardinality as A possible, to return a value that approaches 1 for csegs of greater similarity.

CMEMB$_n$(X,A,B) - counts the number of times the csegs, X (of cardinality n), are mutually embedded in both csegs A and B. (The variable "X" may successively represent more than one cseg-type during the course of the function.) Each cseg X must be embedded at least once in both A and B; then, all instances of X are counted in both A and B. The total number of mutually-embedded csegs of cardinality n is then divided by the number of n-cardinality csubsegs possible in order to return a decimal number approaching 1 as csegs A and B are more similar.

CSIM(A,B) - measures the degree of similarity between two csegs of the same cardinality by comparing specific positions in the upper right-hand triangle of the COM-matrix for cseg A with the corresponding positions in the matrix of cseg B in order to total the number of similarities between them. This sum is divided by the total number of positions compared to return a decimal number that signifies greater similarity between csegs as the value approaches 1.

In addition, **ACMEMB($\underline{A}$,$\underline{B}$)**, **CEMBA($\underline{A}$,$\underline{B}$)**, **CMEMB$_n$(X,$\underline{A}$,$\underline{B}$)**, and **CSIM($\underline{A}$,$\underline{B}$)**, generalize each of the functions above to measure similarity between csegclasses.

APPENDIX
C-SPACE SEGMENT-CLASSES OF CARDINALITIES 2 THROUGH 6

The following table of csegclasses, cardinalities 2 through 6, is a portion of the output of a computer program written in March 1986. The program, written in standard Pascal language, was implemented on a Digital PRO-350 using the Xenix Pascal compiler and editor.

The csegclasses are listed in prime form, grouped by cardinality, and numbered in ascending order by prime form considered as an integer value. An asterisk (*) following the csegclass name indicates identity under retrograde inversion. For referential purposes, the INT_1 of a csegclass is listed at the right of its csegclass representative.

C-space segment classes for cseg cardinality 2

Csegclass/RIinv.	Prime form	INT(1)
c 2-1*	< 0 1 >	< + >

C-space segment classes for cseg cardinality 3

Csegclass/RIinv.	Prime form	INT(1)
c 3-1*	< 0 1 2 >	< + + >
c 3-2	< 0 2 1 >	< + − >

C-space segment classes for cseg cardinality 4

Csegclass/RIinv.	Prime form	INT(1)
c 4-1*	< 0 1 2 3 >	< + + + >
c 4-2	< 0 1 3 2 >	< + + − >
c 4-3*	< 0 2 1 3 >	< + − + >
c 4-4	< 0 2 3 1 >	< + + − >
c 4-5	< 0 3 1 2 >	< + − + >
c 4-6	< 0 3 2 1 >	< + − − >
c 4-7*	< 1 0 3 2 >	< − + − >
c 4-8*	< 1 3 0 2 >	< + − + >

C-space segment classes for cseg cardinality 5

Csegclass/RIinv.	Prime form	INT(1)
c 5-1*	< 0 1 2 3 4 >	< + + + + >
c 5-2	< 0 1 2 4 3 >	< + + + − >
c 5-3	< 0 1 3 2 4 >	< + + − + >
c 5-4	< 0 1 3 4 2 >	< + + + − >
c 5-5	< 0 1 4 2 3 >	< + + − + >

c 5-6	< 0 1 4 3 2 >	< + + − − >
c 5-7	< 0 2 1 4 3 >	< + − + − >
c 5-8	< 0 2 3 1 4 >	< + + − + >
c 5-9	< 0 2 3 4 1 >	< + + + − >
c 5-10	< 0 2 4 1 3 >	< + + − + >
c 5-11	< 0 2 4 3 1 >	< + + − − >
c 5-12	< 0 3 1 4 2 >	< + − + − >
c 5-13*	< 0 3 2 1 4 >	< + − − + >
c 5-14	< 0 3 2 4 1 >	< + − + − >
c 5-15	< 0 3 4 1 2 >	< + + − + >
c 5-16	< 0 3 4 2 1 >	< + + − − >
c 5-17	< 0 4 1 2 3 >	< + − + + >
c 5-18	< 0 4 1 3 2 >	< + − + − >
c 5-19	< 0 4 2 1 3 >	< + − − + >
c 5-20	< 0 4 2 3 1 >	< + − + − >
c 5-21	< 0 4 3 1 2 >	< + − − + >
c 5-22	< 0 4 3 2 1 >	< + − − − >
c 5-23*	< 1 0 2 4 3 >	< − + + − >
c 5-24	< 1 0 3 4 2 >	< − + + − >
c 5-25	< 1 0 4 2 3 >	< − + − + >
c 5-26	< 1 0 4 3 2 >	< − + − − >
c 5-27	< 1 2 4 0 3 >	< + + − + >
c 5-28	< 1 3 0 4 2 >	< + − + − >
c 5-29	< 1 3 4 0 2 >	< + + − + >
c 5-30	< 1 4 0 3 2 >	< + − + − >
c 5-31*	< 1 4 2 0 3 >	< + − − + >
c 5-32	< 1 4 3 0 2 <	< + − − + >

C-space segment classes for cseg cardinality 6

Csegclass/RIinv.	Prime form	INT(1)
c 6-1*	< 0 1 2 3 4 5 >	< + + + + + >
c 6-2	< 0 1 2 3 5 4 >	< + + + + − >
c 6-3	< 0 1 2 4 3 5 >	< + + + − + >
c 6-4	< 0 1 2 4 5 3 >	< + + + + − >
c 6-5	< 0 1 2 5 3 4 >	< + + + − + >
c 6-6	< 0 1 2 5 4 3 >	< + + + − − >
c 6-7*	< 0 1 3 2 4 5 >	< + + − + + >
c 6-8	< 0 1 3 2 5 4 >	< + + − + − >
c 6-9	< 0 1 3 4 2 5 >	< + + + − + >
c 6-10	< 0 1 3 4 5 2 >	< + + + + − >
c 6-11	< 0 1 3 5 2 4 >	< + + + − + >
c 6-12	< 0 1 3 5 4 2 >	< + + + − − >
c 6-13	< 0 1 4 2 3 5 >	< + + − + + >
c 6-14	< 0 1 4 2 5 3 >	< + + − + − >

c 6-15	< 0 1 4 3 2 5 >	< + + − − + >
c 6-16	< 0 1 4 3 5 2 >	< + + − + − >
c 6-17	< 0 1 4 5 2 3 >	< + + + − + >
c 6-18	< 0 1 4 5 3 2 >	< + + + − − >
c 6-19	< 0 1 5 2 3 4 >	< + + − + + >
c 6-20	< 0 1 5 2 4 3 >	< + + − + − >
c 6-21	< 0 1 5 3 2 4 >	< + + − − + >
c 6-22	< 0 1 5 3 4 2 >	< + + − + − >
c 6-23	< 0 1 5 4 2 3 >	< + + − − + >
c 6-24	< 0 1 5 4 3 2 >	< + + − − − >
c 6-25	< 0 2 1 3 5 4 >	< + − + + − >
c 6-26*	< 0 2 1 4 3 5 >	< + − + − + >
c 6-27	< 0 2 1 4 5 3 >	< + − + + − >
c 6-28	< 0 2 1 5 3 4 >	< + − + − + >
c 6-29	< 0 2 1 5 4 3 >	< + − + − − >
c 6-30	< 0 2 3 1 5 4 >	< + + − + − >
c 6-31	< 0 2 3 4 1 5 >	< + + + − + >
c 6-32	< 0 2 3 4 5 1 >	< + + + + − >
c 6-33	< 0 2 3 5 1 4 >	< + + + − + >
c 6-34	< 0 2 3 5 4 1 >	< + + + − − >
c 6-35*	< 0 2 4 1 3 5 >	< + + − + + >
c 6-36	< 0 2 4 1 5 3 >	< + + − + − >
c 6-37	< 0 2 4 3 1 5 >	< + + − − + >
c 6-38	< 0 2 4 3 5 1 >	< + + − + − >
c 6-39	< 0 2 4 5 1 3 >	< + + + − + >
c 6-40	< 0 2 4 5 3 1 >	< + + + − − >
c 6-41	< 0 2 5 1 3 4 >	< + + − + + >
c 6-42	< 0 2 5 1 4 3 >	< + + − + − >
c 6-43	< 0 2 5 3 1 4 >	< + + − − + >
c 6-44	< 0 2 5 3 4 1 >	< + + − + − >
c 6-45	< 0 2 5 4 1 3 >	< + + − − + >
c 6-46	< 0 2 5 4 3 1 >	< + + − − − >
c 6-47	< 0 3 1 2 5 4 >	< + − + + − >
c 6-48*	< 0 3 1 4 2 5 >	< + − + − + >
c 6-49	< 0 3 1 4 5 2 >	< + − + + − >
c 6-50	< 0 3 1 5 2 4 >	< + − + − + >
c 6-51	< 0 3 1 5 4 2 >	< + − + − − >
c 6-52	< 0 3 2 1 5 4 >	< + − − + − >
c 6-53	< 0 3 2 4 1 5 >	< + − + − + >
c 6-54	< 0 3 2 4 5 1 >	< + − + + − >
c 6-55	< 0 3 2 5 1 4 >	< + − + − + >
c 6-56	< 0 3 2 5 4 1 >	< + − + − − >
c 6-57*	< 0 3 4 1 2 5 >	< + + − + + >
c 6-58	< 0 3 4 1 5 2 >	< + + − + − >

c 6-59	< 0 3 4 2 1 5 >	< + + − − + >
c 6-60	< 0 3 4 2 5 1 >	< + + − + − >
c 6-61	< 0 3 4 5 1 2 >	< + + + − + >
c 6-62	< 0 3 4 5 2 1 >	< + + + − − >
c 6-63	< 0 3 5 1 2 4 >	< + + − + + >
c 6-64	< 0 3 5 1 4 2 >	< + + − + − >
c 6-65	< 0 3 5 2 1 4 >	< + + − − + >
c 6-66	< 0 3 5 2 4 1 >	< + + − + − >
c 6-67	< 0 3 5 4 1 2 >	< + + − − + >
c 6-68	< 0 3 5 4 2 1 >	< + + − − − >
c 6-69	< 0 4 1 2 5 3 >	< + − + + − >
c 6-70	< 0 4 1 3 5 2 >	< + − + + − >
c 6-71	< 0 4 1 5 2 3 >	< + − + − + >
c 6-72	< 0 4 1 5 3 2 >	< + − + − − >
c 6-73	< 0 4 2 1 5 3 >	< + − − + − >
c 6-74*	< 0 4 2 3 1 5 >	< + − + − + >
c 6-75	< 0 4 2 3 5 1 >	< + − + + − >
c 6-76	< 0 4 2 5 1 3 >	< + − + − + >
c 6-77	< 0 4 2 5 3 1 >	< + − + − − >
c 6-78	< 0 4 3 1 5 2 >	< + − − + − >
c 6-79*	< 0 4 3 2 1 5 >	< + − − − + >
c 6-80	< 0 4 3 2 5 1 >	< + − − + − >
c 6-81	< 0 4 3 5 1 2 >	< + − + − + >
c 6-82	< 0 4 3 5 2 1 >	< + − + − − >
c 6-83	< 0 4 5 1 2 3 >	< + + − + + >
c 6-84	< 0 4 5 1 3 2 >	< + + − + − >
c 6-85	< 0 4 5 2 1 3 >	< + + − − + >
c 6-86	< 0 4 5 2 3 1 >	< + + − + − >
c 6-87	< 0 4 5 3 1 2 >	< + + − − + >
c 6-88	< 0 4 5 3 2 1 >	< + + − − − >
c 6-89	< 0 5 1 2 3 4 >	< + − + + + >
c 6-90	< 0 5 1 2 4 3 >	< + − + + − >
c 6-91	< 0 5 1 3 2 4 >	< + − + − + >
c 6-92	< 0 5 1 3 4 2 >	< + − + + − >
c 6-93	< 0 5 1 4 2 3 >	< + − + − + >
c 6-94	< 0 5 1 4 3 2 >	< + − + − − >
c 6-95	< 0 5 2 1 3 4 >	< + − − + + >
c 6-96	< 0 5 2 1 4 3 >	< + − − + − >
c 6-97	< 0 5 2 3 1 4 >	< + − + − + >
c 6-98	< 0 5 2 3 4 1 >	< + − + + − >
c 6-99	< 0 5 2 4 1 3 >	< + − + − + >
c 6-100	< 0 5 2 4 3 1 >	< + − + − − >
c 6-101	< 0 5 3 1 2 4 >	< + − − + + >
c 6-102	< 0 5 3 1 4 2 >	< + − − + − >

c 6-103	< 0 5 3 2 1 4 >	< + − − − + >
c 6-104	< 0 5 3 2 4 1 >	< + − − + − >
c 6-105	< 0 5 3 4 1 2 >	< + − + − + >
c 6-106	< 0 5 3 4 2 1 >	< + − + − − >
c 6-107	< 0 5 4 1 2 3 >	< + − − + + >
c 6-108	< 0 5 4 1 3 2 >	< + − − + − >
c 6-109	< 0 5 4 2 1 3 >	< + − − − + >
c 6-110	< 0 5 4 2 3 1 >	< + − − + − >
c 6-111	< 0 5 4 3 1 2 >	< + − − − + >
c 6-112	< 0 5 4 3 2 1 >	< + − − − − >
c 6-113*	< 1 0 2 3 5 4 >	< − + + + − >
c 6-114	< 1 0 2 4 5 3 >	< − + + + − >
c 6-115	< 1 0 2 5 3 4 >	< − + + − + >
c 6-116	< 1 0 2 5 4 3 >	< − + + − − >
c 6-117*	< 1 0 3 2 5 4 >	< − + − + − >
c 6-118	< 1 0 3 4 5 2 >	< − + + + − >
c 6-119	< 1 0 3 5 2 4 >	< − + + − + >
c 6-120	< 1 0 3 5 4 2 >	< − + + − − >
c 6-121	< 1 0 4 2 5 3 >	< − + − + − >
c 6-122	< 1 0 4 3 5 2 >	< − + − + − >
c 6-123	< 1 0 4 5 2 3 >	< − + + − + >
c 6-124	< 1 0 4 5 3 2 >	< − + + − − >
c 6-125	< 1 0 5 2 3 4 >	< − + − + + >
c 6-126	< 1 0 5 2 4 3 >	< − + − + − >
c 6-127	< 1 0 5 3 2 4 >	< − + − − + >
c 6-128	< 1 0 5 3 4 2 >	< − + − + − >
c 6-129	< 1 0 5 4 2 3 >	< − + − − + >
c 6-130	< 1 0 5 4 3 2 >	< − + − − − >
c 6-131	< 1 2 0 4 5 3 >	< + − + + − >
c 6-132*	< 1 2 0 5 3 4 >	< + − + − + >
c 6-133	< 1 2 0 5 4 3 >	< + − + − − >
c 6-134	< 1 2 3 5 0 4 >	< + + + − + >
c 6-135	< 1 2 4 0 5 3 >	< + + − + − >
c 6-136	< 1 2 4 5 0 3 >	< + + + − + >
c 6-137*	< 1 2 5 0 3 4 >	< + + − + + >
c 6-138	< 1 2 5 0 4 3 >	< + + − + − >
c 6-139	< 1 2 5 3 0 4 >	< + + − − + >
c 6-140	< 1 2 5 4 0 3 >	< + + − − + >
c 6-141	< 1 3 0 4 5 2 >	< + − + + − >
c 6-142*	< 1 3 0 5 2 4 >	< + − + − + >
c 6-143	< 1 3 0 5 4 2 >	< + − + − − >
c 6-144	< 1 3 2 5 0 4 >	< + − + − + >
c 6-145	< 1 3 4 0 5 2 >	< + + − + − >
c 6-146	< 1 3 4 5 0 2 >	< + + + − + >

c 6-147*	< 1 3 5 0 2 4 >	< + + − + + >
c 6-148	< 1 3 5 0 4 2 >	< + + − + − >
c 6-149	< 1 3 5 2 0 4 >	< + + − − + >
c 6-150	< 1 3 5 4 0 2 >	< + + − − + >
c 6-151	< 1 4 0 2 5 3 >	< + − + + − >
c 6-152	< 1 4 0 3 5 2 >	< + − + + − >
c 6-153	< 1 4 0 5 2 3 >	< + − + − + >
c 6-154	< 1 4 0 5 3 2 >	< + − + − − >
c 6-155	< 1 4 2 0 5 3 >	< + − − + − >
c 6-156	< 1 4 2 5 0 3 >	< + − + − + >
c 6-157	< 1 4 3 0 5 2 >	< + − − + − >
c 6-158	< 1 4 3 5 0 2 >	< + − + − + >
c 6-159	< 1 4 5 0 2 3 >	< + + − + + >
c 6-160	< 1 4 5 0 3 2 >	< + + − + − >
c 6-161	< 1 4 5 2 0 3 >	< + + − − + >
c 6-162	< 1 4 5 3 0 2 >	< + + − − + >
c 6-163	< 1 5 0 2 4 3 >	< + − + + − >
c 6-164	< 1 5 0 3 4 2 >	< + − + + − >
c 6-165	< 1 5 0 4 2 3 >	< + − + − + >
c 6-166	< 1 5 0 4 3 2 >	< + − + − − >
c 6-167	< 1 5 2 0 4 3 >	< + − − + − >
c 6-168*	< 1 5 2 3 0 4 >	< + − + − + >
c 6-169	< 1 5 2 4 0 3 >	< + − + − + >
c 6-170	< 1 5 3 0 4 2 >	< + − − + − >
c 6-171*	< 1 5 3 2 0 4 >	< + − − − + >
c 6-172	< 1 5 3 4 0 2 >	< + − + − + >
c 6-173	< 1 5 4 0 2 3 >	< + − − + + >
c 6-174	< 1 5 4 0 3 2 >	< + − − + − >
c 6-175	< 1 5 4 2 0 3 >	< + − − − + >
c 6-176	< 1 5 4 3 0 2 >	< + − − − + >
c 6-177*	< 2 0 1 4 5 3 >	< − + + + − >
c 6-178	< 2 0 1 5 4 3 >	< − + + − − >
c 6-179*	< 2 0 4 1 5 3 >	< − + − + − >
c 6-180	< 2 0 4 5 1 3 >	< − + + − + >
c 6-181	< 2 0 5 1 4 3 >	< − + − + − >
c 6-182	< 2 0 5 4 1 3 >	< − + − − + >
c 6-183*	< 2 1 0 5 4 3 >	< − − + − − >
c 6-184	< 2 1 4 5 0 3 >	< − + + − + >
c 6-185*	< 2 1 5 0 4 3 >	< − + − + − >
c 6-186	< 2 1 5 4 0 3 >	< − + − − + >
c 6-187*	< 2 4 0 5 1 3 >	< + − + − + >
c 6-188	< 2 4 1 5 0 3 >	< + − + − + >
c 6-189*	< 2 4 5 0 1 3 >	< + + − + + >
c 6-190	< 2 4 5 1 0 3 >	< + + − − + >
c 6-191*	< 2 5 1 4 0 3 >	< + − + − + >
c 6-192*	< 2 5 4 1 0 3 >	< + − − − + >

NOTES

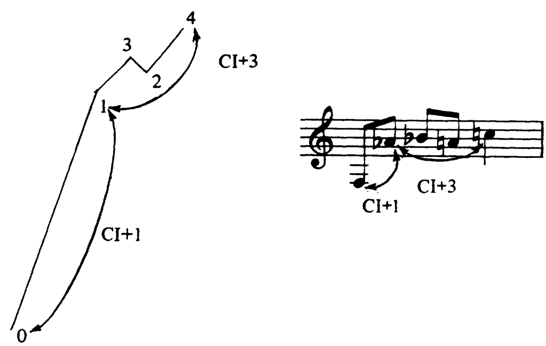
1. A bidimensional model for pitch, distinguishing pitch (or pitch height) from pitch class (called pitch quality or chroma) has existed in the psychological literature since the mid-nineteenth century. Christian Ruckmick ("A New Classification of Tonal Qualities," *Psychological Review* 36 [1929]: 172), for example, cites an M. W. Drobisch article from 1846 ("Uber die mathematische Bestimmung der musikalischen") as the earliest attempt to depict pitch perception as a helical model. This model shows the close perceptual proximity of octaves as distinct from rising pitch height by the vertical alignment of octave-related pitches within each turn of the helix.
2. In recent years, several psychologists have posited representational models for pitch perception on the basis of experimentation, among them Diana Deutsch, Carol Krumhansl and Roger N. Shepard. Shepard's multi-dimensional model for pitch is a double helix wrapped around a helical cylinder, where ascent represents pitch height with octave-related chroma aligned vertically while a downward projection of each pitch produces a circle of fifths model. Further, a vertical plane passing through the double helix model divides those tones which are diatonic to a given key from those which are not. See Shepard's "Structural Representations of Musical Pitch," in Diana Deutsch, ed., *The Psychology of Music* (NY: Academic Press, 1982), pp. 343–390, for an overview of representational models for pitch perception. Shepard notes elsewhere, however, that certain aspects of pitch perception differ markedly among listeners depending upon their musical backgrounds. In experiments undertaken jointly with Krumhansl in 1979, Shepard discovered that musical listeners perceived octave-related pitches as functionally equivalent, whereas subjects with less musical experience did not perceive such an equivalence. See his "Individual Differences in the Perception of Musical Pitch," in *Documentary Report of the Ann Arbor Symposium* (Reston, VA: Music Educators National Conference, 1981), pp. 152–174, for further details of this phenomenon. For purposes of this article, we will therefore assume experienced musical listeners in discussions relating to perceptual issues.
3. See Diana Deutsch, "The Processing of Pitch Combinations," *The Psychology of Music*, pp. 277–289, for an overview of experiments on recognition of melodies distorted by octave displacement or by alteration of interval size. W. J. Dowling and A. W. Hollombe's study, "The Perception of Melodies Distorted By Splitting Into Several Octaves: Effects of Increasing Proximity and Melodic Contour," *Perception and Psychophysics* 21 (1977): 60–64, generalizes Deutsch's findings as published in "Octave Generalization and Tune Recognition," *Perception and Psychophysics* 11 (1972): 411–412, over a number of familiar melodies. See also W. L. Idson and D. W. Massaro, "A Bidimensional Model of Pitch in the Recognition of Melodies," *Perception and Psychophysics* 24 (1978): 551–565 and W. J. Dowling and D. S. Fujitani, "Contour, Interval, and Pitch Recognition in Memory for Melodies," *The Journal of the Acoustical Society of America* 49 (1971): 524–531.
4. W. J. Dowling, "Scale and Contour: Two Components of a Theory of Memory for Melodies," *Psychological Review* 85 (1978): 341–354, and "Mental Structures Through Which Music is Perceived," *Documentary Report of the Ann Arbor Symposium* (Reston, VA: Music Educator's National Conference, 1981), pp. 144–151.
5. W. J. Dowling and D. S. Fugitani in the first of two experiments described in "Contour, Interval, and Pitch Recognition in Memory for Melodies" (*Journal of the Acoustical Society of America* 49 [1971]: 524–431) discovered that listeners were likely to confuse

the exact transposition of a novel non-tonal melody with a second non-tonal melody if the latter retained the same contour. Thus, they concluded that listeners retain non-tonal melodies in memory solely in terms of contour. The authors admitted, however, that their subjects' confusion of same-contour melodies with transpositions of the original melody may have resulted from the severe constraints placed on the intervallic construction of the melodies used in this experiment. Only minor seconds, major seconds, and minor thirds were used (pp. 527–528). See also Dowling, "Mental Structures," p. 146.

6. James C. Bartlett and W. Jay Dowling in "Recognition of Transposed Melodies: A Key-Distance Effect in Developmental Perspective" (*Journal of Experimental Psychology: Human Perception and Performance* 6 [1980]: 501) give a brief overview of several experiments, concluding that "in all of these tasks with unfamiliar melodies, subjects seem to have little trouble reproducing or recognizing the melodic contour, but they have a great deal of trouble with the exact-pitch intervals among the notes." Judy Edworthy, in "Melodic Contour and Musical Structure," *Musical Structure and Cognition* (London: Academic Press, Inc., 1985), confirms these findings. Her experiments involve transposition of novel, tonal melodies to various keys. She concludes that "interval information is well-defined and precise only when the listener is able to establish a key. . . . Contour information is immediately precise but decays rapidly as a melody progresses and its length increases. However, accurate encoding of contour does not depend on the listener's ability to establish a key" (p. 186). In non-tonal contexts, subjects should therefore be able to recognize relationships among contours more quickly and easily than among pitch-class sets, since only the latter requires subjects to perceive intervallic information.
7. Robert Morris, in his *Composition with Pitch Classes: A Theory of Compositional Design* (New Haven: Yale University Press, in press), develops five such spaces. David Lewin's *Generalized Musical Intervals and Transformations* (New Haven: Yale University Press, 1987) posits six temporal and six pitch- and/or pc-related musical spaces (pp. 16–25).
8. John Rahn, in *Basic Atonal Theory* (New York: Longman, 1980) clearly and consistently distinguishes between pitch relationships and pitch-class relationships, effectively separating theoretical concepts which apply only to pitch space from those which operate in pitch-class space.
9. In addition to Robert Morris's *Composition with Pitch Classes*, another important resource is Michael Friedmann's "A Methodology for the Discussion of Contour: Its Application to Schoenberg's Music," *Journal of Music Theory* 29 (1985): 223–248. Friedmann's work raises important issues regarding musical structure, analysis, and perception. His article posits a number of theoretical constructs for comparing and relating musical contours, including the contour adjacency series and related vector, the contour class with its associated vector, and the contour interval succession and array. Although these formulations differ from ours in a number of crucial aspects, his work has greatly influenced our thinking.

   Discussion of musical contour is not without earlier precedents, however, particularly in the writings of music theorist-composers, such as Arnold Schoenberg (*Fundamentals of Musical Composition* [New York: St. Martin's Press, 1967], pp. 113–115), Ernst Toch (*The Shaping Forces in Music* [New York: Criterion Music Corp., 1948], Chapter 5), and Robert Cogan and Pozzi Escot, whose *Sonic Design: The Nature of Sound and Music* (Englewood Cliffs, NJ: Prentice-Hall, Inc., 1976) makes extensive use of contour graphs in musical analysis. See also Cogan's *New Images of Musical Sound* (Cambridge: Harvard University Press, 1984).

10. Morris, Glossary, under the word "c-space."
11. Morris, Definition 1.1.
12. Friedmann defines contour intervals (CIs) as "the distance between one element in a CC (Contour Class) and a later element as signified by the signs + or − and a number. For example, in CC < 0-1-3-2 >, the CI of 0 to 3 is +3, and the CI of 3 to 2 is −1" (p. 246). He readily acknowledges that the contour interval is "infinitely expandable or contractable in pitch space," and that "a larger CI contains a great number of intervening pitches in the registral order of the musical unit . . . [and] is by no means necessarily a larger interval in pitch space" (p. 230). Although we find such a concept interesting, it seems counterintuitive from the perspective of a listener's perceptions, since a contour interval of +3 may be considerably smaller in pitch space than a CI of +1. For example, the cseg < 0 1 3 2 4 > may be realized as follows:



In this case, CI + 3 (measured from contour pitches 1 to 4) is only a major third, while CI + 1 is a minor tenth. Other musical realizations of this cseg may produce even larger differences in CI size. Further, Friedmann uses the contour interval, contour interval array, and associated vectors as an equivalence criterion (pp. 231 and 234), and to compare similarities among contours in his analyses (pp. 240 ff). Since we choose not to define intervals in c-space, our equivalence criteria and similarity relations differ markedly from Friedmann's in concept.

13. We use a slightly different definition than Morris, since we refer to all contours as c-segments, not as c-sets.
14. Note that our definitions do not account as yet for repeated tones within a musical contour. This is a separate issue which will be addressed at a later point in the article.
15. Morris, Definition 1.2.
16. Friedmann, pp. 226–227.
17. The term INT is used to be consistent with Morris's terminology for matrices in p- and pc-space, where the integers appearing in each diagonal give information about a set's intervallic structure, including properties of invariance. Thus the term INT is retained here, even though we do not define intervals in c-space.
18. We rephrase Morris's Definition 1.4 slightly to conform with our terminology: the inversion of a cseg P, of cardinality n, is the cseg IP. Each IP_m equals $(n - 1) - P_m$ where the subscript m denotes order positions with the cseg P.

19. Morris, Chapter 2.
20. More formally:

Let [cp(1) . . . cp(n)] be a cseg with cps numbered in time from 1 to n.
Let "n" equal the cardinality of the cseg;
Let "x" equal an ordinal position within the cseg, ranging from 1 to n (thus, "cp(x)" is a particular c-pitch, located "xth" from the left).

1) If necessary, translate the cseg to normal form,
2) If $(n - 1) - cp(n) < cp(1)$, then invert the cseg,
3) If $cp(n) < cp(1)$, then retrograde the cseg.

21. The design of these functions is modelled, in part, upon the similarity measures for pitch-class sets previously formulated by David Lewin, Robert Morris and John Rahn. See, in particular, Lewin's "Forte's Interval Vector, my Interval Function, and Regener's Common-Note Function," *Journal of Music Theory* 21 (1977): 194–237; Morris's "A Similarity Index for Pitch-Class Sets," *Perspectives of New Music* 18 (1979/80): 445–460; and Rahn's "Relating Sets," in the same volume, pp. 438–498.
22. We are following John Rahn's example in designing functions to return a decimal value approaching "1" as similarity increases. See his "Relating Sets."
23. As previously mentioned, the entries in the lower left-hand triangle of the COM-matrices used here simply mirror (with inverse values) those in the upper right-hand triangle. We therefore base our similarity measurement upon compared positions in the upper triangles alone.
24. Rahn, "Relating Sets," p. 490.
25. This total number of comparisons between right triangles is sigma(n); which we define as:

$$\sum_{S=1}^{n-1} (S)$$

(in other words, the summation of an arithmetic series from 1 to $(n - 1)$, where n equals the cardinality of the cseg).
26. We choose this method of comparing csegs of unequal cardinality over an expansion and generalization of the CSIM measurement for two reasons. First, the embedding relation is easier to hear and therefore is intuitively more satisfying. Second, any generalization of CSIM to csegs of unequal cardinality would, in effect, create another type of embedding function, since it would involve comparing matrices of unequal size (thus embedding one matrix within another and systematically shifting the position of the embedded smaller matrix to make comparisons with each position of the larger matrix).
27. Rahn, *Basic Atonal Theory*, p. 122.
28. Rahn, "Relating Sets," p. 492. Rahn generalizes David Lewin's embedding function as formulated in Lewin, "Forte's Interval Vector," pp. 194–237.
29. More formally:

$$\text{ACMEMB(A,B)} = \frac{\sum_{n=2}^{c} \text{CMEMB}_n\text{(X,A,B)}}{2^{\#A} + 2^{\#B} - (\#A + \#B + 2)}$$

where c = cardinality of the larger of the 2 csegs,

n = cardinality of x,

x = mutually embedded cseg, and

stands for "cardinality of."

The numerator of this fraction loops through the $\text{CMEMB}_n(X,A,B)$ function successively for cardinalities 2 through the cardinality of the larger cseg. The denominator divides this figure by the total number of csegs possible ($2^{\#A} + 2^{\#B}$) minus the one-note csubsegs (#A + #B) and minus the null set for each (2).

30. Friedmann, pp. 234–236.

31. The introduction of repeated notes into contour theory, as formulated to this point, strikes at the heart of the distinction between pitch space and contour space. Because our definition of c-space, following Morris, disregards the exact intervals between c-pitches and chooses to leave this distance undefined, the perception of a repeated note must be seen as a pitch-space rather than a c-space phenomenon. In considering analytical applications of contour theory, we must therefore depart slightly from our previous c-space definition in order to accommodate those segments in which pitches are repeated.

32. In symmetrically-structured csegs of odd cardinality (i.e., < c b r x r b c > or < 1 3 2 0 2 3 1 >), the composite label will reflect the cseg's symmetry. For example, the COM-matrix for the repeated-note cseg < 1 0 2 0 1 > is shown below with the two matrices which determine its composite label:

$$\begin{array}{c|ccccc} & 1 & 0 & 2 & 0 & 1 \\ \hline 1 & 0 & - & + & - & \boxed{0} \\ 0 & + & 0 & + & \boxed{0} & + \\ 2 & - & - & 0 & - & - \\ 0 & + & 0 & + & 0 & + \\ 1 & 0 & - & + & - & 0 \end{array} \qquad \begin{array}{c|ccccc} & 2 & 0 & 4 & 1 & 3 \\ \hline 2 & 0 & - & + & - & \boxed{+} \\ 0 & + & 0 & + & \boxed{+} & + \\ 4 & - & - & 0 & - & - \\ 1 & + & - & + & 0 & + \\ 3 & - & - & + & - & 0 \end{array} \qquad \begin{array}{c|ccccc} & 3 & 1 & 4 & 0 & 2 \\ \hline 3 & 0 & - & + & - & \boxed{-} \\ 1 & + & 0 & + & \boxed{-} & + \\ 4 & - & - & 0 & - & - \\ 0 & + & + & + & 0 & + \\ 2 & + & - & + & - & 0 \end{array}$$

rc5-28/28   c5-28   also c5-28

In cases such as these, the related csegs that determine the composite label belong to the same c-space segment class. The composite label reflects this dual relationship by listing the csegclass's ordinal number twice.

33. The maximum possible value for CSIM(A,B) between cseg A with repeated notes and cseg B without, is equal to $\frac{\text{sigma(n)} - \text{r}}{\text{sigma (n)}}$, where r is the total number of cp repetitions. Such a comparison cannot therefore return a value of "1."

34. Such a pattern will always result between inversionally-related csegs in which adjacent cps add to an odd index number, in this case, 5. Other patterns of invariance between inversionally-related contours may be predicted using the T_nI cycles. See Daniel Starr, "Sets, Invariance, and Partitions," *Journal of Music Theory* 22 (1978): 1–42, for a detailed examination of this subject.