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Author(s): Fred Lerdahl
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Tonal Pitch Space

FRED LERDAHL
University of Michigan

Models of pitch space have been developed in music psychology to account for perceived proximity among pitches, chords, or regions. This article introduces a different model that (1) treats pitches, chords, and regions within one framework, (2) correlates with the experimental data, and (3) connects in interesting ways with a variety of music theories.

Introduction

A number of cognitive psychologists, notably Longuet-Higgins (1962/1987) and Shepard (1982), have proposed theories of tonal pitch relations expressed in the form of topological models. These theories are intended to capture the sense of proximity and distance among pitch configurations that listeners bring to bear when hearing tonal pieces.

A topological approach to pitch relations has a longer tradition in music theory than in psychology. German musicians in the eighteenth century, in attempting to express degrees of modulation among related keys, developed circles on which all the major and minor keys were placed. Adjacent moves on these circles represented close modulations and nonadjacent moves, more distant modulations. (Werts (1983) gives an historical review.) These primitive circles culminated in the elegant pitch space of Weber (1824), which corresponds to Schoenberg's (1954) chart of regions.[1]

There is a complementary relationship between these topological models and various twentieth-century tonal theories that focus on structure in actual pieces, such as Schenker (1935), Meyer (1973), and Lerdahl and Jackendoff (1983, *A Generative Theory of Tonal Music,* henceforth *GTTM*).

Requests for reprints may be sent to Fred Lerdahl, School of Music, The University of Michigan, Ann Arbor, Michigan 48109.

1. As Weber (1824) was well-known to German and Austrian musicians throughout the nineteenth century, we can surmise that Schoenberg was aware of Weber's space but did not bother to attribute it. Schoenberg (1954) was designed not as a theoretical treatise but as a textbook, and it was assembled from notes after his death.

The connection can be explained through the distinction between a *tonal hierarchy* and an *event hierarchy* (Deutsch, 1984; Bharucha, 1984a).

A tonal hierarchy is a nontemporal mental schema that listeners utilize in assigning event hierarchies to pitch sequences. For example, listeners of Classical music understand chromatic inflections within a diatonic framework, and they employ distance from the tonic on the circle of fifths in determining harmonic domination or subordination. Such schemas arise from listening experience through the filter of our general musical cognitive capacity. Exposure to a different musical idiom would lead to a different tonal hierarchy. In addition—and this may be less obvious—a tonal hierarchy arises from the intrinsic organization of the pitch materials, such as described in Balzano (1982). For instance, the asymmetrical diatonic scale allows unambiguous "position-finding" (Browne, 1981) in a way that, say, the whole-tone and octatonic scales do not; nor would the latter two scales permit the gradual introduction of new pitch classes along the cycle of fifths. The topological models mentioned above can be viewed as representations of the tonal hierarchy for Classical music, where superordinate elements appear near the referential center of a space and less stable elements appear farther away from the center.

An event hierarchy, in contrast, is part of the structure that listeners infer from temporal musical sequences. This kind of hierarchy has long been familiar in the music-theoretic literature (although the specifics vary significantly from theory to theory) and will not be dwelt upon here. *GTTM* develops in detail two kinds of event hierarchies: *time-span reduction,* which expresses the relative structural importance of events as heard within rhythmic units; and *prolongational reduction,* which expresses recursive patterns of tension and relaxation among events.

The tonal hierarchy—hence an implicit pitch space—appears in *GTTM* in incomplete form under the rubric of verbally stated *stability conditions.* The theory claims that listeners cannot infer complex event hierarchies without having access to such conditions, which are the main source for ranking the relative stability of events within embedded temporal regions. These regions in turn derive directly or indirectly from the grouping and metrical structures. Figure 1 illustrates schematically that the time-span reduction derives from the rhythmic components and the stability conditions, and the prolongational reduction derives from the time-span reduction and the stability conditions. In short, both a rhythmic framework and criteria for pitch stability are needed if the listener is to hear events in a dominating–subordinating manner.

These remarks suggest, incidentally, that tonal and event hierarchies are interdependent developmentally. Exposure to real music is a prerequisite for internalizing a tonal schema, yet an event hierarchy cannot be con-

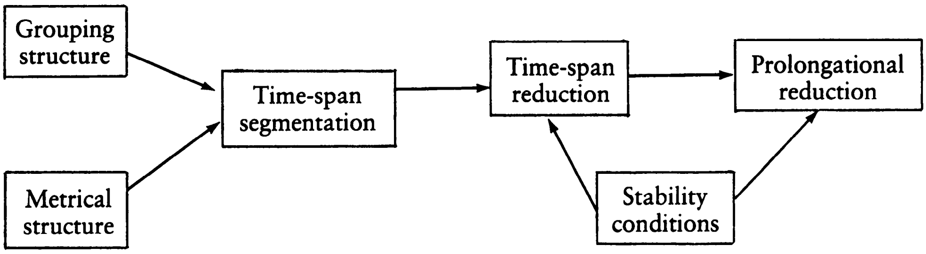

Fig. 1. The form of generative music theory.

structed without such a schema. They "bootstrap" one another into existence (Deutsch, 1984).

In what follows, I equate pitch space, the tonal hierarchy, and stability conditions. After a brief review of recent topological models, I propose at length a rather different model of tonal pitch space. Experimental support for the model is then introduced. Finally, the model is brought to bear on aspects of various music theories, including the sketchiness of *GTTM*'s treatment of stability conditions.

Problems of Current Models

There are two general problems with current tonal pitch spaces: they are too symmetrical and they only address one level of pitch description.

Symmetrical topological models arise because cognitive theorists want to capture certain regularities of the tonal system. First, the semitone interval between adjacent members of the chromatic scale is constant, resulting in the transpositional invariance of any interval. Second, the octave recurs periodically, creating the perception of pitch classes (henceforth "pcs"). Third, the basic interval of harmonic motion, the perfect fifth, traverses all 12 pcs before returning to the original pc, producing the chromatic circle of fifths. A move by any of these three intervals—a semitone, an octave, or a fifth—results in a cognitively proximate pitch. Topological modeling of cognitive distance therefore incorporates a metric of distance along a number of dimensions. Following Shepard (1982), if the semitone and the octave spaces are modeled, the result is a helix. Modeling the semitone and the fifth spaces produces the double helix shown in Figure 2. If the octave and fifth circles are combined, the result is a four-dimensional torus. Finally, adding semitone space to the torus creates a five-dimensional double helix wrapped around a helical cylinder. Inclusion of other interval cycles, such as the major third, would add further dimensions.

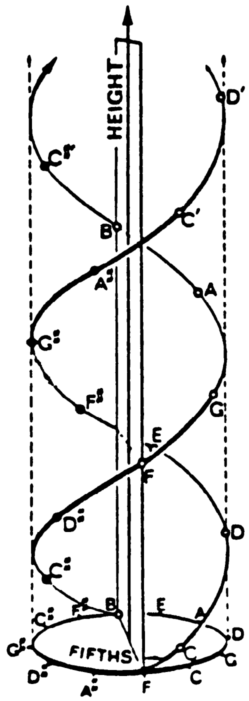

Fig. 2 Shephard's (1982) double helical model of pitch space. (Redrawn with permission of publisher.)

But the diatonic system is less symmetrical than these models suggest. Note, for example, that the double helix in Figure 2 is made up of two strands of whole-tone scales. Such a feature may be relevant to Debussy, but it has nothing to do with diatonic tonal music, the subject of Shepard's inquiry. Similarly, the major third cycle (as well as the minor third cycle) begins to appear only in the course of nineteenth-century chromaticism, as an alternative organizational principle to the diatonic system. Even the emphasis on the chromatic circle of fifths is suspect insofar as it causes neglect of the equally fundamental but irregular diatonic circle of fifths, with its single diminished fifth. A similar situation arises with Longuet-Higgin's (1962/1987) space, shown in Figure 3, where the horizontal axis consists of a fifth cycle and the vertical axis of a major third cycle. A third axis, not shown, represents octave space. As with Shepard, the space is filled with symmetrical interval cycles.[2]

2. Note that the diagonals yield semitone and minor third cycles (although Longuet-Higgins is not directly concerned with this). Through a different route, Balzano (1982) arrives at an equivalent space, rotated so that the perfect fifth cycle rather than the minor third cycle forms a diagonal. Unlike Balzano, Longuet-Higgins is much concerned with just intonation, a subject I prefer to avoid here.

A♯	E♯	B♯	Fx	Cx	Gx
F♯	C♯	G♯	D♯	A♯	E♯
D	A	E	B	F♯	C♯
B♭	F	C	G	D	A
G♭	D♭	A♭	E♭	B♭	F
E♭♭	B♭♭	F♭	C♭	G♭	D♭

Fig. 3. Longuet-Higgins' (1962/1987) model of pitch space. (Redrawn with permission of the author.)

The second and deeper problem concerns levels of pitch description. The spaces of Shepard and of Balzano (1982) refer to the relative proximity of individual *pitch classes* within a tonal framework, whereas the eighteenth-century and Weber/Schoenberg spaces refer to the relative proximity of tonal *regions*. From this perspective, Longuet-Higgins' treatment is multi-leveled: he derives his space from pcs but applies it to regions (it is inappropriate for describing pc proximity). In between these two categories, there is the question of the relative proximity of *chords*.[3] Many models address just one or another of these levels.[4] A comprehensive model must incorporate all three levels into one framework, representing perceived proximity at each level and showing how the levels interconnect. Much of the following presentation will be concerned with this issue.

A Model of Tonal Pitch Space

THE BASIC SPACE

There is no a priori requirement that intuitions about pitch, chord, and key distance be modeled topologically. We do not know how the brain performs such calculations. I therefore feel free to introduce a nontopological

3. The level of chord proximity within a region is in effect addressed in Piston (1941/1978) in his "Table of Usual Root Progressions" (Chapter 3). A usual progression would be between chords that are close in a spatial representation, an uncommon progression between spatially more distant chords. Also see the flow diagram in Werts (1983, Chapter 2), which expresses similar relationships in another guise. Lewin (1987, Chapter 2) introduces a space equivalent to that of Longuet-Higgins, but discusses it at the level of chord (root) progression across regions rather than at the level of regions per se. He points out a precedent in Riemann (1893).

4. Exception must be made not only for Longuet-Higgins but for the experimental work of Krumhansl and her associates, which has been clear about these distinctions. Her research program, summarized in Krumhansl (1983), has proceeded from the level of pcs (Krumhansl, 1979), to that of chords within a tonal region (Krumhansl, Bharucha, & Kessler 1982), to that of tonal regions (Krumhansl & Kessler, 1982). As will be seen, much of the following discussion relates to this work.

tonal pitch space—one that works, so to speak, more like a cash register. This space is musically obvious but has surprising explanatory power. The basic space is given in Figure 4, oriented toward a tonic chord in C major, notated in Roman-numeral fashion as "I/I," or "I of I"; chords are designated in plain print, regions in boldface. Later on I will discuss how the space shifts for other chords and regions, using equivalent topological representations where feasible.

The idea of this space comes from Deutsch and Feroe (1981), who speak of different pitch "alphabets"—chiefly the chromatic, diatonic, and triadic—that the listener "highly overlearns" from previously heard musical surfaces (see also Deutsch, 1982). Deutsch and Feroe show that coding via these alphabets provides a convincing account of how listeners learn and remember pitch sequences; parsimonious encoding means a small memory load. As their formalism suggests, these alphabets are hierarchically related as in Figure 4, where each level elaborates into more pcs at the next smaller level. (Or, conversely, the superordinate pcs at one level continue to the next larger level). Level *a* is octave space, *b* "open-fifth" space, *c* triadic space, *d* diatonic space, and *e* chromatic space. Pcs at larger levels are more stable than pcs at smaller levels. In short, the space is reductional (in the music-theoretic sense) rather than topological in character.[5]

Observe that this space circumvents the excessive symmetry in the spaces of Shepard and Longuet-Higgins. Although chromatic space expresses an equal-interval cycle, the other levels in Figure 4 describe the asymmetric patterns appropriate to diatonic music. In particular, and unlike the topological approaches mentioned above, this space represents the diatonic scale directly. (It should be remembered in this connection that the diatonic scale is far more ancient than the chromatic scale.) Finally and most importantly, this space allows unified treatment of pc, chord, and regional proximity.

PITCH-CLASS PROXIMITY

We begin with pc proximity within the framework of I/I, turning afterwards to the relative proximity to I/I of other chords and regions. A numerical representation will prove useful, so Figure 4 is translated into Figure 5,

5. Deutsch and Feroe do not mention open-fifth space, but include a "seventh-chord space" that would appear either between levels *c* and *d* of Figure 4 or as an extension within level *c*. (I prefer the latter alternative; see Figure 16i below.) Seventh-chord space will mostly be disregarded here, on the ground that in Classical music the interval of a seventh generally behaves as a local dissonance governed by voice-leading principles. In consequence seventh chords have little independent status. (For certain later styles—Wagner in his *Tristan*-chord mode, Debussy and Ravel, jazz—it makes more sense to incorporate an independent seventh-chord level.)

level *a*:	C												(C)
level *b*:	C							G					(C)
level *c*:	C				E			G					(C)
level *d*:	C		D		E	F		G		A		B	(C)
level *e*:	C	D♭	D	E♭	E	F	F♯	G	A♭	A	B♭	B	(C)

Fig. 4. The basic space, oriented to I/I.

level *a*:	0											
level *b*:	0							7				
level *c*:	0				4			7				
level *d*:	0		2		4	5		7		9		b
level *e*:	0	1	2	3	4	5	6	7	8	9	a	b

Fig. 5. A numerical representation of the basic space, oriented to I/I.

where C = 0, C♯ (or D♭) = 1, D = 2, . . . B = 11. (For notational convenience, 10 = a and 11 = b). This notation highlights the group-theoretic basis of the space: level *a* provides a closed (cyclic) number of elements, with adjacent elements separated by the same interval. The fixed orientations of pc0 and I/I are arbitrary.

The distance of a pc from pc0 can be calculated both vertically and horizontally. The vertical dimension gives the depth of embedding of a pc, counting the number of levels down that a pc first appears. Figure 6 shows the "embedding distance" for each pc within the space of I/I. With chordal and regional changes, of course, the pc embedding would shift accordingly.

Distance along the horizontal dimension can best be explained through the notions of *step* and *skip*. In traditional usage a step occurs between adjacent members of the chromatic or diatonic scales (a chromatic or a diatonic step), and an "arpeggiation" takes place between adjacent members of a triad. It is more illuminating, however, to think of an arpeggiation as stepwise motion in triadic space. In similar fashion, one can speak of a step in open-fifth space or octave space. A leap of two octaves, on the other hand, is a skip in octave space. In sum, a step is adjacent motion along any

Pitch class:	0	1	2	3	4	5	6	7	8	9	a	b
Embedding distance:	0	4	3	4	2	3	4	1	4	3	4	3

Fig. 6. Pitch-class embedding distance.

level of the hierarchy, and a skip is nonadjacent motion—two or more steps—along any level.[6]

The notion of "step distance" makes sense only in terms of specific pitches rather than of pcs. Assuming pitches 0 to 11 are contained within an octave, the step distance of a pitch from p0 ("pitch 0") can be calculated horizontally at a number of levels. For example, p4 has a step value of 1 in triadic space, 2 in diatonic space, and 4 in chromatic space. P6 has a step value of 6 in chromatic space, but in the harmonic context of V/V−V−I it has a value of 2: one step from 6 to 7, another step (at a higher level) from 7 to 1. Hence it is pointless to assign a fixed value for step distance comparable to the embedding distance in Figure 6, since step distance typically implicates chords and regions.

The ideas of embedding and step distance shed light on the musical treatment of consonance and dissonance. Given a certain orientation of the space, more deeply embedded pitches are understood in terms of their adjacent superordinate context. Moving down in the space means increasing dissonance, to be relieved by moving up again. Stepwise motion ensures the coherence of these moves. If a skip to a more embedded pitch occurs, this second pitch is located through subsequent stepwise motion to a third pitch that in turn lies a step away from the first at a superordinate level.

CHORD PROXIMITY WITHIN A REGION

For the tonal hierarchy it is best to regard a chord (triad) as a *Klang* abstracted away from specific register or spacing (Riemann, 1893). Chords are represented at level *c* in the space, with level *b* outlining the fifth in the chord and level *a* supplying the root.

There are two factors in calculating the proximity of any two chords within a tonal region: the diatonic circle of fifths and common tones. These factors originate in the top two levels of the basic space. Both are familiar in the music-theoretic literature. The circle-of-fifths factor is typically justified on the ground that root motion by fifths is basic to cadences and is pervasive at all levels of tonal organization. (There are psychoacoustical and group-theoretic justifications as well: the prominence of the third partial and the traversal of the entire chromatic via the fifths cycle.) The common-

6. The step/skip idea, and indeed the whole question of musical space, became a preoccupation when I was working on prolongational "timbral hierarchies" through digital synthesis (Lerdahl, 1987a). I found that timbral stability conditions could not be realized without constructing at least a two-dimensional array within which timbral events "traveled." Perhaps there is an underlying "spatial" level of mental representation common to visual, musical (pitch and timbre), and even semantic understanding (Shepard & Cooper, 1982; Jackendoff, 1983).

tone factor is justified on the ground that the sharing of pcs—of octave space—is a strong associative link between two chords.

The diatonic circle-of-fifths operation takes place in triadic, open-fifth, and octave space, leaving the content of diatonic space intact. The *chord-circle rule* can be stated as "move the pcs at levels *a–c* four diatonic steps to the right or left (mod 12) on level *d*." For example, applying the rule once to the right on I/I in Figure 5 produces Figure 7a. Applying the rule two and three times to the right on I/I results in Figures 7b and 7c, respectively. (The triad in Figure 7a is labeled V/I, or more simply as V; and so on.) The number of applications of the rule in any given instance measures the circle distance between two chords. Repeated application of the rule results in the complete *diatonic* circle of fifths, shown in Figure 8 with only the pc numbers for chordal roots given. Note the diminished fifth between b and 5, resulting from the two half steps at level *d* between b and 0 and between 4 and 5.

In an extension of the step/skip distinction, a step can be said to occur between adjacent chords in Figure 8 and a skip between nonadjacent chords. For example, a progression from I to V is a step, and a progression from I to ii or from I to vi is a skip.

(a)

							7				
		2					7				
		2					7				b
0		2		4	5		7		9		b
0	1	2	3	4	5	6	7	8	9	a	b

V/I

(b)

		2									
		2							9		
		2			5				9		
0		2		4	5		7		9		b
0	1	2	3	4	5	6	7	8	9	a	b

ii/I

(c)

									9		
				4					9		
0				4					9		
0		2		4	5		7		9		b
0	1	2	3	4	5	6	7	8	9	a	b

vi/I

Fig. 7. Applications of the chord-circle rule to generate other chords within a region.

0(I)

5(IV) 7(V)

2(ii)

b(vii°)

9(vi)

4(iii)

Fig. 8. The diatonic circle of fifths, or the "chord circle."

These shifts also cause pcs to return, forming common-tone relationships with I. In Figure 7, V has one common tone with I, ii has no common tones, and vi has two common tones. Root motion by thirds is proximate because it maximizes common-tone relationships. However, it is not enough merely to say that two chords share one or two pcs. The strength of a duplication depends on hierarchical position, with the root of a chord being most important, the fifth less so, and the third least so. These weightings are articulated in the space by the number of times a pc appears in it. Conversely, the common-tone distance between two chords depends on the number of *distinctive* pcs between the two. In order to include the differences in weight of the root, fifth, and third in the new chord, its distinctive pcs must be counted *at all levels*. As the underlined numbers in Figure 7 indicate, V has four distinctive pcs in relation to I; ii has six distinctive pcs; and vi again has four distinctive pcs. This will be our measure of common-tone distance.

How do these two factors interact to measure the overall distance between two chords in a region? It might be sufficient just to associate a pair of numbers (j, k) with chord distance, where $j =$ the shortest number of steps on the circle of fifths and $k =$ the number of distinctive pcs. The distance from I to V would then be described as (1,4); to ii as (2,6); and to vi as (3,4). This notation reveals, for example, that even though ii is closer to I on the circle of fifths, vi shares more common tones with I; and that even though vi shares two common tones with I, and V shares only one common tone, $k = 4$ for both because the root of V is a common tone and the root of vi is not.

It is desirable, however, to take the added step of combining the two factors to produce one overall measure of distance. The simplest way to do this is additive: $d(\text{chord}) = j + k$, where $d =$ shortest distance. This procedure fits intuition surprisingly well and will be followed here. Figuring from I, then, $d(\text{V}) = 1 + 4 = 5$, $d(\text{ii}) = 2 + 6 = 8$, and $d(\text{vi}) = 3 + 4 = 7$. If the circle-of-fifths rule is similarly applied to the left, the same values result for IV, vii°, and iii, as shown in Figure 9.[7]

Figure 10 summarizes d for each triad within **I**. The results are more or less as one might anticipate: V and IV are equally close to I; vi and iii are next; ii and vii° are more distant.

Distance d is invariant under transposition. Figure 11 illustrates by comparing IV to vi, resetting the values for vi to 0. Just as with vi in comparison to I, $j = 3$, $k = 4$, and $d = 7$. This procedure works in the general case, a significant point in itself about the space. It also suggests a way to enrich the

7. Differentiation could be made here by assigning minus numbers for leftward movement on the circle and then adding by using the absolute value; for example, $d(\text{IV/I}) = |-1| + 4 = 5$. I will not bother with this refinement.

(a)
```
        5
0       5
0       5       9
0   2   4 5   7   9   b
0 1 2 3 4 5 6 7 8 9 a b
```
d(IV) = 1 + 4 = 5

(b)
```
                      b
          5           b
    2     5           b
0   2   4 5   7   9   b
0 1 2 3 4 5 6 7 8 9 a b
```
d(vii°) = 2 + 6 = 8

(c)
```
        4
        4             b
        4     7       b
0   2   4 5   7   9   b
0 1 2 3 4 5 6 7 8 9 a b
```
d(iii) = 3 + 4 = 7

Fig. 9. Derivation of some chord distances within I.

Chord:	I	ii	iii	IV	V	vi	vii°
d:	0	8	7	5	5	7	8

Fig. 10. Summary of chord distances within I.

```
                  9
        4         9
0       4         9
0   2   4 5   7   9   b
0 1 2 3 4 5 6 7 8 9 a b
```
d(vi) = 0 + 0 = 0

```
          5
0         5
0         5       9
0   2   4 5   7   9   b
0 1 2 3 4 5 6 7 8 9 a b
```
d(IV) = 3 + 4 = 7

Fig. 11. Example of d's transpositional invariance. d(vi) is reset to 0.

concept of chordal step, for which we have so far had only a circle-of-fifths account (Figure 8). In Figure 10 root motion by thirds is next in proximity to motion by fifths ($d = 7$ as opposed to $d = 5$). These common-tone relationships can be used to generate a second axis, projecting the circle in Figure 12, where stepwise motion again takes place between adjacent chords. Because of d's transpositional invariance, these steps can be made equidistant from one another, disregarding the major or minor quality of any particular third motion. (This equidistance is quite different from Longuet-Higgin's major-thirds vector.)

Figure 13 combines the two circles in Figures 8 and 12. The vertical axis represents the fifths circle and the horizontal axis the thirds circle. Since both axes are circular, the topology for the space should be toroidal rather

0(I)
9(vi) 4(iii)
7(V)
5(IV)
b(vii°)
2(ii)

Fig. 12. The common-tone circle (within a region).

vii°	ii	IV	vi	I	iii	V
iii	V	vii°	ii	IV	vi	I
vi	I	iii	V	vii°	ii	IV
ii	IV	vi	I	iii	V	vii°
V	vii°	ii	IV	vi	I	iii
I	iii	V	vii°	ii	IV	vi
IV	vi	I	iii	V	vii°	ii

Fig. 13. The chordal torus, or "chordal space," created by combining Figures 8 and 12.

than just two dimensional. (Imagine a piece of paper wrapped around in a circle both horizontally and vertically at the same time.) The chords on the horizontal axis are more widely spaced, reflecting the larger value for d for chords a third apart. Diagonal "steps" are still larger and might be called "chordal skips."

It may be objected that the values in Figure 10 from which Figure 13 derives do not adequately mirror real musical behavior. First of all, ii often functions in its first inversion as a dominant-preparation voice-leading variant of IV, not as a "real" ii, in which case $j = 1$ rather than 2. Second, to avoid the diminished fifth between 5 and b, iii is often approached by rightward motion, in which case $j = 4$ rather than 3. This value would reflect the typically greater remoteness (in major) of the harmonic as opposed to the melodic third degree. Third, the characterization of vii° favors its behavior in a circle-of-fifths sequence rather than its common voice-leading dominant function, in which latter case $j = 1$ rather than 2 (since Rameau, vii° has been characterized as a V^7 with an absent fundamental). The revised overall values for these chords would then be $d(\text{ii}) = 1 + 6 = 7$, $d(\text{iii}) = 4 + 4 = 8$, and $d(\text{vii}°) = 1 + 6 = 7$. Alternative values for these chords might be selected for different contexts.

But I would argue that chordal space need not directly reflect these voice-leading factors. Voice-leading belongs to event hierarchies, to which the tonal hierarchy is not the sole input. I prefer to stick with the original values for d if only because they permit the construction of the regular chordal space of Figure 13, which has further uses below.

A second objection might be that Figures 10 and 13 depend on a formula ($d = j + k$) that compares, so to speak, apples and oranges. Why should the steps on circle of fifths and the number of distinctive pcs combine in a simple additive fashion? Why not another formula, perhaps one that includes constants ($d = aj + bk$) or some sort of vector analysis? These questions continue below when a second circle-of-fifths rule is introduced; then there will be two apples and one orange. In response, both here and below, I merely point out that a simple additive model leads to attractive theoretical results and (as will be seen) correlates with the experimental data. But probably a better quantification can be found.

CHORD PROXIMITY ACROSS REGIONS

The next stage is to evaluate the distance between chords that are in different regions. We begin with regional shifts—that is, with changes in diatonic collection.[8] Distance is calculated by moving on the circle of fifths at level d. The *region-circle rule* can be stated as "move the pcs at level d seven chromatic steps to the right or left (mod 12) on level e." As shown in Figure 14, applying the rule once to the right changes the C major diatonic collection (Figure 14a) to the G major diatonic collection (Figure 14b). A special feature of the diatonic system is that a move on the regional circle causes just one pc change, and that this change is to an adjacent pc in chromatic space. Thus in Figure 14, pc5 shifts to pc6. The voice-leading ramifications of this feature are profound: a mere half-step alteration in a line can change the listener's tonic orientation.

The number of applications of the region rule is one measure of the distance between two regions. Repeated application of the rule produces the complete *chromatic* circle of fifths, shown in Figure 15 with only the roots given. Analogous to Figure 8, regional stepwise motion can be thought of provisionally in terms of adjacency on the chromatic circle.

It is important to see that a circle-of-fifths operation must apply at two conceptual levels, once for chords within a region and once for regions. By way of illustration, the distance from C major to C minor is 0 on the chord

8. Contrasting musical terminology is widely used for chromatic moves at different levels of event hierarchies: "applied dominant" for momentary regional shifts, "tonicization" for intermediate shifts, and "modulation" for global shifts. For present purposes these distinctions need be considered *only* as a matter of hierarchical level.

(a)													(b)											
0		2		4	5		7		9		b		0		2		4		6	7		9		b
0	1	2	3	4	5	6	7	8	9	a	b		0	1	2	3	4	5	6	7	8	9	a	b

Fig. 14. An application of the region-circle rule to generate the G major collection from the C major collection.

0
5 7
a 2
3 9
8 4
1 b
6

Fig. 15. The chromatic circle of fifths, or the "region circle."

circle (Figure 8) but 3 on the region circle (Figure 15); conversely, the distance from C major to A minor is 3 on the chord circle but 0 on the region circle. The parallel and relative minor are equally proximate to I/I, but in opposite ways.

This example suggests enlarging the above formula for overall distance: $d(\text{chord}) = i + j + k$, where i = steps on the region circle and, as before, j = steps on the chord circle and k = distinctive pcs. Until now i has always been 0. The simplest way to calculate i is to count the number of changes in flats and sharps in the key signature. To gauge chord-circle distance within the new region, j is calculated first to the new tonic and then from it. For example, for iv/vi, the value for j is not 2, as it would be directly from I/I (iv/vi = ii/I). Instead, $j = 3 + 1 = 4$; that is, three circle steps up to vi and one back to iv. The value for k will usually be larger than before because distinctive pcs will now appear at the diatonic level as well. As above, this formula is transpositionally invariant.

There is always a question as to how to treat the minor mode, which supposedly has three versions: the natural, harmonic, and melodic. It is better, however, to view this matter only in terms of scale-degree voice-leading tendencies. The raised seventh degree appears as the leading-tone when rising to the tonic, and as a result is part of V and vii° (it is not part of III, certain harmony texts notwithstanding). A raised sixth happens only in voice-leading situations where an augmented second with the raised seventh would otherwise arise. These are all context-dependent deviations within a natural-minor framework, so I will regard the natural minor as basic. However, for V and vii° the raised seventh will be assumed, increasing *k* by one from what it would otherwise be.

A consequence of this view of the minor mode is that *i* becomes the same for relative major/minor collections. This temporarily leads, in the manner of Schoenberg's first attempt at describing tonal space (Schoenberg 1911/1978, Chapter 9), to a "double" circle, where each position in Figure 15 is filled with a major mode and its associated minor mode. On the other hand, *j* shifts as before, so the overall result is not equivalent. For example, $i = 1$ for both I/IV and i/ii, but $j = 1$ for I/IV and $j = 2$ for i/ii.

Figure 16 gives various examples of interregional chord distances. As above, *d* is reckoned in relation to I/I. Figures 16a and 16b juxtapose the dominant chord heard as dominant (V/I, or "V of I") and as tonicized (I/V, or "I of V"). Because *i* as well as *j* shifts one time, and because pc5 changes to pc6 in diatonic space, the overall distance of I/V from I/I is two values greater than that of V/I. Figures 16c and 16d similarly juxtapose an A major triad as V of d minor and as I of A major; this time the difference is 3, because of the inclusion of pc1 (because it is part of the chord) in the d minor collection. Such distinctions in chord context are crucial to musical understanding—V/ii really feels closer to I/I than I/VI does—so it is important that the model be able to express these differences.

Figures 16e and 16f compare i/ii and i/iii; i/iii turns out slightly closer to i/i. Figure 16g adds V/iii, for which *d* is larger than for i/iii. However, Figures 16h and 16i similarly compare I/IV and V/IV, to highlight the easily neglected fact that another chord of a second region is sometimes closer to I/I than its local tonic is. (The seventh has been included at the triadic level in Figure 16i to indicate how sevenths might best be treated when they have harmonic status; think of the beginning of Beethoven's First Symphony. Exclusion of the seventh would decrease *d* by one.)

Figures 16j and 16k juxtapose tonic chords in the relative and parallel minor regions. The values for *i* and *j* are reversed (as suggested above), and the value for *k* is identical despite the contrasting distribution of distinctive pcs; so the overall result is the same.

Finally, Figures 16l and 16m give characteristic "mixture" chords, iv/I and ♭II/I (the Neapolitan), in which it is understood that the region does *not* shift; rather, certain pcs are "borrowed" from the parallel mode. Thus *i* and *j* do not change from their diatonic equivalents, and *k* increases. (Compare Figure 16l (iv/I) with Figures 9a (IV/I) and 16h (I/IV).) In the same vein, $i = 0$ for I/i at the end of a movement in minor (the Picardy third), whereas $i = 3$ when the parallel major region is established for the second theme group in the recapitulation of a movement in minor. The former is a local

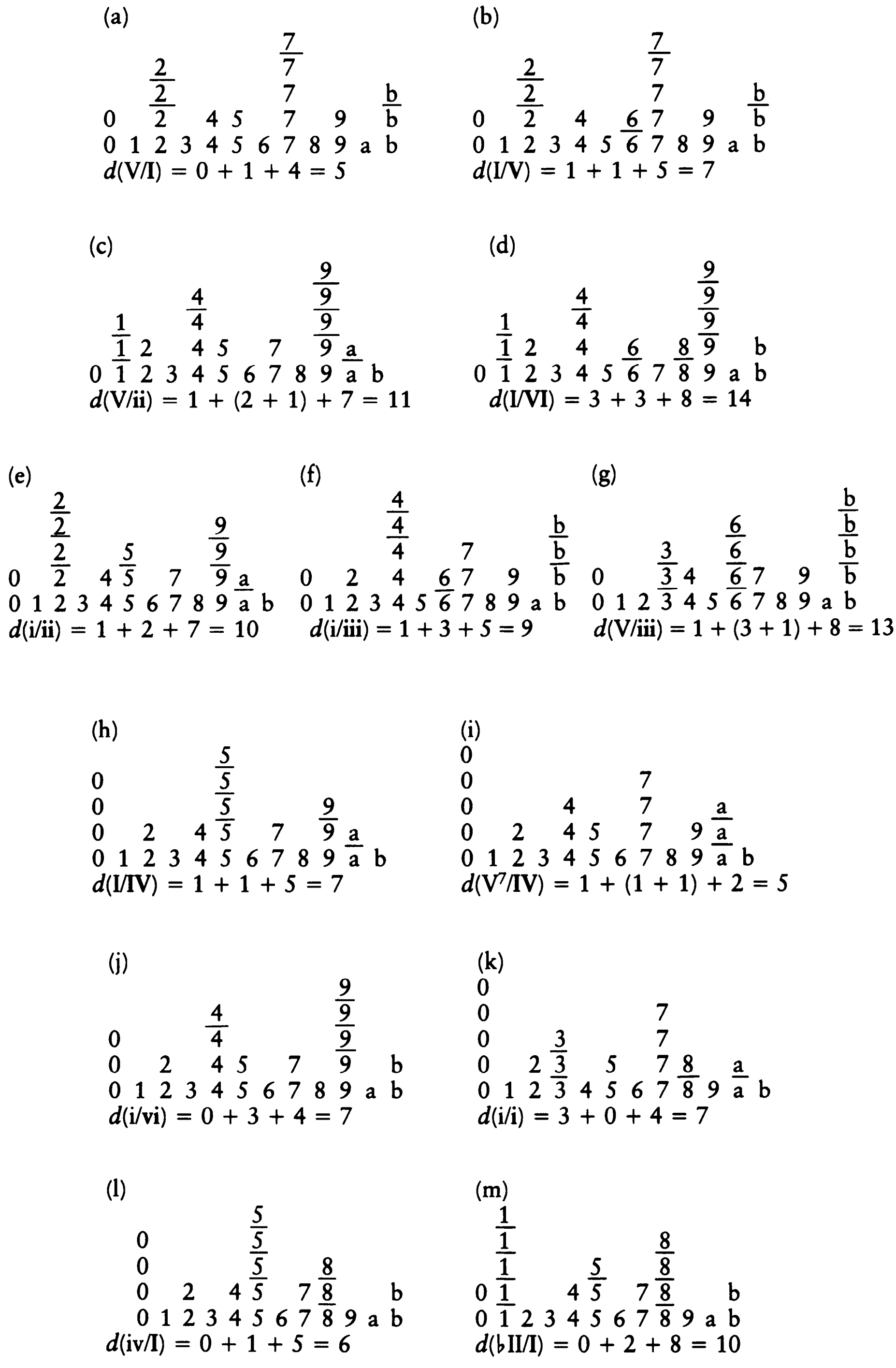

Fig. 16. Examples of chord distances across regions.

inflection, the latter a reorientation often requiring careful preparation by the composer.

The formula for *d* works for chords in nearby regions but often goes awry for chords in more distant regions. One reason for this is that *i* counts only circle-of-fifths relationships between regions. As with chordal space (Figure 13), regional space must be enriched to include another axis. However, as we have seen, *d* varies both for the same chord in different regions (Figure 16a–b, c–d) and for different chords within the same region (Figure 16f–g, h–i). It is necessary to address a more abstract level, that of the region as a whole.

REGIONAL PROXIMITY

Let us define the proximity of a secondary region simply in terms of the *distance of its local tonic*. This step is justified because one strongly hears chords in relation to their local tonic as well as to one overall governing tonic. The distance of a local tonic is a simplified but salient indicator of the distance of the region it represents.

Regional space can now be developed in the same way as toroidal chordal space. First the local tonics are selected that have the smallest values for *d*. These are I/V, I/IV, i/vi, and i/i, for all of which $d = 7$ (although *i*, *j*, and *k* differ for each; see Figure 16b, h, j, and k). The circle of fifths (I/V and I/IV) is placed on the vertical axis, and relative and parallel minor relationships (i/vi and i/i) on the horizontal axis. Then *d* is recursively transposed to generate two cycles, in fifths and minor thirds, together forming a torus. Because $d = 7$ in all directions, the elements on this torus are equally spaced, unlike the chordal torus. The result is Figure 17a. Figure 17b translates the note names of Figure 17a into Roman numerals, with "i/♯ii" abbreviated as "♯ii," and so on, all oriented toward I/I.

Since the chordal and regional spaces are both projected by fifths and thirds axes, the two spaces can be combined as in Figure 18. A fragment of regional space is shown, and for each such region a representative portion of chordal space is given. Each regional designation doubles for "I" or "i" at the chordal level, for which various major/minor adjustments are also made. Observe that the chordal pattern is repeated at a larger level in the regional pattern. Moreover, the proximity of chords across regions emerges directly in this representation. In I, for example, V is closest to V; conversely, in V, IV is closest to I (IV/V = I/I); and so on throughout the space, horizontally and diagonally as well as vertically. This visualization is misleading only to the extent that, according to the calculations for *d*, V/I is actually closer to I/V than IV/V is, and so forth; that is, the chords within regions should overlap those in an adjacent region. But the representation nicely illustrates the recursive and hierarchical nature of tonal pitch space.

(a)

d♯	F♯	f♯	A	a	C	c
g♯	B	b	D	d	F	f
c♯	E	e	G	g	B♭	b♭
f♯	A	a	C	c	E♭	e♭
b	D	d	F	f	A♭	a♭
e	G	g	B♭	b♭	D♭	d♭
a	C	c	E♭	e♭	G♭	g♭

(b)

♯ii	♯IV	♯iv	VI	vi	I	i
♯vi	VII	vii	II	ii	IV	iv
♯i	III	iii	V	v	♭VII	♭vii
♯iv	VI	vi	I	i	♭III	♭iii
vii	II	ii	IV	iv	♭VI	♭vi
iii	V	v	♭VII	♭vii	♭II	♭ii
vi	I	i	♭III	♭iii	♭V	♭v

Fig. 17. The regional torus, or "regional space," created by combining the fifths cycle and the relative and parallel major-minor cycle. (a) represents the regions by key; (b) represents them as subregions oriented to C major.

III	V	vii°	iii	V	vii°
VI	**iii**	III	vi	**V**	iii
ii°	iv	VI	ii	IV	vi
III	V	vii°	iii	V	vii°
VI	**vi**	III	vi	**I**	iii
ii°	iv	VI	ii	IV	vi
III	V	vii°	iii	V	vii°
VI	**ii**	III	vi	**IV**	iii
ii°	iv	VI	ii	IV	vi

Fig. 18. A two-dimensional representation of the combined chordal and regional tori.

As it stands, continuing the pattern horizontally in Figure 18 necessitates skipping parallel regions. This difficulty can be circumvented by adding a further dimension to Figure 17, forming the "stairs" sketched in Figure 19. Here VI lies directly below vi, i directly above I, and so on. Regions a fifth or minor third away remain on one stair. Intraregional chords, in a transformed representation of Figure 18, could then be mapped into Figure 19, with iii/I lying directly beneath ♭III/i, and so forth, yielding a vertical correspondence between "mixed" versions of the same scale degree and preserving the general pattern of Figure 18. Because each chordal region forms a torus, different distances would subsist between these vertical chord pairs, with the pair I–i being the closest. But at this point the geometry becomes hard to visualize—we are dealing with stairs within a regional torus, within which there are numerous "satellites" of chordal tori—so I will not attempt it.

PIVOT CHORDS AND PIVOT REGIONS

Figure 18 suggests a further reason why the above formula for *d* works for nearby but not distant regions. Nearby regions share chords that can function as *pivot chords* from one region to another. But as chromatic changes infiltrate more distant regions, direct pivot chords disappear, requiring intermediate pivots that have not yet been taken into account.

Pivots can be thought of as points of tonic reorientation along a stepwise path. So we return again to the step/skip notion. Regional stepwise motion might at first be thought of just in terms of vertical or horizontal adjacency in Figure 17. For example, **VI** is two steps to the west but three steps to the north from **I**. So far this makes sense, since it is indeed a shorter route to **VI** via **vi** than via **V** and **II**. However, this conception would make **ii** as well as **VI** and **II** two steps (rather than one step) away from **I**, an intuitively undesirable result. This problem could be rectified by including diagonal motion as stepwise; but then **v**, too, would be a step from **I**—even though intuitively **v** is accessed from **V** or **i** instead.

These and other difficulties can be resolved by calculating steps in regional space via "tonic pivots." Consider Figure 20a, which corresponds to the regional level of Figure 18. As Figure 20a indicates, the tonic pivots from **I** are **ii**, **iii**, **IV**, **V**, and **vi**; that is, these are regions whose local tonics are also chords in **I**. The diagonal steps to **v** and **iv** are excluded, as is the diminished-chord seventh degree, which is not adjacent even diagonally. The value of *d* is given for each region in parentheses. (Note the slight asymmetry due to d(**iii**) = 9 and d(**ii**) = 10.) Figure 20b provides in reverse the tonic pivots from **i**. Again the values of *d* are included, this time with d(**i**) reset to 0.

Let us call six-regional units such as Figures 20a and 20b *pivot regions*. These are analogous to pivot chords, but at a larger level. In terms of Figure 17, these units slide stepwise up and down the fifths axis, but for horizontal steps they must move in their entirety, linked by parallel regions. Figure 21a illustrates by shifting the pivot region of Figure 20a northward one notch and eastward to the entirely nonoverlapping adjacent unit of Figure 20b. The same moves emerge more perspicuously in Figure 21b, which is based on the "stair" representation of Figure 19 (parallel regions are on adjacent stairs).[9]

If a local tonic is not in the base pivot region, the pivot region can be moved stepwise until it is, each time adding 7—the step value in all vertical

9. There is a resemblance between these stairs of pivot regions and the blocks (or "compact groups") that Longuet-Higgins uses to move around in his space (Figure 3). His blocks move up and down the fifths cycle and flip-flop for parallel major–minor relationships. However, the details and even the levels of analysis are different.

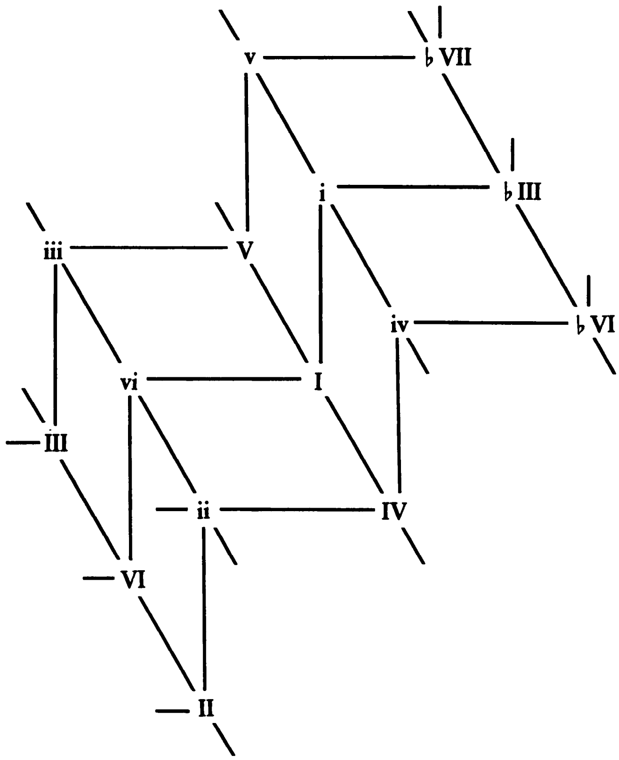

Fig. 19. A "stair" representation of a portion of the regional torus. The distance between I and i, for example, remains the same as in Figure 17. It is possible to imagine including the chordal tori as "satellites" within this representation (compare Figure 18).

(a)		(b)	
iii(9)	V(7)	v(7)	♭VII(10)
vi(7)	I(0)	i(0)	♭III(7)
ii(10)	IV(7)	iv(7)	♭VI(9)

Fig. 20. Representative pivot regions, together with their distance values from I and i, respectively.

and horizontal directions—to the total distance. This operation can be stated as $D = d1 + d2 \ldots. + dn$, where D = total distance, $d1$ = the intermediate distance from the starting point to the first pivot, $d2$ = the intermediate distance from the first pivot to the second, and so forth. For convenience, the chord designation for $d2 \ldots. dn$ can be reset each time to I (or

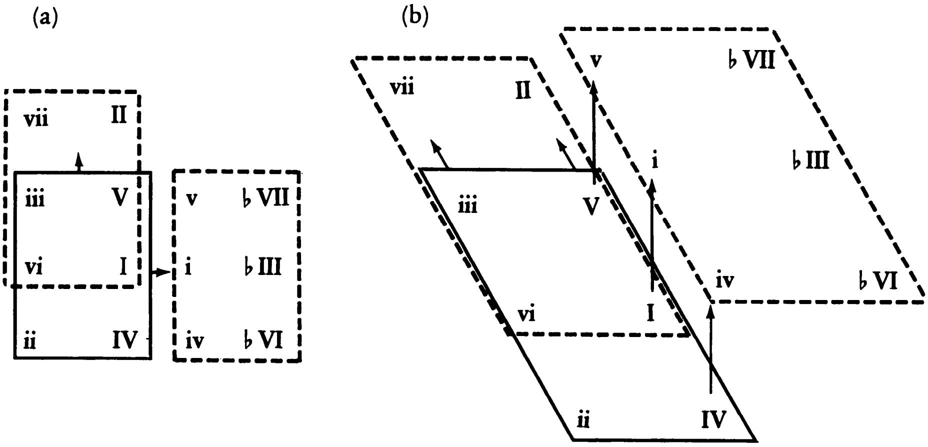

Fig. 21. The shifting of pivot regions to adjacent pivot regions, (a) in a flat representation, and (b) in the "stair" representation.

i); thus V to v is reset as I to i, and so on. For example, $D(\mathrm{v}) = d1(\mathrm{V}) + d2(\mathrm{i}) = 7 + 7 = 14$; or, by the other route, $D(\mathrm{v}) = d1(\mathrm{i}) + d2(\mathrm{v}) = 7 + 7 = 14$. Another example: $D(\mathrm{III}) = d1(\mathrm{iii}) + d2(\mathrm{I}) = 9 + 7 = 16$. A more complicated example: $D(\sharp\mathrm{i}) = d1(\mathrm{iii}) + d2(\mathrm{I}) + d3(\mathrm{vi}) = 9 + 7 + 7 = 23$; or, in more normal terminology, "move from I to iii, then to iii's parallel major, III, then to III's relative minor." But if ♯i is understood as ♭ii, then the derivation might be $D(\flat\mathrm{ii}) = d1(\mathrm{i}) + d2(\flat\mathrm{VI}) + d3(\mathrm{IV}) + d4(\mathrm{i}) = 7 + 9 + 7 + 7 = 30$, which covers a trip from C to c to A♭ to D♭ to d♭.

The proximity of all regions can be calculated in this manner, using the values within a pivot region (as shown in Figure 20) in conjunction with the formula for D. Figure 22a lists the smallest value for D in relation to I for all the major and minor regions. In Figure 22b these same values are placed in the context of Figure 17, to indicate the regional stepwise paths.

The values in Figure 22 generally seem adequate—with a few exceptions. For instance, $D(\flat\mathrm{II})$ is too large. Perhaps ♭II is just odd: the Neapolitan has a special history as a replacement for ii° in minor, letting ♭II insinuate itself as a pivot chord; and in the nineteenth century, ♭II appeared extensively in connection with the enharmonic treatment of the German augmented-sixth

(a)

Region:	I	i	♭II	♯i	II	ii	♭III	♭iii	III	iii	IV	iv
D:	0	7	23	23	14	10	14	21	16	9	7	14
Region:	♯IV	♯iv	V	v	♭VI	♭vi	VI	vi	♭VII	♭vii	VII	vii
D:	30	21	7	14	16	23	14	7	14	21	23	16

(b)

	♯IV(30)					
	VII(23)	vii(16)	II(14)			
♯i(23)	III(16)	iii(9)	V(7)	v(14)		
♯iv(21)	VI(14)	vi(7)	I(0)	i(7)	♭III(14)	♭iii(21)
		ii(10)	IV(7)	iv(14)	♭VI(16)	♭vi(23)
			♭VII(14)	♭vii(21)	♭II(23)	

Fig. 22. (a) Summary of regional distances from I. (b)Representation of regional distances in regional space.

chord (G^6 = V^7/♭II), a chromatic shortcut that lies outside the purview of the model. On the other hand, a related problem—because ♭VI lies in ♭II's path—is that *D*(III) and *D*(♭VI) seem slightly large compared to *D*(II) or *D*(♭VII). The difficulty here lies with the idealization toward local tonic *chords* as the basis for measurement in regional space. The ease of modulating to III or ♭VI arises rather from their tonic chords' sharing of a common *tone* with I (unlike II or ♭VII). A full solution to regional proximity, then, demands a partial abandonment of this idealization.

Let us continue with the regional space of Figures 17 and 18. There is one more step to take. Once the region in question falls within the shifted pivot region, the value of any other, nontonic chord in the region can be calculated in similar fashion. This can be done by moving to the regional tonic and then summing *j* and *k*—that is, by thinking of the nontonic chord solely in terms of its local tonic. But a smaller overall value can be obtained by calculating the nontonic chord directly from a regional pivot, provided that the tonic to which it refers belongs in the pivot region in question. This was the method used above, for example, for V/ii in Figure 16c. Taken in two stages, *D*(V/ii) = *d1*(ii) + *d2*(V/i) = 10 + 7 = 17, which is too high a value (*d*(I/VI) = 14); but figured directly, *d*(V/ii) = 11, which is about right. This shortcut is possible only because the ii to which this V refers belongs in the base pivot region. For a more remote case, say to arrive at an f♯ chord in the region of c♯, one can count directly from the intermediate E region, since an f♯ chord is a pivot chord within E. Thus *D*(iv/♯i) = *d1*(iii) + *d2*(I) +

$d3(\text{iv/vi}) = 9 + 7 + (0 + (1 + 3) + 6) = 9 + 7 + 10 = 26$. It is a matter of a pivot chord within a pivot region.

The overall model needs further refinement. In addition to occasional infelicities in the numerical calculations, the topological representations are incomplete. I hope at least to have presented features that a comprehensive model should incorporate. Let us now briefly review the experimental evidence for the model.

Experimental Support

The relevant psychological literature confirms the basic pc space of Figures 4 and 5. Deutsch (1980) shows that pitch sequences structured parsimoniously by the chromatic, diatonic, and triadic alphabets are more easily processed than sequences not so structured. Lake (1987), in a study reminiscent of Carlsen (1981), has found that subjects, after hearing two tones in a tonal context, spontaneously sing continuations that are to a large extent interpretable in terms of the space. Krumhansl (1979), in testing for the stability of tones (pcs) in a tonal context, has derived a three-dimensional cone, shown in Figure 23, that matches the stability configuration in Figure

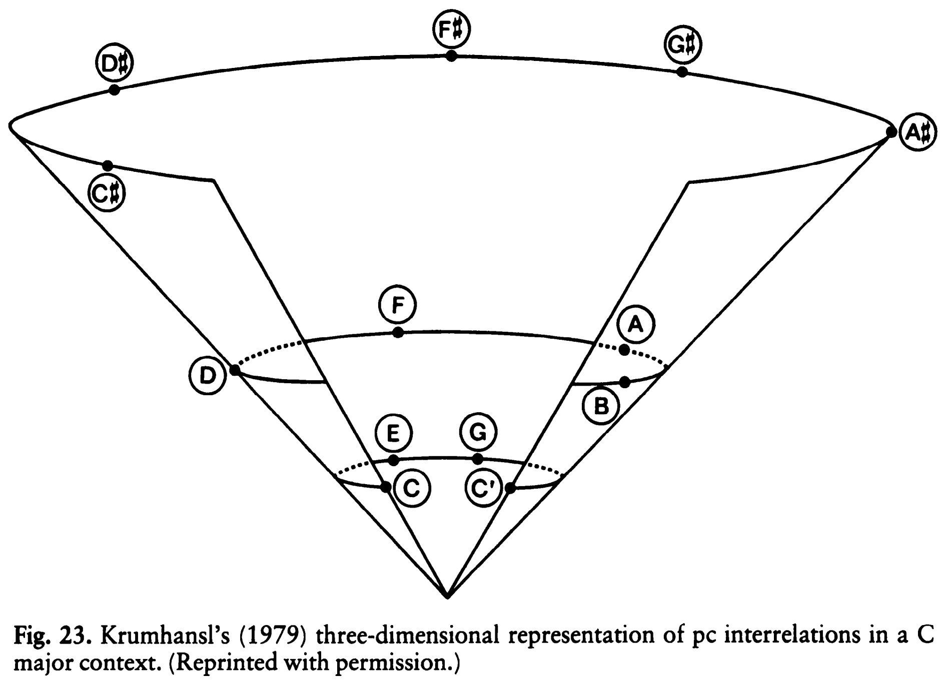

Fig. 23. Krumhansl's (1979) three-dimensional representation of pc interrelations in a C major context. (Reprinted with permission.)

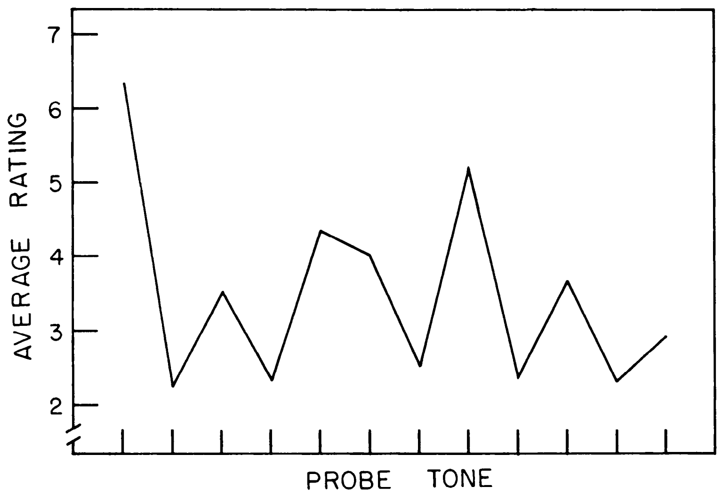

Fig. 24. Krumhansl and Kessler's (1982) C major key profile. (Copyright 1982 by the American Psychological Association. Adapted by permission of the publisher and author.)

4 (also see Krumhansl & Shepard, 1979). In addition, the values for pc embedding distance in Figure 6 correspond closely to the major key profile in Krumhansl and Kessler (1982), given here in Figure 24. In their experiment, the "fit" of each pc was rated by listeners who were primed with a tonal context.

In the minor key profile, however, Krumhansl and Kessler found that pc3 rated higher than pc7. This result is puzzling, since the hierarchical relationship between the third and fifth of a triad is the same in its major and minor forms. Their result allows them to derive the relative major key as closely related to the referential minor key, but this seems to be a confusion of levels. As suggested in this model (through the variable *i*), the relative major is so close to the tonic minor because they share the same diatonic collection.

The notions of embedding and step distance at the pc and pitch level relate to Krumhansl's (1979) finding of asymmetry in judgment in pairs of tones, whereby an unstable pitch is found to be closer to a stable pitch than vice versa. In a I/I context, for example, p11 ("pitch 11") is psychologically closer to p12 than p12 is to p11. In Figure 5 this is reflected in the step and embedding context of a pitch pair: p11 "gravitates" in diatonic space toward the adjacent and weightier p12, which also belongs to superordinate levels. In Bharucha's (1984b) terms, p11 is unstable and requires subsequent "anchoring" to the proximate and more stable p12. (This asymmetry is familiar to musicians in the form of incomplete neighbors or appoggiaturas: the dissonance may be unprepared but must resolve by step to the succeeding pitch, to which it "belongs.") Of course, if the orientation of the space were toward another chord, say V/I, the opposite would be the case—p12 would require anchoring in p11. In similar fashion, in a I/I context and in chromatic space, p10 gravitates toward p9 or p11; in a I/I context and in triadic space, p4 gravitates toward p0; and so on, throughout various harmonic and regional contexts.

We turn now to chords and regions. The values for d for chords within a region (Figure 10) correlate well with Krumhansl, Bharucha, and Kessler's (1982) multidimensional scaling solution, given here in Figure 25, for the perceptual relatedness of the various diatonic triads to the tonic. Exception must be made, however, for d(iii), for which the value $j = 4$ should evidently be used (see the remarks above regarding iii). With this proviso, it would appear that listeners intuitively employ the chord circle and common tones along the lines of the formula $d = (i +)\, j + k$.

The values for chords across regions are supported in a general way by experiments reported in Krumhansl, Bharucha, and Kessler (1982), with respect to chords heard in the context of the closely related keys of C major, G major, and A minor; and by experiments reported in Bharucha and Krumhansl (1983), with respect to chords heard in the context of the distantly related keys of C major and F♯ major (also see Krumhansl, Bharucha, & Castellano, 1982). Further testing needs to be done, however, on evaluations of specific chords as heard in different regions.

Moving up to the level of abstract regions, we find that Krumhansl and Kessler (1982) have established a regional space almost identical to Figure 17. They projected the space in Figure 26 by intercorrelating rating profiles

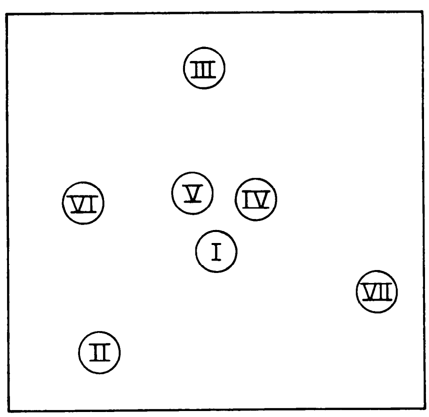

Fig. 25. Krumhansl, Bharucha, and Kessler's (1982) multidimensional scaling solution of the relatedness of chords within a key. (Copyright 1982 by the American Psychological Association. Adapted by permission of the publisher and author.)

of individual tones (such as the one shown in Figure 24) and applying multidimensional scaling. The fifths axis moves from southwest to northeast, the thirds axis from northwest to southeast.[10]

There is no direct data regarding the stairs of pivot regions, these being an innovation of the present model. However, there is tantalizing evidence in Krumhansl and Kessler's (1982) finding that listeners behave differently in modulating to close and distant regions. Subjects reorient themselves more slowly to distant regions; and once they do, they tend to lose their orientation to the first region. Evidently shifting from one stair to another requires more effort. Krumhansl and Kessler also show that different paths taken from one region to another can be significant, as reflected in the formula for D above.

Taken as a whole, empirical studies support the model. A different sort of support comes from earlier music theories. After considering these, I conclude by returning to the model's relevance for the *GTTM* theory.

Theoretical Connections

SCHENKER, MEYER, AND THE BASIC SPACE

Perhaps the fundamental voice-leading procedure in Schenker's mature theory (1935/1979) is the *Zug,*[11] which may be defined as a diatonic progression bounded by pitches of the prolonged harmony. This operation works at any level in an event hierarchy, and unites line and harmony into one conception. The *Urlinie* itself is a special case of the *Zug* at the highest structural level: over a prolongation of the tonic harmony, it outlines a downward diatonic span from a tonic chord-tone to the tonic tone.

The idea of the *Zug* can be generalized in terms of the basic space in Figure 5. Let us say that melodic *completeness* consists of stepwise motion *at any level,* such that the boundary pitches of the line are superordinate in the space. Assuming the context I/I, the progression p0–p1–p2, for example, is complete at the diatonic but not at the triadic level; p0–p2–p4 (forming a *Zug*) is complete at the triadic level; p0–p4–p7 is complete at the open-fifth level; p0–p7–p12 is complete at the octave level.

10. Do not be misled in Figure 26 by the closer visual proximity of iii to I than of V and IV. This results from the translation of a four-dimensional solution to a two-dimensional format (C. Krumhansl, personal communication, 1987).

11. This multipurpose word is usually translated in this technical context as "linear progression," although "span" gives more of its energy. As with other key Schenkerian concepts, I prefer the original German.

It is common for lines to be complete at a level more than one embedding step above. For instance, the chromatic line p4–p5–p6–p7 is complete at the triadic level; the diatonic scale is complete at the octave level. Although most Classical melodies explore completeness at the diatonic and triadic levels, some acquire their distinctiveness by emphasizing other levels and combinations of levels. Beethoven's Ninth Symphony begins with an exploration of open-fifth space, eventually filling it in with triadic and diatonic elaborations. The motivic material of the first movement of the *Eroica* Symphony often bypasses diatonic space in favor of a combination of triadic and chromatic spaces (Epstein, 1979). The famous C♯ in measure 7 motivates a number of dramatic passages in its varied search for completeness.

Completeness occurs not only at the musical surface of a piece but at underlying reductional levels. Incomplete pitches always reduce out, leaving a residue of completion at a higher level in the space. A *Zug* eventually collapses into a representation of the harmony embodied in its endpoints. Other Schenkerian voice-leading procedures can also be viewed in terms of completeness at underlying levels. For instance, the *Anstieg* (initial ascent) rises diatonically within triadic endpoints; the *Bassbrechung,* which unites with the *Urlinie* to form the all-important *Ursatz,* projects fifths within octave space.

The notion of completeness accounts as well for important aspects of Meyer's (1973) theory of melodic implication (see also Narmour, 1977). Via the Gestalt principle of good continuation, Meyer claims that a diatonic step implies continuation in the same direction, and likewise for an arpeggiation. (Such an implication can be realized or not.) As with the *Zug,* this principle generalizes to all levels of Figure 5. In fact, the basic space offers one way of constraining the number of implications, an area where implication–realization theory has been criticized. If, say, a diatonic line reaches completion at the triadic level, there would be no further implication of continuation in the same direction. Hooked up to a thorough-going reductional theory, the theory would then be able to invoke a small number of implications at any point in a given reductional level (Jackendoff & Lerdahl, in preparation).

SECHTER, RIEMANN, AND CHORDAL SPACE

Much ink was spilled by nineteenth-century fundamental-bass theorists over which root progressions are and are not acceptable (Wason, 1985). The influential Sechter (1853) restricted harmonic progressions to root motion by fifths and thirds, excluding motion by seconds, such as I to ii. In other words, he allowed only vertical or horizontal adjacent motion in the chordal space of Figure 13, whose "core" is reproduced in Figure 27. If di-

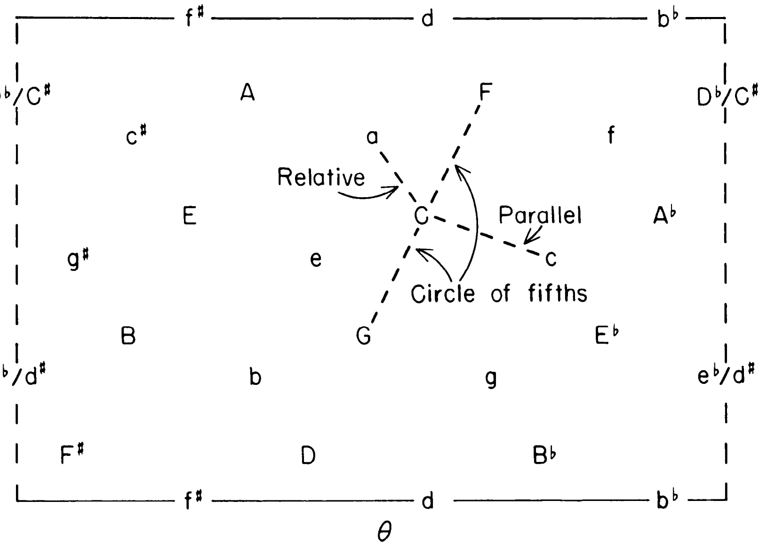

Fig. 26. Krumhansl and Kessler's (1982) toroidal representation of interkey distances. (Copyright 1982 by the American Psychological Association. Reprinted by permission of the publisher and author.)

iii	V	vii°
vi	I	iii
ii	IV	vi

Fig. 27. The "core" of the chordal torus (from Figure 13).

agonal motion in chordal space counts as a skip, he permitted only stepwise chordal motion.

Progressions such as I–ii happen in real music, of course. For these Sechter was obliged to interpose "intermediate fundamentals," or, conversely, to relate more than one chord to a single fundamental. Thus, I–ii became I–(vi)–ii. These maneuvers led to bizarre harmonic glosses, especially in the hands of his pupil, the composer Bruckner. Instead of rejecting Sechter's reasoning entirely, however, I suggest that his real mistake was to have confused the tonal and event hierarchies. Having intuited the chordal space of Figure 13, he went on to apply it inappropriately to real musical sequences. There is no reason why chord progressions should not include skips.[12]

12. Fifty years later, with his more hierarchical and linear approach, Schenker eliminated unwanted progressions by treating them as voice-leading. This was an advance but not, I

Chordal space also bears on Riemann's (1893) functional theory. For each of his three harmonic pillars, I, IV, and V, he conceived of harmonic substitutions a root-third away that retain the original function of tonic, subdominant, or dominant. A substitute a third below (a *Parallelklang*) gives tonic function to vi, subdominant function to ii, and dominant function to iii; a substitute a third above (a *Leittonwechselklang*) gives tonic function to iii, subdominant function to vi, and dominant function to vii°. Thus iii and vi can vary in function depending on context.

All of these relationships are implicit in Figure 27. The vertical axis yields the three harmonic pillars. On the horizontal axis, I is flanked by vi and iii, IV by ii and vi, and V by iii and vii°.

SCHOENBERG AND REGIONAL SPACE

The regional space of Figure 17 is identical to the Weber/Schoenberg space. (Krumhansl & Kessler point out the same correspondence with respect to Figure 26.) The method used here of generating the space presumably illuminates why Weber and Schoenberg thought it up in the first place. They must have conceived of regional distance in terms of regional tonics; and the equal proximity of I/V, I/IV, i/vi, and i/i is explicated by the formula for *d*.

The concept of musical space preoccupied Schoenberg throughout his career. He invoked it not only in his treatises on tonal harmony (Schoenberg 1911/1978; 1954/1969) but in his central essay on 12-tone composition (Schoenberg 1941/1975). A case can be made that just as he constructed his large-scale 12-tone forms by analogy with tonal forms, so he built 12-tone space by analogy with tonal pitch space. Each pc has multiple positions in the space, depending at any particular point on set orientation instead of on scale and chord orientation. The chordal level emerges largely from subsets of the row, roughly in the way that triads are subsets of scales (only "roughly," because there is no analogue to the reductional relationship between triads and scales). The regional level affords the best correspondence: within the framework of hexachordal combinatoriality, two or more sets create a "12-tone regional unit" that transposes to proximate or distant regions, where distance is measured by interval cycles of transposition (as with *i* above) and by invariance of subsets across transposition, forming "common subsets" instead of common tones (by analogy with *k*

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think, a complete solution. Even voice-leading chords are chords, having a place in the overall event hierarchy and therefore in the tonal hierarchy. (On the other hand, in some cases, such as parallel six-three progressions, the chords *are* pure voice-leading.)

above). These regional units facilitate a sort of modulation within the 12-tone universe.[13]

For reasons discussed elsewhere (Lerdahl, 1987b), moves within 12-tone space are not as cognitively transparent as those within tonal space. But the analogies are interesting, and they help to explain how Schoenberg was able to erect such an intellectually rich system.

Incidentally, the so-called Schoenberg–Schenker controversy (see, e.g., Dahlhaus, 1973) can be somewhat defused with the realization that Schenker was primarily interested in event hierarchies and Schoenberg in the tonal hierarchy. Of course, Schoenberg also had a much more vertical conception of events than did Schenker; here the difference is real.

PITCH SPACE AND GENERATIVE MUSIC THEORY

There are three subjects I want to touch on briefly with regard to pitch space and generative music theory.

First, as was pointed out at the beginning, pitch space and stability conditions are in principle the same thing. To the extent that the above model works, it can therefore replace the verbally stated stability conditions in *GTTM* (see especially pp. 224–225). The relevant reductional preference rules (TSRPR 2 and PRPR 3) can then refer to distance measurements in the space. In the general case and all else being equal, given pitch events x and y in time-span or prolongational region r, x will dominate y if x is more proximate to the referential center of the space, and conversely if y is more proximate. This formulation works at any reductional level. It is significant here that pitch space, too, is hierarchical in character. This permits a correspondence between levels of the event and tonal hierarchies that otherwise would be impossible.

Second, an acknowledged limitation of the *GTTM* theory is its sketchy account of voice-leading, an area where Schenker is strong. This is only a practical limitation, however, requiring not revision but extension of the theory. The correct solution, I believe, is not to borrow in a literal fashion Schenker's numerous voice-leading techniques, many of which I take to be merely colorful descriptions of hierarchical relations, but to adopt their essence through the above generalization of the *Zug*. That is, the notion of completeness, as fleshed out through the proposed pitch space, provides the basis for an account of voice-leading intuitions. (A related shortcoming of the *GTTM* theory, its inability to give separate structural descriptions for different contrapuntal lines, requires a different sort of solution that lies beyond present concerns.)

13. Lewin (1972) discussed such relationships in connection with Schoenberg's Violin Fantasy, Mead (1985) in connection with Schoenberg's Violin Concerto.

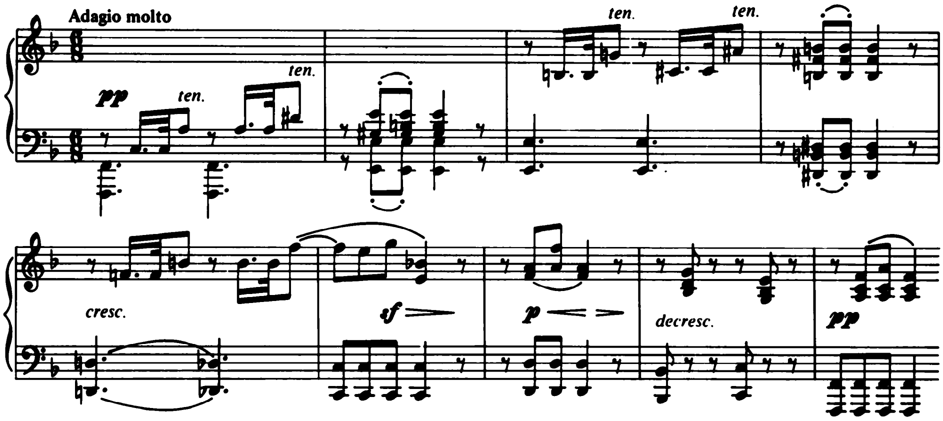

Fig. 28. Measures 1–9 of the second movement of Beethoven's *Waldstein* Sonata, op. 53.

Third, having a theory of the tonal hierarchy permits a richer description of event hierarchies. Let me illustrate with the opening phrase of the second movement of Beethoven's *Waldstein* Sonata, given in Figure 28. Smith (1986) demonstrates that a linear analysis alone fails to capture the passage's striking harmonic specificity. To rectify this shortcoming, he adds a leveled Roman-numeral analysis to his linear graph. This step is useful but does not really convey a sense of the phrase's extraordinary harmonic progression.

A pitch-space description can help. With I/I reset to F major, Figure 29 repeats part of Figure 17b, showing the regional route traveled in the course of the phrase. A number of diagonal steps leads to a distant, tentative ♯IV in measure 4, after which the harmony skips back to vii°/V–A^6–V^7/I in measures 5 and 6. This "map" visualizes the harmonic journey far better than Roman numerals can.[14]

But such a map has meaning only in conjunction with a hierarchical analysis. So, following the spirit of Smith's argument, I append a *GTTM*-style reduction of the passage in Figure 30.[15] The time-span reduction (TSR) appears in the upper staves of the musical notation, the prolongational reduction (PR) in the tree and lower staves. Nonessential voices are reduced out at underlying levels. The octave transfer at underlying levels in measures 1 and 2 is justified by the motivic parallelism with measure 3 and by the re-

14. Longuet-Higgins (1962/1987), Carpenter (1983), Werts (1983), and others have employed maps such as Figure 29, although not in connection with a reductional analysis.

15. It is beyond my purpose here to discuss the derivation of this analysis or its deviations from the theory as presented in *GTTM*. For a rather similar Schenkerian analysis, see Salzer (1952).

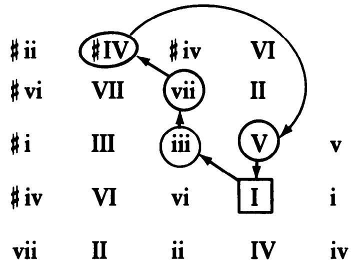

Fig. 29. The regional path in measures 1–9 of the *Waldstein* Sonata. (The space is taken from the regional torus of Figure 17, oriented to F major.)

sulting voice-leading pattern of parallel tenths governing measures 1–5—chromatically at level *b* of the prolongational reduction, diatonically at level *a*. (Incidentally, the chromatically descending bass is a good example of completeness within open-fifth space, and is a counterpart to measures 1–13 in the first movement.)

The prolongational reduction reveals that measures 2–4 prolong **VII**, but that at a deeper level this region exists within the framework of a passing motion from I to V. Measures 6–8 are a compensating V^7 prolongation, which in turn falls within the I prolongation of measures 1 and 9. Thus the abnormal progression to ♯IV^6 is embedded within a normal voice-leading framework. The ♯IV^6 is all the more salient because it abuts an intermediate-level grouping boundary; yet it is hierarchically quite subordinate. The expressive force of the passage can be understood only through this combination of harmonic distance and hierarchical position.

The double representation in Figures 29 and 30 is lacking in at least two respects. First, the regional space of Figure 29 could be enriched by the equivalent of Figure 18, so that, for instance, not just iii but its actual chord, V/iii, is shown. Second, the harmonic map and the reduction should coexist in one graph. Both steps present notational problems. I imagine prolongational trees reaching down to events in some *N*-dimensional pitch space, uniting the tonal and event hierarchies in one overall representation.[16]

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16. The idea for this article appears in embryo in one section of Lerdahl (1987b). While preparing this article, I benefited greatly from correspondence and discussions with Diana Deutsch, Carol Krumhansl, Andrew Mead, and William Rothstein. Figures 6 and 10 were suggested by Krumhansl.

Fig. 30. The time-span and prolongational reductions of measures 1–9 of the *Waldstein* Sonata.

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