

Moserware



A Stick Figure Guide to the Advanced Encryption Standard (AES)

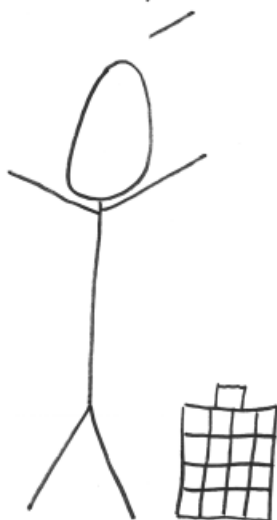
Sep 22, 2009

(A play in 4 acts. Please feel free to exit along with the stage character that best represents you. Take intermissions as you see fit. Click on the stage if you have a hard time seeing it. If you get bored, you can [jump to the code](#). Most importantly, enjoy the show!)

Act 1: Once Upon a

Time...

I handle petabytes* of data every day. From encrypting juicy Top Secret intelligence to boring packets bound for your WiFi router, I do it all!

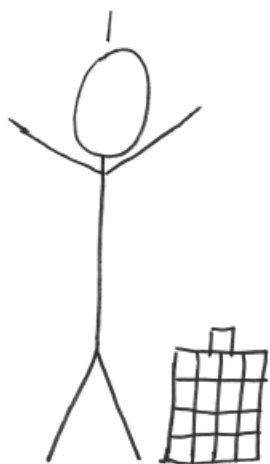


* 1 petabyte \approx a lot

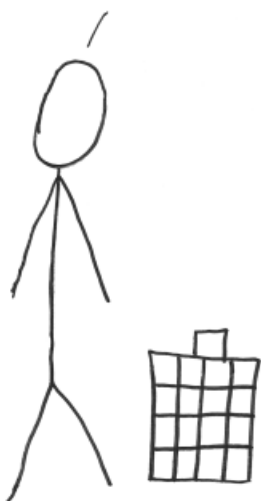
... and still no one seems to care about me or my story.



I've got a better-than-Cinderella story as I made my way to become king of the block cipher world.



Whoa! You're still there. You want to hear it? Well let's get started...



Once upon a time,* there was no good way for people outside secret agencies to judge good crypto.

EBG13 vf terrng!

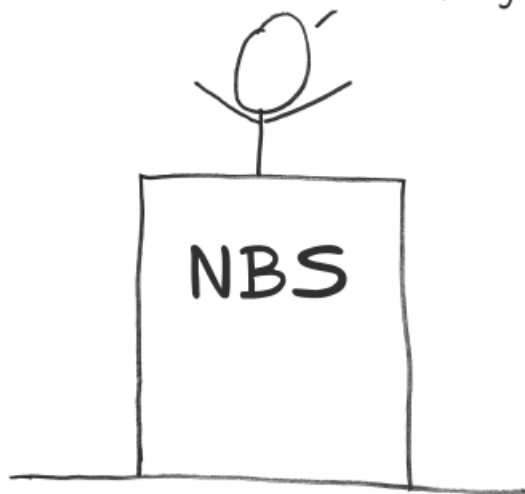


Double ROT13
is better!

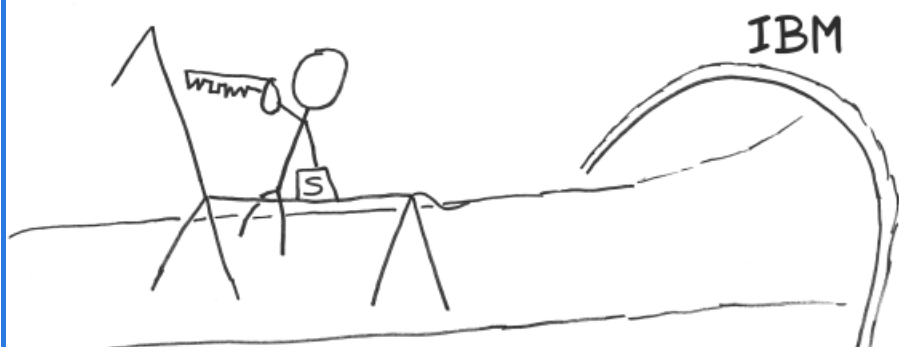
* ~ pre-1975 for the general public

A decree went throughout the
land to find a good, secure, algorithm.

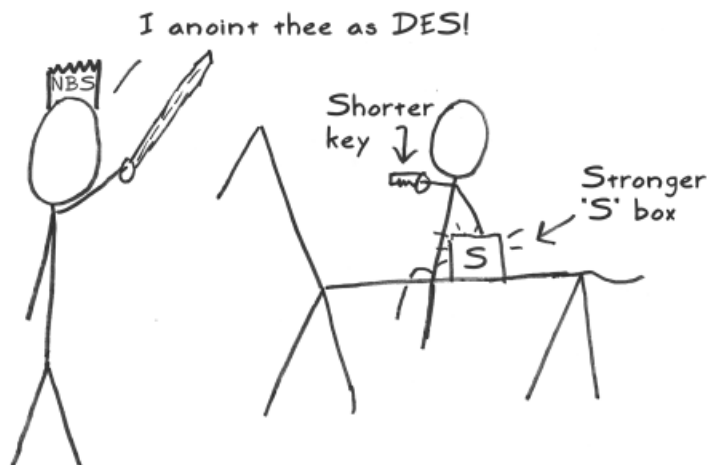
We need a good cipher!



One worthy competitor named
Lucifer came forward.



After being modified by the National Security Agency (NSA), he was anointed as the Data Encryption Standard (DES).



DES ruled in the land for over 20 years. Academics studied him intently. For the first time, there was something specific to look at. The modern field of cryptography was born.

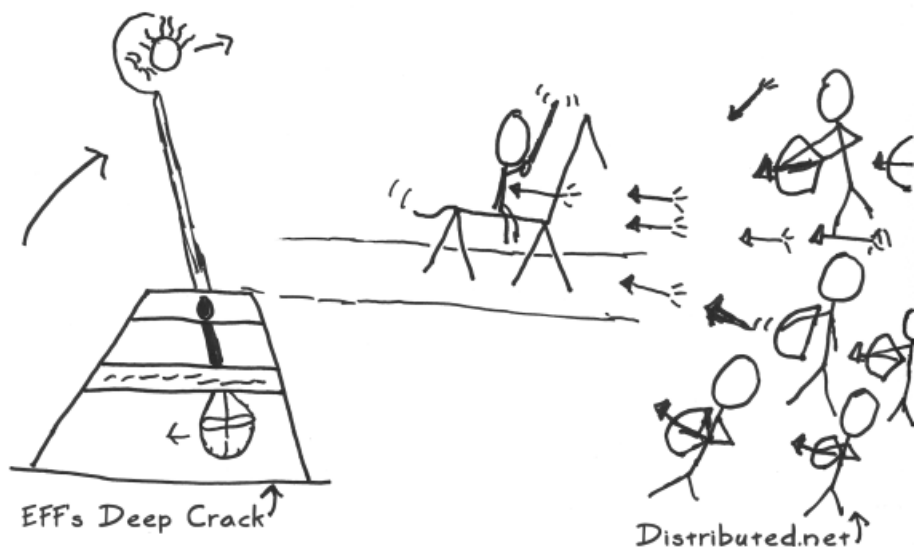
'... to the best of our knowledge, DES is free from any statistical or mathematical weakness.'



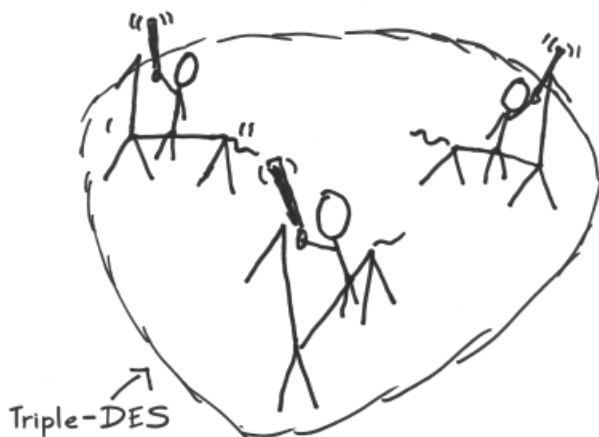
Check out that Feistel network!



Over the years, many attackers challenged DES. He was defeated in several battles.



The only way to stop the attacks was to use DES 3 times in row to form 'Triple-DES.' This worked, but it was awfully slow.



Another decree went out* ...



We need something at least as strong as Triple-DES, but it has to be fast and flexible.

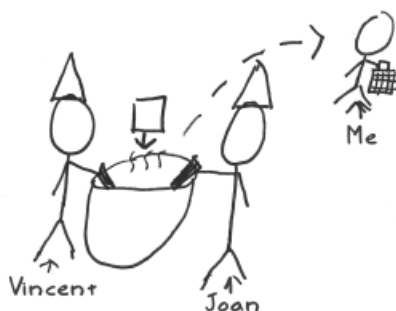
NIST

* ~early 1997

This call rallied the crypto wizards to develop something better.

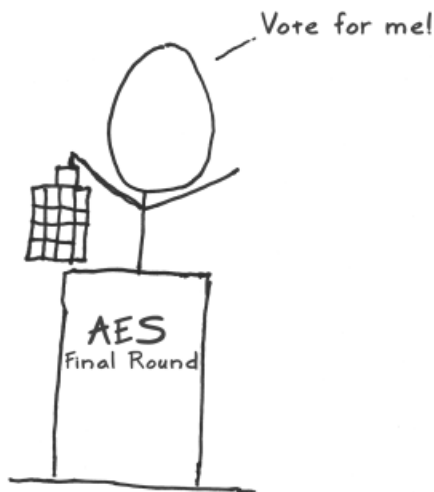


My creators, Vincent Rijmen and Joan Daemen, were among these crypto wizards. They combined their last names to give me my birth name: Rijndael.*



* That's pronounced 'Rhine Dahl' for the non-Belgians out there.

Everyone got together to vote and...



I won!!

	Rijndael	Serpent	Twofish	MARS	RC6
General Security	2	3	3	3	2
Implementation Difficulty	3	3	2	1	1
Software Performance	3	1	1	2	2
Smart Card Performance	3	3	2	1	1
Hardware Performance	3	3	2	1	2
Design Features	2	1	3	2	1
Total	16	14	13	10	9

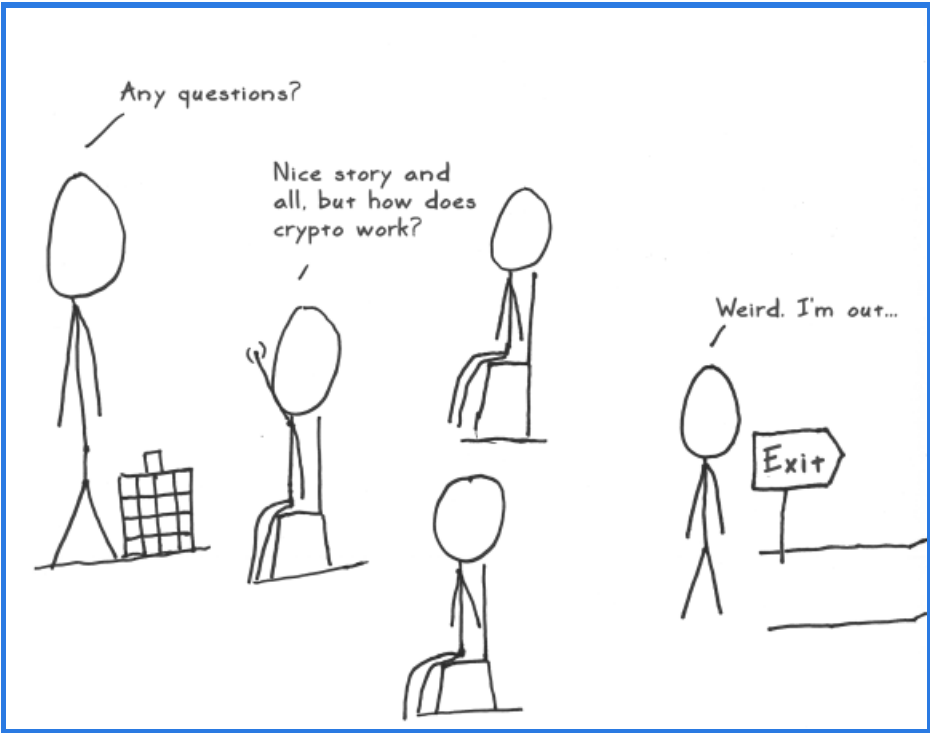
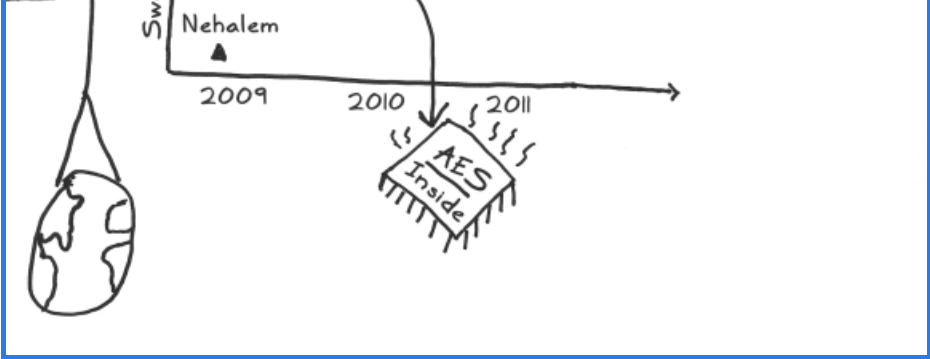
...and now I'm the new king of the crypto world. You can find me everywhere. Intel is even putting native instructions for me in their next chip to make me smokin' fast!

Intel Processor Roadmap

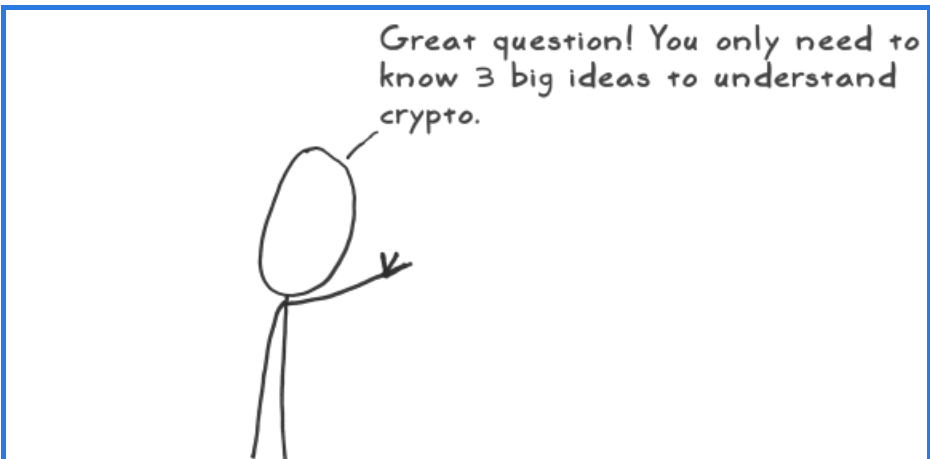
Sandy Bridge

Westmere

Security



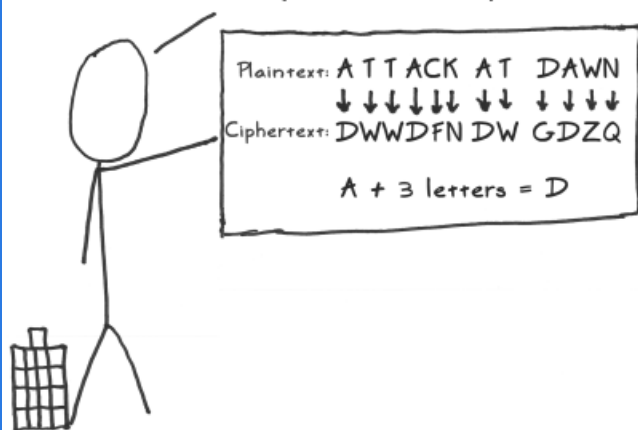
Act 2: Crypto Basics





Big Idea #1: Confusion

It's a good idea to obscure the relationship between your real message and your 'encrypted' message. An example of this 'confusion' is the trusty ol' Caesar Cipher:



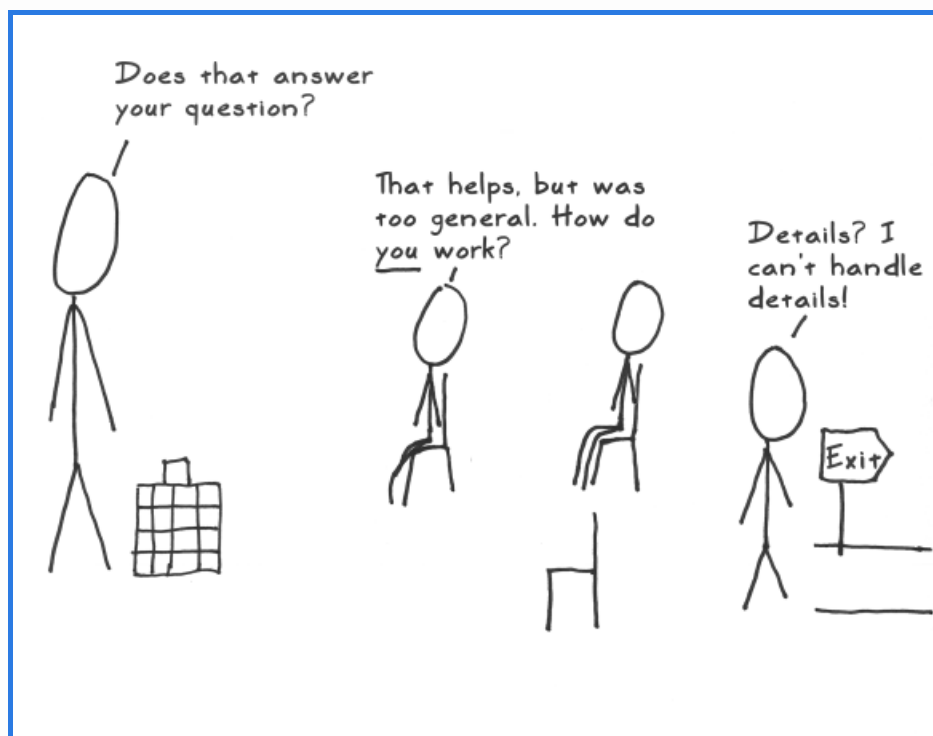
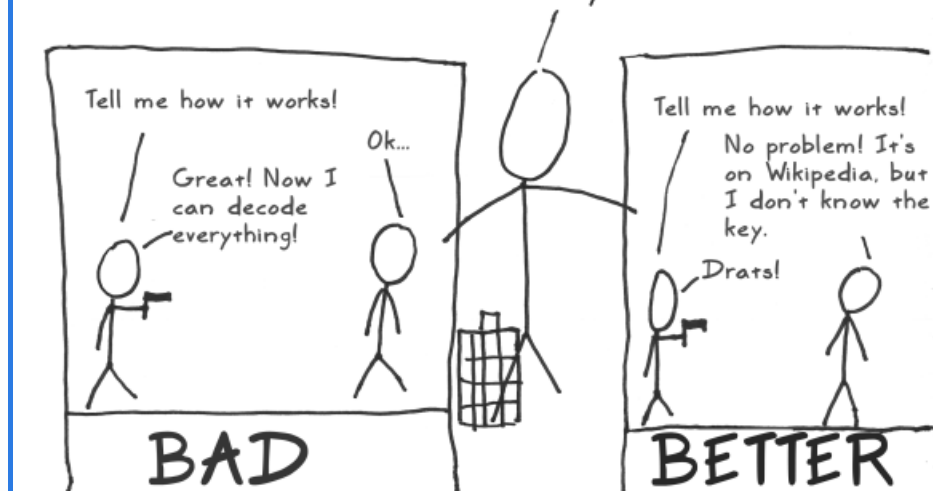
Big Idea #2: Diffusion

It's also a good idea to spread out the message. An example of this 'diffusion' is a simple column transposition:



Big Idea #3: Secrecy Only in the Key

After thousands of years, we learned that it's a bad idea to assume that no one knows how your method works. Someone will eventually find that out.



Act 3: Details

I'd be happy to tell you
how I work, but you have
to sign this first.



Uh... what's that?



Foot-Shooting Prevention Agreement

I, _____, promise that once
Your Name

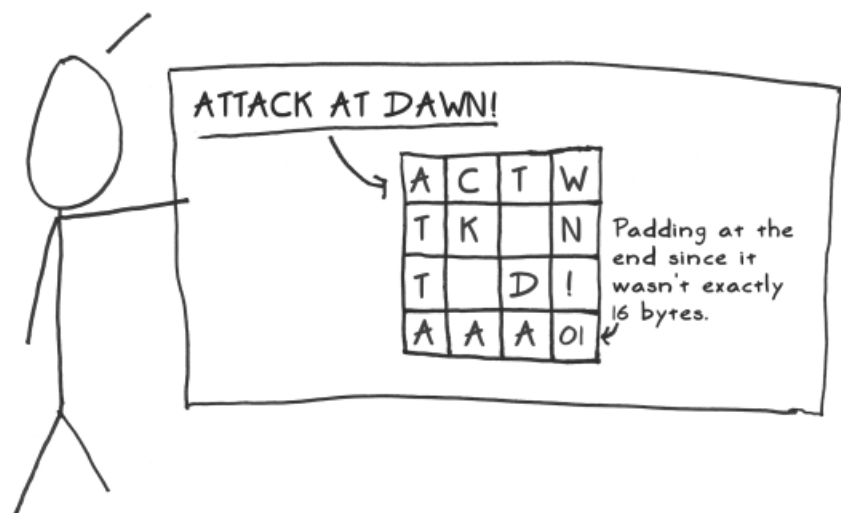
I see how simple AES really is, I will
not implement it in production code
even though it would be really fun.

This agreement shall be in effect
until the undersigned creates a
meaningful interpretive dance that
compares and contrasts cache-based,
timing, and other side channel attacks
and their countermeasures.

Signature

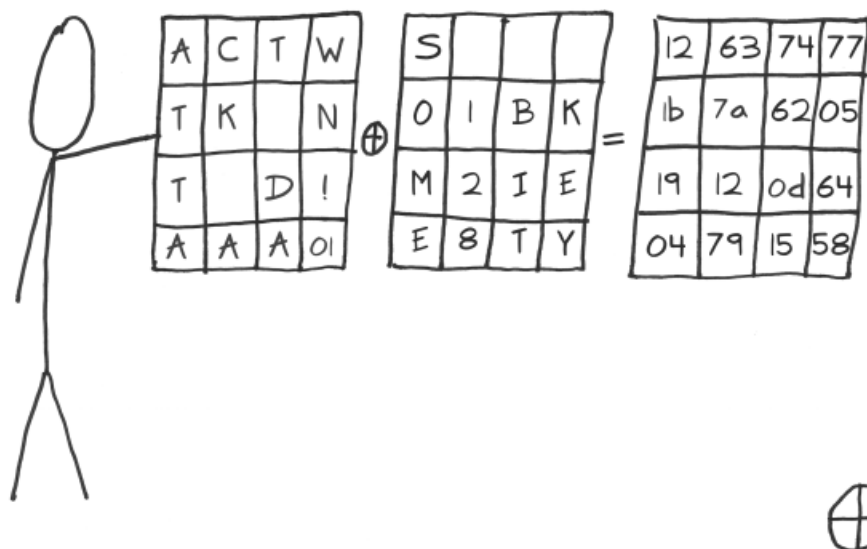
Date

I take your data and load it into this 4x4 square.*



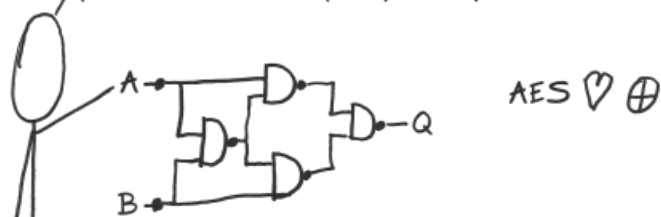
* This is the 'state matrix' that I carry with me at all times.

The initial round has me xor each input byte with the corresponding byte of the first round key.



A Tribute to XOR

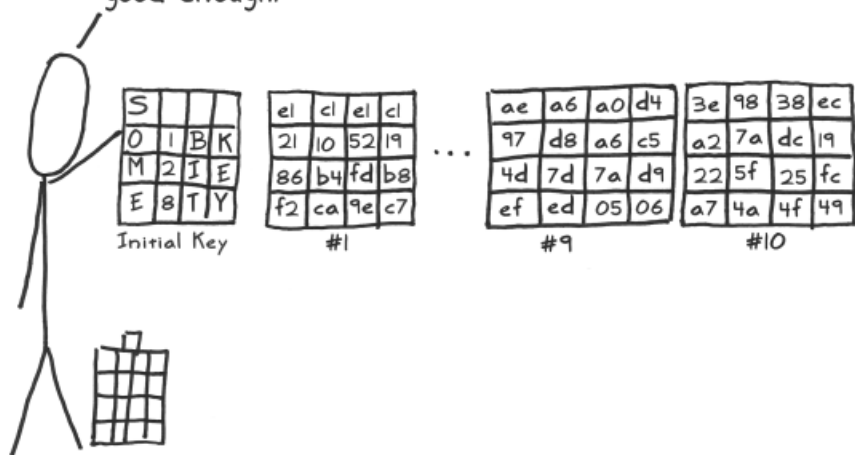
There's a simple reason why I use xor to apply the key and in other spots: it's fast and cheap - a quick bit flipper. It uses minimal hardware and can be done in parallel since no pesky 'carry' bits are needed.





Key Expansion: Part 1

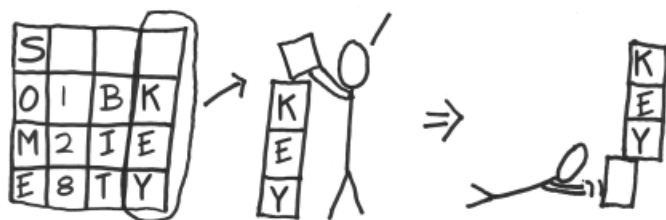
I need lots of keys for use in later rounds. I derive all of them from the initial key using a simple mixing technique that's really fast. Despite its critics,* it's good enough.



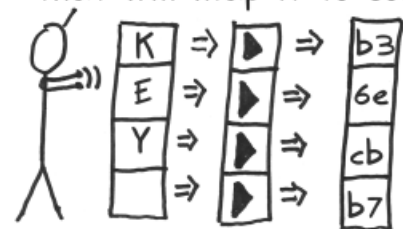
* By far, most complaints against AES's design focus on this simplicity.

Key Expansion: Part 2a

① I take the last column of the previous round key and move the top byte to the bottom:

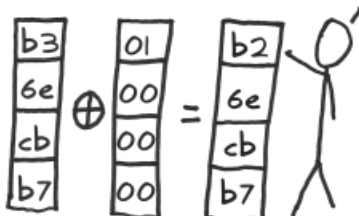


② Next, I run each byte through a substitution box that will map it to something else:

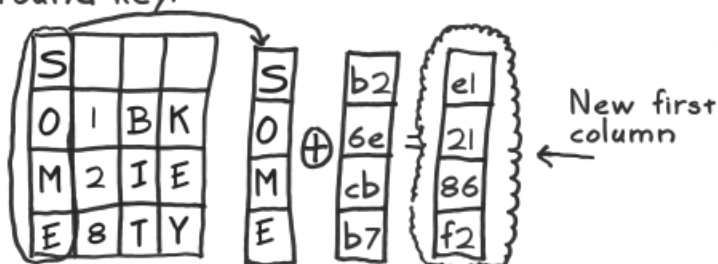


Key Expansion: Part 2b

- ③ I then xor the column with a 'round constant' that is different for each round.



- ④ Finally, I xor it with the first column of the previous round key:



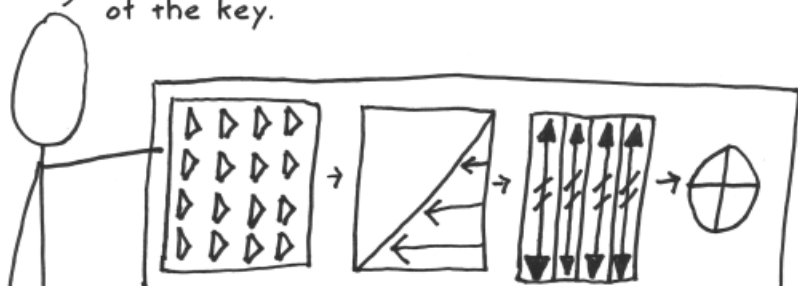
Key Expansion: Part 3

The other columns are super-easy,* I just xor the previous column with the same column of the previous round key.



* Note that 256 bit keys are slightly more complicated.

Next, I start the intermediate rounds. A round is just a series of steps I repeat several times. The number of repetitions depends on the size of the key.

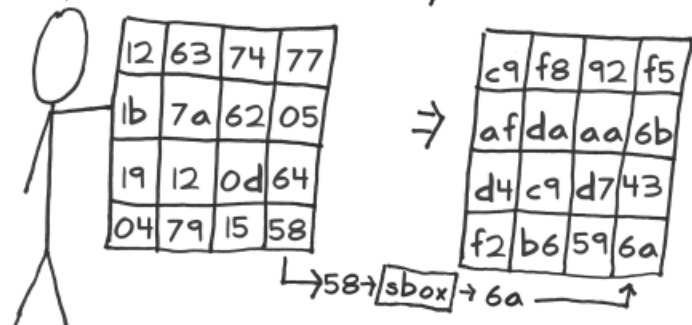


Intermediate Round ↗

Round Reperitions	Key Size
9	128
11	192
13	256

Applying Confusion: Substitute Bytes

I use confusion (Big Idea #1) to obscure the relationship of each byte. I put each byte into a substitution box (sbox), which will map it to a different byte:



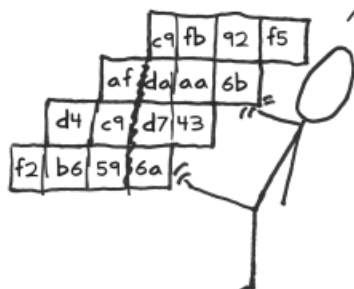
Denotes
← 'confusion'



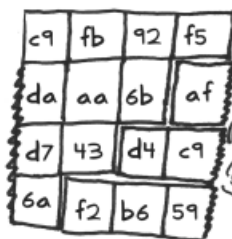
Applying Diffusion, Part 1: Shift Rows

Next I shift the rows to the left

Hiiii yaah!



...and then wrap them around the other side



Denotes
← 'permutation'



Applying Diffusion, Part 2: Mix Columns

c9	fb	92	f5
da	aa	6b	af
d7	43	d4	c9
6a	f2	06	59

41	b9	e0	8b
6e	83	95	a9
18	da	8b	38
99	00	65	d0

I take each column and mix up the bits in it.



Applying Key Secrecy: Add Round Key

At the end of each round, I apply the next round key with an xor:



41	b9	e0	8b
6e	83	95	a9
18	da	8b	38
99	00	65	d0

\oplus

e1	c1	e1	c1
21	10	52	19
86	b4	fd	b8
f2	ca	9e	c7

=

a0	78	01	4a
4f	93	c7	b0
9e	6e	76	80
6b	ca	fb	17

$$d0 \oplus c7 = 17$$



In the final round, I skip the 'Mix Columns' step since it wouldn't increase security* and would just slow things down:



Final Round

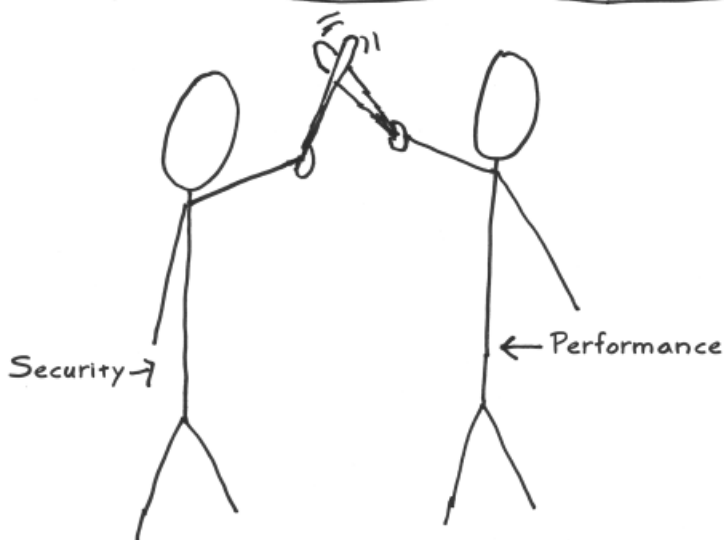


*The diffusion it would provide wouldn't go to the next round.

...and that's it. Each round I do makes the bits more confused and diffused. It also has the key impact them. The more rounds, the merrier!

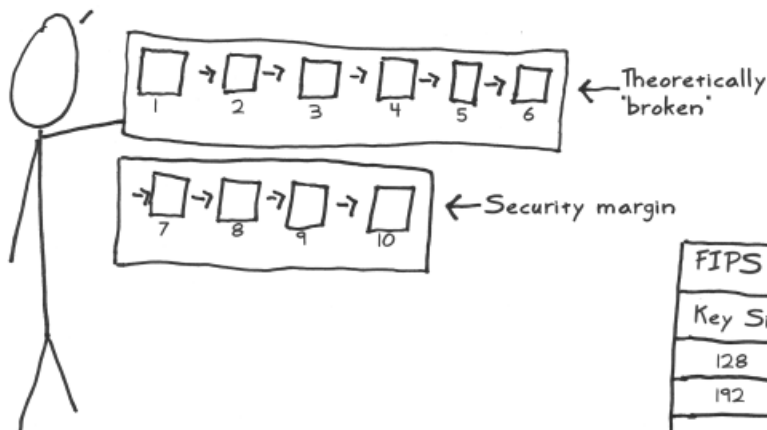


Determining the number of rounds always involves several tradeoffs.



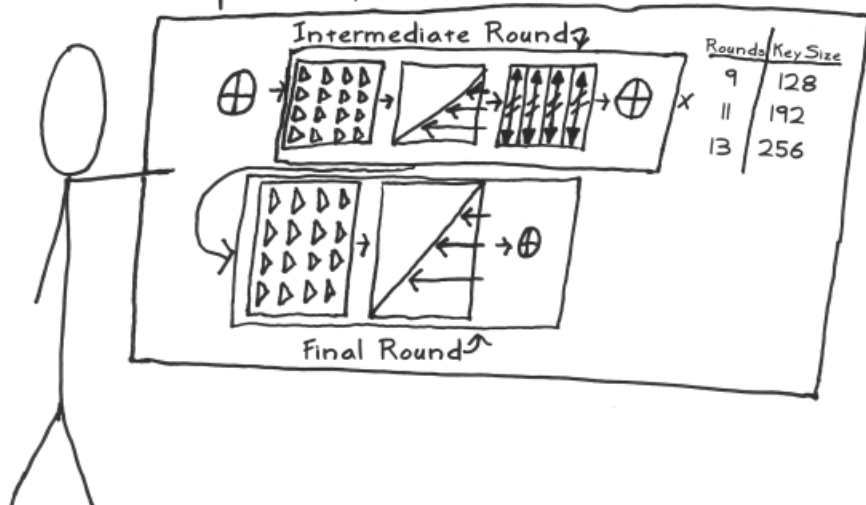
'Security always comes at a cost to performance' - Vincent Rijmen

When I was being developed, a clever guy was able to find a shortcut path through 6 rounds. That's not good! If you look carefully, you'll see that each bit of a round's output depends on every bit from two rounds ago. To increase this diffusion 'avalanche,' I added 4 extra rounds. This is my 'security margin.'



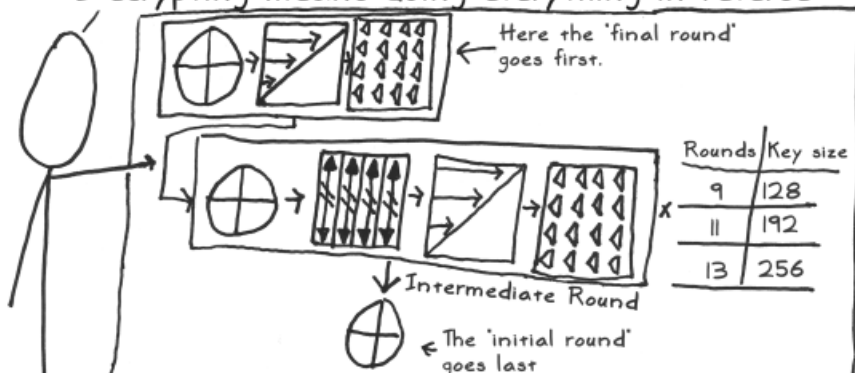
FIPS 197 Spec	
Key Size	Rounds
128	10
192	12
256	14

So in pictures, we have this:

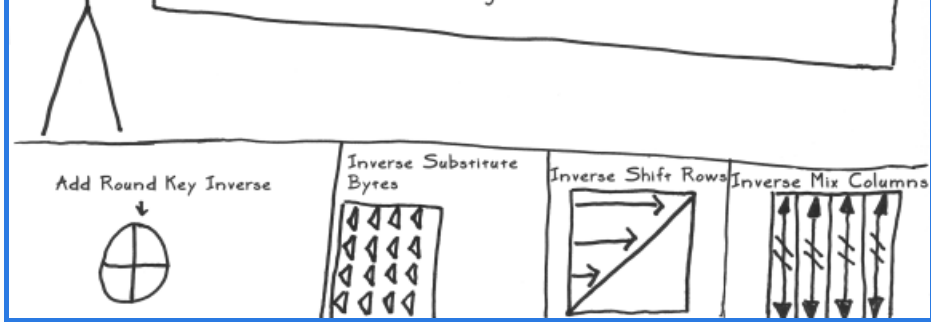


Rounds	Key Size
9	128
11	192
13	256

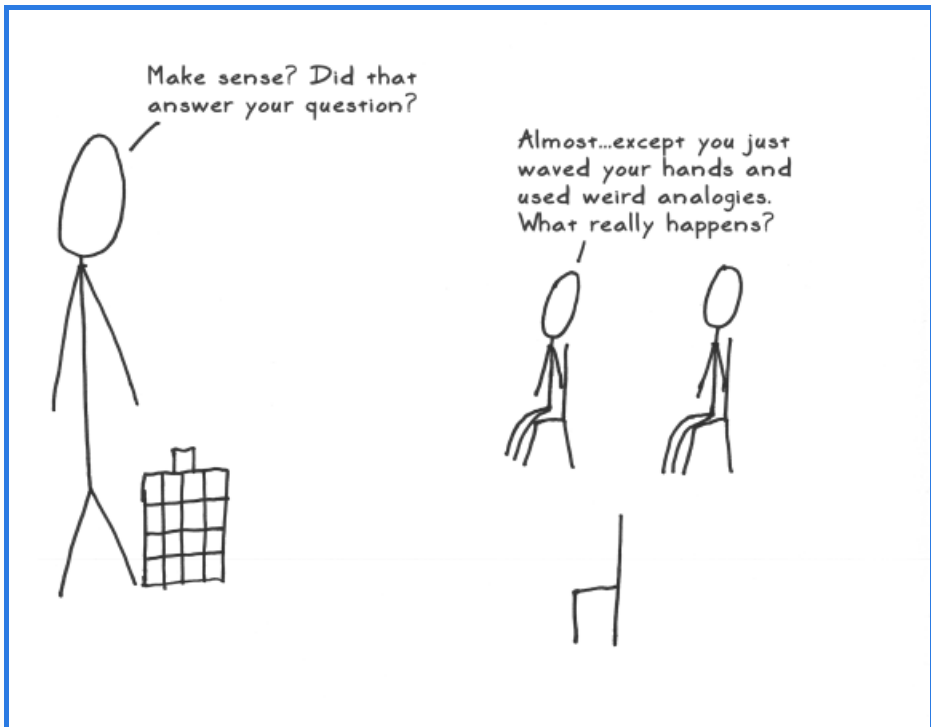
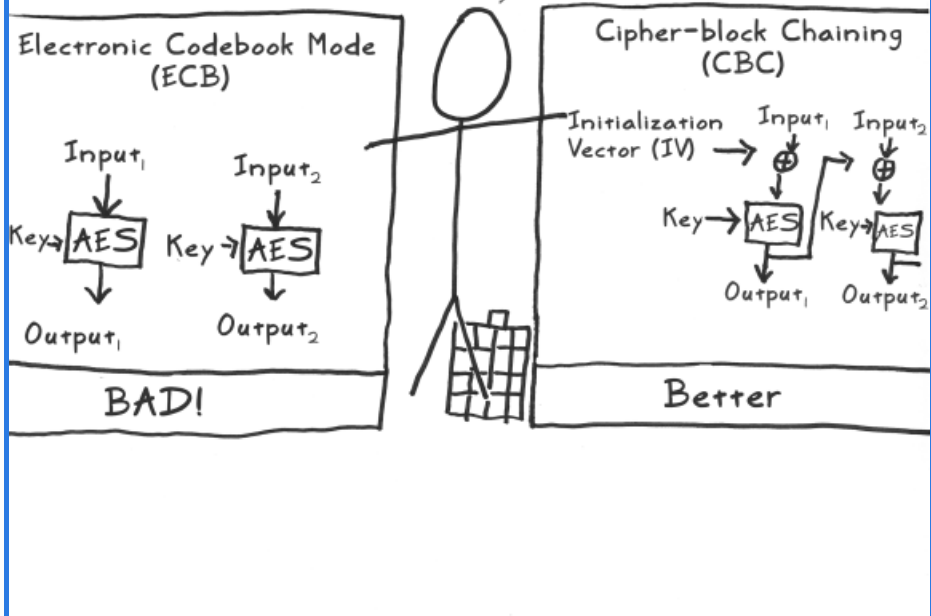
Decrypting means doing everything in reverse

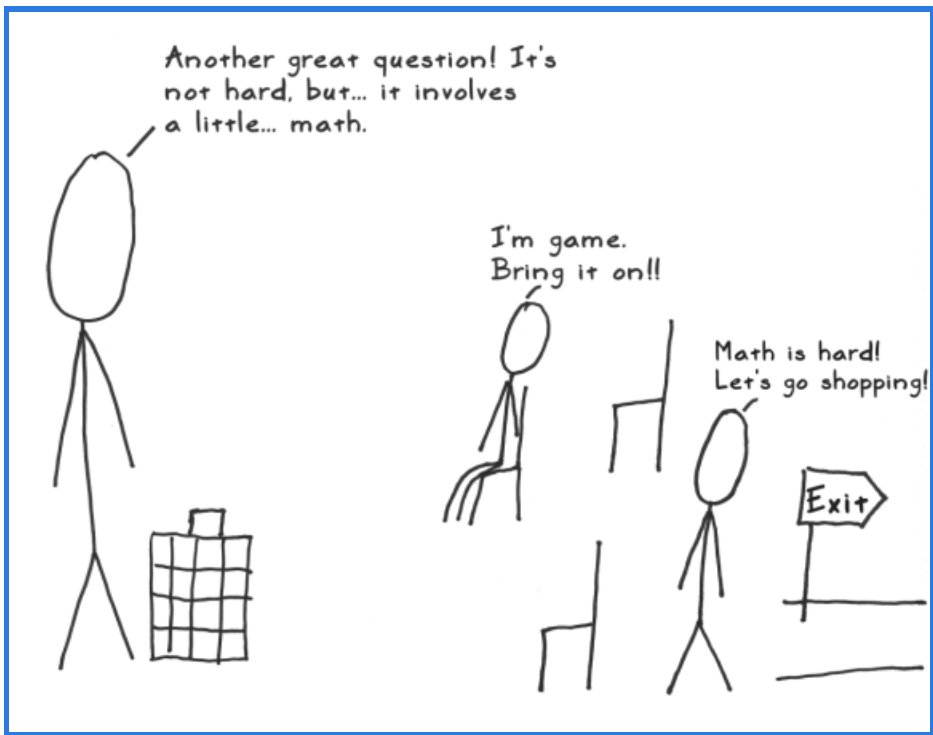


Rounds	Key size
9	128
11	192
13	256

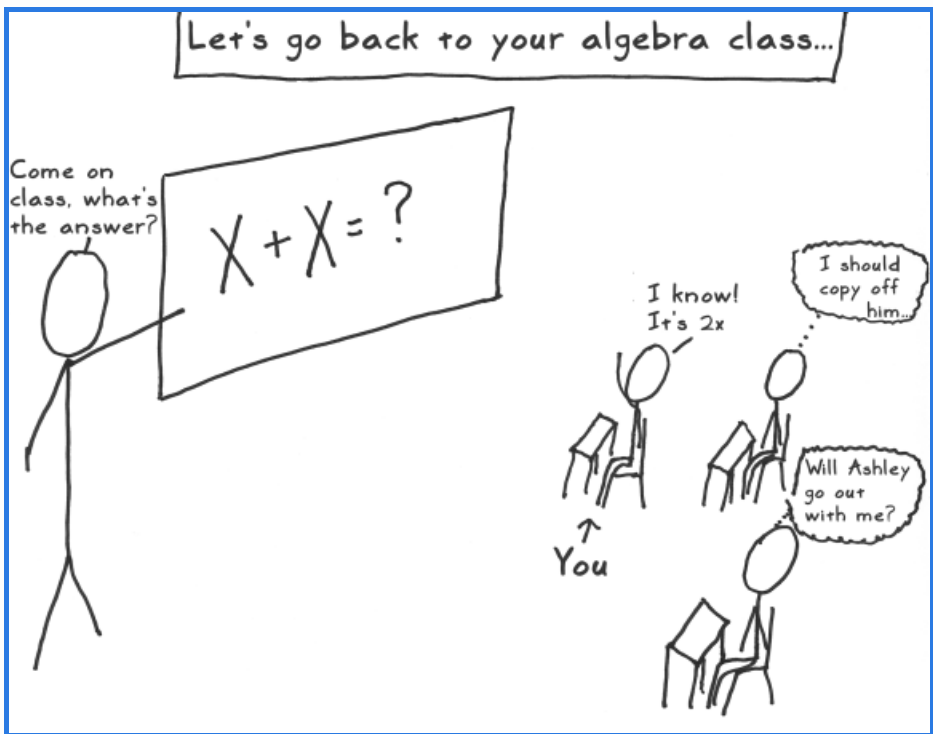


One last tidbit: I shouldn't be used as-is, but rather as a building block to a decent 'mode.'

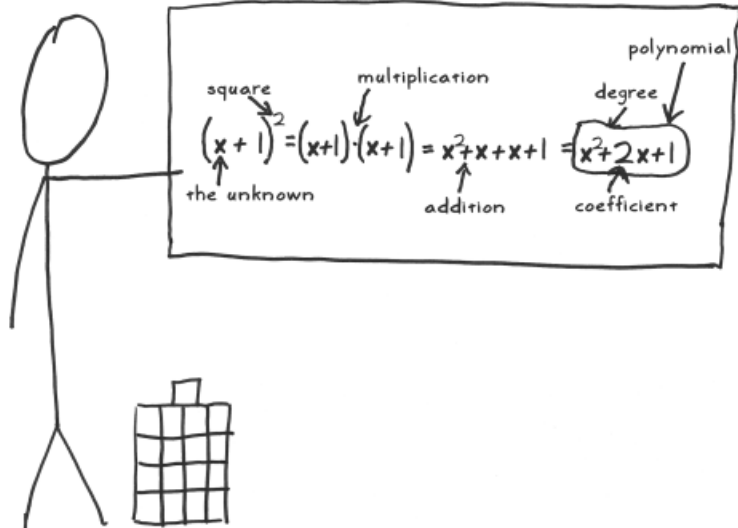




Act 4: Math!



Reviewing the Basics...



We'll change things slightly. In the old way, coefficients could get as big as we wanted. In the new way, they can only be 0 or 1:

Old Way

$$123x^2 + 45x^2 + 678x + 9x + 10$$

$$= 168x^2 + 687x + 10$$

Big coefficients

New Way

$$x^2 \oplus x^2 \oplus x^2 \oplus x \oplus x \oplus 1$$

$$= x^2 \oplus 1$$

The 'new' add*

Small coefficients

$$x^2 \oplus x^2 \oplus x^2 = (x^2 \oplus x^2) \oplus x^2$$

$$= 0 \oplus x^2$$

$$= x^2$$

*Nifty Fact: In the new way, addition is the same as subtraction (e.g. $x \oplus x = x - x = 0$)

Remember how multiplication could make things grow fast?

$$(x^7 + x^5 + x^3 + x) \cdot (x^6 + x^4 + x^2 + 1)$$

$$= x^{7+6} + x^{7+4} + x^{7+2} + x^{7+0} + x^{5+6} + x^{5+4} + x^{5+2} + x^{5+0}$$


$$+ x^{3+6} + x^{3+4} + x^{3+2} + x^{3+0} + x^{1+6} + x^{1+4} + x^{1+2} + x^{1+0}$$

$$= x^{13} + x^{11} + x^9 + x^7 + x^{11} + x^9 + x^7 + x^5 + x^9 + x^7 + x^5 + x^3 + x^7 + x^5 + x^3 + x$$


$$= x^{13} + x^{11} + x^{11} + x^9 + x^9 + x^9 + x^7 + x^7 + x^7 + x^7 + x^5 + x^5 + x^5 + x^3 + x^3 + x$$

$$= x^{13} + 2x^{11} + 3x^9 + 4x^7 + 3x^5 + 2x^3 + x$$

Big and yucky!




With the 'new' addition, things are simpler, but the x^{13} is still too big. Let's make it so we can't go bigger than x^7 . How can we do that?




$$x^{13} \oplus 2x^{11} \oplus 3x^9 \oplus 4x^7 \oplus 3x^5 \oplus 2x^3 \oplus x$$



$$\Rightarrow x^{13} \oplus 0x^{11} \oplus x^9 \oplus 0x^7 \oplus x^5 \oplus 0x^3 \oplus x$$

$$= x^{13} \oplus x^9 \oplus x^5 \oplus x$$


We use our friend, 'clock math*', to do this. Just add things up and do long division. Keep a close watch on the remainder:




4 o'clock + 10 hours = 2 o'clock

\Rightarrow  + 10 hours = 

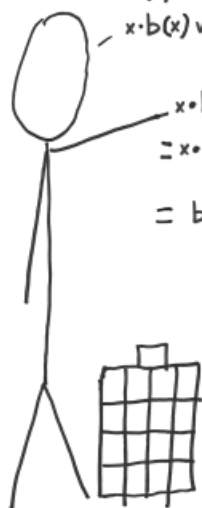
\Rightarrow 4 + 10 = 14

\Rightarrow
$$\begin{array}{r} 12 \overline{) 14} \\ \underline{-12} \\ 2 \end{array}$$
 R(2)



*This is also known as 'modular addition.' Math geeks call this a 'group.' AES uses a special group called a 'finite field.'

We can do 'clock' math with polynomials. Instead of dividing by 12, my creators told me to use $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$. Let's say we wanted to multiply $x \cdot b(x)$ where $b(x)$ has coefficients $b_7 \dots b_0$:



$$\begin{aligned} x \cdot b(x) &= x \cdot (b_7x^7 \oplus b_6x^6 \oplus b_5x^5 \oplus b_4x^4 \oplus b_3x^3 \oplus b_2x^2 \oplus b_1x \oplus b_0) \\ &= b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x \end{aligned}$$

↑ Eeek! x^8 is too big. We must make it smaller.

* Remember that each b_n (e.g. b_7) is either 0 or 1.

We divide it by $m(x) = x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1$ and take the remainder:



$$\begin{array}{r} x^8 \oplus x^4 \oplus x^3 \oplus x \oplus 1 \overline{) b_7x^8 \oplus b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x} \\ \oplus \quad b_7x^8 \qquad \qquad \qquad \oplus b_7x^4 \oplus b_7x^3 \oplus b_7x \oplus b_7 \\ \hline b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus (b_3 \oplus b_7)x^4 \oplus (b_2 \oplus b_7)x^3 \\ \oplus b_1x^2 \oplus (b_0 \oplus b_7)x \oplus b_7 \end{array}$$

Remainder

$$\rightarrow b_6x^7 \oplus b_5x^6 \oplus b_4x^5 \oplus b_3x^4 \oplus b_2x^3 \oplus b_1x^2 \oplus b_0x$$

Note how the b 's are shifted left by 1 spot.

$$\oplus b_7 \cdot (x^4 \oplus x^3 \oplus x \oplus 1)$$

↑ This is just b_7 multiplied by a small polynomial.

Now we're ready for the hardest blast from the past: logarithms. After logarithms, everything else is cake! Logarithms let us turn multiplication into addition:



$$\log(x \cdot y) = \log(x) + \log(y)$$

$$\text{So... } \log(10 \cdot 100) = \log(10^1) + \log(10^2)$$

$$= 2 + 1 = 3$$

In reverse:

$$\begin{aligned}\log^{-1}(1) &= 10^1 = 10 \\ \log^{-1}(2) &= 10^2 = 100 \\ \log^{-1}(3) &= 10^3 = 1,000\end{aligned}$$

$$\Rightarrow 10 \cdot 100 = 1,000$$

We can use logarithms in our new world. Instead of using 10 as the base, we can use the simple polynomial of $x \oplus 1$ and watch the magic unravel.*

$$(x \oplus 1)^1 = x \oplus 1$$

$$(x \oplus 1)^2 = (x \oplus 1)(x \oplus 1) = x^2 \oplus x \oplus x \oplus 1 = x^2 \oplus 1$$

$$(x \oplus 1)^3 = (x \oplus 1)^2 \cdot (x \oplus 1) = x^3 \oplus x^2 \oplus x \oplus 1$$

So...

$$\log_{x \oplus 1}(x \oplus 1) = 1, \log_{x \oplus 1}(x^2 \oplus 1) = 2, \log_{x \oplus 1}(x^3 \oplus x^2 \oplus x \oplus 1) = 3$$

*If you keep multiplying by $(x \oplus 1)$ and then take the remainder after dividing by $m(x)$, you'll see that you generate all possible polynomial below x^8 . This is very important!

Why bother with all of this math?* Encryption deals with bits and bytes, right? Well, there's one last connection: a 7th degree polynomial can be represented in exactly 1 byte since the new way uses only 0 or 1 for coefficients:

$$x^4 \oplus x^3 \oplus x \oplus 1$$

$$= 0x^7 \oplus 0x^6 \oplus 0x^5 \oplus 1x^4 \oplus 1x^3 \oplus 0x^2 \oplus 1x \oplus 1$$

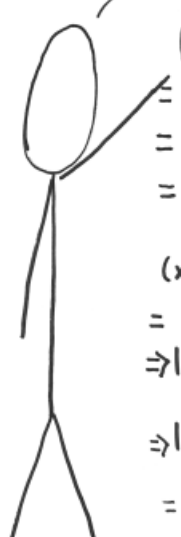
$$= \begin{array}{ccccccc} \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{array}$$

$$\begin{array}{c} 1 \\ \downarrow \end{array} \quad \begin{array}{c} 1011_2 = 11_{10} = b_{16} \leftarrow \text{hexadecimal} \end{array}$$

$$= 1b \leftarrow \text{A single byte!!}$$

*Although we'll work with bytes from now on, the math makes sure everything works out.


With bytes, polynomial addition becomes a simple xor. We can use our logarithm skills to make a table for speedy multiplication.*



$$\begin{aligned}
 & (x^4 \oplus x^3 \oplus x \oplus 1) \oplus (x^7 \oplus x^5 \oplus x^3 \oplus x) \\
 &= \underset{\downarrow}{1b} \oplus \underset{\downarrow}{aa} \leftarrow \text{byte xor} \\
 &= \underset{\downarrow}{b1} \\
 &= x^7 \oplus x^5 \oplus x^4 \oplus 1 \\
 \\
 & (x^4 \oplus x^3 \oplus x \oplus 1) \cdot (x^7 \oplus x^5 \oplus x^3 \oplus x) \\
 &= \underset{\downarrow}{1b} \cdot \underset{\downarrow}{aa} \xrightarrow{\text{logarithm table lookup}} \\
 &\Rightarrow \log(1b) + \log(aa) = c8 + 1f = e7 \\
 &\Rightarrow \log^{-1}(e7) = 8c \xrightarrow{\text{inverse table lookup}} 1b \cdot aa \\
 &= x^7 \oplus x^3 \oplus x^2
 \end{aligned}$$

* We can create the table as we keep multiplying by $(x \oplus 1)$.


Since we know how to multiply, we can find the 'inverse' polynomial byte for each byte. This is the byte that will undo/invert the polynomial back to 1. There are only 255* of them, so we can use brute force to find them:



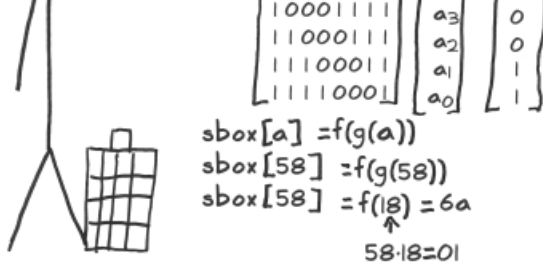
$$\begin{aligned}
 & (x^4 \oplus x^3 \oplus x \oplus 1) \cdot ? = 1 \\
 & \underset{\downarrow}{1b} \cdot \underset{\downarrow}{cc} = 1 \\
 & \quad \quad \quad \uparrow \text{found using a brute force for-loop}
 \end{aligned}$$

* There are only 255 instead of 256 because 0 has no inverse.

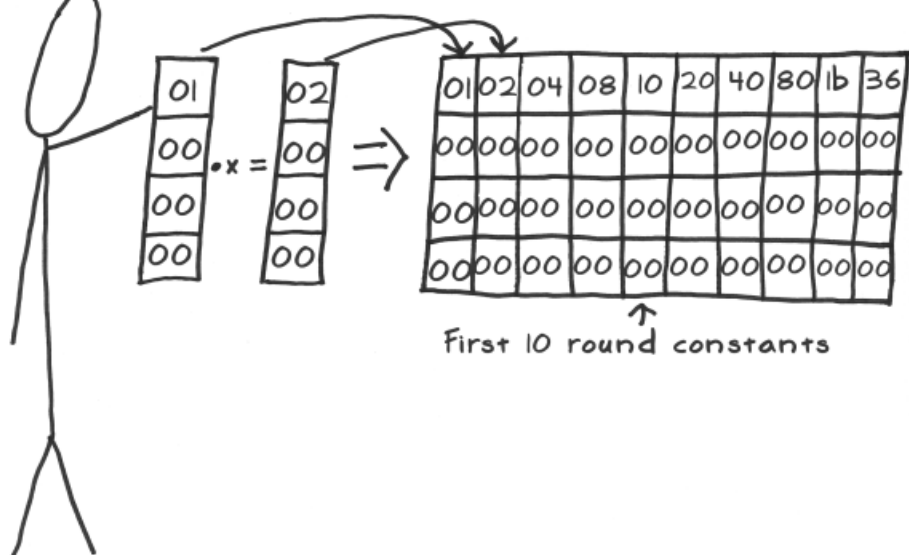
Now we can understand the mysterious s-box. It takes a byte 'a' and applies two functions. The first is 'g' which just finds the byte inverse. The second is 'f' which intentionally makes the math uglier to foil attackers.



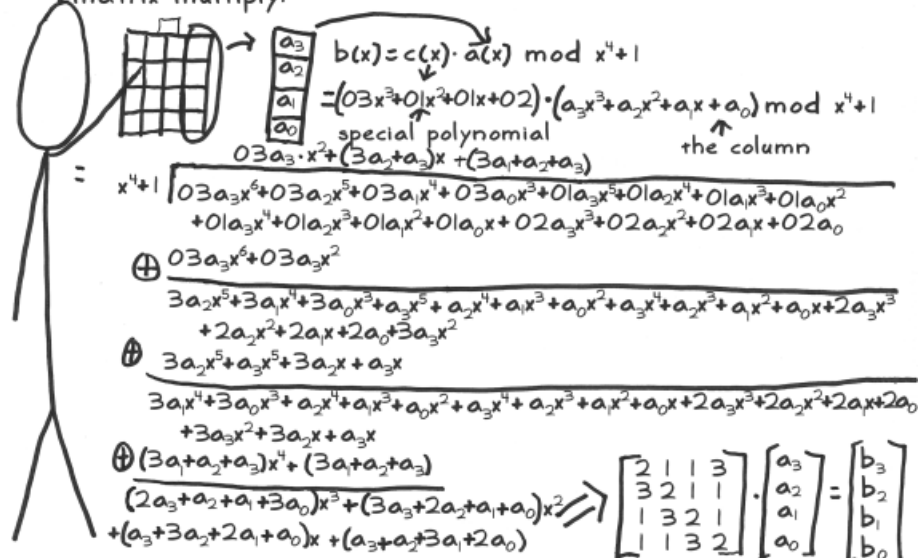
$$\begin{aligned}
 & g(a) = a^{-1} \\
 & f(a) = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} a_7 \\ a_6 \\ a_5 \\ a_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}
 \end{aligned}$$

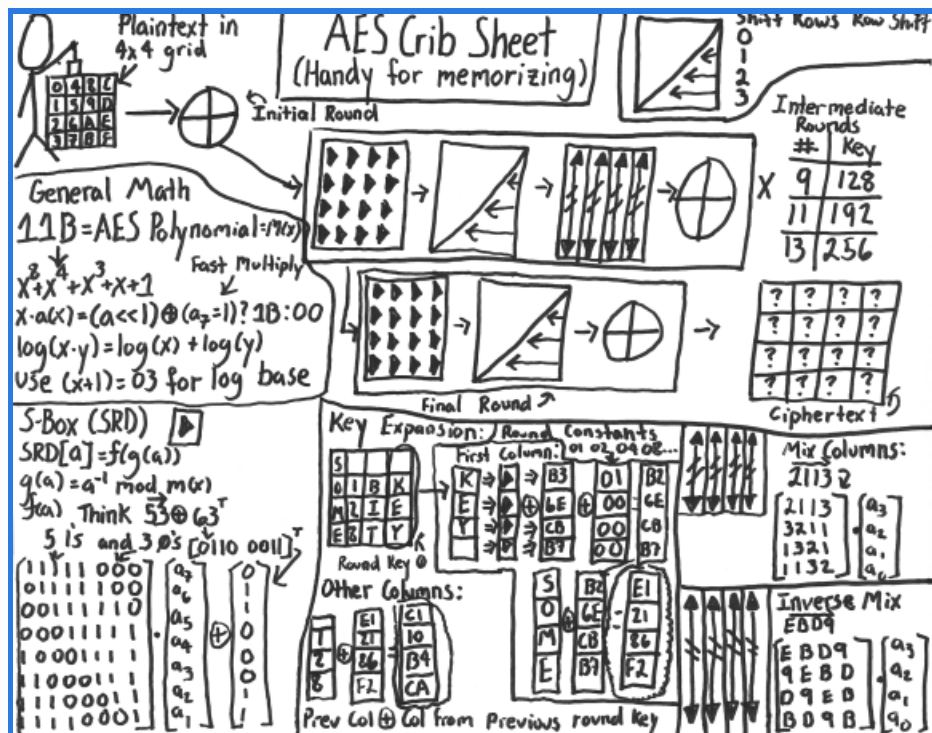


We can also understand those crazy round constants in the key expansion. I get them by starting with '1' and then keep multiplying by 'x':



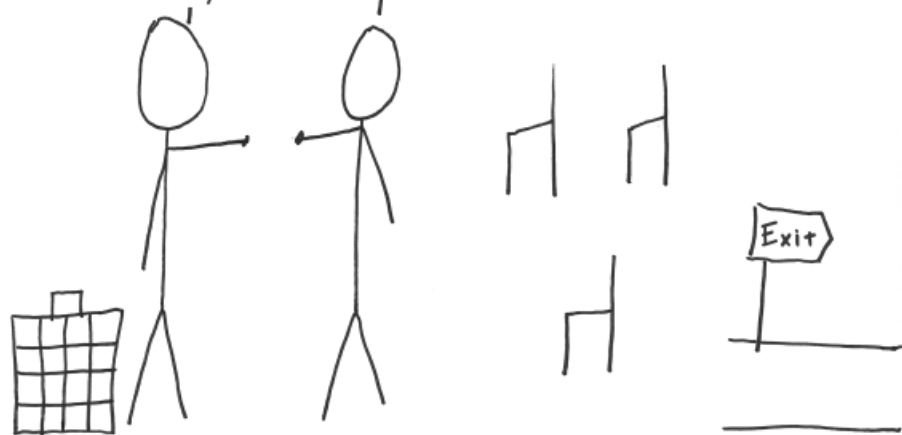
Mix Columns is the hardest. I treat each column as a polynomial. I then use our new multiply method to multiply it by a specially crafted polynomial and then take the remainder after dividing by x^4+1 . This all simplifies to a matrix multiply:



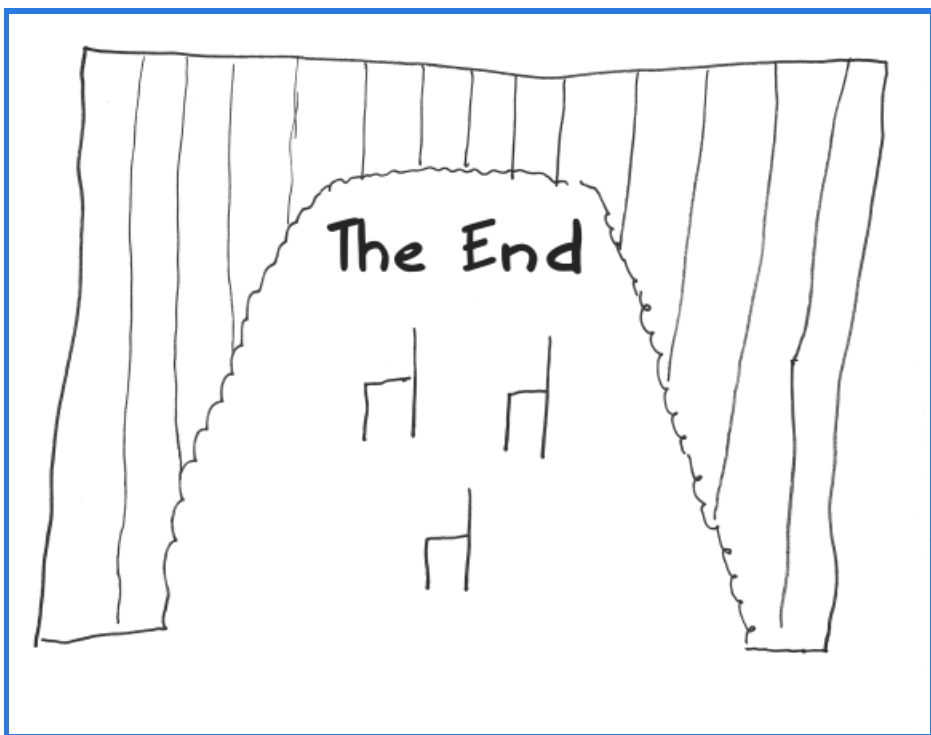
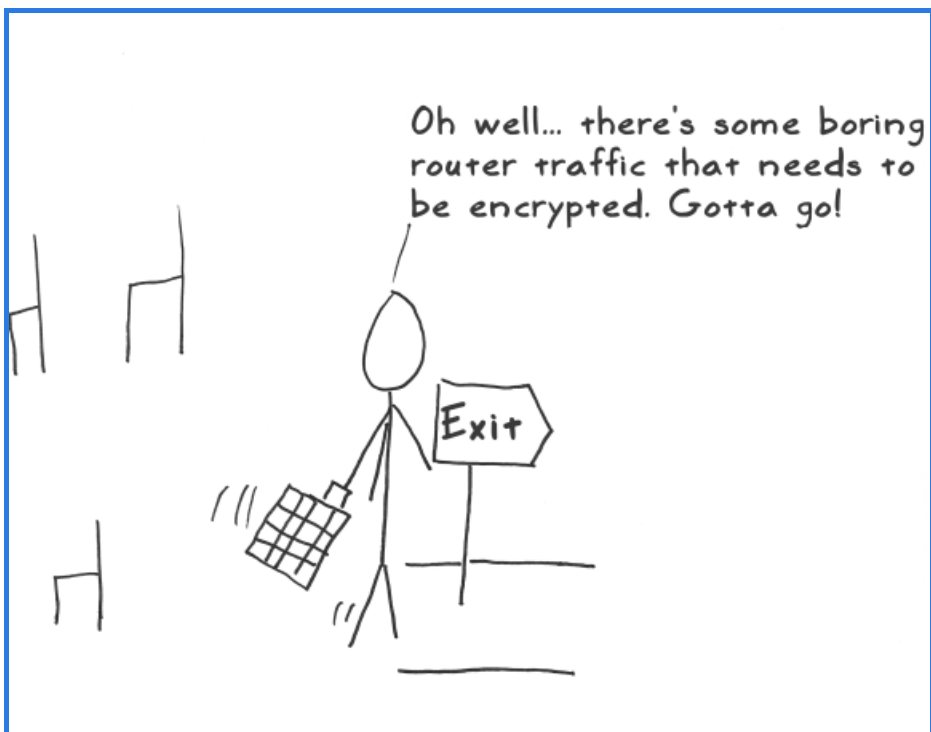
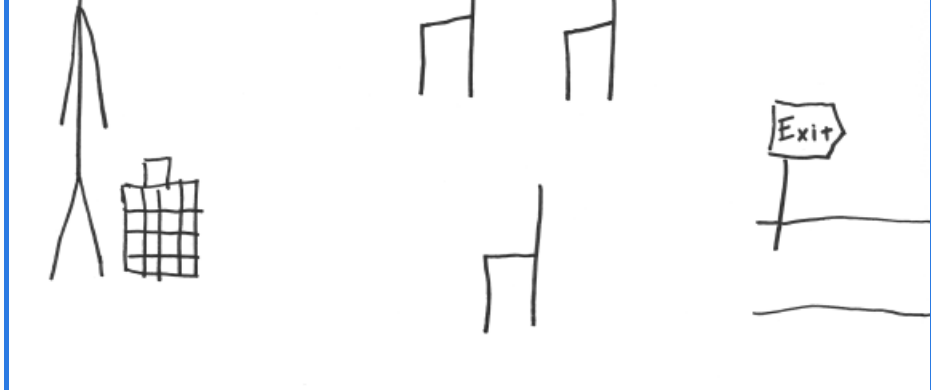


Whoa... I think I get it now. It's relatively simple once you grok the pieces. Thanks for explaining it. I gotta go now.

My pleasure.
Come back anytime!



But there's so much more to talk about: my resistance to linear and differential cryptanalysis, my Wide Trail Strategy, impractical related-key attacks, and... so much more... but no one is left.

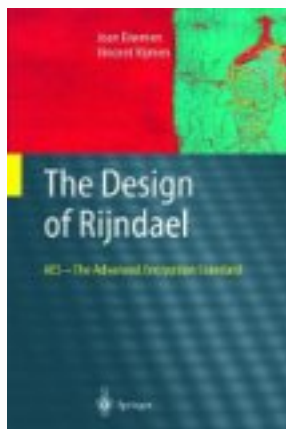


Epilogue

I created a heavily-commented AES/Rijndael implementation to go along with this post and [put it on GitHub](#). In keeping with the Foot-Shooting Prevention Agreement, it shouldn't be used for production code, but it should be helpful in seeing exactly where all the numbers came from in this play. Several resources were useful in creating this:

- [The Design of Rijndael](#)

is *the* book on the subject, written by the Rijndael creators. It was helpful in understanding specifics, especially the math (although some parts were beyond me). It's also where I got the math notation and graphical representation in the left and right corners of the scenes describing the layers ([SubBytes](#), [ShiftRows](#), [MixColumns](#), and [AddRoundKey](#)).



- The [FIPS-197](#) specification formally defines AES and provides a good overview.
- [The Puzzle Palace](#), especially [chapter 9](#), was helpful while creating Act 1. For more on how the NSA modified DES, see [this](#).

- More on Intel's (and now AMD) inclusion of native AES instructions can be found [here](#) and in detail [here](#). - Other helpful resources include [Wikipedia](#), [Sam Trenholme's AES math series](#), and [this animation](#).

Please leave a comment if you notice something that can be better explained.

Update #1: Several scenes were updated to fix some errors mentioned in the comments.

Update #2: By request, I've created a slide show presentation of this play in both [PowerPoint](#) and [PDF](#) formats. I've licensed them under the [Creative Commons Attribution License](#) so that you can use them as you see fit. If you're teaching a class, consider giving extra credit to any student giving a worthy interpretive dance rendition in accordance with the Foot-Shooting Prevention Agreement.

