How Do Persistent Earnings Affect the Response of Consumption to Transitory Shocks? Expressing the Earnings Process of Guvenen, Karahan, Ozkan, and Song 2021 as a Sum of Monthly Earnings

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I examine under which conditions the yearly earnings specification of Guvenen, Karahan, Ozkan, and Song 2021 could write as a sum of monthly earnings, which is what people receive in practice. The specification I assume in Section 3 is more general than the specification of Guvenen, Karahan, Ozkan, and Song 2021 (because it does not make assumptions about the distributions from which the shocks are drown for instance), but I try to exhibit monthly processes that are consistent with the exact specification of Guvenen, Karahan, Ozkan, and Song 2021 because it has been shown to fit the data well, and it is the one I use in the numerical simulations. I identify two cases in which the yearly earnings assumed in Guvenen, Karahan, Ozkan, and Song 2021 can rewrite as the sum of monthly earnings.

The first case is straightforward: if the shocks occur once per year, at the beginning of the year, monthly earnings are the same every month $m \in [1:12]$ and equal to one twelfth of annual earnings. Their sum immediately writes up as the process proposed in Guvenen, Karahan, Ozkan, and Song 2021:

$$y^i_{t,m} = \frac{1}{12} y^i_t = \frac{1}{12}(1-\nu^i_t)(e^{p^i_{t-1}})^{\rho} e^{\eta^i_t} e^{\varepsilon^i_t} e^{\alpha^i} e^{g(t)}$$

The second case allows persistent shocks to occur at different points in time over the year (or alternatively transitory shocks to occur at different points in time over the year). The other components are constant over the year. Thus, the monthly earnings only differ because of the monthly value of the persistent shock $e^{\eta^i_{t,m}}$:

$$y^i_{t,m} = \frac{1}{12}(1-\nu^i_t)(e^{p^i_{t-1}})^{\rho} \underbrace{e^{p^i_{t,m}}}_{\text{Monthly}} e^{\varepsilon^i_t} e^{\alpha^i} e^{g(t)}$$

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With probability p_η, the persistent shock occurs at the beginning of the year and is drawn from a given distribution $\mathscr{S}^S$ (indexing it with S for soon). The monthly persistent shock is then:

$$e^{\eta^i_{t,m}} = s^i_t \;\; \forall m \;\in [1:12]$$

In contrast, with probability $(1-p_\eta)$, the persistent shock occurs later on, say on month m^η, and is drawn from a given distribution $\mathscr{L}^L$ (indexing it with L for late). The monthly persistent shock is then:

$$e^{\eta^i_{t,m}} = \begin{cases} 1 \;\; \forall m \;\in [1:m^\eta - 1] \\ l^i_t \;\; \forall m \;\in [m^\eta - 1:12] \end{cases}$$

Annual earnings are:

$$y^i_t = \sum_{m=1}^{12} y^i_{t,m} = \begin{cases} e^{\eta^i_{t-1}}(1-v^i_t)(e^{p^i_{t-1}})^\rho e^{\varepsilon^i_t} e^{\alpha^i} e^{g(t)} s^i_t \;\text{ with prob. } p_\eta \\ e^{\eta^i_{t-1}}(1-v^i_t)(e^{p^i_{t-1}})^\rho e^{\varepsilon^i_t} e^{\alpha^i} e^{g(t)} \left(\frac{m^\eta-1}{12} + \frac{13-m^\eta}{12} l^i_t\right) \;\text{ with prob. } (1-p_\eta), \end{cases}$$

with s^i_t drawn from a a lognormal distribution $\mathscr{S}^S = \text{Lognormal}(\mu_{\eta,1}, \sigma^2_{\eta,1})$, and $(\frac{m^\eta-1}{12} + \frac{13-m^\eta}{12} l^i_t)$ drawn from a log-normal distribution $\text{Lognormal}(\mu_{\eta,2}, \sigma^2_{\eta,2})$ (so l^i_t is drawn from a three-parameter log-normal $\mathscr{L}^L = \text{Lognormal}(-\dfrac{m^\eta-1}{12}, \mu_{\eta,1} + ln(\dfrac{12}{13-m^\eta}), \sigma^2_{\eta,1}))$. This corresponds exactly to the annual earnings specification proposed by Guvenen, Karahan, Ozkan, and Song 2021.