PPHA 44330: Energy and Environmental Economics, Fall 2020
Problem set #2, due before class on Thursday, December 3

For all questions, please turn in (via email) a single pdf file with your answers. It's ok to turn in a scanned copy of hand-written answers, so long as you write legibly (you can use a tool like Microsoft Office Lens to use your phone to scan hand-written work to pdf).

1. Fishing and extinction. Consider a salmon fishery with a single owner. The fishery is small relative to the world market for salmon, so that the fishery owner takes the salmon price p as constant and exogenous. Denote the extraction rate at time t by y_t, and denote the stock at t by x_t.

   The extraction cost, $c(y)$, is such that $c'(y) \in (0, p)$ and $c''(y) > 0$. Extraction cost is not a function of stock. The rate of change of the fish stock is given by $\dot{x}_t = f(x_t) - y_t$, where $f(0) = 0$, $f'(0) > 0$, $f''(\cdot) < 0$, and $\exists \bar{x} > 0$ s.t. $f(\bar{x}) = 0$. The initial resource stock is given by $x_0 > 0$. Denote the resource owner's discount rate by r. The time horizon is infinite.

   (a) Write down the resource owner's current value Hamiltonian and first order conditions.

   (b) Write down a necessary and sufficient condition on $f'(0)$ such that the fishery will not be driven to extinction in the limit as t goes to infinity.

   (c) Assuming your condition from (b) holds, draw the phase diagram for this problem in x-y space, noting the steady-state stock and extraction rate, along with the stable arm.

   (d) Now suppose that at time T, government regulation will force the fishery to close. Continue to assume that your condition from (b) holds. Prove that at the optimum, $x_T = 0$. That is, the resource will be fished to extinction.

   (e) Re-draw your phase diagram. Suppose the initial stock $x_0 = \bar{x}$. Continuing to assume that the fishery closes at T, draw the optimal time paths for the resource stock and extraction rate. Intuitively, why is $\dot{y}_t > 0$ for t near T?

2. Common pool with an exhaustible resource. Consider an oil reserve with initial stock S that is being extracted by N firms. The resource is a common pool and is sufficiently small that the constant world oil price P is exogenous to the firms' extraction. The marginal cost of extraction faced by any one firm is strictly increasing in its rate of extraction but does not depend on the quantity of reserves remaining in the pool (i.e., there are no stock effects).

   (a) As usual, denote the remaining stock at any given time by x_t. In continuous time, write the first order necessary conditions for optimal extraction by firm i, assuming that the firms are playing a Markov Perfect Equilibrium.

   (b) Interpret your first order conditions and the implied MPE. How do the equilibrium paths of extraction and (p-MC) compare to the social optimum (where the social optimum is the same as what would be obtained in a competitive equilibrium in which each firm had private ownership of its share of reserves)?

3. Highway congestion externalities. Suppose N drivers commute daily to Manhattan across the Hudson River. There are two alternatives. The George Washington (GW) bridge provides a direct route, but it tends to get crowded. Daily commute time in hours for drivers who use the bridge, z, is $z = \alpha x$, where x is the number of drivers using the bridge and α is a constant. The alternative route takes 1 hour daily and is uncongestible. Suppose that all drivers value time at ω per hour and that N is so large that there are always drivers taking the alternate route. Assume that each driver simultaneously chooses which route to take into the city. Non-integer number of drivers are acceptable throughout this question.

   (a) In the Nash equilibrium, how many drivers take the GW bridge?

   (b) What is the Pareto efficient number of drivers on the bridge (i.e., the number on the bridge that would minimize the total person-hours lost commuting over the available routes)?

   (c) At the Pareto optimum, what is the externality caused by the marginal driver on the bridge?

   (d) Show that imposing a "Pigouvian toll" equal to the externality measured in (c) causes the Nash equilibrium outcome to be Pareto optimal. Who is better off at this equilibrium relative to that from (a)?

4. Targeting Pigouvian taxes. Consider a simple model of a good that is produced with a polluting technology. The private benefit from consuming a quantity q of the good is given by $U(q) = aq - (b/2)q^2$, where both a and b are strictly greater than zero, and the social damage is given by τe, where τ is a non-negative parameter and e is the amount of emissions. The good is produced by a large number of identical price-taking firms using inputs $k_1,\ldots,k_n$ and emissions e under the production function $f(k,e)$. $f(k,e)$ exhibits constant returns to scale, with $f_i > 0$ and $f_{ii} < 0$ for all inputs i, as well as $f_e > 0$ and $f_{ee} < 0$. The price of input k_i is r_i, and all input prices are constant and exogenous.

   (a) Derive the optimal tax on emissions t^*.

   (b) Suppose that emissions are unobservable and that only total output can be taxed. Show that an efficient outcome can only be reached if $\tau = 0$. Explain your answer.

   (c) Suppose that emissions are unobservable and that only the inputs $k_1,\ldots,k_n$ can be taxed. Show that an efficient outcome can only be reached if $\tau = 0$. Explain your answer.

   (d) Suppose that emissions are unobservable and that both output and the inputs can be taxed. Show that careful selection of these taxes can achieve the efficient outcome. Explain your answer.

   (e) Suppose that the parameter a in the private benefit function is unknown to the regulator and can take a high or low value with equal probability. Discuss the relative benefits of taxing emissions versus a using an emissions quantity standard (implemented via a cap-and-trade scheme).

5. Permit market dynamics. Firms pollute at an aggregate rate of E_0 per year. Suppose the government hopes to restrict cumulative emissions over a 10-year period to a pre-specified target $\bar{S} < 10E_0$ using a cap-and-trade mechanism. After this decade, permits will be worthless because a tax will replace this program.

   At each instant $t \in [0,10]$, each firm can abate its pollution at a cost, but if any emissions remain they must be covered by surrendering permits to the regulator. One permit covers one unit of emissions. Permits must be surrendered as the pollution occurs (continuous compliance). The higher the permit price, the less firms rely on permits and the more they resort to abatement. In particular, if the permit price is $p(t)$, in aggregate permits will be surrendered at the rate $D(p(t))$, with $D(0) = E_0$, $D(\cdot)$ weakly decreasing, and $D(p) > 0$ for any $p \geq 0$. Marginal abatement costs are given by $C'(E_0 - D(p))$, so that $p(t) = C'(E_0 - D(p(t)))$ at all times at t.

   Permits can be costlessly stored over time but, unlike cash in the bank, earn no interest. Assume that the unchanging, continuous-time interest rate is r. Each firm is well-informed about the regulations in place and maximizes wealth with perfect foresight about permit prices in the future.

(a) Write an equation determining the initial permit price ($p(0)$) if the government grandfathers, at $t = 0$, $\bar{S}$ permits.

(b) Suppose that the government is concerned that the price path $p(t)$ in part (a) will be too high toward the end of the decade. To prevent this, it therefore announces at the outset its willingness to sell to the market at a pre-specified ceiling price p^c strictly between the initial and the final price in part (a) as many permits as people want (it can always print them!). Explain why in the new equilibrium the price path will start lower than in part (a), cumulative emissions will reach $\bar{S}$ before the decade ends, and the government will provide Δ additional permits at the rate $D(p^c)$ over the final interval of length τ. Write equations defining τ and Δ.

(c) Suppose that the government has only a finite stock $G > 0$ of permits with which to defend the ceiling at p^c. Write down a condition on G such that there will be a speculative attack that suddenly exhausts the government's permit stock. Explain your answer.

(d) Write down a condition on G such that there be a speculative attack and that there will not be a measurable interval of time over which the permit price equals p^c. Explain your answer.

(e) Suppose instead that the government is concerned that prices in part (a) would be too low toward the beginning of the decade. The government therefore stands ready at any time to purchase at a floor price of p^f between the initial and the final price in part (a) any of the grandfathered $\bar{S}$ permits. Describe the new equilibrium price path. Write an equation defining β, the cumulative number of permits the government would buy back in the new equilibrium.

6. Clean backstop technology. Suppose that the demand for transportation fuel is perfectly elastic, so that the marginal utility of consumption is constant and equal to p (which is the fuel price per gallon in a market context). There are two potential sources of fuel: crude oil and biofuel.

Crude oil is extracted from an underground stock with an initial size $S_0 > 0$ (in gallons). Denote the instantaneous rate of crude oil production at time t as y_t (expressed in units of gallons per year). The extraction cost $c_y(y_t)$ is strictly positive, strictly increasing, and strictly convex. It also satisfies $c_y'(0) \in (0, p)$.

Biofuel is farmed and refined continuously, and it cannot be stored. Denote the instantaneous rate of biofuel production at time t as b_t (expressed in units of gallons per year). Its production cost $c_b(b_t)$ is strictly positive, strictly increasing, and strictly convex. It also satisfies $c_b^{'}(0) \in (0, p)$.

Consumers view crude oil fuel and biofuel as perfect substitutes. Denote the discount rate used by all agents and the social planner by r. The time horizon is infinite.

(a) The social planner's problem is to choose time paths for y_t and b_t that maximize welfare. Write down the planner's objective function and constraints. Write down the current value Hamiltonian and all necessary conditions.

(b) Sketch out a graph showing the optimal extraction and production paths for oil and biofuel.

(c) Now suppose that the horizon is finite: at time T all cars will be banned, and nobody will demand fuel anymore. How small does T need to be such that some of the initial stock will be left in the ground? Explain.

(d) Return to the infinite horizon version of the problem. Suppose that oil generates CO2 emissions at a rate e_y while biofuel generates emissions at rate e_b (both in units of tons per gallon). $0 \le e_b < e_y$. Each ton of CO2 emitted creates environmental damages equal to τ.

Suppose that extractors of oil and producers of biofuel are perfectly competitive (and that the oil stock is divided among the resource owners so that there is no common pool problem). Write down the optimal Pigouvian taxes on oil and biofuel, and show that they result in the social planner's optimal outcome.

(e) Suppose instead that there is a low carbon fuel standard (LCFS). This standard states that the following condition must be satisfied:

$$\frac{e_y y_t + e_b b_t}{y_t + b_t} \le \sigma$$

Where $\sigma \in [e_b, e_y]$ Show that the LCFS can achieve the first best outcome only if $e_b = 0$ and $c_y^{'}(0) \ge p - \tau e_y$. Explain.

7. Mustangs and Civics. Consider a world with only two types of car drivers: economics faculty (F) and economics grad students (S). The overpaid faculty all drive identical, brand new, Ford Mustang GT's that emit φ_F grams of NO_x per gallon of gasoline and consume $e_F > 0$ gallons of gasoline per mile driven. The grad students can only afford to drive 1993 Honda Civics, which emit $\varphi_S > \varphi_F > 0$ grams of NO_x per gallon of gasoline (the Civics' catalytic converters have rusted out). The Civics each consume $e_S = e_F$ gallons per mile.

   There are equal numbers of faculty and grad students. Environmental damages equal τ dollars per gram of NO_x emitted. Let m_F and m_S denote the miles driven by faculty and students. Miles driven are a function of the total price of gasoline, which consists of the sum of the pre-tax price p with the per-gallon gasoline tax t.

   (a) Suppose that faculty and grad students have identical demands for miles driven. In particular, they have identical (and non-zero) responses of miles driven to a change in the price of gasoline. Prove that a gasoline tax t alone cannot achieve the first-best welfare outcome. Write down an expression for the second-best optimal tax t on gasoline to address the externality. Explain your answer. (Even if you cannot derive the second-best optimal gasoline tax, try to intuitively discuss what you think it should be and why.)

   (b) Continue to assume that faculty and grad students have identical demands for miles driven, with identical (and non-zero) responses of miles driven to a change in the price of gasoline. Suppose that $\varphi_S = 2\varphi_F$. What percentage of the first-best welfare gain is achievable with the second-best optimal policy?

   (c) Now suppose that the faculty are so overpaid that they do not care about the price of gasoline at all. That is, faculty miles driven m_F is perfectly inelastic to the tax-inclusive gasoline price. Students' miles m_S continue to have a non-zero response to the tax-inclusive price. How does the inelasticity of m_F change the optimal gasoline tax relative to part (a)? Show that the first-best can now be attained with an optimal gasoline tax. Explain intuitively why this is the case.