

Problem3_Jeanne

November 15, 2020

```
[1]: import numpy as np
      from matplotlib import pyplot as plt
      from scipy.sparse import csr_matrix
      import time
      from scipy import sparse
      import numpy.linalg as lin
      from scipy.stats import norm
```

0.1 Drilling and Real Options

```
[2]: D = 3000000
      X = 100000
      P = 50 # given
          = 0.05
          = 1/(1+ )
      P1 = P + np.random.normal(0, 4)
```

```
[3]: # Solve for the optimal Trigger price

      # a) set up the state space as a vector of possible oil prices
      P_grid = np.linspace(0,80,81)

      Profits = P_grid*X - D
```

```
[4]: # b) Define the state transition matrix T that gives, for each row i, the proba
      ↪ the next period's price will
      # be "in" the col state j

      T_price = np.zeros((81, 81))

      # Cutoff grid = middle of price grid
      P_grid_inter = np.zeros(81)
      for j in range(81-1):
          P_grid_inter[j] = (P_grid[j] + P_grid[j+1])/2
      P_grid_inter[80] = np.inf

      # Compute price transition matrix
```

```

for i in range(81):
    Pcdf = norm.cdf(P_grid_inter, P_grid[i], 4)
    Ppdf = np.zeros(81)
    for k in range(1,81):
        Ppdf[k] = Pcdf[k] - Pcdf[k-1]
    Ppdf[0] = Pcdf[0]
    T_price[i,:] = Ppdf

np.round(np.sum(T_price, axis=1), 5) == 1

```

```

[4]: array([ True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True,
          True,  True,  True,  True,  True,  True,  True,  True,  True])

```

```

[5]: # c) Use VFI to compute the value function and the control function
      # report the "trigger price" at which drilling is optimal

```

```

### Initiate vectors & numbers
iter = 0
epsi = 1
tol = 1e-8
maxiter = 1000
N = len(P_grid)
Vp_new = np.zeros(N)
Vp = np.ones(N)
now = np.zeros(N)

### We already have the utility (profit) matrix from before
# Profit

### VFI
iter = 0
while (epsi > tol) & (iter < maxiter):
    # For each starting price
    Vold = np.copy(Vp)
    nowold = np.copy(now)
    for ik in range(N):
        #isgreater0 = 1*(Profits[ik] > 0)
        EVnext = np.dot(Vp, T_price[ik,:])
        #print(Vnext)

```

```

drilltoday = Profits[ik] + 0
drilltomorrow = * EVnext
Vp[ik] = max(drilltoday, drilltomorrow)
now[ik] = (drilltoday >= drilltomorrow)
#print(Vnext)

epsi = lin.norm(Vp - Vold)

if epsi < tol:
    break

iter=iter+1
print("iter is ", iter, "and norm is ", epsi)

triggerprice = np.min(np.where(now==1)[0])

```

```

iter is 1 and norm is 20718343.201135807
iter is 2 and norm is 324449.14590140415
iter is 3 and norm is 212902.59418132566
iter is 4 and norm is 160704.1519463266
iter is 5 and norm is 127941.58381819326
iter is 6 and norm is 104912.50216275868
iter is 7 and norm is 87677.44608631416
iter is 8 and norm is 74210.19012209596
iter is 9 and norm is 63435.880899050564
iter is 10 and norm is 54684.11687461095
iter is 11 and norm is 47515.45515555741
iter is 12 and norm is 41616.07051441495
iter is 13 and norm is 36680.072939797894
iter is 14 and norm is 32504.20361376454
iter is 15 and norm is 28935.436267139743
iter is 16 and norm is 25856.30800791992
iter is 17 and norm is 23176.28043707488
iter is 18 and norm is 20825.39417044625
iter is 19 and norm is 18749.3555251835
iter is 20 and norm is 16905.71464841143
iter is 21 and norm is 15260.931742649898
iter is 22 and norm is 13788.161193765172
iter is 23 and norm is 12465.600101125046
iter is 24 and norm is 11275.268690887553
iter is 25 and norm is 10202.11444467888
iter is 26 and norm is 9233.355274136571
iter is 27 and norm is 8357.997302540183
iter is 28 and norm is 7566.479168055524
iter is 29 and norm is 6850.407507439281
iter is 30 and norm is 6202.357966145947
iter is 31 and norm is 5615.72329892391

```

iter is 32 and norm is 5084.595413681863
iter is 33 and norm is 4603.672032495395
iter is 34 and norm is 4168.181373622395
iter is 35 and norm is 3773.8201922104276
iter is 36 and norm is 3416.7018787034517
iter is 37 and norm is 3093.3122681878767
iter is 38 and norm is 2800.4714810932505
iter is 39 and norm is 2535.300581670813
iter is 40 and norm is 2295.1921663349326
iter is 41 and norm is 2077.7842220185826
iter is 42 and norm is 1880.9367550596628
iter is 43 and norm is 1702.7108045840455
iter is 44 and norm is 1541.349535320677
iter is 45 and norm is 1395.2611632540188
iter is 46 and norm is 1263.0035103765488
iter is 47 and norm is 1143.2700167975033
iter is 48 and norm is 1034.877062865281
iter is 49 and norm is 936.7524730082068
iter is 50 and norm is 847.9250882372476
iter is 51 and norm is 767.5153067491955
iter is 52 and norm is 694.726502540636
iter is 53 and norm is 628.8372409026584
iter is 54 and norm is 569.1942174606004
iter is 55 and norm is 515.2058542944992
iter is 56 and norm is 466.33649280253803
iter is 57 and norm is 422.10112846863024
iter is 58 and norm is 382.06063766688777
iter is 59 and norm is 345.8174511410406
iter is 60 and norm is 313.0116328949684
iter is 61 and norm is 283.31732696254755
iter is 62 and norm is 256.43953792709783
iter is 63 and norm is 232.11121416264655
iter is 64 and norm is 210.09060559675214
iter is 65 and norm is 190.15887037885557
iter is 66 and norm is 172.1179071821219
iter is 67 and norm is 155.7883920159907
iter is 68 and norm is 141.00800037164333
iter is 69 and norm is 127.62979730139678
iter is 70 and norm is 115.52077964705268
iter is 71 and norm is 104.56055610006236
iter is 72 and norm is 94.64015211285457
iter is 73 and norm is 85.66092789231207
iter is 74 and norm is 77.53359880791938
iter is 75 and norm is 70.17734854663301
iter is 76 and norm is 63.51902625551286
iter is 77 and norm is 57.49241973312278
iter is 78 and norm is 52.03759748130113
iter is 79 and norm is 47.100313103104305

iter is 80 and norm is 42.631466148565266
iter is 81 and norm is 38.58661406323984
iter is 82 and norm is 34.92553040454254
iter is 83 and norm is 31.611804941213098
iter is 84 and norm is 28.612481670371352
iter is 85 and norm is 25.897731158434745
iter is 86 and norm is 23.440553955159537
iter is 87 and norm is 21.21651213396591
iter is 88 and norm is 19.203486293876136
iter is 89 and norm is 17.381455609051255
iter is 90 and norm is 15.732298740135612
iter is 91 and norm is 14.239613631076386
iter is 92 and norm is 12.88855439829153
iter is 93 and norm is 11.665683693651882
iter is 94 and norm is 10.558839071856886
iter is 95 and norm is 9.557012035388194
iter is 96 and norm is 8.65023855369495
iter is 97 and norm is 7.829499968869549
iter is 98 and norm is 7.086633302319711
iter is 99 and norm is 6.414250070823047
iter is 100 and norm is 5.805662804302519
iter is 101 and norm is 5.254818535769077
iter is 102 and norm is 4.756238600676304
iter is 103 and norm is 4.3049641481254435
iter is 104 and norm is 3.8965068221645645
iter is 105 and norm is 3.5268041210189325
iter is 106 and norm is 3.192178993061749
iter is 107 and norm is 2.889303265364006
iter is 108 and norm is 2.6151645424039924
iter is 109 and norm is 2.3670362451462066
iter is 110 and norm is 2.1424504929756183
iter is 111 and norm is 1.939173558477506
iter is 112 and norm is 1.7551836509189542
iter is 113 and norm is 1.5886508077286867
iter is 114 and norm is 1.4379186938269535
iter is 115 and norm is 1.3014881278864685
iter is 116 and norm is 1.1780021715394555
iter is 117 and norm is 1.0662326333950412
iter is 118 and norm is 0.9650678532867675
iter is 119 and norm is 0.8735016459243952
iter is 120 and norm is 0.7906232934618751
iter is 121 and norm is 0.7156084873903753
iter is 122 and norm is 0.6477111299789713
iter is 123 and norm is 0.5862559135970185
iter is 124 and norm is 0.5306316043096204
iter is 125 and norm is 0.4802849621428088
iter is 126 and norm is 0.4347152392035216
iter is 127 and norm is 0.3934691984276473

iter is 128 and norm is 0.3561366064950927
iter is 129 and norm is 0.3223461529191219
iter is 130 and norm is 0.2917617577262206
iter is 131 and norm is 0.26407922810242174
iter is 132 and norm is 0.23902323325997776
iter is 133 and norm is 0.2163445662167307
iter is 134 and norm is 0.1958176645306006
iter is 135 and norm is 0.17723836755330083
iter is 136 and norm is 0.1604218852811625
iter is 137 and norm is 0.14520096096569918
iter is 138 and norm is 0.13142420698654572
iter is 139 and norm is 0.11895459965952972
iter is 140 and norm is 0.10766811615105731
iter is 141 and norm is 0.09745250086222718
iter is 142 and norm is 0.08820614929369308
iter is 143 and norm is 0.07983709711941527
iter is 144 and norm is 0.0722621055878352
iter is 145 and norm is 0.06540583382017502
iter is 146 and norm is 0.05920008918744841
iter is 147 and norm is 0.05358314925947835
iter is 148 and norm is 0.048499148006628266
iter is 149 and norm is 0.043897519842446386
iter is 150 and norm is 0.03973249685776639
iter is 151 and norm is 0.03596265383164016
iter is 152 and norm is 0.03255049572393065
iter is 153 and norm is 0.029462085211911602
iter is 154 and norm is 0.026666704944558795
iter is 155 and norm is 0.024136552012498466
iter is 156 and norm is 0.021846461434492866
iter is 157 and norm is 0.019773656171093612
iter is 158 and norm is 0.017897519807080292
iter is 159 and norm is 0.016199392421098986
iter is 160 and norm is 0.014662384394754584
iter is 161 and norm is 0.013271208572354464
iter is 162 and norm is 0.012012028391857466
iter is 163 and norm is 0.010872319930817321
iter is 164 and norm is 0.009840747704815873
iter is 165 and norm is 0.008907051736300062
iter is 166 and norm is 0.008061945308455888
iter is 167 and norm is 0.0072970230756689595
iter is 168 and norm is 0.006604677151656469
iter is 169 and norm is 0.005978021493655297
iter is 170 and norm is 0.005410823238376705
iter is 171 and norm is 0.004897441016130946
iter is 172 and norm is 0.004432768904223601
iter is 173 and norm is 0.0040121851303194424
iter is 174 and norm is 0.003631506666683425
iter is 175 and norm is 0.003286947071263274

iter is 176 and norm is 0.0029750796238297945
iter is 177 and norm is 0.0026928022783934977
iter is 178 and norm is 0.0024373075861386607
iter is 179 and norm is 0.0022060543253070936
iter is 180 and norm is 0.0019967426164235634
iter is 181 and norm is 0.0018072904898384728
iter is 182 and norm is 0.0016358137023765823
iter is 183 and norm is 0.0014806067108542262
iter is 184 and norm is 0.0013401258330469064
iter is 185 and norm is 0.0012129739102217062
iter is 186 and norm is 0.0010978862514150472
iter is 187 and norm is 0.0009937181278034805
iter is 188 and norm is 0.0008994335312232665
iter is 189 and norm is 0.000814094712444214
iter is 190 and norm is 0.0007368529625299514
iter is 191 and norm is 0.0006669398718325957
iter is 192 and norm is 0.0006036602534922707
iter is 193 and norm is 0.0005463845813516837
iter is 194 and norm is 0.0004945432919192665
iter is 195 and norm is 0.0004476207343357973
iter is 196 and norm is 0.00040515021739170333
iter is 197 and norm is 0.00036670931491020013
iter is 198 and norm is 0.0003319157281615347
iter is 199 and norm is 0.000300423347728333
iter is 200 and norm is 0.00027191897914753395
iter is 201 and norm is 0.00024611913555323334
iter is 202 and norm is 0.00022276723668839568
iter is 203 and norm is 0.00020163095596395747
iter is 204 and norm is 0.00018250007835633872
iter is 205 and norm is 0.00016518438786159077
iter is 206 and norm is 0.00014951162120890695
iter is 207 and norm is 0.0001353257978811548
iter is 208 and norm is 0.000122486054464186
iter is 209 and norm is 0.00011086451959183883
iter is 210 and norm is 0.00010034560077172664
iter is 211 and norm is 9.082476004926643e-05
iter is 212 and norm is 8.220725833564871e-05
iter is 213 and norm is 7.440735130812205e-05
iter is 214 and norm is 6.734753846696534e-05
iter is 215 and norm is 6.0957582309622063e-05
iter is 216 and norm is 5.5173891111391466e-05
iter is 217 and norm is 4.9938923691964254e-05
iter is 218 and norm is 4.520073086063461e-05
iter is 219 and norm is 4.091202674826635e-05
iter is 220 and norm is 3.703029021093769e-05
iter is 221 and norm is 3.351679547351546e-05
iter is 222 and norm is 3.0336705051592927e-05
iter is 223 and norm is 2.7458358849036658e-05

iter is 224 and norm is 2.4853079318568644e-05
iter is 225 and norm is 2.249505184926565e-05
iter is 226 and norm is 2.0360726954295227e-05
iter is 227 and norm is 1.8428883352268395e-05
iter is 228 and norm is 1.6680269698749603e-05
iter is 229 and norm is 1.5097632229353942e-05
iter is 230 and norm is 1.3665208740659164e-05
iter is 231 and norm is 1.2368662578950016e-05
iter is 232 and norm is 1.1195132838480591e-05
iter is 233 and norm is 1.0132893916445375e-05
iter is 234 and norm is 9.171446214223954e-06
iter is 235 and norm is 8.301255071289588e-06
iter is 236 and norm is 7.513608652940778e-06
iter is 237 and norm is 6.800787579033828e-06
iter is 238 and norm is 6.155522874930071e-06
iter is 239 and norm is 5.571455970443868e-06
iter is 240 and norm is 5.0428547284398665e-06
iter is 241 and norm is 4.564365349824255e-06
iter is 242 and norm is 4.131271355188454e-06
iter is 243 and norm is 3.739276076394871e-06
iter is 244 and norm is 3.384497811896766e-06
iter is 245 and norm is 3.0634103762559527e-06
iter is 246 and norm is 2.7727345326635144e-06
iter is 247 and norm is 2.509659039131833e-06
iter is 248 and norm is 2.271529854002473e-06
iter is 249 and norm is 2.0559924597073396e-06
iter is 250 and norm is 1.8609074422341361e-06
iter is 251 and norm is 1.6843874768531079e-06
iter is 252 and norm is 1.5245416895895386e-06
iter is 253 and norm is 1.3798838464878412e-06
iter is 254 and norm is 1.248970610133471e-06
iter is 255 and norm is 1.1304957747837032e-06
iter is 256 and norm is 1.0232200729389682e-06
iter is 257 and norm is 9.261516136588563e-07
iter is 258 and norm is 8.38223106322964e-07
iter is 259 and norm is 7.587517533860814e-07
iter is 260 and norm is 6.867434397317076e-07
iter is 261 and norm is 6.215612196475778e-07
iter is 262 and norm is 5.62614302829287e-07
iter is 263 and norm is 5.092300724705563e-07
iter is 264 and norm is 4.6092074776496825e-07
iter is 265 and norm is 4.1720013857581535e-07
iter is 266 and norm is 3.7765637628390193e-07
iter is 267 and norm is 3.417552148261515e-07
iter is 268 and norm is 3.0932426364287045e-07
iter is 269 and norm is 2.7998884569366514e-07
iter is 270 and norm is 2.534134280927512e-07
iter is 271 and norm is 2.294143477658272e-07


```

iter is 272 and norm is 2.0765540761958677e-07
iter is 273 and norm is 1.879104719844502e-07
iter is 274 and norm is 1.7008005668316078e-07
iter is 275 and norm is 1.5394336040748632e-07
iter is 276 and norm is 1.3935057170488374e-07
iter is 277 and norm is 1.2611822807669944e-07
iter is 278 and norm is 1.1416070706437738e-07
iter is 279 and norm is 1.0335207747384763e-07
iter is 280 and norm is 9.348842348550168e-08
iter is 281 and norm is 8.464688858155026e-08
iter is 282 and norm is 7.66128762509854e-08
iter is 283 and norm is 6.932139434744217e-08
iter is 284 and norm is 6.28157082208881e-08
iter is 285 and norm is 5.683332044803126e-08
iter is 286 and norm is 5.140602208220706e-08
iter is 287 and norm is 4.649778006586554e-08
iter is 288 and norm is 4.210555369033167e-08
iter is 289 and norm is 3.813753272291183e-08
iter is 290 and norm is 3.4531231846200205e-08
iter is 291 and norm is 3.124162335309172e-08
iter is 292 and norm is 2.8308075993485597e-08
iter is 293 and norm is 2.558133987019569e-08
iter is 294 and norm is 2.3150414130389888e-08
iter is 295 and norm is 2.0947667723409683e-08
iter is 296 and norm is 1.8981125750966715e-08
iter is 297 and norm is 1.718449341335556e-08
iter is 298 and norm is 1.5582702880286358e-08
iter is 299 and norm is 1.4077020671193046e-08
iter is 300 and norm is 1.2750367861338948e-08
iter is 301 and norm is 1.1568671932486237e-08
iter is 302 and norm is 1.044772790276879e-08

```

```

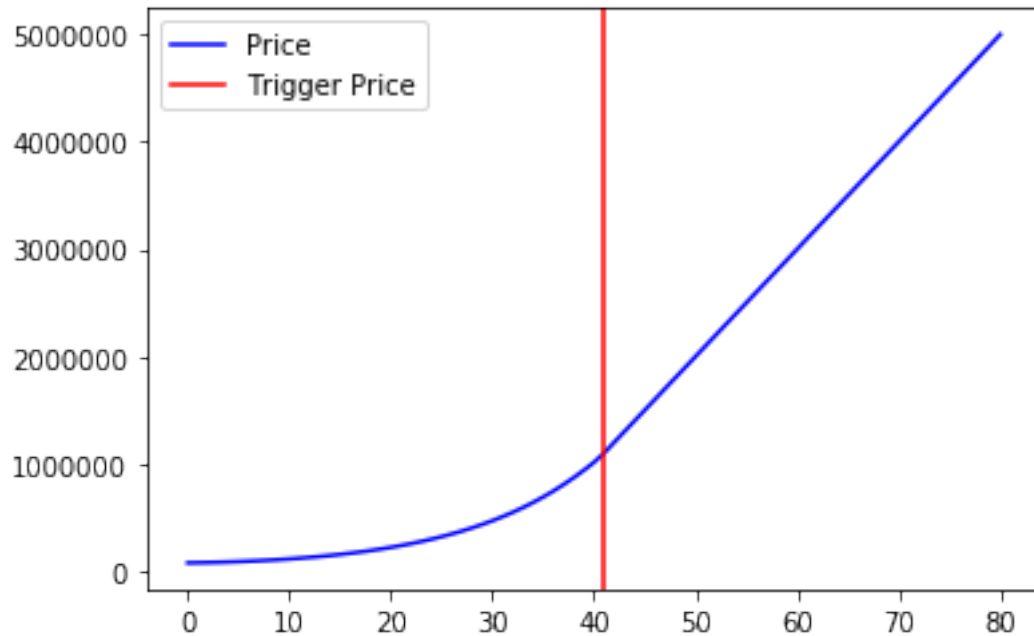
[6]: plt.plot(P_grid, Vp, color="blue", label="Price")
      plt.axvline(x=triggerprice, color="red", label="Trigger Price")
      plt.legend()
      plt.show

```

```

[6]: <function matplotlib.pyplot.show(*args, **kw)>

```



The price curve is a straight line after the trigger price because from that price onwards, the drilling option is not valuable anymore as the drilling will happen for these prices, so future prices are not relevant anymore. The uncertainty about future prices is what gives value to the drilling option.

[]: