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The Sizes and Types of Cities

By J. V. Henderson*

This paper presents a general equilibrium model of an economy where production and consumption occur in cities. The paper solves for equilibrium and optimum city sizes, discussing under what situations the equilibrium size differs from the optimum. Optimum city sizes are defined as those which maximize potential welfare of participants in the economy. Equilibrium city sizes are determined by the location or investment decisions of laborers and capital owners, each attempting to maximize their own perceived welfare.

Some of the basic concepts underlying the model are contained in the following propositions. We observe population agglomeration or cities because there are technological economies of scale in production or consumption and because these activities are not space or land intensive (relative to agriculture). Scale economies may occur at the final output level, at the marketing level, or at the intermediate input level, such as in transportation systems or capital and labor market development.

Given the existence of scale economies, what limits city size? The following argument is developed by Edwin Mills, and I utilize his basic argument in this paper. Mills assumes urban production of traded goods to occur in a central business district (CBD). In addition to traded goods, housing is produced in the city and workers commute to the CBD from their sites

surrounding the CBD. As city size and the area devoted to housing increase spatially, the average distance a worker commutes necessarily increases as does congestion. That is, average per person commuting costs rise with city size. Efficient city size occurs where these increasing per person resource costs offset the resource savings due to scale economies in traded good production.

Why do cities vary in size? This question pertains basically to Section IV of paper, since in the main body of the paper cities will all be the same size and type. City sizes vary because cities of different types specialize in the production of different traded goods, exported by cities to other cities or economies. If these goods involve different degrees of scale economies, cities will be of different sizes because they can support different levels of commuting and congestion costs. But why do cities specialize?

Provided there are no positive production benefits or externalities from locating two industries together, locating the production of the two goods in the same city only works to raise total production costs. Laborers employed in the two industries contribute to rising per person commuting costs, but scale economy exploitation occurs only with labor employment *within* each industry. If we locate the industries together, there are higher average per person commuting resource costs for a *given* level of scale economy exploitation or industry employment within either industry than if we locate the industries in separate cities. This is one reason why cities will tend to specialize in the production of different traded goods. To be weighed against the specialization advan-

tage are the transportation costs of trade between specialized cities. Goods such as retailing services are not traded between cities because of high transportation costs. Note that cities will probably specialize in bundles of goods, where, within each bundle, the goods are closely linked in production. They may use a common specialized labor force or a common intermediate input.

Throughout this paper, it is assumed that capital and labor are scarce resources available in fixed supply to the national economy. The economy is defined as a region or country within which these factors are perfectly mobile. Factors move between cities in the economy to equalize appropriate measures of factor rewards. The economy is situated on a flat featureless plain large enough so that land per se has zero opportunity cost and is never a scarce resource.

As stated above, I deal mainly with an economy where there is only one type of city. Each city produces and exports the same traded good at a fixed price to other regions or countries. In return the cities import another consumption good at a fixed price. In Section IV, I outline complications of the model presented in my thesis that incorporate multiple types of cities trading with each other in the same economy (where the terms of trade may be endogenous). Although these complications are interesting, they are not needed to develop the basic ideas in this paper.

I. The Model of a City

The model of a single representative city is presented in this section and solved for factor reward equations (which may refer to either or both factor prices and utility levels). Factor rewards will be a function of city employment of capital and labor and the fixed price of the city's export good. In Section III, the factor reward equations will be used to solve equilibrium

and optimum city size for all cities in the economy subject to the economy endowment of resources. City size indicates the allocation of the economy's factors to each city, the number of cities, and the prevailing level of factor rewards which are equalized between cities (for equilibrium in factor markets). Given that there is only one type of city for most of the paper, city sizes will turn out to be all identical. We now turn to our representative city and develop the factor reward equations.

A. Production Conditions

Our representative city produces a traded good X_1 under conditions of increasing returns to scale, external to the firm but internal to the industry and city. These scale economies are responsible for the urban agglomeration discussed above. The industry production function is

$$(1) \qquad X_1^{1-\rho_1} = L_1^{\alpha_1} K_1^{\beta_1} N_1^{\delta_1},$$

$$\alpha_1 + \beta_1 + \delta_1 = 1, \quad 0 < \rho_1 < 1$$

where L_1, N_1, and K_1 are inputs of home or land sites, labor, and capital, respectively. The variable ρ_1 represents the degree of increasing returns to scale; hence $(\alpha_1 + \beta_1 + \delta_1)/(1 - \rho_1) > 1$. As stated above, X_1 is sold by the city at a fixed price set in national or international markets.

Under this externality specification (as explained in an article by John Chipman), the *firm* views itself as having a constant returns to scale production function. Therefore the private marginal product of, say, labor in the industry is $\delta_1 X_1/N_1$ rather than the social marginal product $[\delta_1/(1-\rho_1)]X_1/N_1$. This preserves exhaustion of firm revenue by factor payments. Atomistic competition is ensured since any entering firm benefits from the existing level of externalities or industry scale economies, i.e., firm size is unimportant in the model. Later in the paper I briefly discuss the fact that since social and private marginal

products differ, factor allocations may not be strictly Pareto optimal.

The second good produced is X_3, housing services for workers living in the city. Since housing services are a nontraded good, their price will vary with city size and will be determined in the model. The production function for housing is

$$(2) \qquad X_3 = N_3^{\delta_3} K_3^{\beta_3} L_3^{\alpha_3}, \qquad \delta_3 + \beta_3 + \alpha_3 = 1$$

The third good produced in the city is sites, an intermediate input in X_3 and X_1 production. In a spatial model, a site used in the production of housing is produced with an input of raw land and labor (time) inputs of commuting needed for travel to the CBD from a spatial location in the city. These commuting costs of producing sites escalate as city size increases and average commuting inputs or distance and congestion increase. In addition, increased use of sites in X_1 production competes with sites for residential use and therefore contributes to rising commuting distances and costs.

For mathematical simplicity without crucial omissions in economic reasoning, the spatial world is collapsed into a non-spatial world in this paper. Rather than explicitly having spatial dimensions or commuting in the paper, I simply assume sites are produced with labor inputs subject to decreasing returns to scale or rising per site labor inputs as city size increases. I hypothesize the model works qualitatively "as if" it were a spatial model. With no spatial dimensions, people will have identical housing consumption but the average resource costs of sites and housing will rise as city size increases. Also, there is no separate class of land owners in the model.[1]

[1] The other crucial aspect of the commuting phenomenon in a spatial model is land rents. Residential location theory as developed by Richard Muth and Mills tells us there is a spectrum of commuting costs and land rents in a city. Land rents act as a rationing device so

Sites are produced with labor or commuting time inputs and used in X_1 and X_3 production. That is,

$$(3) \qquad (L_1 + L_3)^{1-z} \equiv L^{1-z} = N_0 \qquad z < 0$$

where L and N_0 are sites and labor inputs (raw land is not specified separately in (3) since its opportunity cost is zero). The variable z represents the degree of decreasing returns to scale. (Furthermore z is assumed to increase in absolute value with city size. Specifically it is assumed $1/(1-z) = N^m$, $-1 < m < 0$ where N is city population. The reason for this assumption is essentially algebraic and is mentioned later.) The diseconomies of scale are assumed to be external to the individual, and so again while factor payments exhaust revenue, factor allocations to sites are not strictly Pareto optimal. This problem is commented on briefly later. Intuitively the externality exists because when a laborer enters a city he imposes higher average commuting costs on other city inhabitants (see James Buchanan and Charles Goetz).

Given the production functions for the three goods produced in the city, the production side of the model is completed by the resource and intermediate input employment equations where N and K are city population and capital stock.

that people who live nearer the CBD and experience lower commuting costs pay higher rents to offset their cost advantage relative to those further from the CBD. The actual land itself involves no resource costs if its opportunity cost is zero. The land rents are a transfer from renter to landowner reflecting the relative "scarcity" of a location. In a nonspatial world there is no role for a rationing device or spectrum of land rents and landowners. Rising resource costs of commuting are captured but the location scarcity principle is not represented. However, given land rents are essentially a transfer from renter to landowner, our results concerning equilibrium and Pareto optimum city size are unaffected. But to the extent that rising land rents induce further substitution away from homesite inputs in housing and X_1 production, the resource costs of the commuting phenomenon are "under-represented" in our model.

(4) $N_0 + N_1 + N_3 = N$

$K_1 + K_3 = K$

$L_1 + L_3 = L$

B. Consumption Conditions

To close the model, consumption conditions must be specified in order to derive the demand equations for three consumer goods. In addition to goods produced in their own city, X_1 and X_3, city inhabitants consume a good X_2 imported from other economies at fixed price q_2. Consumers have identical tastes and maximize logarithmic linear utility functions subject to their income and prices q_1, q_2, and q_3. Income spent in the city and city demand for X_1, X_2, and X_3 is determined as follows.

Laborers live in the cities where they work and spend their income. Capital owners are not constrained to live in the city where their capital rentals are earned. They may live in the countryside, in other cities, or in other countries. Cities may be net borrowers or lenders with respect to the proportion of capital rentals earned versus spent in the city. Given these problems and varieties of situations, two alternative polar assumptions are made. These assumptions play a crucial analytical role later in the paper.

Assumption A. All capital owners live in the cities of this economy and also work as laborers. For simplicity it is assumed capital ownership is evenly divided among laborers. If the cities in our economy have the same K/N ratio in production (which they will as long as they are identical or until Section IV), they are neither *net* borrowers nor net lenders. However, they may be gross borrowers or lenders since capital owners need not invest in the city they live in.

Assumption B. Capital owners are a separate group of people who do not work as laborers. They avoid the high cost of living or housing in cities (see below) by living in the countryside or other countries. (Since we have fixed the supply of capital to the economy in this paper, living in the countryside makes more sense.) *No capital rentals are spent in the cities of this economy.*

Summarizing the consumption conditions, we have individuals maximizing utility, $U = x_1^a x_2^b x_3^c$ where x_i is individual consumption of x_i, subject to, for Assumptions A and B, respectively, either $y = p_N + p_K \ K/N$ or $y = p_N$ where p is factor price. From this optimization process, we may obtain expressions for the indirect utility function of an individual and the aggregate demand equations for the city (the sum of individual demands). Where Y is city income which equals yN, these expressions are

(5) $X_1^C = aY/q_1, \quad X_2^C = bY/q_2, \quad X_3^C = cY/q_3$

(6) $U = a^a b^b c^c y q_1^{-a} q_2^{-b} q_3^{-c}$

Equation (6) for the indirect utility function is used extensively throughout the paper. In (5), the superscript $C(P)$ refers to goods consumed (produced) in the city. This distinction is crucial for the balance of trade equation for the city $X_1^P q_1 - X_1^C q_1 = X_2^C q_2 + k p_K K$ where $k = 1$ under Assumption B and $k = 0$ under Assumption A if cities are not net lenders or borrowers.

II. Solution of the Model for a City

From the consumption and production equations of the model, city output, exports, factor prices, and the price of sites and housing can be solved for in terms of city employment of capital K and labor N and the fixed prices of traded goods, q_1 and q_2. City employment of capital and labor will be determined in Section III when we solve for city size. In this section we simply explore how equilibrium factor prices, housing prices, and, in particular, utility levels vary as we vary K and N. To solve for equilibrium movements of factor and housing prices as we vary city employment

of K and N, we solve our model of a city. We combine the supply and demand side by combining the full employment equations (4) for our representative city with the private marginal product equations, determining factor prices in the X_3, X_1, and L industries, the consumer demand equations for X_1 and X_3 from equation (5), and the cost functions derived from (1), (2), and (3). The method of solution is detailed in my thesis as is the derivation of all equations in the model. Given the solutions in terms of K and N for housing prices, wage rates (p_N), and capital rentals (p_K), we may substitute these variables into (6) to solve for utility levels as a function of city employment of capital and labor.

Assumption B. In determining equilibrium city size, we must consider the location decision of a laborer. A laborer will choose to live in the city that he perceives as maximizing his utility. From equation (6) his utility is a function of just two variables: q_3, the price of housing or the city cost of living and p_N, the city wage rate. Both of these as explained can be solved for in terms of city employment of K and N. Equation (6) also defines the welfare of laborers and is used in solving for optimum city sizes. Below we present the expression for utility of laborers in terms of K and N called U_N which we will use to help solve for both equilibrium and optimum city sizes under Assumption B.

For capital owners, the distinction between variables governing investment decisions and those reflecting the benefits of such decisions will be crucial. Capital owners do not have to live in cities where their capital is employed and under Assumption B do not live in cities at all. In determining equilibrium city size, we assume capital owners invest to maximize capital rentals. Under Assumption B (only), capital rentals also reflect the welfare of investment decisions of capital

owners. Since their cost of living or housing is independent of their investment decisions and the size and cost of living in cities in this economy, p_K is the only variable in equation (6) in terms of the benefits to capital owners of their investment decisions.

Therefore, under Assumption B, we use an expression for the utility of laborers, U_N, to analyze the location decisions of laborers in solving for equilibrium city size. We use p_K to analyze the investment decisions of capital owners. We also use these same variables to solve for optimum city sizes.

Assumption A. Now since laborers are also capital owners and all factor payments are spent in the cities of our economy, things are not so simple. There are two basic decisions determining equilibrium city size. One is an investment decision and the other a location decision. Since laborers can invest their capital in any city in the economy, not just the one they live in, their investment decision is divorced from their location decision of which city to live in. Therefore when we examine the capital market in determining equilibrium city size, we will·assume laborers seek to maximize capital rentals which then must be equalized between cities for capital markets to be in equilibrium.

In making location decisions, laborers seek to maximize utility as a function of the variables in (6): namely q_3, the price of housing, and $p_N + \overline{p_K} \, \overline{K/N}$, the per laborer income, where p_K is exogenous to the location decision[2] and $\overline{K/N}$ is the fixed amount of capital *owned* by each laborer. Per laborer utility is then

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[2] If one assumes capital is physically tied to the owner as for, say, a small business owner, then he will move his business to maximize an index of utility—the capital rentals deflated by a cost of living. For example a small business owner in New York will demand a higher return on his capital than if he lived in Albany, simply due to cost of living differences.

$$U = a^a b^b c^c (p_N + \overline{p_K} \, \overline{K/N}) q_1^{-a} q_2^{-b} q_3^{-c}$$

For expositional simplicity we split per laborer utility levels into the sum of two parts:

$$U_N = a^a b^b c^c \, p_N q_1^{-a} q_2^{-b} q_3^{-c}$$

plus

$$U_K = a^a b^b c^c \, \overline{p_K} \, \overline{K/N} q_1^{-a} q_2^{-b} q_3^{-c}$$

The level U_N is utility from labor income and U_K is utility from capital income. Again, since as explained above, p_N, p_K, and q_3 can be expressed in terms of city employment of K and N, U_N and U_K can similarly be expressed in terms of K and N.

In summary, to solve for equilibrium city size under Assumption A we depict investment decisions as investors or laborers seeking to maximize capital rentals or p_K. Location or migration decisions are depicted by laborers seeking to maximize the sum of U_N and U_K where $\overline{p_K}$ in U_K is exogenous to the location decision.

In determining optimum city sizes, we are not concerned with separate investment and location decisions. Instead we are concerned with the simultaneous determination of p_K and utility levels that maximizes utility or welfare of laborers in the economy. Since laborers are the only participants in our economy, we simply seek to maximum their welfare which is the sum of U_N and U_K where p_K is no longer exogenous to the location or any other decision. The precise meaning of these statements will become apparent below; but, to repeat, we seek to maximize U_N plus U_K, *given* the determination of U_N, U_K, and p_K through simultaneous location and investment of labor and capital in cities in the economy.

The following equations are given and used in subsequent analysis of city size. As explained above, housing prices and factor prices in terms of city employment of K and N are obtained by combining the city full employment equations, the industry marginal product equations, the consumer demand equations, and industry cost functions. The exponents of the equations contain production and consumption parameters including z, the degree of decreasing returns to scale in homesite production (where $1/(1-z) = N^m, 0 > m > -1$). For example, the expression for p_K depicts the equilibrium movement of capital rentals or private marginal product equations (for example, $p_K = q_1 B_1 X_1/K_1$) in the X_1 and X_3 industries, where the equilibrium movement of p_K is determined by the production and consumption conditions of our model. With nonconstant returns to scale characterizing production functions, equilibrium p_K is a function of the city K/N ratio, the scale of output, and the degrees of increasing (ρ_1) and decreasing (z, where $1/1-z = N^m$) returns to scale.

Under Assumption A, in addition to p_K, we present expressions for the utility U_N from wage income (p_N) and the utility U_K from capital income ($p_K \, \overline{K/N}$).[3] As explained above these are obtained by substituting expressions for p_N, P_K, and q_3 into (6). We write the equations in na-

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[3] The expressions for p_N and q_3 under Assumption A are

$$\log p_N = \log (C_N q_1) + N^m \frac{\alpha_1}{\rho_1 - 1} \log t$$
$$+ \frac{\alpha_1(1 - N^m) - \rho_1}{\rho_1 - 1} \log N$$
$$+ \frac{-\beta_1}{\rho_1 - 1} \log K/N$$
$$\log q_3 = \log (C_q q_1) + N^m \frac{(\alpha_1 - \alpha_3(1 - \rho_1))}{\rho_1 - 1} \log t$$
$$+ \frac{\beta_3 - \beta_1 - \beta_3 \rho_1}{\rho_1 - 1} \log N$$
$$+ \frac{(\alpha_1 - \alpha_3(1 - \rho_1))(1 - N^m) - \rho_1}{\rho_1 - 1} \log K/N$$

where C_N and C_q are constant.

$$(7) \quad \log U_N = \log (W_N q_2^{-b} q_1^{1-c-a}) + N^m \frac{(\alpha_1(1-c) + c\alpha_3(1-\rho_1))}{\rho_1 - 1} \log t$$

$$+ \frac{(-\beta_1 - c\beta_3 + c\beta_1 + c\beta_3\rho_1)}{\rho_1 - 1} \log (K/N)$$

$$+ \left(\frac{(\alpha_1(1-c) + c\alpha_3(1-\rho_1))(1-N^m) - \rho_1(1-c)}{\rho_1 - 1} \right) \log N$$

$$(8) \quad \log p_K = \log (C_K q_1) + N^m \frac{\alpha_1}{\rho_1 - 1} \log t + \frac{1 - \rho_1 - \beta_1}{\rho_1 - 1} \log K/N + \frac{\alpha_1(1 - N^m) - \rho_1}{\rho_1 - 1} \log N$$

$$(9) \quad \log U_K = \log \overline{K/N} + \log (W_K q_2^{-b} q_1^{1-c-a}) + N^m \left(\frac{\alpha_1(1-\rho_1) + c\alpha_3(1-\rho_1)}{\rho_1 - 1} \right) \log t$$

$$+ \left(\frac{1 - \rho_1 - \beta_1 + c\beta_1 + c\beta_3(\rho_1 - 1)}{\rho_1 - 1} \right) \log K/N$$

$$+ \left(\frac{(\alpha_1(1-c) + \alpha_3 c(1-\rho_1))(1-N^m) - \rho_1(1-c)}{\rho_1 - 1} \right) \log N$$

tural logarithmic form as (7), (8), and (9). The coefficients W_N, C_K, and W_K are constants defined in my thesis and are not relevant to our discussion. In addition,

$$t = \left(\alpha_1 + \alpha_3 \frac{c}{1-c} \right)^{-1}$$

$$\cdot \left(\alpha_1 + \delta_1 + (\alpha_3 + \delta_3) \frac{\iota}{.1-c} \right) > 1$$

Note that in (9), we distinguish between the fixed $\overline{K/N}$ in ownership describing the quantity of capital owned by each individual and the variable city K/N ratio in production which is determined in the model. (In this paper since there is only one type of city, in the following it will turn out all cities are identical in size and economic characteristics and, hence, $\overline{K/N}$ will equal K/N in production.) By inspection it can be seen $\partial U_N/\partial(K/N) > 0$ and $\partial p_K/\partial(K/N)$, $\partial U_K/\partial(K/N) > 0$ if $\rho_1 + \beta_1 < 1$. That is, normal factor ratio effects on factor rewards prevail unless the degree of increasing returns to scale ρ_1 is very large.

Under Assumption B, we are only concerned with the expressions for utility from spending labor income and for capital rentals. The expressions for U_N and p_K are

identical to (7) and (8) except the constants W_N, C_K, and t are replaced by W'_N, C'_K, and s^{-1} where $s = [(1 - \delta_3 c - \alpha_3 c)\alpha_1 \cdot (\alpha_1 + \delta_1)^{-1} - \alpha_3 c] < 1$. It can be shown that normally $s^{-1} > t$.

A. *Utility and Capital Rental Paths*

The equations presented above are a function of the K/N ratio in the city, a measure of scale of city output or N, a variety of production consumption parameters, and prices, q_1 and q_2. We want to see how factor rewards vary with city size or N so that in Section III we may determine equilibrium and optimum city sizes or N. To do this, we isolate the scale effect from the factor ratio effect and any effect of changing q_1 and q_2. We take the derivative of the above equations with respect to N, or city size, holding K/N, q_1 and q_2 constant. We will show later our analysis is neutral or unaffected by changes in K/N, q_1, and q_2. Using the derivatives of factor rewards, the values that factor rewards assume with different city sizes will be summarized in *factor reward paths*. The derivatives of (7), (8), and (9) are shown in (10), (11), and (12). These equations are

$$(10) \qquad \frac{\partial p_K}{\partial N} = N^{m-1} p_K \frac{\alpha_1 m}{\rho_1 - 1} \left[\log t - 1/m + \frac{\alpha_1 - \rho_1}{\alpha_1 - m} N^{-m} - \log N \right]$$

$$(11) \qquad \frac{\partial U_K}{\partial N} = U_K N^{m-1} \left(\frac{m(\alpha_1 - c\alpha_1 - c\alpha_3(\rho_1 - 1))}{\rho_1 - 1} \right)$$
$$\cdot \left[\log t - \frac{1}{m} + \frac{(1-c)(\alpha_1 - \rho_1) + c\alpha_3(1 - \rho_1)}{m(\alpha_1 - c\alpha_1 - c\alpha_3(\rho_1 - 1))} N^{-m} - \log N \right]$$

$$(12) \qquad \frac{\partial U_N}{\partial N} = \frac{U_N}{U_K} \frac{\partial U_K}{\partial N}$$

analyzed in my thesis and here we just summarize the relevant results.

The sign of the derivatives is given by the sign of the expressions in the square brackets in each equation. If N is small, the expressions are all positive indicating that initially capital rentals and utility levels rise as city size rises. As N increases, either the derivatives remain positive or become negative, depending on whether the signs of the third terms in the square brackets are positive or negative.

A sufficient condition (necessary for p_K) that *both* capital rentals and utility levels rise to maximum and then decline[4] or that the derivatives eventually become negative is that $\alpha_1 \geq \rho_1$. The variable α_1 represents the intensity with which the resource input, land sites, is used in X_1 production, and ρ_1 is the degree of scale economies in X_1 production. If $\alpha_1 \geq \rho_1$, factor rewards attain a maximum and decline because the benefits of agglomeration (ρ_1) are eventually offset by scale diseconomies in site production where the level of site production rises as α_1 rises. This net change in efficiency will be reflected in factor prices p_K and p_N which will reach a maximum and then decline. Moreover,

[4] The reason why we specified $z(1/(1-z) = N^m)$ as variable, occurred because if z is fixed the derivatives in (10)–(12) are either always positive or always negative. That is, factor rewards never climb to a maximum and then decline, the situation we are interested in. Alternative to z varying, is ρ_1 varying or both varying. Our choice is arbitrary.

consumption benefits such as U_N or U_K of spending marginal products are *further* limited because to obtain U_N or U_K, factor prices are deflated by q_3^{-c}. The cost of housing q_3 rises with city size as sites become more expensive. This effect is reflected in the c and α_3 parameters in equation (11) where they represent the share of housing in consumption and of homesites in housing production.

If $\alpha_1 < \rho_1$, either the utility levels may reach a maximum while capital rentals rise indefinitely or both utility and capital rentals may rise indefinitely. Due to space limitations, we only discuss the case where capital rentals and utility levels both rise to a maximum and then decline.

The changes in factor rewards with city size are illustrated in Figure 1. We draw utility levels and capital rentals on the same diagram for convenience although they are measured in different units on the vertical axis. The city sizes where factor rewards are maximized, $N(p_K^*)$, $N(U_N^*)$, and $N(U_K^*)$, may be determined by solving equations (10)–(12) equated to zero. The maximum $N(U_N^*) = N(U_K^*) \equiv N(U_K^*, \ U_N^*)$ but it can be shown that $N(U_K^*, \ U_N^*) < N(p_K^*)$. This is not surprising since, say U_K^* is p_K^* *further* deflated by q_3^{-c}, the price of housing which rises with city size.

Two further comments are in order. Given $\alpha_1 \geq \rho_1$ from equations (7)–(9), the following normal results can be shown to
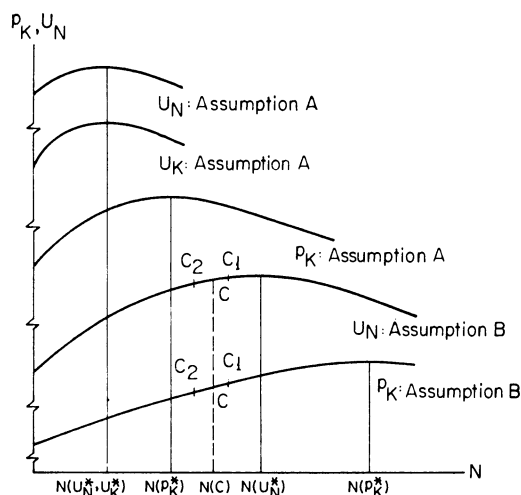
FIGURE 1. UTILITY AND CAPITAL RENTAL
PATHS UNDER ASSUMPTIONS A AND B

prevail: $\partial p_K/\partial(K/N)$, $\partial U_K/\partial(K/N) < 0$, $\partial U_N/\partial(K/N) > 0$, and all derivatives with respect to q_1 are positive. A change in K/N or q_1 shifts the factor reward paths up or down as indicated by the sign of the derivations. Further, from (10)–(12) equated to zero, it can be seen that the city sizes where the factor reward paths attain a maximum are invariant with respect to changes in K/N and q_1.

To examine factor reward paths under Assumption B, *log t* is replaced by *− log s* in equations (10) and (12). The above discussion in terms of relative values of α_1 and ρ_1 and the shape of the utility and capital rental paths applies directly. However, given the values of t and s cited above, because *− log s* has replaced the smaller *log t* in equations (10)–(12), the city sizes where U_N and p_K are maximized are larger than under Assumption A. This is not surprising. Under Assumption B, no capital rentals are spent in cities nor thus devoted to increasing the demand for housing produced and consumed in the city. Since the amount of housing and hence number of sites produced relative to X_1 is smaller, the cost of sites rises more slowly to offset the

benefits of agglomeration. The variables U_N and p_K achieve a maximum at a larger city size under Assumption B.

III. City Size

In this section, the utility and capital rental paths derived above are used to solve for city size. Optimum city size is found by maximizing welfare of participants in the economy. Equilibrium city size is determined given atomistic optimization behavior in the investment and location decisions of capital owners and laborers.

To initiate the process of city formation, we start with one city in the economy producing X_1 and then increase the size of the economy. This does not mean we have a growth model per se since we have no savings behavior, technological change, etc. It is an artificial and simple method of solving for city size. However it does yield solutions for optimum city sizes and does serve to reveal the possibility of inadequate functioning of the market forces and signals that determine equilibrium city sizes. In this paper we do not discuss how the economy K/N ratio is determined. Presumably there are underlying growth relationships in the economy and well-defined saving and investment behavior. With only one type of city under discussion in this paper as explained above, changes in the K/N ratio do not affect the shapes of the factor reward paths and as we will see below do not affect when new cities form. We just assume a K/N ratio which may or may not change (given macro-economic conditions in the economy) and a growing economy.

A. *Stability Conditions*

Two or more cities cannot be of sizes such that they are both on the rising part of the utility and capital rental paths. Only if there is a *single* city, can a city be on the rising part of both factor reward

paths. For example, in Figure 1 under Assumption B, suppose there are two cities of size $N(C)$. A random movement of a small amount of capital and labor from one city to the other would move us from C to C_1 in the receiving city and from C to C_2 in the losing city. Since U_N and p_K both rise in the receiving city this will induce further movements of factors to the receiving city. That is, the initial equilibrium is unstable with respect to factor movements. However it is stable to have two cities on the rising part of the U_N path and falling part of the p_K path. *Throughout the paper we rule out unstable solutions as possible optimal* (and of course equilibrium) *solutions.* The above stability arguments are properly developed in my thesis.

B. *Optimum City Size*

We now determine optimum city size. The discussion serves only to solve for optimum city size and does not reflect or indicate behavior on the part of capital owners and laborers. There is initially one city in the economy and the economy is growing. We want to know when it is optimal to form a second, then a third, etc., city. For the initial discussion the K/N ratio and q_1 are held constant by assumption. In solving for optimum city size Assumptions A and B play a crucial role. We solve first for Assumption A.

Under Assumption A, all capital owners work as laborers in this economy and hence endure the cost of living in the city when spending their income. Although capital owners invest to maximize capital rentals, we are concerned with their utility from spending capital rentals (and labor wages) when solving for optimum city size. As explained above, to solve for optimum city size we maximize laborers' utility from wage and rental income or we maximize the vertical sum of the U_N and U_K paths.

In Figure 2, holding K/N constant as city size grows beyond $N(U_N^*, U_K^*)$, the
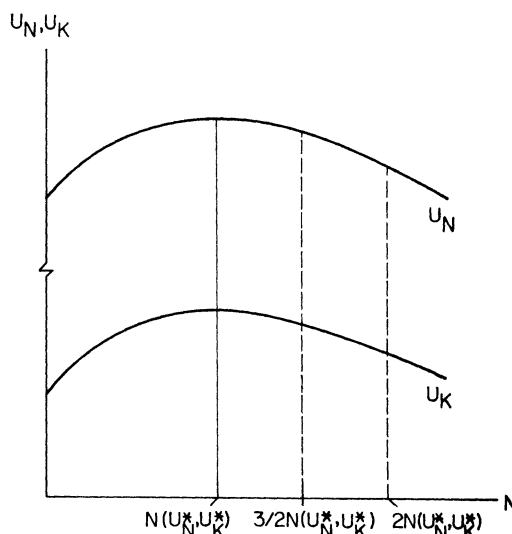
FIGURE 2. OPTIMUM CITY SIZE: ASSUMPTION A

city size of maximum U_N and U_K, a second city should form when N equals twice $N(U_N^*, U_K^*)$. The new solution then has two cities of size $N(U_N^*, U_K^*)$, resulting in stability in factor markets, equalization of factor rewards between cities, and full employment of factors in the economy. If two cities formed before $2N(U_N^*, U_K^*)$, resulting in city sizes less than $N(U_N^*, U_K^*)$ on the rising part of the factor payment paths, stability would not prevail in factor markets. Note that dividing utility into the sum of utility from capital and labor income presents no problems in analysis since these paths attain a maximum at the same point and hence their sum attains a maximum at $N(U_N^*, U_K^*)$. As the two cities of size $N(U_N^*, U_K^*)$ continue to grow, a third city of size $N(U_N^*, U_K^*)$ should form from the two cities when they reach size $3/2N(U_N^*, U_K^*)$. In general, a $n+1$ city should form when the n cities reach size $(n+1/n) N(U_N^*, U_K^*)$. If $n \rightarrow \infty$, which will be called the *large sample case*, city size will approach $N(U_N^*, U_K^*)$ where U_N and U_K are maximized. From equations (11) and (12), N equals $N(U_N^*, U_K^*)$ can be solved from:

(13) $\log t - 1/m$

$$+ \frac{(1-c)(\alpha_1-\rho_1)+c\alpha_3(1-\rho_1)}{m(\alpha_1-c\alpha_1-c\alpha_3(\rho_1-1))} N^{-m}$$

$$- \log N = 0$$

A change in the K/N ratio as the economy grows would not affect city size. Regardless of K/N, U_N and U_K always attain a maximum at the same city size and hence optimal city sizes as well as equation (13) are unaffected.

In the discussion here and in particular for the discussion of market equilibrium to follow, we are discussing abstract solutions in which new cities form instantaneously. In the real world this would not happen, of course. For example, assume a growing economy with one city. When it is appropriate to form a second city in the economy, the second city would start off very small growing over time until the two cities were the same size. Given in reality the nonmalleability of capital, the actual population decline in the first city when a second city forms might be very small. This would be particularly true if growth in the economy was accompanied by technological change that increased efficient city sizes. Then the first city might not decline in size, but the second city would grow more rapidly over time until the two cities were the same size.

Note finally the city size $N(U_N^*, U_K^*)$ indicates the *maximum benefits of scale economies or welfare for our economy.* The benefits of scale economies are limited by the costs of agglomeration or the rising costs of homesites or commuting in a spatial world. In a certain sense, at $N(U_N^*, U_K^*)$ we approach a constant returns to scale case in production. Doubling the size of the economy would only double the number of cities and would bring no further scale economy benefits.

Under Assumption B, if capital owners live in the countryside or abroad in other countries, the price they pay for housing is independent of the urban cost of living. Therefore they maximize utility by maximizing their only variable in equation (6), p_K. To solve for optimum city sizes the p_K and U_N paths are utilized in the same fashion as the paths for Assumption A. The difference here is that the points where capital owners want to form cities indicated by $N(p_K^*)$ in Figure 3 are *different* than the points where laborers want to form cities indicated by $N(U_N^*)$. How are these differences reconciled? Before proceeding we note as mentioned above that although U_N and p_K are measured in different units, we draw them on the same diagram. Hence the vertical axis measures capital rentals in dollars or utility levels in utility units.

If the initial city has reached twice $N(U_N^*)$ in Figure 3, laborers would be better off if two cities of size $N(U_N^*)$ formed and capital owners worse off. The size of the initial city should increase beyond twice $N(U_N^*)$ *if from the increase in earnings of capital owners we can compensate laborers for their loss in utility from not forming a second city.* In other words, we employ a Pareto optimality criterion to define optimal city sizes. Our criterion includes capital owners, whether they live in the econ-

FIGURE 3. OPTIMAL CITY SIZE: ASSUMPTION B

omy (the countryside) or in other regions or other countries. Second, it is important to understand that our compensation mechanism is used only to depict a Pareto optimal solution as might be administered by an omniscient ruler. I do not envision groups of laborers and capital owners bribing each other to attain Pareto optimal solutions in the free market. Market behavior of capital owners and laborers and market solutions are discussed in the next section.

Suppose the initial city size moves beyond twice $N(U_N^*)$ to $N(E)$ where it is optimal to form a second city, yielding two cities of size $N(E')$ where $N(E) = 2N(E')$. At $N(E)$, we can no longer compensate laborers from the earnings of capital owners for not forming two cities of size $N(E')$. As illustrated in Figure 3, the loss to capital owners of two cities forming is $K(p_K(E) - p_K(E'))$ and the gain to laborers is $N(U_N(E') - U_N(E))$. The compensation that we could give to individual laborers from capital owners for not forming a second city is $M(K)$ and the compensation needed by a laborer for not forming a second city is $M(N)$ where

$$(14) \quad M(K) = K/N(p_K(E) - p_K(E'))$$

$$U_N(E') - U_N(E) = a^a b^b c^c q_1^{-a} q_2^{-b} q_3^{-c} M(N)$$

or

$$(15) \quad M(N) = a^{-a} b^{-b} c^{-c} q_1^a q_2^b q_3^c$$
$$\cdot (U_N(E') - U_N(E))$$

From equation (6), the q_3^{-c} in (15) is used to deflate the compensation $M(N)$ where q_3 is the price of housing in city size $N(E)$. The variable $M(N)$ is the income subsidy to laborers needed to raise utility levels in city size $N(E)$ to those in the smaller city size $N(E')$. (Note that the calculation of p_N and q_3 and hence $U_N(E)$ will be affected by $M(N)$ since the demand for housing in city size $N(E)$ will rise if income is subsidized by $M(N)$.) At $N(E)$, two cities of

size $N(E')$ form because $M(N) \geq M(K)$, where both $M(N)$ and $M(K)$ are measured in dollars.

After two cities of size $N(E')$ form, the economy continues to grow with additional optimal size cities forming via our compensation mechanism. Of particular interest is the large sample case where the number of cities is very large and hence when an additional city forms the changes in city size of existing cities are minimal. Since city size changes are minimal, the changes in U_N and p_K and the compensation that could be made from the earnings of capital owners and needed by laborers in equations (14) and (15) can be expressed in derivative form. In Figure 3, $N(J)$ is picked as the optimal point to form an additional city and optimal city size approaches $N(J)$. Note that $N(U_N^*) \leq N(J) \leq N(p_K^*)$. At J, $M(N) \geq M(K)$ or by directly substituting in (14) and (15)

$$(16) \quad M(N) = \mid a^{-a} b^{-b} c^{-c} q_1^a q_2^b q_3^c (\partial U_N / \partial N) \mid$$
$$\geq \mid K/N(\partial p_K / \partial N) \mid = M(K)$$

From equation (15), $a^{-a} b^{-b} c^{-c} q_1^a q_2^b q_3^c$ converts $\partial U_N / \partial N$ to dollars. Substituting in $\partial U_N / \partial N$ and $\partial p_K / \partial N$ from equations (10) and (12) for Assumption B, (16) becomes equation (17). Solving (17) for N would yield $N = N(J)$, the optimum city size. A change in the K/N ratio would not affect optimum city size or equation (17), just as it would not affect $N(U_N^*)$ and $N(p_K^*)$.

Our compensation mechanism defines Pareto optimal solutions. Until $N(J)$ or optimal city size is reached, we can move city sizes towards $N(J)$ and take from the income of the gaining group of factors and compensate the losing group, such that both parties benefit by moving closer to optimum city size $N(J)$. Alternatively one can view the process as maximizing a hypothetical total benefit curve (or "the size of the pie" given that factor incomes are determined by marginal productivity con-

$$(17) \quad \left| \left[C'_N \frac{m(\alpha_1 - \alpha_1 c - c\alpha_3(\rho_1 - 1))}{\rho_1 - 1} \right] \right.$$

$$\left. \cdot \left[-\log s - \frac{1}{m} + \frac{(1 - c)(\alpha_1 - \rho_1) + c\alpha_3(\rho_1 - 1)}{m(\alpha_1 - c\alpha_1 - c\alpha_3(\rho_1 - 1))} N^{-m} - \log N \right] \right|$$

$$\gtreqless \left| C'_K \frac{\alpha_1 m}{\rho_1 - 1} \left[-\log s - \frac{1}{m} + \frac{\alpha_1 - \rho_1}{\alpha_1 m} N^{-m} - \log N \right] \right|$$

ditions). Total benefits are the "sum" (converted into common units) of utility levels plus capital rentals each weighted by factor endowments. Benefits are maximized where $N(a^{-a}b^{-b}c^{-c}q_1^a q_2^b q_3^c) \; \partial U_N / \partial N + K \; \partial p_K / \partial N = 0$ as in (16) at $N(J)$. Before $N(J)$, gains in capital income contributing to total benefits exceed losses in utility levels by moving towards $N(J)$. After $N(J)$, losses in utility levels contributing to total benefits exceed gains in capital rentals. We draw in the hypothetical total benefit curve in Figure 3. Optimal city size and maximum benefits of agglomeration occur at $N(J)$.

Three other points are of interest before we turn to market equilibrium solutions. First, since $N(U_N^*)$ under Assumption B lies beyond $N(U_N^*)$ for Assumption A as discussed in Section II, and since optimal city size lies beyond (rather than at) $N(U_N^*)$ under Assumption B, city sizes under Assumption B are larger than under Assumption A. Secondly, as should be obvious, under both Assumptions A and B if capital rental and utility paths rise indefinitely, a possibility mentioned in Section II, the optimum number of cities will be one.

Finally, before leaving the discussion on optimality, we mention a problem raised when we specified the production side of the model. The production of X_1 and L involved economies of scale external to the firm and hence, following Chipman, these industries should be respectively subsidized and taxed to correct for divergences of private from social costs. This problem is independent of our analysis, and from my thesis, it appears the effect of these taxes on city size is not large and one can assume in our discussion that they have been accounted for.

C. City Formation and Size: A Market Economy

We now solve for equilibrium city size in the economy. In our *initial naive* solution, the market economy is characterized by atomistic behavior of capital owners, firms, and laborers. For the initial discussion we deal only with Assumption B. The market behavior of factor owners is depicted by laborers moving between cities to maximize utility levels and capital owners investing to maximize capital rentals. Therefore we use the U_N and p_K paths to solve for equilibrium city size.

Initially it is the behavior of firms that determines city size and formation in our economy. Starting with a single city in the economy, a second city forms when a firm sees it is profitable to leave the first city, hire factors competitively, and set up a second city. However, because scale economies are external to the firm, individual firms act unaware of the scale economies that could accrue to their own size of operation. When they move to form a second city they hire an arbitrarily small amount of factors and initially set up an arbitrarily small firm size and city. (With external economies of scale and firms having linear homogeneous subjective production functions, firm size is indeterminate.)

In Figure 4, a firm can hire small

amounts of capital and labor away from the initial city when it reaches size $N(E)$. In the new arbitrarily small city of size N(small), the entrepreneur will *initially* operate with a lower K/N ratio, explaining the shifts in the U_N and p_K paths. (The firm could operate (inefficiently) without the paths shifting at N(small) when the initial city size is slightly larger than $N(E)$ at existing K/N ratios in production by paying laborers less than their marginal product and capital more than its marginal product.) Given the shifts in the paths and the difference in scale of operations, the entrepreneur can pay capital rentals and utility levels equal to or greater than competitive ones. The greater than specification allows him to earn profits for setting up the new city. These profits will encourage other firms to come to the new city.[5] Factors will flow from the old to the new city until the two cities are of equal size, $1/2\ N(E)$ with the same K/N ratio.[6] (Note these factor flows are ensured because, in general, the rising parts of the factor reward paths are steeper than the declining parts, so that factor rewards in the initial city rise more slowly than those in the new city as factors flow from the initial to the second city.) Finally we note capital rentals and utility levels are both higher at $1/2\ N(E)$ than at $N(E)$.

[5] In a more sophisticated model there would be a speed of adjustment problem here. Suppose a firm does not instantaneously go out and form a second miniature city at point E in Figure 4. If our initial city size proceeds slightly beyond E, then two or more separately located small firm/cities become profitable at a point beyond E. This raises the possibility of three cities forming from the initial one. To avoid this problem, we assume that a firm acts as soon as the initial city reaches size E.

[6] Under Assumptions A and B, it is sometimes possible for factor rewards to be equalized with different K/N ratios in cities of the same type. For example, in Figure 3, our two cities could be of different sizes such that, with different K/N ratios and corresponding relative shifts in p_N and U_N, the curves p_K and U_N could be equalized between cities. In general, such solutions are ruled out as being unstable with respect to random factor movements.
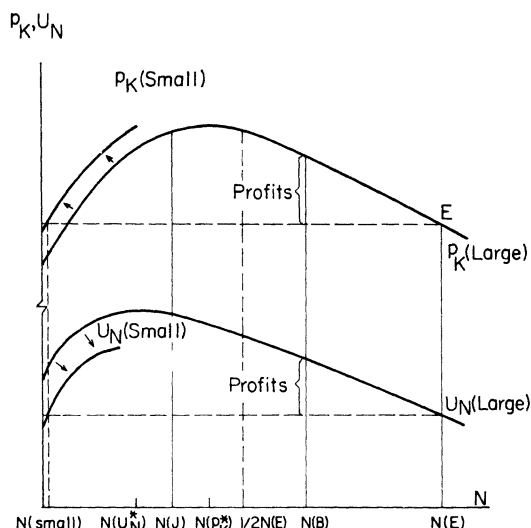

FIGURE 4. EQUILIBRIUM CITY SIZE

At $1/2\ N(E)$ the two cities continue to grow until they both reach size $N(E)$. At $N(E)$, by the above process, a third city forms. The resulting equilibrium has three cities of size $2/3\ N(E)$. As the economy grows new cities continue to form and the lower bound on *equilibrium city size approaches* $N(E)$, the point of city formation. Then for example, in the large sample case where the number of cities formally approaches infinity, *equilibrium* city size is at $N(E)$ in Figure 4. In contrast, under Assumption B, *optimum* city size lies between $N(U_N^*)$ and $N(p_K^*)$ at $N(J)$.[7]

Does divergence between equilibrium and optimum city sizes persist in a more sophisticated model?[8] The depiction of

[7] It has been pointed out several times to me that if the falling parts of the paths were very steep, market conditions would dictate cities splitting at a smaller size than pictured in Figure 4. This does not help the generalized rule for city formation since utility and capital rentals at $N(E)$ regardless of where $N(E)$ occurs are always the same as these factor records in an arbitrarily small city. Although the divergence from optimum city size might be smaller, the factor reward loss is just as bad.

[8] Note that our formation mechanism in terms of dynamics is naive, although it serves to reveal some of

firms or entrepreneurs acting myopically seems naive. Although scale economies are external to the firm, certainly there are entrepreneurs who will grasp the concepts of agglomeration benefits and disbenefits and be willing to initiate cities by moving industries, not just an arbitrarily small firm to form a new city. To facilitate our discussion, we introduce the "city corporation." The purpose of this exercise is to show there may be market forces ensuring that an equilibrium such as $N(E)$ would not persist. It does not pretend to deal with the dynamics of city formation!

D. The City Corporation

Suppose we are at $N(E)$ in Figure 4 in the large sample case. If a city corporation were to hire factors into a city restricted to size less than $N(E)$, factor rewards that could be paid in that city would be higher than competitive factor rewards. For example, at size $N(B)$, the corporation could hire factors competitively and have left over as profit the amounts indicated in Figure 4. Other entrepreneurs would follow the initial one, hiring factors and setting up new cities. Competition between entrepreneurs for factors to set up new cities should drive up factor prices and eliminate profits in the city corporation industry. Until cities are of size $N(J)$ where total benefits are maximized in Figure 3, by definition of $N(J)$, profits can be made by restricting city size. In other words, *the city corporation industry works "as if" the compensation mechanism used in the discussion of optimum city size is in effect*. If our city corporation industry is competitive and has adequate information, we will approach optimum city size $N(J)$.

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the forces at work. For example, technological change in industrial plants and transportation systems may be continually increasing optimum city sizes. Even if market forces leave us city sizes at $N(E)$, dynamic elements keep shifting optimum city sizes $N(J)$ out towards $N(E)$.

Note, however, to achieve this solution, the city corporation must be able to restrict city size. In the real world, either the corporation must own all land in the city or it must control land development and usage.

It seems likely that land developers play a crucial role in the real world and in a more sophisticated model than ours would play a more intricate role. For example, if our model allowed for suburbs, land developers would form suburbs as our core type or Mills-type cities grew in size. Suburbs would allow for (a) the release of pressure to form a completely new city due to rising commuting costs and (b) a mechanism for a completely new (economic) city to form where the "suburb" or our new city would be economically independent of the old city of the type in our model, without suburbs. By economically independent, we mean there would be little cross-commuting between the core city and suburb and interdependence in input and output markets would be weakened.

While there may be market forces ensuring attainment of optimal city sizes under Assumption B, this is *not* true under Assumption A. Equilibrium city size under Assumption A is determined in much the same way as for B. Laborers invest their capital throughout the cities of the economy so as to maximize capital rentals. On the other hand, they locate so to maximize the sum of U_N and U_K where p_K is exogenous to the labor location decision. To solve for equilibrium city size, we therefore use the p_K path to depict investment forces at work and the vertical sum of the U_N and U_K paths to depict the location or labor migration forces at work. We assume the existence of a city corporation mechanism and confine our discussion to the large sample case.

Parallel to the situation under Assumption B, with a city corporation mechanism, equilibrium city size will lie beyond

$N(U_N^*, U_K^*)$ at point $N(J)$ in Figure 5 where the decline in total utility or the sum of the decline in U_N plus U_K converted to dollar units becomes greater than the rise in $p_K \overline{K/N}$. The rise in p_K is given by equation (12). The decline in total utility equals $\partial U_N/\partial N$ plus $\partial U_K/\partial_N$ where p_K is fixed in the latter derivative, since laborers view $\overline{p_K}$ as exogenous to their location decision. The expression for $\partial U_N/\partial N$ is given by equation (14), while

$$\partial U_K/\partial N = a^a b^b c^c q_1^{-a} q_2^{-b} \overline{p_K} \overline{K/N} \partial(q_3^{-c})/\partial N$$

where $\overline{p_K}$ is given in (8); $\partial q_3/\partial N$ may be obtained from footnote 3, and $\overline{K/N}$ is the fixed ownership ratio of capital to labor. In making location decisions, laborers seek to maximize deflated real income or U_N plus U_K. In making investment decisions, laborers seek to maximize p_K. In trading off these two decisions or forces, city corporations maximize profits when the losses in location income of increasing city size are no longer exceeded by the gains in investment income.[9] Therefore equilibrium city size occurs at $N(J)$ where parallel to equation (16), we have

$$(18) \quad a^{-a} b^{-b} c^{-c} q_1^a q_2^b q_3^c \left| (\partial U_N/\partial N + \partial U_K/\partial K) \right|$$
$$> \left| \partial p_K/\partial N \cdot \overline{K/N} \right|$$

The city corporation mechanism fails to solve the problem of laborers in their role as investors investing to increase city size to maximize capital rentals. This investment behavior inadvertently prohibits the attainment of optimum city size at $N(U_N^*, U_K^*)$. It is worth examining this result from another angle.

[9] Note that in the interests of profit maximization, city corporations could internalize the "externality" that occurs when individuals make location decisions with p_K viewed as being exogenous. In that case, $\partial U_K/\partial N$ in (18) would simply be the expression in equation (13). This of course does not help solve the problem that by investing to maximize capital rentals, laborers inadvertently create a nonoptimal city size. It just alters the quantitative solution.
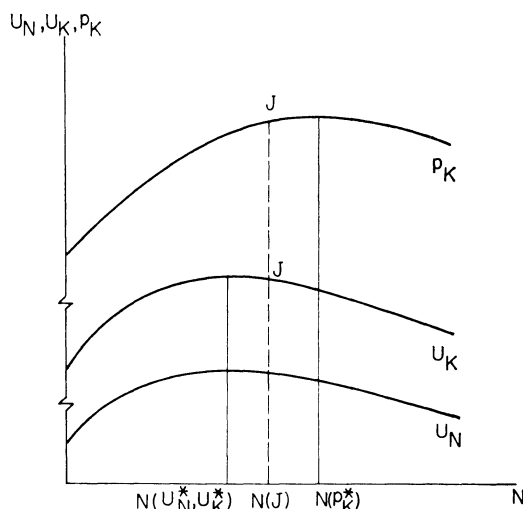

FIGURE 5. EQUILIBRIUM CITY SIZE: ASSUMPTION A

Point $N(J)$ is the equilibrium city size because no new profits can be made by entrepreneurs forming cities of a size different than $N(J)$ such as $N(U_N^*, U_K^*)$. For example, if a city corporation formed a city of size $N(U_N^*, U_K^*)$, it would raise the utility level of laborers *living* in the city. However, the capital rentals the city corporation could pay out would simultaneously fall. All investors could earn higher capital rentals which they are seeking to maximize in cities of size $N(J)$ and hence would not invest in a city of size $N(U_N^*, U_K^*)$. Similarly city size would not be bigger than $N(J)$ since the utility levels the city corporation could pay out would fall. As under Assumption B, by definition of $N(J)$ or the total benefit curve in Figure 3, the "sum" of utility levels from location decisions and capital rentals from investment decisions are maximized at $N(J)$ and no further profits can be made by changing city size.

IV. Extensions

In this section, we briefly outline complications that arise when we introduce a second (or more) type of city into the

model. Other extensions, not discussed here, include introduction of natural resources and transport costs of intercity trade.

Our second type of city specializes in the production of another type of traded good, say, X_2. The development of the utility and capital rental paths is the same as before. Equilibrium in the economy is depicted by both equilibrium in factor markets with equalized capital rentals and utility levels and equilibrium in output markets where markets clear and trade is balanced between the two types of cities. Different types of cities differ in size because production parameters, in particular α_i and ρ_i, differ between the traded goods of each type of city. Therefore, the shapes of factor reward paths determining city size will be different. Although utility levels will be equalized between cities, wage rates and housing prices will vary with city type and size.

Minor complications arise in the discussion of city formation.[10] When a new city of a particular type forms, factors from both types of cities will flow to it, since factor rewards will be affected throughout the economy. Equations (14)–(16) in this paper would have to be appropriately adjusted.

Other complications arise under Assumption A because the cost of living varies between types of cities. In equilibrium, capital rentals are equalized between all cities by investment behavior. Given these two facts, people living in larger cities will demand higher wages, not only because wage income is deflated by higher costs of living relative to smaller cities, but capital rentals are also deflated

[10] Another complication arises when the economy is very small. The thrust of the argument is contained in our discussion of stability. When the economy is very small, it *may* be unstable to have in the economy two different types of cities both on the rising part of their factor reward paths.

by higher costs of living. If we then allow capital owners still working as laborers to own different amounts of capital, there arises an incentive for laborers with relatively large dividend income to live in smaller cities or towns to enjoy the lower cost of living. In some sense, our distinction between Assumption B where capital owners live in the countryside and Assumption A may become rather fuzzy.

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