ECON 36740 / PPHA 44330 - Lectures 1-2
Valuing Non-Market Goods: Hedonics

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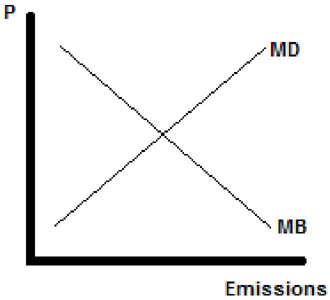
Roadmap

- I. Background
- II. Willingness-to-Pay
- III. Hedonics
  - A. Setup
  - B. Consumers
  - C. Producers
  - D. Equilibrium
  - E. Empirics
  - F. Welfare

Motivation

- Why do we need values of non-market goods?
  - Many environmental issues can be conceived of as externalities.
  - If the social planner corrects externalities with Pigovian taxes or optimal emissions limits then she needs estimates of individual's valuations.
  - Tremendous practical importance–without numbers we have to guess about policy.
- Consider ozone:



Basic Problem

- How do we estimate demand for goods with no explicit market?
- Never mind Hicksian vs. Marshallian demand – we do not observe price!

Basic Problem

Economists have developed a number of techniques to deal with this problem.

1. Household Production Function Methods.
    - Infer valuations from consumption of substitutes/complements to environmental goods.
    - Travel costs (e.g. expenses to get to natural park).
    - Averting behavior (e.g. air filters).
2. Health Effects Approach.
    - Builds on Becker's human capital model.
    - Views health as an argument in utility function degraded by exposure to environmental hazards.
3. Locational Equilibrium Approach.
    - Structural approach, very assumption-driven.
4. Direct Elicitation of Preferences/Contingent Evaluation.
5. Hedonics.
    - Use existing markets to infer valuations for differentiated goods

Basic Insight of Hedonics

- Price of land $P = PDV$ of stream of rents.
  - If $P < PDV$, someone will bid up the price until $P = PDV$ and vice-versa.
- Environmental goods may affect productivity of land, producer's value, or consumer's value.
  - (e.g. views, water and air pollution).
- Can land price differentials be used to measure the benefits to the household of improvements in air or water quality?

Basic Insight of Hedonics

- Ridker (1967) was first to try this:
  - "If the land market were to work perfectly, the price of a plot of land would be the sum of the present discounted streams of benefits and costs derivable from it. If some of its costs rise (e.g., if additional maintenance and cleaning costs are required) or some of its benefits fall (e.g., if one cannot see the mountains from the terrace) the property will be discounted in the market to reflect people's evaluation of these changes. Since air pollution is specific to locations and the supply of location is fixed, there is less likelihood that the negative effects of pollution can be significantly shifted onto other markets. We should therefore expect to find that the majority of effects reflected in this market, and we can measure them by observing associated changes in property values."

Basic Insight of Hedonics

- This raised three questions:
  1. Do environmental variables such as air pollution systematically affect land prices?
  2. If yes, is knowledge of this relationship sufficient to predict changes in land prices when, say, air pollution levels change?
  3. Do changes in land prices accurately measure the underlying welfare changes?
- Ridker (1967) stimulated large literature on the proper theoretical interpretation of observed relationship between air pollution and property value.
- Hedonic price theory provides a powerful connection between basic consumer theory and the observed relationship.

Willingness-to-Pay

- Before diving into the theory of hedonics, it is useful to derive a clear interpretation of willingness-to-pay (WTP) for an amenity where an explicit market does not exist.
- Define WTP as the maximum amount that a consumer is willing to pay for a change Δ in an amenity.

Willingness-to-Pay

- Assume some level of amenity attached to consumption of market good (e.g. air quality or quality of public schools attached to houses).
  - q quantity of market good, which has amenity attached
  - r_e amount of amenity (e.g. environmental quality)
  - y income
  - x quantity of composite good consumed; price normalized to 1
  - p price of market good
- Consumer exercises choice over q and x, but cannot directly choose r_e:

$$\max_{q,x} u(q, x, r_e)$$

$$\text{s.t. } pq + x = y.$$

- Optimal q^*, x^* are functions of p, y, r_e.

Willingness-to-Pay

- We want to derive the payment that makes consumer indifferent between having and not having a particular change Δ in amenity.
- Consider a small change in r_e holding u constant.
- Take total derivatives of utility and budget constraint:

$$du = \frac{\partial u}{\partial q} dq + \frac{\partial u}{\partial r_e} dr_e + \frac{\partial u}{\partial x} dx$$

$$dy = qdp + pdq + dx.$$

Willingness-to-Pay

- We want to determine how changes in q and r_e can be compensated with changes in y.
- Set $du = 0$, assume $dp = 0$ (overall price level unchanged).
- Rearrange to get:

$$-dx = \frac{\partial u/\partial q}{\partial u/\partial x}dq + \frac{\partial u/\partial r_e}{\partial u/\partial x}dr_e$$

$$-dx = pdq - dy.$$

- Combining,

$$\underbrace{-dy}_{\text{payment}} = \underbrace{\frac{\partial u/\partial q}{\partial u/\partial x}dq}_{\text{value of }\Delta\text{in quantity}} + \underbrace{\frac{\partial u/\partial r_e}{\partial u/\partial x}dr_e}_{\text{value of }\Delta\text{in amenity/quality}} - \underbrace{pdq}_{\Delta\text{in expenditure on good}}.$$

Willingness-to-Pay

- Recall from basic consumer theory that at the optimum q, x,

$$\frac{\partial u/\partial q}{\partial u/\partial x} = p.$$

- Substitute this expression into the equation for the payment on the previous slide, :

$$\underbrace{-\frac{dy}{dr_e}}_{\Delta \text{in income that holds } u \text{constant}} = \underbrace{\frac{\partial u/\partial r_e}{\partial u/\partial x}}_{\text{MRS between } r_e \text{and } x\text{, slope along an indifference curve}}$$

- This is a theoretically correct welfare measure of the “price” or maximum that consumers would be willing to pay for a desirable change in r_e.
- **Potential for Great Practical Value**: If we can measure how people trade off r_e and x, holding utility constant (i.e,. the *MRS*), then we have WTP.
- But, where are we going to get this slope? It can be difficult to observe indifference curves. We will show that hedonics offers the promise of finding the MRS.

Hedonics Notation

- Consider a differentiated good with many different characteristics, considered by consumers to be a single commodity.
- Think of property market, but there are many other examples (e.g. cars).
- Describe a property by its characteristics $z = (z_1, \ldots, z_k)$.
- Assume z completely describes the services provided by the property and define z so all z_i are goods (not bads).
- The choice of a property is equivalent to the choice of z. (Like choosing between shopping carts filled with goods.)

Hedonic Price Schedule

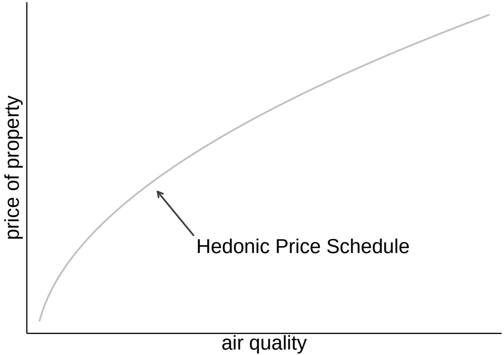
- Define the price of a house as

$$p = p(z)$$

  where $p(z)$ is the **hedonic price schedule**.
- $p = p_t$ is the rental rate of a house in a period t and the purchase price is given by $\sum_{t=1}^{\infty} p_t/(1+r)^t$.
- Marginal prices may not be constant:
  - Two loaves of bread costs twice as much as one loaf of bread due to arbitrage. Not true here; one house with two bedrooms may not cost the same as two houses with one bedroom each.
  - Typically, marginal price declines as quantities increase.

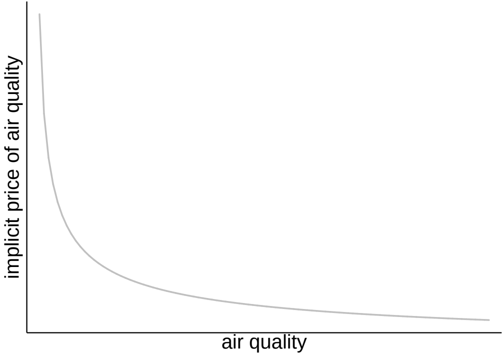
Hedonic Price Schedule

- Write price schedule for characteristic i as $p(z_i; z_{-i})$ where z_{-i} is the vector of remaining characteristics held constant.
- Price rises with air quality.
- Marginal effect declines as air quality improves.



Hedonic Price Schedule

- Declining marginal cost can be seen by plotting the implicit price function, $p_{z_i}(z_i, z_{-i}) = \partial p(z)/\partial z_i$.
- Key point is that this does not have to be constant.
- This proves to be a painful issue for empirical estimation of welfare measures.



Hedonic Price Schedule

- What determines the HPS?
- Key insight of Rosen (1974): HPS results from interactions of suppliers and demanders and represents a market clearing equilibrium.
- We return to how equilibrium is achieved, but for now begin with how households facing such an HPS make residential location decisions.

Consumer's Problem

- Rosen makes the following assumptions:
  - Consumers are price-takers.
  - Each household rents one property.
  - Well-defined utility function.
- Consumer's problem is:

$$\max_{x,z} u(z, x; s)$$

$$\text{s.t.} y = x + p(z)$$

where
  - z is vector of house characteristics.
  - x is amount of a composite good consumed.
  - s is vector of demand shifters.
  - y is household income.

Consumer's Problem

- Form the Lagrangian

$$\mathcal{L} = u(z, x; s) + \lambda(y - x - p(z)).$$

- Consumer has first-order conditions

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial z_i} &: \quad \frac{\partial u}{\partial z_i} = \lambda \frac{\partial p}{\partial z_i} \text{ for all } i \\ \frac{\partial \mathcal{L}}{\partial x} &: \quad \frac{\partial u}{\partial x} = \lambda \\ \frac{\partial \mathcal{L}}{\partial \lambda} &: \quad y = x + p(z). \end{aligned}$$

- These imply

$$\textbf{(*)} \qquad \underbrace{\frac{\partial u / \partial z_i}{\partial u / \partial x}}_{\text{MRS}} = \underbrace{\frac{\partial p}{\partial z_i}}_{\text{implicit price}} .$$

- This is a big achievement! We can interpret this as MWTP!

Bid Functions

- To illustrate this, Rosen defines the **bid function**, which corresponds to an indifference curve.
- Recall

$$u = u(z, x; s)$$

defines an indifference curve; bundles of (z, x) that yield u.

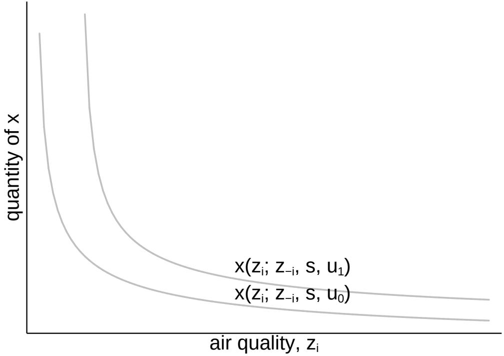
- We can solve for x:

$$\textbf{(**)} \quad x = x(z; s, u).$$

- This tells us how much x to purchase to get utility level u given some value of z.
- x is an indifference curve $\Rightarrow$ slope of x with respect to z_i holding z_{-i} *and* u constant is the MRS between x and z_i.

Bid Functions

Indifference curves between x and z_i:



- **Hint about what is coming next**: x has a price of \$1, so we can read off the WTP for Δ's in z in terms of money, instead of quantities of a good.

Bid Functions

- Rosen wanted to be able to show interactions of suppliers and consumers, and how they produce $P(z)$. This motivates the next step
- It can be difficult to wrap your mind around this, but the main thing is that Rosen has to get out of X, Z space and into $\$, Z$ space. So, Rosen invented the bid function.

Bid Functions

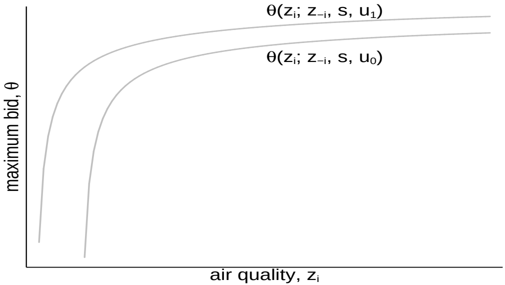
- Define the **bid function**:

$$\textbf{(***)} \quad \theta(z; s, u) = y - x(z; s, u).$$

- $\theta(z; s, u)$ is the maximum amount a household could pay for house with characteristics z given that utility is held constant at u and has demand shifters s. [NB: Also assume income is constant at y.] It is the WTP for z.
- There is a one-to-one relationship between the bid functions, $\theta(z; s, u)$, and the indifference curves, $x(z; s, u)$.

Bid Functions

Now let's look at bid functions between θ and z_i graphically:



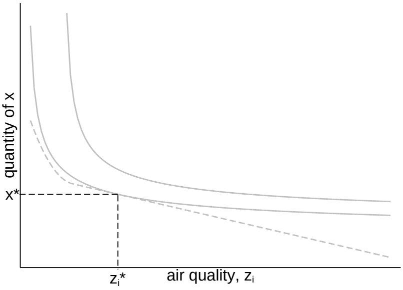
- Specifically θ is the maximum bid for a property with characteristic z_i that results in the level of utility u, given y, z_{-i} and s. Alternatively, it is the maximum amount a household would pay for a property with z_i, such that they can achieve u with y.
- Bid curves still define indifference relationships; they are combinations of z_i and payments θ for $z_i$that yield equal utility.
- NB: $u_0 > u_1$because if we fix θ, then $z_i$is higher with u_0

Bid Functions

- We can interpret the slope as MWTP for z_i since the price of x is 1. Movements along θ reveal the change in x since u is constant.
- Slope of the bid curve is the MRS (i.e. $= \frac{\partial u/\partial z_i}{\partial u/\partial x}$), except with the opposite sign. This is the LHS of **(*)** derived earlier. $\Rightarrow$ Slope is an economically interpretable parameter.
- However, this is not yet helpful empirically because we don't observe θ.

Bid Functions

- What is the relationship between the household's location decision (i.e., choice of air quality) and the bid function?
- Recall that the budget constraint is $x = y - p(z)$, generally non-linear since the hedonic price schedule is non-linear.
- Can find the consumer's optimal bundle (x^*, z_i^*) by tangency between highest indifference curve and budget constraint.



Bid Functions

- We can rewrite the budget constraint as

$$p(z) = y - x(z; s, u) = \theta(z; s, u)$$

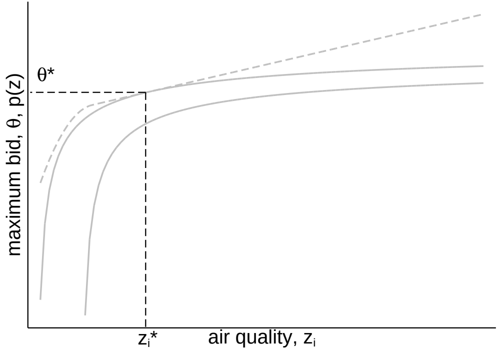
  and graph on the bid curve graph.
- Note that the budget constraint is just the hedonic price schedule and this is POTENTIALLY OBSERVABLE!.
- Also note that the tangency condition is

$$\underbrace{\frac{\partial u/\partial z_i}{\partial u/\partial x}}_{\text{MRS}} = \underbrace{\frac{\partial p}{\partial z_i}}_{\text{implicit price}},$$

  which we showed earlier determines the consumer's optimum.
- The household values the last unit of z at the implicit price it faced and the chosen bundle is purchased at market price.

Bid Functions

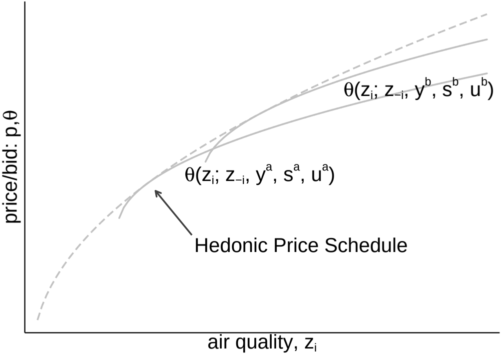
To see this graphically:



- At z^*, the household values the last unit of z, at the implicit price it faces (i.e. the market price). At levels of $z < z^*$, the consumer gains by increasing level of z.
- From a practical standpoint, it is easy to observe P(z) and it may be feasible to observe $P(z_i)$ through multivariate regression (with a credible research design).

Multiple Households

- Now consider multiple households with differing income y and tastes s.
- We can see that households locate at different places on the HPS.
- HPS forms an *upper envelope* to the bid curves for all consumers in the market.



Supply Side

- The supply side is very similar to the demand side.
  - Each landlord builds 1 house.
  - Rents it out at price $p(z)$.
  - Incurs per-period costs $c(z; \bar{z}, r)$ (land purchase, maintenance).
- $\bar{z}$ is exogenous characteristics of the house at point of purchase including
  - structural characteristics (e.g. protection against noise)
  - location/neighborhood/environment characteristics
- Can think of landlord purchasing house for $p(\bar{z})$ and improving it.
- $z \neq \bar{z}$ if the landlord makes investments or lets the house depreciate.
- r is a vector of cost shifters.

Offer Functions

- Landlord solves

$$\max_z p(z) - c(z; \bar{z}, r),$$

yielding FOC

$$\frac{\partial p}{\partial z_i} = \frac{\partial c}{\partial z_i} \text{ for all } i.$$

- Define profit function to be

$$\pi(z; \bar{z}, r) = p(z) - c(z; \bar{z}, r).$$

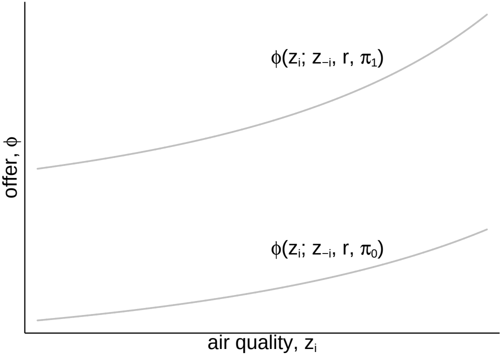
- Analogously to consumer case define **offer function** to be

$$\phi(z; \bar{z}, r, \pi) = \pi + c(z; \bar{z}, r).$$

- Offer is rent the landlord needs to get profit π, if he provides a house with characteristics z.
- Holding z constant, higher prices translates into higher profit.

Offer Functions

- Holding z_i constant, higher prices translates into higher π one-for-one because we have held everything else constant.
- Notice initial conditions $\bar{z}_i$ have effect. (NB: In the below graph $\bar{z}_i$ should come after the ";").
- Also, note that $\pi_1 > \pi_0$.

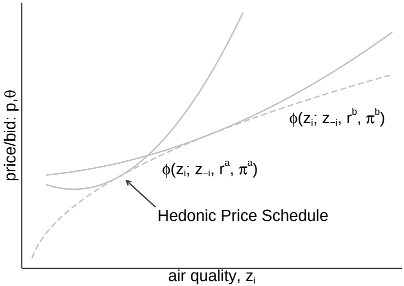


Offer Functions

- Firm chooses offer curve with the highest π such that they are on the HPS.
- Tangency condition is

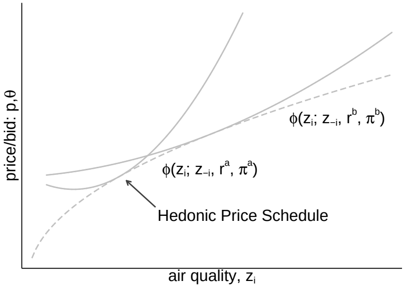
$$\frac{\partial p}{\partial z_i} = \frac{\partial c}{\partial z_i} \text{ for all } i,$$

which defines the firm's optimum.



Offer Functions

- Offer curves differ across landlords based on ability to produce z.
- Different landlords produce different houses. In particular, some are better able to produce certain characteristics.
- The HPS forms the lower envelope to the set of landlords' offer curves.

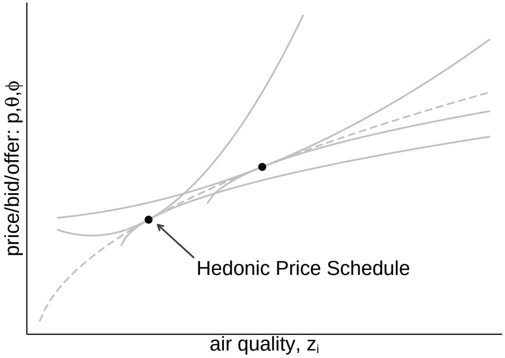


Notes on Producers

1. Landlords may not be able to change characteristics very much (e.g. air pollution, maybe can buy filters). Offer curve may cover a small region of z. As an extreme example, the distance to a park cannot be changed, so the offer curve would be a single point in this case.
2. Our focus is on the consumer side.

Market Equilibrium

- Now we have looked at consumer and producer sides separately, taking prices as given.
- We now combine them to form the equilibrium HPS.



Market Equilibrium

Key points:

1. Along HPS, households cannot increase utility and producers cannot increase profits.
2. Each coincidence of bid and offer curves pairs a landlord and a consumer. Since neither can do better, this is a market equilibrium.
3. $$\underbrace{\frac{\partial u/\partial z_i}{\partial u/\partial x}}_{\text{MRS}} = \underbrace{\frac{\partial p}{\partial z_i}}_{\text{price}} = \underbrace{\frac{\partial c}{\partial z_i}}_{MC}.$$

This is truly the power of economics! It takes a potentially observed relationship (the HPS), uses an economic theory to interpret it that will in principle allow for welfare analysis. (Hedonics provides a DGP.)

Hedonics

Further key points:

1. Assumes costless moving, everyone can optimize.
2. No gaps in range of available models.
3. If amenity cannot be influenced by landlords, then supply of properties with certain characteristics is fixed. Then aggregate supply is perfectly inelastic and equilibrium HPS will be determined solely by aggregate demand for properties. (Think of Leontief offer curves.)

Empirical Interlude

- Rosen proposed following two-step approach (ignoring supply):
  1. Estimate HPS using a non-linear function f of characteristics:
  $$p = \alpha + f(z) + \varepsilon.$$
  2. Take $\partial p(z)/\partial z$ and call it "price". With prices and quantities, can estimate bid function(uncompensated). How? Estimate
  $$\frac{\partial p(z)}{\partial z} = \alpha' + \delta z + \varepsilon'.$$
  Note ε is tastes and income, which we *hope* to observe and control for. Then get slopes at various values of z and with this, back out the bid functions.

Empirical Interlude

- One immediate problem!
- Suppose
$$p(z) = \beta_1 z + \beta_2 z^2.$$
- Then $\partial p(z)/\partial z = \beta_1 + 2\beta_2 z$ so Rosen's second step is a regression of $\beta_1 + 2\beta_2 z$ on z.
- This is just a regression of z on z! There is no new information gained.
- Unless you are fortunate enough to know functional forms, then the exercise is over.
- For example, if $p(z)$ is a cubic in z, then this is feasible, but the results involve a great deal of arbitrariness since we have to assume the correct functional form.

Welfare: Partial Equilibrium

How do we evaluate a change from z_i^1 to z_i^2?

1. If change is small, knowledge of HPS is sufficient since the slope is MWTP. Sum over all affected individuals:

$$\Delta W_{z_i^1 \text{to } z_i^2} = \sum_i \frac{\partial p(z_i)}{\partial z} \Delta z_i.$$

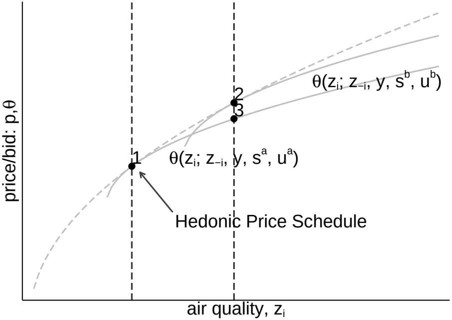
2. If change is large, need full WTP functions.
   - These can be determined with knowledge of θ, the bid function.
   - Why? θ gives combinations of z and x that leave u unchanged.
   - Then

$$\Delta W_{z_i^1 \text{to } z_i^2} = \sum_i \theta(z_i^1; z_{-i}, y_i, s_i, u_i) - \theta(z_i^2; z_{-i}, y_i, s_i, u_i).$$

   - Linchpin for this analysis is that we need θ!
   - But even setting aside the above issues, this is extremely difficult!

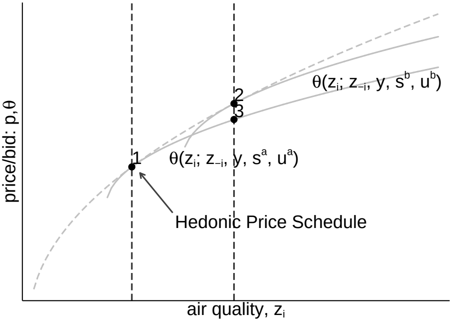
Welfare: Partial Equilibrium

- Why is it difficult? At it's heart, problem is we only see 1 point on each person's bid function.
- Consider two people with identical income, but different tastes.
- Only get to see the point where the person actually consumed. Cannot infer θ from these two points, this will overestimate the welfare change.



Welfare: Partial Equilibrium

- We can use points 1 and 2, but really we need 1 and 3.
- Other marginal prices only observed for individuals with other tastes and characteristics and provide no info on the original consumer's bid for different quantities of the characteristic.
- Cannot separate the HPS from the demand shifter!



Welfare: Partial Equilibrium

- Econometrically, Rosen's second step is

$$\frac{\partial p(z)}{\partial z} = \alpha' + \delta z + \varepsilon',$$

  where $\varepsilon' = v + s$, where s is the unobserved taste shifter.
- If $Cov(z, s) \neq 0$ then we will get biased results.
- But, people sort based on price.
- Epple (1987), Bartik (1987).

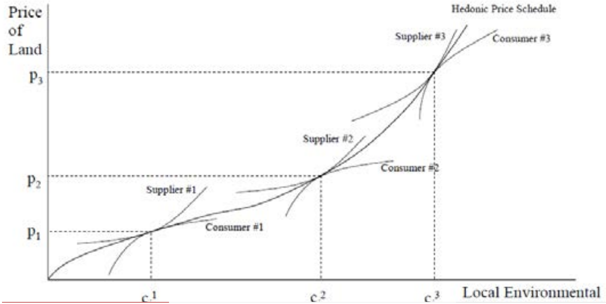
Welfare: Partial Equilibrium

Common solutions:

1. Single taste/income type. Then the bid curve is traced out by suppliers, so the HPS is θ.
2. Constant MWTP. Estimated effect surely biased upwards.
3. Multiple markets.
    - Idea is to find cases where individuals with identical preferences and incomes face different implicit prices (e.g. zoning restrictions).
    - Requires segmented markets within or across cities because there must be separate HPS's.
    - For exogenous reasons, people face different supplies.
    - Compute implicit prices in each market, regress on quantities as in Rosen.
4. Impose structure. Assume functional form on effect of income and that tastes can be measured and controlled for.
5. Each individual's MWTP decreases linearly from its observed point to highest attainable level of amenity where MTWP is zero.

Welfare: General Equilibrium

- Partial equilibrium approach understates gains of a large improvement because agents will make changes that make themselves even better off than the partial equilibrium approach suggests. (Otherwise would not have shifted.)
- Includes relocation (potentially a very important issue that has been ignored thus far) and shift in the HPS.



Welfare: General Equilibrium

Consider an increase in environmental quality from c_1 to c_3. What happens?

- Price rises from p_1 to p_3 for the improved tract of land.
- Consumer 1's valuation has increased but he values the improvement at much less than p_3. $p_3 > MWTP$, the neighborhood is too expensive given their tastes and income. 1 is worse off.
- 1 moves to a new neighborhood with originally chosen and optimal values of p and z.
- Some 3's will move to the newly improved tract.

Welfare: General Equilibrium

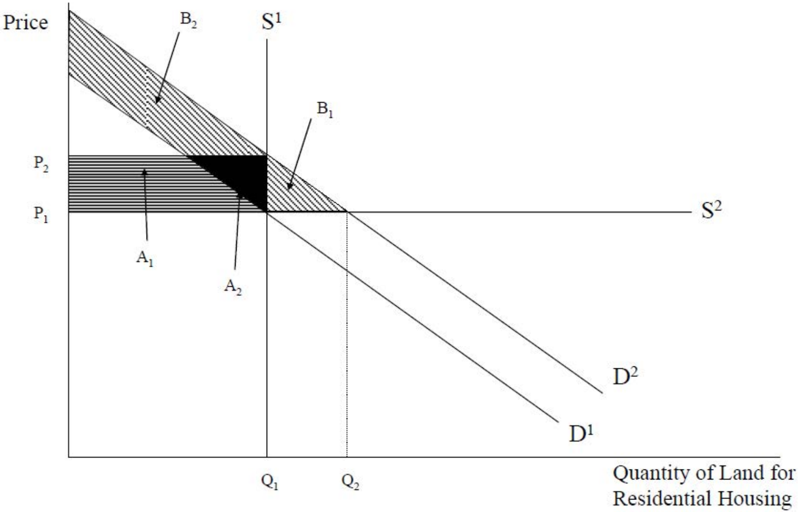
- Due to taste-based sorting, residents of improved area have higher incomes and greater taste for environmental quality. (Marginal resident less tolerant of exposure to pollution.)
- Land owners near the site are the only beneficiaries.
  - **Fixed supply of residential land**: higher rental rates are a pure benefit to existing landowners since the change in amenity is costless. All adjustments occur through price.
  - **Elastic supply of residential land**: non-residential land can be converted to residential land, then the increase in amenity is a pure benefit for all landowners, higher prices cause some conversion to residential land. Benefits to landowners are smaller than gain in price because of conversion costs. The size of the benefits determines the shape of the supply curve and those owners' welfare.

Welfare: General Equilibrium

Summarizing, there are four predictions:

1. p increases unless supply perfectly elastic.
2. Taste-based sorting.
3. Supply of land increases.
4. Entire welfare gain accrues to landowners.

Welfare: General Equilibrium



Welfare: General Equilibrium

Cases:

1. Perfectly inelastic supply.
    - $D^1 \rightarrow D^2$ (due to cleaner environment and migration).
    - Welfare gain is $A_1 + A_2 = \Delta P * Q_1$.
2. Perfectly elastic supply.
    - All adjustment through Q.
    - Welfare gain is $B_2 + A_2 + B_1$, depends on slope of D.
    - Previous hedonic work almost exclusively looks at prices!
3. Neither perfectly elastic or inelastic supply.
    - Adjustments through both Q and P.
    - Welfare gain depends on slopes of S and D.

Welfare: Empirics

Can we learn about the welfare effects of Superfund clean-ups from decennial Census data? Lots of empirical difficulties.

1. Consistent estimation of HPS proven to be extremely challenging.
2. Estimation of even a single individual's bid function is difficult because it is impossible to observe the same individual facing two sets of prices in a cross-section. Further complicated by sorting.
3. Welfare calculations requires estimates of bid functions for *all* consumers and cost functions for *all* suppliers.
4. Individual-level data frequently unavailable.

Welfare: Empirics

What can be done with Census data?

- Infeasible to see people over time. Census tracts are smallest unit.
- Assume tract-level demand and supply for residential land, which are determined by bid and offer functions of local consumers and supplies.
- Census data useful for testing sorting.
- Limitation of Census data is that it only occurs decenially.
- Many environmental clean-ups affect time path of environmental quality differently.
    - Superfund clean-up: announcement, clean-up, remediation completion.
    - Do you want impact of clean-up or whole process?

Bottom Line

- What do I do? One thing is accurate estimation of nonlinear surface is difficult in and of itself.
- Hedonics, especially for land, can be very useful for obtaining valuations of nonmarket goods; these valuations are essential for evaluating the optimal public policy.
- A nice feature of land markets is that land is immovable and in fixed supply, so it can be used to value amenities (e.g. parks, schools, commute times, crime, air quality, water quality, etc.)