BUSN 33946 & ECON 35101

International Macroeconomics and Trade

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CHICAGO BOOTH

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Today: Economic geography →trade costs for goods

Three models:

workers also customers

- Krugman (1991): The breakthrough paper :circularity →market access
- Helpman (1998): A re-formulation more popular in empirical and quantitative exercises
- Allen and Arkolakis (2014): A substantial generalization to almost all continuous geographies

Evidence:

- Davis & Weinstein: "Bones, Bombs, and Breakpoints"
- Redding & Sturm: "The Costs of Remoteness: Evidence from German Division and Reunification"

“Increasing Returns and Economic Geography”

↳ Krugman 1991

- The second of Krugman’s pair of Nobel-winning papers
- Apply tools from ’70s theoretical IO and ’80s trade models to
  Dixit Stiglitz
  look at the geographic concentration of industry
- General story about IRS manufacturing vs CRS agriculture
  industrializat.
  rather than industry localization
- Circular causation: “manufactures production will tend to concentrate where there is a large market, but the market will be large where manufactures production is concentrated”
- Formalize this unoriginal story
- Comparative statics: Agglomeration depends on $\overbrace{\text{transport costs}}^{\tau}$, $\underbrace{\text{economies of scale}}_{\sigma}$, and manufacturing share [these are all exogenous parameters in Krugman’s account]
- Key restrictions: Immobile peasants and only two locations

σ = CES pref ⇒ economies of scale bc CRS etc

Krugman (JPE 1991): "II. A Two Region Model"

μ = cobb Douglas manufacture share

$$U = C_M^{\mu} C_A^{1-\mu} \tag{1}$$

$$C_M = \left[\sum_{i=1}^{N} c_i^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{2}$$

$$L_1 + L_2 = \mu \tag{3}$$

Normalizat !

$$L_{Mi} = \alpha + \beta x_i \tag{4}$$

$$p_1 = \frac{\sigma}{\sigma - 1} \beta w_1 \tag{5}$$

= markup × MC

$$\frac{p_1}{p_2} = \frac{w_1}{w_2} \tag{6}$$

$$(p_1 - \beta w_1) x_1 = \alpha w_1 \tag{7}$$

$$x_1 = x_2 = \frac{\alpha}{\beta}(\sigma - 1) \tag{8}$$

$$\frac{n_1}{n_2} = \frac{L_1}{L_2} \tag{9}$$

peasants ≠ workers → no occupat choice

- Immobile peasants, mobile workers, and "clever" choice of units in (3)
- Usual Dixit-Stiglitz monopolistic competition setup, but L_i endogenous
- Freely traded CRS good but still wages w_1 and w_2
- As in Krugman (1980), all action on extensive margin
- Iceberg trade costs: Only $\tau < 1$ arrives , send $1/\tau$ [inverse of modern notation]

Wages of peasants = numeraire
= price agr
↳ in both regions

III. Short-Run Equilibrium

c_{ij} is consumption in i of a variety from j; z_{1i} is relative expenditure in i on varieties from 1; that's awkward notation; eqlbm $w_i, z_{1i}|L_i$ $-1/\sigma$

relative exp = relative prices

$$\frac{c_{11}}{c_{12}} = \left(\frac{p_1\tau}{p_2}\right)^{-\sigma} = \left(\frac{w_1\tau}{w_2}\right)^{-\sigma} \tag{10}$$

$$z_{11} = \left(\frac{n_1}{n_2}\right)\left(\frac{p_1\tau}{p_2}\right)\left(\frac{c_{11}}{c_{12}}\right) = \left(\frac{L_1}{L_2}\right)\left(\frac{w_1\tau}{w_2}\right)^{-(\sigma-1)} \tag{11}$$

$$z_{12} = \left(\frac{L_1}{L_2}\right)\left(\frac{w_1}{w_2\tau}\right)^{-(\sigma-1)} \tag{12}$$

$$w_1 L_1 = \mu\left[\left(\frac{z_{11}}{1+z_{11}}\right)Y_1 + \left(\frac{z_{12}}{1+z_{12}}\right)Y_2\right] \tag{13}$$

$$w_2 L_2 = \mu\left[\left(\frac{1}{1+z_{11}}\right)Y_1 + \left(\frac{1}{1+z_{12}}\right)Y_2\right] \tag{14}$$

$$Y_1 = \frac{1-\mu}{2} + w_1 L_1 \tag{15}$$

$$Y_2 = \frac{1-\mu}{2} + w_2 L_2 \tag{16}$$

III. Long-Run Equilibrium

⇒ endogenous labor free = what's different trade model

- L_i is endogenous, not fixed. Let $f \equiv L_1/\mu$.
- Workers care about real wages ω_i, not nominal wages w_i.

$$P_1 = \left[f w_1^{-(\sigma-1)} + (1-f)\left(\frac{w_2}{\tau}\right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \tag{17}$$

$$P_2 = \left[f \left(\frac{w_1}{\tau}\right)^{-(\sigma-1)} + (1-f)\left(w_2\right)^{-(\sigma-1)} \right]^{-1/(\sigma-1)} \tag{18}$$

$$\omega_1 = w_1 P_1^{-\mu} \cdot 1^{1-\mu} \tag{19}$$

numeraire

$$\omega_2 = w_2 P_2^{-\mu} \tag{20}$$

$\omega_1 = \omega_2$ → populat° > 0 in both regions

$\omega_1 \gtrless \omega_2$ → pop = 0 ✓ one region

→ savings on trade cost

⇒ agglomerat° force

- Given w_i, greater f (or) lowers P_1/P_2 and therefore raises ω_1/ω_2.
- A race between home market effect (firms want to be close to customers) and price index effect (consumers want to be close to firms) (convergence) and competition for sales to peasants (divergence)
- See Figure 1 for numerical example varying τ

saving ↓ & → higher demand → higher pay → wages to equilibrium + trade models

HME = trade cost will ↓ & SR ↳ see

Not sufficient!..

IV. Necessary Conditions for Mfg Concentration

Suppose there are n manufacturing firms and all are in region 1.

Their value of sales is V_1 and a potential defector's value of sales is V_2.

Defecting firm must pay workers wage premium to compensate for cost of living. (bc have to import all your goods)

$$\frac{Y_2}{Y_1} = \frac{1-\mu}{1+\mu} \tag{21}$$

$$V_1 = \left(\frac{\mu}{n}\right)(Y_1 + Y_2) \tag{22}$$

$$\frac{w_2}{w_1} = \left(\frac{1}{\tau}\right)^{\mu} \tag{23}$$

assume defector = infinitesimal

just peasants

$$V_2 = \left(\frac{\mu}{n}\right)\left[\left(\frac{w_2}{w_1\tau}\right)^{-(\sigma-1)} Y_1 + \left(\frac{w_2\tau}{w_1}\right)^{-(\sigma-1)} Y_2\right] \tag{24}$$

$$\frac{V_2}{V_1} = \frac{1}{2}\tau^{\mu(\sigma-1)}\left[(1+\mu)\,\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}\right] \tag{25}$$

Defection profitable if $V_2/V_1 > w_2/w_1 = \tau^{-\mu}$.

$$Y_2 = \frac{1-\mu}{2}$$

$$y_1 = \frac{1-\mu}{2} + w_1 L_1 = \overbrace{\frac{1-\mu}{2}}^{Y_2} + \mu(y_1 + Y_2)$$

$$\frac{Y_2}{y_1} = \frac{1-\mu}{1+\mu}$$

IV. Necessary Conditions for Mfg Concentration

Defection profitable if $V_2/V_1 > \tau^{-\mu} \iff \nu < 1$.

$$\nu = \frac{1}{2}\tau^{\mu\sigma}\left[(1+\mu)\,\tau^{\sigma-1} + (1-\mu)\tau^{-(\sigma-1)}\right] \tag{26}$$

$$\frac{\partial \nu}{\partial \mu} = \nu\sigma(\ln\tau) + \frac{1}{2}\tau^{\sigma\mu}\left[\tau^{\sigma-1} - \tau^{-(\sigma-1)}\right] < 0 \tag{27}$$

peasant populat° ↓ diverging less profitable

ambiguous

$$\frac{\partial \nu}{\partial \tau} = \frac{\mu\sigma\nu}{\tau} + \frac{\tau^{\sigma\mu}(\sigma-1)\left[(1+\mu)\,\tau^{\sigma-1} - (1-\mu)\tau^{-(\sigma-1)}\right]}{2\tau} \tag{28}$$

$$\frac{\partial \nu}{\partial \sigma} = \ln(\tau)\left\{\mu\sigma + \frac{1}{2}\tau^{\mu\sigma}\left[(1+\mu)\,\tau^{\sigma-1} - (1-\mu)\tau^{-(\sigma-1)}\right]\right\}$$

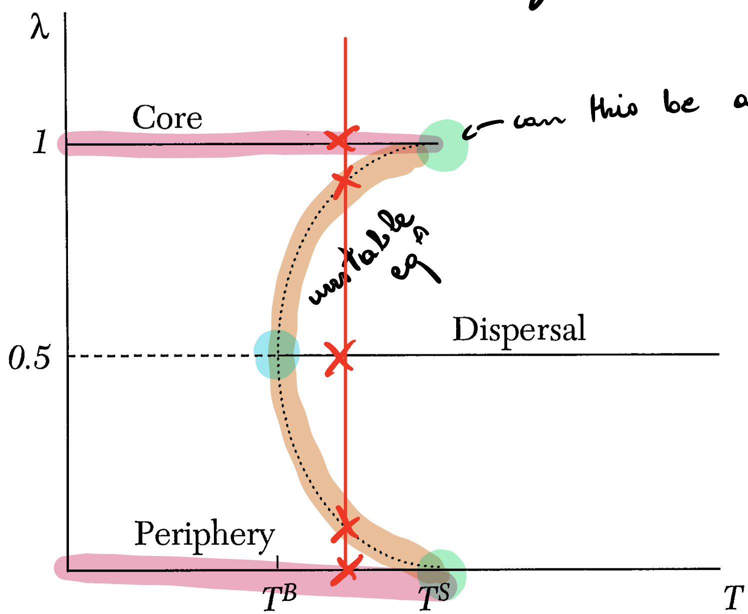
$$= \ln(\tau)\left(\frac{\tau}{\sigma}\right)\left(\frac{\partial \nu}{\partial \tau}\right) \tag{29}$$

Note $\tau = 1 \quad \Rightarrow \quad \nu = 1$

The bifurcated "tomahawk" diagram

× Five potential eq^n at ×

λ = share in region 1 = f



← can this be an equilibrium

Figure 2. Agglomerated and Dispersed Equilibria as a Function of Trade Costs

- Krugman (1991) established a "sustain point" ν necessary for concentration
- There is also a "break point" that is sufficient for concentration to be possible

sustain point

break point ≠

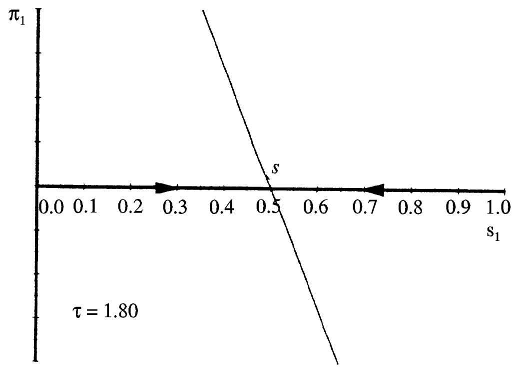
$T = 1/\tau$ trade costs

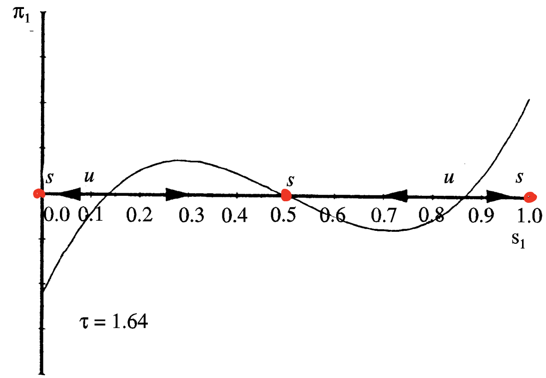
T high : peasants strong ancre bc high trade cost → incentives for defect → dispersion

T low :

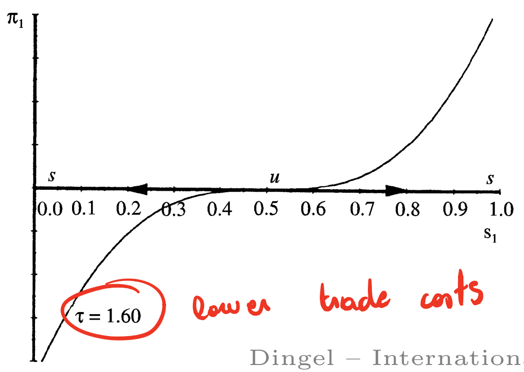
More on that diagram and stability

- If firms “move faster” than firms, same conclusions about break and sustain points (Puga 1999)
- There are at most two interior asymmetric steady states. If they exist, they’re unstable. (Robert-Nicoud 2005)





$\tau \geq 1$



lower trade costs

Beyond Krugman (1991)

- More than two regions: lines, circles (Fujita Krugman Venables *The Spatial Economy*)
- Intermediate inputs (V, KV)
- Alternative demand systems: Ottaviano Tabuchi Thisse

Table 1. Three classes and 13 years of economic geography models

Vertical product differentiation
Matsuyama

	Underlying model Krugman (1980)	Flam and Helpman (1987)
Factor migration	**CP** (core-periphery) Krugman (1991b)	**FE** (Footloose Entrepeneur) Forslid and Ottaviano (2003)
Input-output linkages (a.k.a. vertical linkages)	**CPVL** (core-periphery, vertical linkages) Krugman and Venables (1995)	**FCVL** (footloose capital, vertical linkages) Robert-Nicoud (2002) **FEVL** (Footloose entrepreneur, vertical linkages) Ottaviano (2002) and Ottaviano and Robert-Nicoud (2003)
Factor accumulation		**CC** (constructed capital) Baldwin (1999)

labor can't move
but input/output
linkages

Not necessarily suitable
to take to the data

Robert-Nicoud (*JEG*, 2005)

“Of Hype and Hyperbolas”

Neary (*JEL* 2001) on Fujita, Krugman, Venables monograph:

- “the model used throughout the book has a number of special features that make it less suitable for addressing some issues” → lot of special functional form
- “Though σ starts as a taste parameter, it ends up as an index of returns to scale
- “the Dixit-Stiglitz model has almost nothing to say about individual firms”
- “while costs may be fixed they are never sunk, so firms, industries, and even cities are always free to move”
- The “iceberg” assumption: “Of all industries, it seems to be characterized by very high ratios of fixed to variable costs”
- Davis (1998): This doesn’t work with comparable agricultural trade costs
- “The book deals in turn with regions, cities and countries, but there is nothing intrinsic to the models which conclusively identifies these units.”

Davis & Weinstein: “Bones, Bombs, and Breakpoints”

E.G competes with three theories

Assess three competing theories: locational fundamentals, increasing returns, and random growth

What can we empirically do about econ geography

- Bones: In data on Japanese regional population over 8,000 years, there has always been a great deal of variation in measures of regional population density. Zipf’s Law holds even in 6,000 BC.
- Bombs: Does a temporary shock have permanent effects? After the Allied bombing in WWII, most cities returned to their relative position in the distribution of city sizes within about 15 years

TABLE 4—MATCH BETWEEN THEORIES AND PREDICTIONS

Stylized fact	Increasing returns	Random growth	Locational fundamentals
Large variation in regional densities at all times	−	+	+
Zipf’s Law	−	+	+ ⇒ rev
Rise in variation with Industrial Revolution	+ ← change magnitudes around	−	−
Persistence in regional densities	?	?	+
Mean reversion after temporary shocks	?	−	+

Gabaix QJE 1999
↳ Gibrat’s law + reflecting barrier ⇒ Zipf’s law

Bombs vs canoes

Gibrat's law: growth rate independent from initial stock ⇒ growth is iid
↳ constant of literature on cities

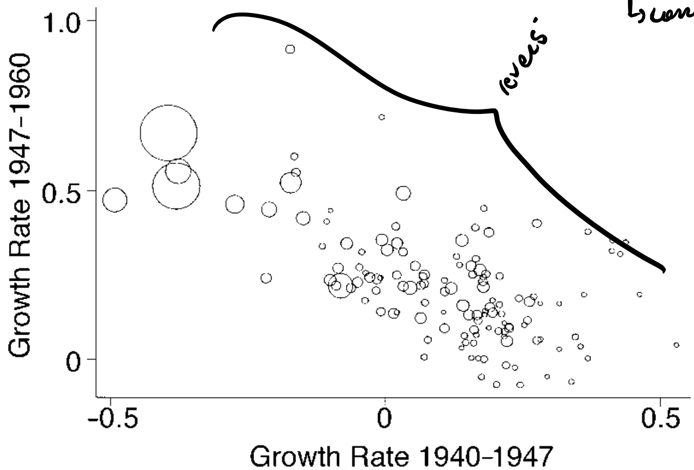


FIGURE 1. EFFECTS OF BOMBING ON CITIES WITH MORE THAN 30,000 INHABITANTS

Note: The figure presents data for cities with positive casualty rates only.

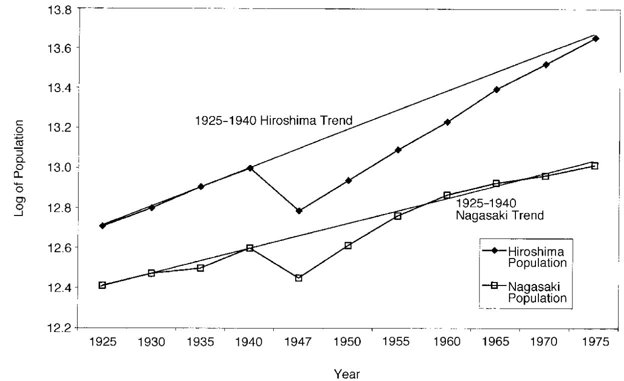


FIGURE 2. POPULATION GROWTH

- Also see Miguel and Roland (*JDE* 2011): "even the most intense bombing in human history did not generate local poverty traps in Vietnam"
- On the other hand, see Bleakley and Lin (*QJE* 2012) on "Portage and Path Dependence"

Helpman (1998): Overview

Couple small Deviations from Krugman 91
⇒ significant ≠ in results

- Vocab: centripetal = agglomeration, centrifugal = dispersion
- Replace freely traded agriculture with non-traded fixed factor (housing) ⇒ Cobb-Douglas preferences over housing and varieties
- Replace location-bound peasant income with assumption that housing is owned equally by all individuals (regardless of location) who spend income where they live
- These two deviations flip the comparative statics for trade costs in Krugman (1991)! "While in Krugman's model low transport costs lead to agglomeration and high transport costs lead to dispersion, in my model, low transport costs lead to dispersion and high transport costs lead to agglomeration."
- The first deviation (freely traded ag) is the key
- Note difference in equilibria when manufactures are freely traded

Helpman: Housing Fixed Factor (Free) ⇒ allocat proportionate to housing
K91: Free trade ⇒ locat doesn't matter

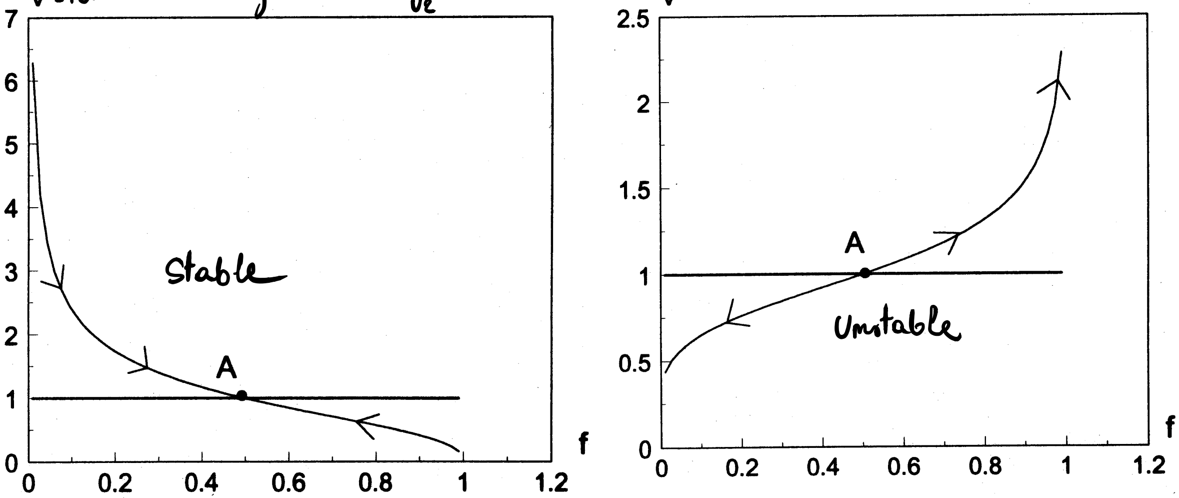
Helpman (1998): Setup

- β is Cobb-Douglas expenditure share on housing $1-\mu$
- ϵ is elasticity of substitution across differentiated varieties σ
- $t > 1$ is the iceberg trade cost $t = \frac{1}{\tau}$
- $f = N_1/N$ is the population share of region 1
- $v = u_1/u_2$ is the relative utility level of 1

Helpman (1998)

- When $t \to 1$, only housing prices matter: Go to less populous place
- When $\beta\epsilon > 1$, housing prices relatively more than traded prices
- When $\beta\epsilon < 1$, differentiated products are poor substitutes and demand for housing is low. In this case, trade costs matter. everybody wants to be in bigger region

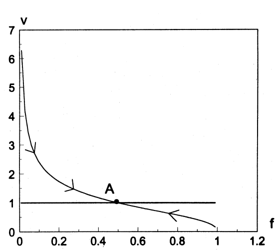
v = relative utility level $\frac{v_1}{v_2}$



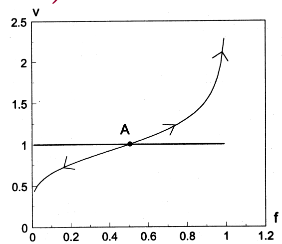
t close to 1 or $\beta\varepsilon > 1$

t unbounded and $\beta\varepsilon < 1$

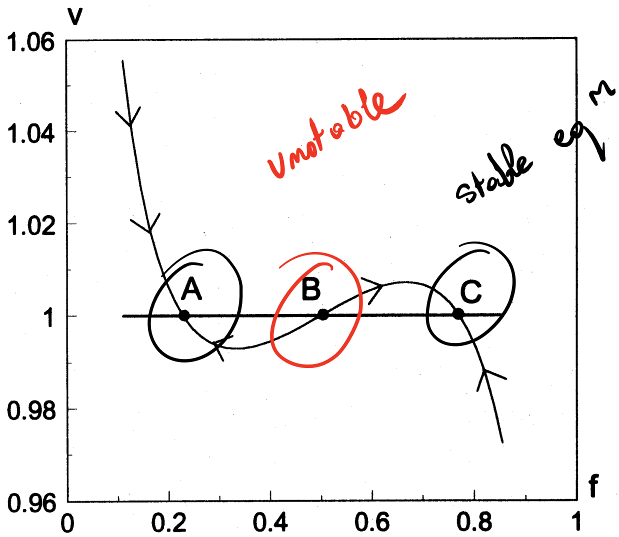
Helpman (1998): The “tomahawk” reverses



t close to 1 or $\beta\varepsilon > 1$

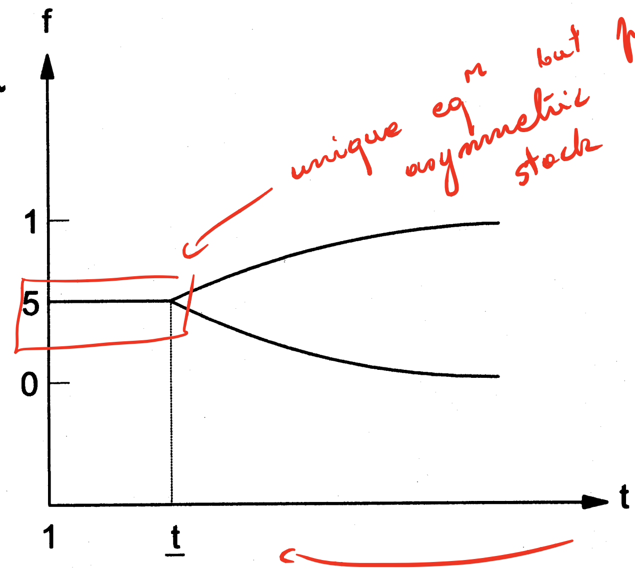


t unbounded and $\beta\varepsilon < 1$



ε=2 , β=0.4 , $\beta\varepsilon < 1$

t=6



unique eqm but potentially asymmetric housing stock

$\beta\varepsilon < 1$

lowering trade cost: fm uneven to even bc housing matters

Redding and Sturm (2008)

- "We exploit the division of Germany after the Second World War and the reunification of East and West Germany in 1990 as a source of exogenous variation to provide evidence for the causal importance of market access for economic development."
- "The key idea behind our empirical approach is that West German cities close to the new border experienced a disproportionate loss of market access relative to other West German cities."
- Extend Helpman (1998) to a multi-region version (assume that $\beta\epsilon > 1 \Rightarrow$ unique equilibrium)
- Distinctive prediction: Beyond market-access effet, "the greater dependence of small cities on markets in other cities implies that this effect will be particularly pronounced for small cities."
- Difference-in-differences is proximate vs distant West German cities before and after division

Redding and Sturm (2008): Difference-in-differences

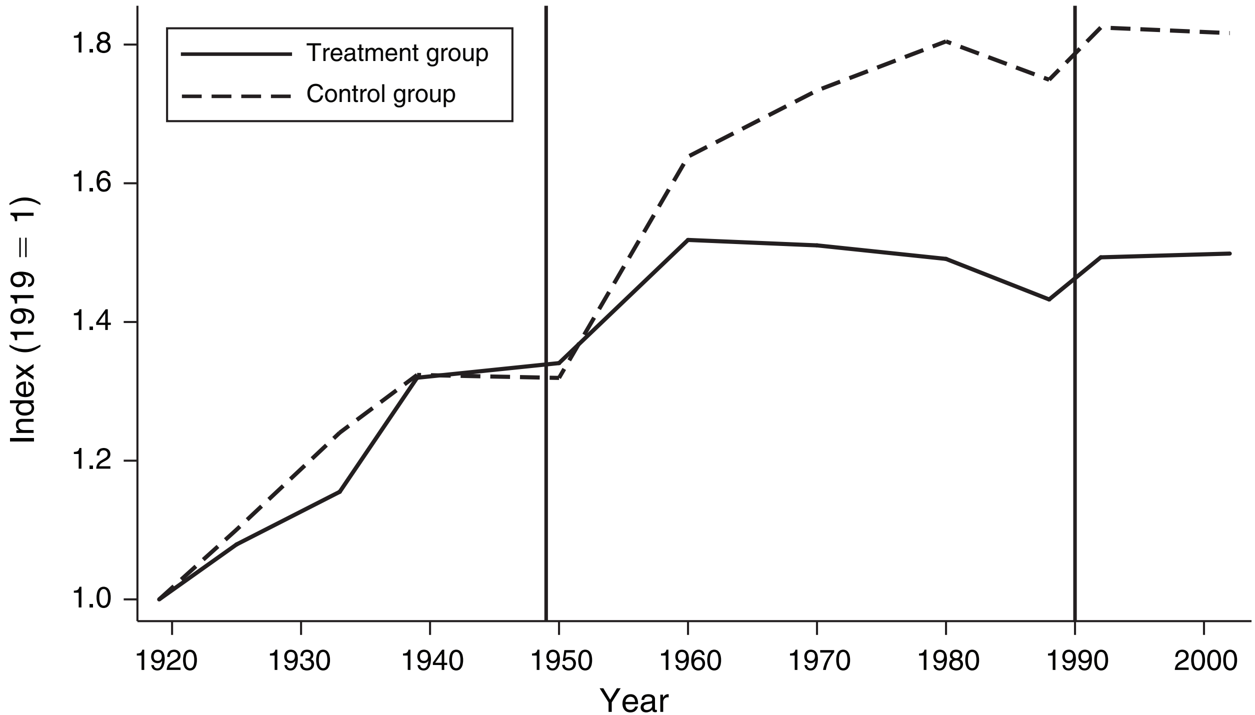


Figure 3. Indices of Treatment and Control City Population

Redding and Sturm (2008): Quantitative model fit

- Model can explain "the quantitative magnitude of the relative decline of small and large cities along the East-West German border relative to other West German cities"
- Choose parameter values to minimize the distance between moments in the simulation and data
- Finding "plausible parameter values" is "further evidence that their relative decline is indeed due to a loss of market access"

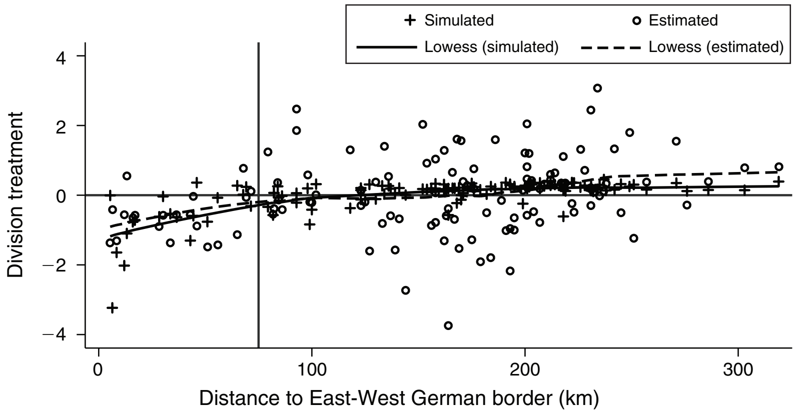


FIGURE 6. SIMULATED AND ESTIMATED DIVISION TREATMENTS

Notes: Simulated division treatments based on the parameter configuration that minimizes the sum of squared deviations between the simulated and estimated division treatments for small and large cities. Lowess is a locally weighted linear least squares regression of the division treatment against distance to the East-West German border (bandwidth 0.8).

Allen and Arkolakis (*QJE*, 2014)

- The stylized nature of geography (e.g., line or circle) in many economic-geography models (FKV and similar) makes them awkward to take to data
- Authors develop a quantitative framework (more general than Redding-Sturm) that extends new economic geography to much broader class
- Derive sufficient conditions for existence and uniqueness of equilibrium in spatial geography models (with a continuum of locations)
- Illustrative quantitative exercise: What was the welfare effect of the interstate highway system?

Allen and Arkolakis (2014): Model – Geography

- Continuum of locations, $i \in S$ (S is a closed and bounded set of a finite dimensional Euclidean space)
- Location $i \in S$:
  - Endowed with differentiated variety (Armington assumption)
  - Productivity: $A(i) = \bar{A}(i)L(i)^{\alpha}$ (exogenous part: $\bar{A}(i)$)
  - Amenity: $u(i) = \bar{u}\,(i)\,L(i)^{\beta}$ (exogenous part: $\bar{u}(i)$)
- $\alpha \geq 0$, $\beta \leq 0$. Spillovers are local (only affect i)
- For all $i, j \in S$, symmetric iceberg bilateral trade cost $T\,(i, j)$
- Together, $\bar{A}$, $\bar{u}$, and T comprise the *geography* of S
- A geography is regular if $\bar{A}$, $\bar{u}$, and T are continuous and bounded above and below by strictly positive numbers.

Can't have ∞ trade costs bc no place with 0 trade
a populat°

Allen and Arkolakis (2014): Model – Workers

- CES preferences over differentiated varieties with elasticity of substitution $\sigma > 1$
- Can choose to live/work in any location (free mobility)
- Receive wage $w(i)$ for their inelastically supplied unit of labor
- Welfare in location i is

$$W(i) = \left(\int_{s \in S} q(s,i)^{\frac{\sigma-1}{\sigma}} \, ds \right)^{\frac{\sigma}{\sigma-1}} u(i)$$

where $q(s,i)$ is the per capita quantity consumed in location i of the good produced in location s and $u(i)$ is the local amenity.

Allen and Arkolakis (2014): Model – Production

- Labor is the only factor of production, $L(i)$ is the density of workers.
- Productivity of worker in location i is $A(i)$
- Price of good from i is $\frac{w(i)}{A(i)} T(i,j)$ in location j
- Trade flows: $X(i,j) = \left(\frac{T(i,j)w(i)}{A(i)P(j)}\right)^{1-\sigma} w(j)L(j)$
- Price index: $P(j)^{1-\sigma} = \int_S T(s,j)^{1-\sigma} A(s)^{\sigma-1} w(s)^{1-\sigma} ds$

Allen and Arkolakis (2014): Model – Equilibrium

- A spatial equilibrium is a distribution of economic activity (functions w and L) such that:
  - Markets clear, i.e. $w(i)\,L(i) = \int_S X(i,s)\,ds$,
  - Welfare is equalized, i.e. $W \in \mathbb{R}_{++}$ such that for all $i \in S$, $W(i) \leq W$, with equality if $L(i) > 0$,
  - The aggregate labor market clears, i.e. $\int_S L(s)\,ds = \bar{L}$.
- A spatial equilibrium is regular if L and w are continuous and strictly positive (i.e. every location is inhabited).
- A spatial equilibrium is point-wise locally stable if $\frac{dW(i)}{dL(i)} < 0$ for all $i \in S$ (i.e. no small number of workers can increase welfare by moving to another location).

AA ‘14: Solving for equilibrium

Plugging the expression for trade flows and indirect utility function into the goods market clearing condition yields:

$$L(i)w(i)^{\sigma} = \int_S W(s)^{1-\sigma} T(i,s)^{1-\sigma} A(i)^{\sigma-1} u(s)^{\sigma-1} L(s) w(s)^{\sigma} ds$$

Combining the price index with the indirect utility function yields:

$$w(i)^{1-\sigma} = \int_S W(s)^{1-\sigma} T(s,i)^{1-\sigma} u\left(i\right)^{\sigma-1} A(s)^{\sigma-1} w(s)^{1-\sigma} ds$$

We are looking for functions $w(i)$ and $L(i)$ that solve these equations and will look for equilibria in which every location is inhabited (regular equilibrium); hence we consider $W(s) = W \ \forall s$

AA '14: Solving for equilibrium without spillovers

When there are no productivity nor amenity spillovers ($\alpha = \beta = 0$) and welfare is equalized across space, the previous equations can be written as:

$$x = \lambda A x$$
$$y = \lambda A' y$$

where $x(i) = L(i)w(i)^{\sigma}$, $y(i) = w(i)^{1-\sigma}$, $\lambda = W^{1-\sigma}$, and
$A(i,j) = T(i,j)^{1-\sigma} A(i)^{\sigma-1} u(j)^{\sigma-1}$.
We know that $A(i,j) > 0 \; \forall i,j$. This pair of equations "can be viewed as a linear system of equations for which extensions of standard results in linear algebra guarantee the existence and uniqueness of a positive solution."

AA '14: Solving for equilibrium with spillovers

When there are productivity or amenity spillovers and welfare is equalized across space, the previous equations yield:

$$L(i)^{1-\alpha(\sigma-1)}w(i)^{\sigma} = W^{1-\sigma}\int_S T(i,s)^{1-\sigma}\bar{A}(i)^{\sigma-1}\bar{u}(s)^{\sigma-1}L(s)^{1+\beta(\sigma-1)}w(s)^{\sigma}ds$$

$$w(i)^{1-\sigma}L(i)^{\beta(1-\sigma)} = W^{1-\sigma}\int_S T(s,i)^{1-\sigma}\bar{A}(s)^{\sigma-1}\bar{u}(i)^{\sigma-1}w(s)^{1-\sigma}L(s)^{\alpha(\sigma-1)}ds$$

If trade costs are symmetric, the system can be written as:

$$A(i)^{\sigma-1}w(i)^{1-\sigma} = \phi L(i)w(i)^{\sigma}u(i)^{\sigma-1} \quad (1)$$

$$L(i)^{\tilde{\sigma}\gamma_1} = K_1(i)W^{1-\sigma}\int_S T(s,i)^{1-\sigma}K_2(s)\left(L(s)^{\tilde{\sigma}\gamma_1}\right)^{\frac{\gamma_2}{\gamma_1}}ds, \quad (2)$$

where $K_1(i)$ and $K_2(i)$ are functions of $\bar{A}(i)$ and $\bar{u}(i)$, γ_1, γ_2, and $\tilde{\sigma}$ are functions of α, β, and σ.

The last equation is a Hammerstein non-linear integral equation

AA '14: Existence and uniqueness (with spillovers)

Theorem 2: Consider any regular geography with endogenous productivity and amenities with T symmetric. Define $\gamma_1 = 1 - \alpha(\sigma - 1) - \beta\sigma$, and $\gamma_2 = 1 + \alpha\sigma + (\sigma - 1)\beta$. If $\gamma_1 \neq 0$, then:

1. There exists a regular equilibrium.
2. If $\gamma_1 < 0$, no regular equilibria are point-wise locally stable.
3. If $\gamma_1 > 0$, all equilibria are regular and point-wise locally stable.
4. If $\frac{\gamma_2}{\gamma_1} \in [-1, 1]$, the equilibrium is unique.

Note that

$$W(i) = \frac{\left(\int_S T(i,s)^{1-\sigma} P(s)^{\sigma-1} w(s) L(s) ds\right)^{\frac{1}{\sigma}}}{P(i)} \bar{A}(i)^{\frac{\sigma-1}{\sigma}} \bar{u}(i) L(i)^{-\frac{\gamma_1}{\sigma}}$$

hence parts 2 and 3 follow from $\text{sign}\left(\frac{dW(i)}{dL(i)}\right) = -\text{sign}(\gamma_1)$

Sufficient conditions for uniqueness satisfied only if no net spillovers, i.e. $\alpha + \beta \leq 0$

AA '14: Existence and uniqueness (with spillovers)

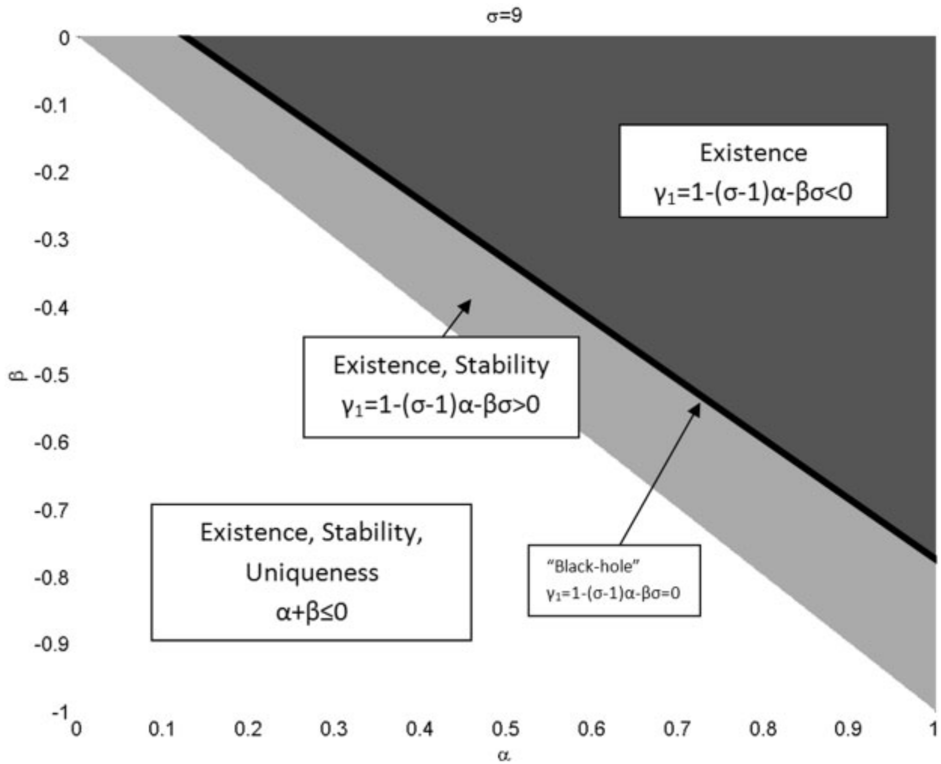


FIGURE I

Equilibria with Amenity and Productivity Spillovers

This figure shows the regions of values for the productivity spillover α and the amenity spillover β for which there exists an equilibrium, for which there exists a point-wise locally stable equilibrium, and whether that equilibrium is unique. The elasticity of substitution σ is chosen to equal 9.

AA '14: Geographic component T

- Apart from everything above, Allen and Arkolakis (2014) introduce the "fast-marching method" into spatial economics
- Suppose S is a compact surface (e.g., line, plane, cow)
- Let $\tau : S \to \mathbb{R}_+$ be a continuous function where $\tau(i)$ is the instantaneous cost of traveling over location i
- Trade cost $T(i,j) = f(t(i,j)), f' > 0, f(0) = 1$ is the total iceberg trade cost incurred along least-cost route from i to j

$$t(i,j) = \min_{g \in \Gamma(i,j)} \int_0^1 \tau(g(t)) \left\| \frac{dg(t)}{dt} \right\| dt$$

where $g : [0,1] \to S$ is a path and Γ is set of possible continuous once-differentiable paths

AA ‘14: Eikonal equations and FMM

- Previous equation is oft-studied in physics (wave propagation)
- A necessary condition for its solution is the following eikonal partial differential equation

$$\|\nabla t(i,j)\| = \tau(j)$$

  where the gradient is taken with respect to the destination j.
- One solution algorithm for this is the fast marching method
- “FMM can be interpreted as a generalization of Dijkstra to continuous spaces”
- Treb’s website has an example of implementing FMM in Matlab
- Julia is popular with people who solve PDEs: `http://juliadiffeq.org/`
- I have not yet tried EikonalInv.jl, but I hope to soon

Application to the US economy

- Estimate the geography of the United States.
- Estimate bilateral trade costs
- Given trade costs, identify (composite) productivities and amenities
- Quantify the importance of geographic location.
- Perform counterfactual exercise: remove the Interstate Highway System.
- Note: Cannot identify α, β, σ; they do analysis for a large variety of (α, β) while assuming $\sigma = 9$.

AA '14: Estimating trade costs

Goal: Find trade costs that best rationalize the bilateral trade flows observed in 2007 Commodity Flow Survey (CFS). Three step process:

- Using Fast Marching Method and observed transportation network, calculate the (normalized) distance between every CFS area for each major mode of travel (road, rail, air, and water).
- Using a discrete-choice framework and observed mode-specific bilateral trade shares, estimate the relative cost of each mode of travel.
- Using a gravity model and observed total bilateral trade flows, pin down normalization (and incorporate non-geographic trade costs).

Note: The CFS only covers agriculture, manufacturing, mining, and wholesaling. Where do tradable services show up in this model?

AA '14: Estimating trade costs

- For any $i, j \in S$, assume $\exists$ traders t choosing mode $m \in \{1, ..., M\}$ of transit where cost is: $\exp\left(\tau_m d_m(i,j) + f_m + \nu_{tm}\right)$
- Then mode-specific bilateral trade shares are:

$$\pi_m(i,j) = \frac{\exp\left(-a_m d_m(i,j) - b_m\right)}{\sum_k \left(\exp\left(-a_k d_k(i,j) - b_k\right)\right)}, \tag{3}$$

where $a_m = \theta\tau_m$ and $b_m = \theta f_m$.

- Combined with model, yields gravity equation:

$$\ln X_{ij} = \frac{\sigma - 1}{\theta} \ln \sum_m \left(\exp\left(-a_m d_{mij} - b_m\right)\right) + (1 - \sigma)\beta' \ln \mathbf{C}_{ij} + \delta_i + \delta_j \tag{4}$$

- Estimate a_m and b_m using bilateral trade share eq (3), θ using gravity eq (4). Assume $\sigma = 9$.

AA '14: Trade cost estimates

Table II: Estimated mode-specific relative cost of travel

	All CFS Areas				Only MSAs			
Geographic trade costs	Road	Rail	Water	Air	Road	Rail	Water	Air
Variable cost	0.5636***	0.1434***	0.0779***	0.0026	0.4542***	0.1156***	0.0628***	0.0021
	(0.0120)	(0.0063)	(0.0199)	(0.0085)	(0.0233)	(0.0210)	(0.0265)	(0.0176)
Fixed cost	0	0.4219***	0.5407***	0.5734***	0	0.34***	0.4358***	0.4621***
	N/A	(0.0097)	(0.0236)	(0.0129)	N/A	(0.0235)	(0.0375)	(0.0264)
Estimated shape parameter (θ)	14.225***				17.6509***			
	(0.3375)				(1.4194)			
Non-geographic trade costs								
Similar ethnicity	-0.0888***				-0.0803***			
	(0.0153)				(0.0275)			
Similar language	0.063***				0.0286			
	(0.0223)				(0.0359)			
Similar migrants	-0.0191				-0.0135			
	(0.0119)				(0.0203)			
Same state	-0.2984***				-0.3104***			
	(0.0101)				(0.0176)			
R-squared (within)	0.4487				0.4113			
R-squared (overall)	0.6456				0.5995			
Observations with positive bilateral flows	9601	9601	9601	9601	3266	3266	3266	3266
Observations with positive mode-specific bilateral flows	9311	1499	78	1016	3144	340	26	471

AA '14: Estimating productivities and amenities

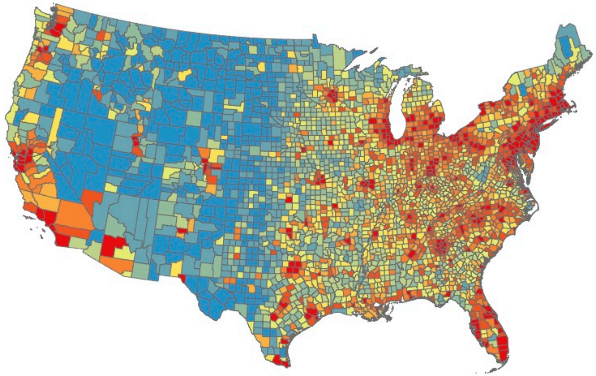
- Given data on wages and population across space, productivities A and amenities u can be recovered.
- To see this, plug (1) into the indirect utility function (after substituing the price index into it). This yields:

$$u(i)^{1-\sigma} = \frac{W^{1-\sigma}}{\phi} \int_S T(s,i)^{1-\sigma} w(i)^{\sigma-1} w(s)^{\sigma} L(s) u(s)^{\sigma-1} ds$$

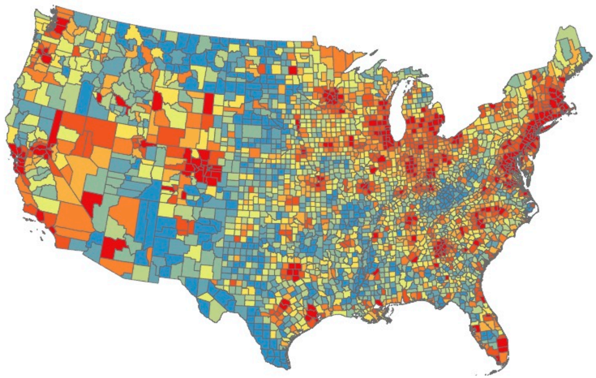
- Again, this is a Hammerstein non-linear integral equation, which can be uniquely solved for $u(i)$.
- Then A(i) can be recovered from (1).
- Note: $\bar{A}$ and $\bar{u}$ cannot be identified without knowledge of α and β.

AA ‘14: Population and wages – data

Figure 12: United States population density and wages in 2000



Population density

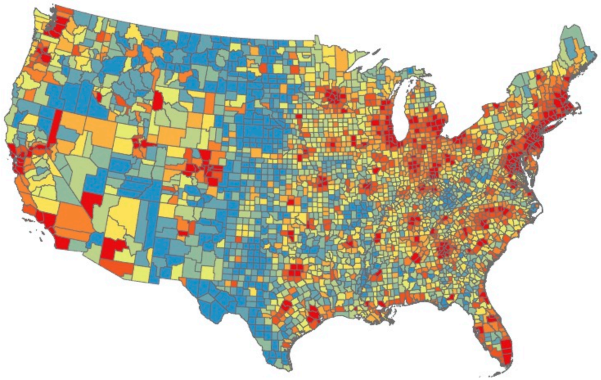


Wages

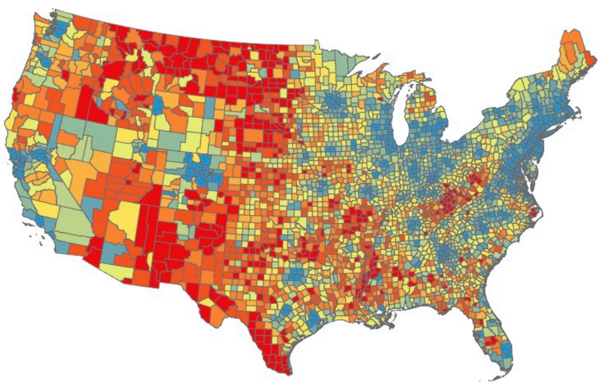
Notes: This figure shows the relative population density (top) and wages (bottom) within the United States in the year 2000 by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles. (Source: MPC (2011a)).

AA '14: Estimated composite productivity and amenity

Figure 13: Estimated composite productivity and amenity



Composite productivity

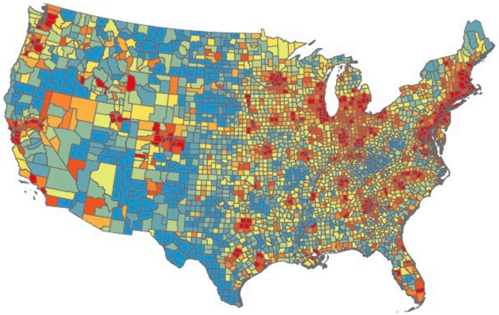


Composite amenity

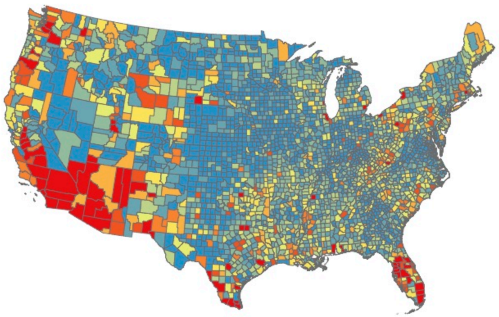
Notes: This figure shows the estimated composite productivity (top) and amenity (bottom) by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

AA ‘14: Estimated exogenous productivity and amenity

Figure 14: Estimated exogenous productivity and amenity



Exogenous productivity

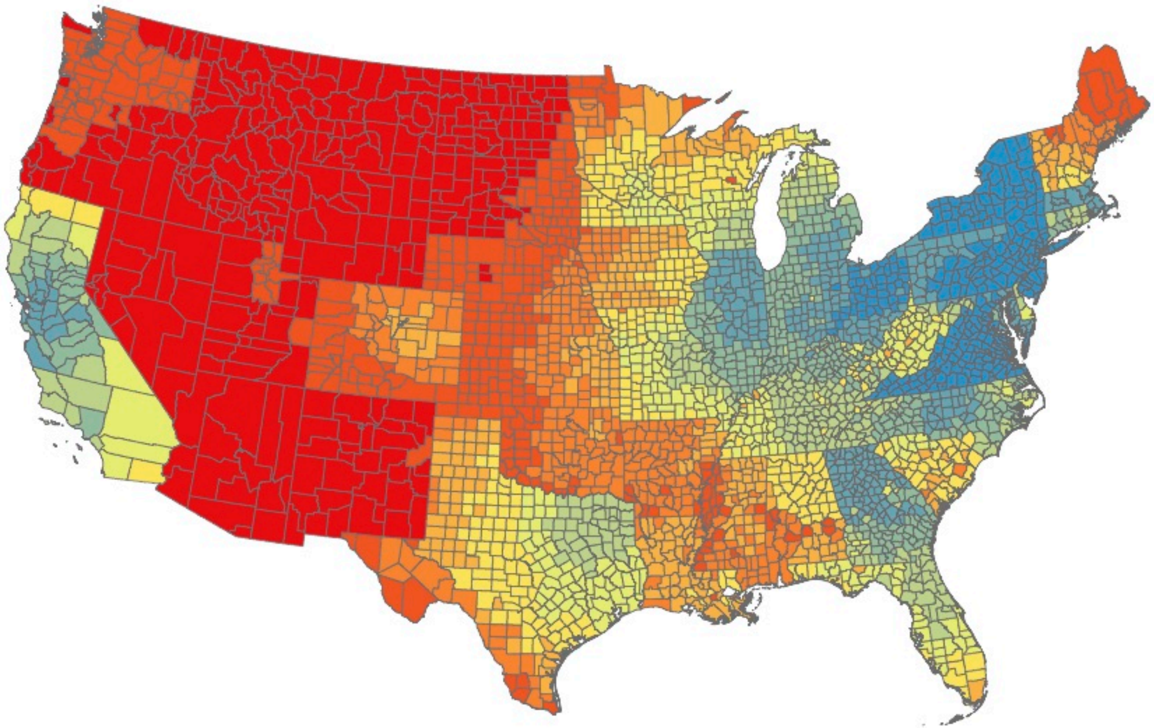


Exogenous amenity

Notes: This figure shows the estimated exogenous productivity $\bar{A}$ (top) and amenity $\bar{u}$ (bottom) by decile assuming $\alpha = 0.1$ and $\beta = -0.3$. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

AA '14: Estimated price index

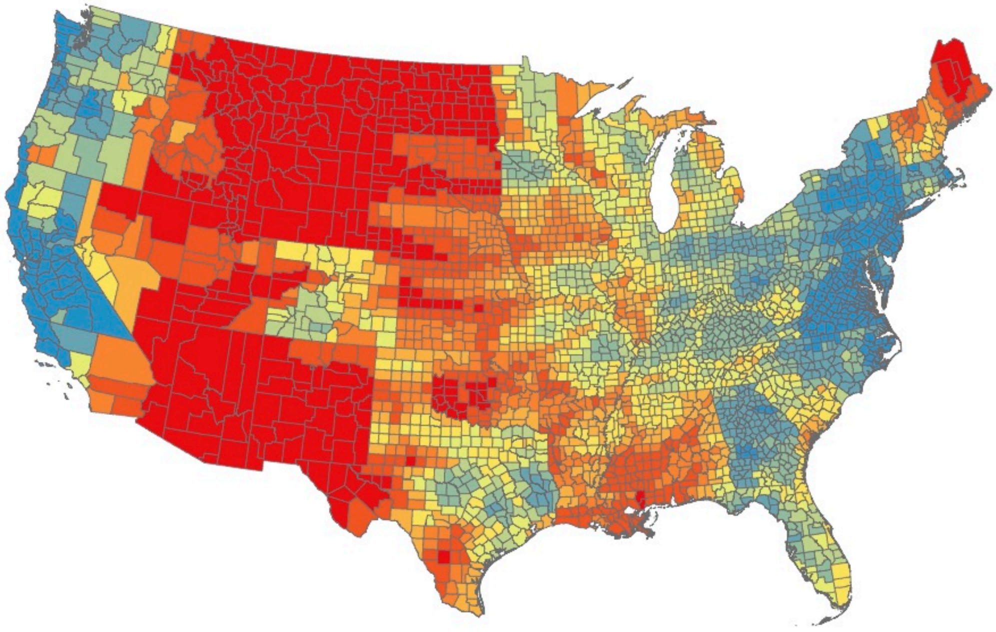
Figure 15: Estimated price index



Notes: This figure shows the estimated price index by decile. The data are reported at the county level; red (blue) indicate higher (lower) deciles.

Estimated increase in price index from removing IHS

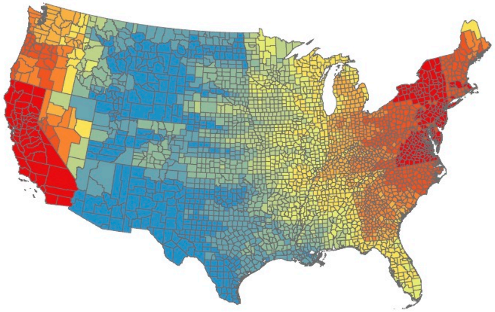
Figure 17: Estimated increase in the price index from removing the Interstate Highway System



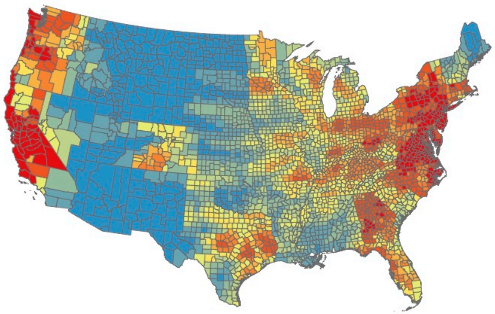
Notes: This figure depicts the estimated increase in the price index (by decile) across space from removing the Interstate Highway System (IHS), holding wages and productivities constant at the 2000 U.S. levels. Red (blue) indicate higher (lower) deciles (e.g. the removal of the IHS disproportionately increased the economic remoteness in red regions).

Estimated change in population from removing IHS

Figure 18: Estimated change in the population from removing the Interstate Highway System



$\alpha = 0$, $\beta = 0$



$\alpha = 0.1$, $\beta = -0.3$

Notes: This figure shows the estimated change in population (in deciles) from the removal of the Interstate Highway System (IHS). The top map reports the estimated population changes when there are no spillovers (i.e. $\alpha = \beta = 0$), while the bottom map reports the estimated population changes when spillovers are chosen to approximately match those from the literature (i.e. $\alpha = 0.1$ and $\beta = -0.3$). Red (blue) indicates higher (lower) deciles (e.g. the removal of the IHS increased the relative population in red areas).

Next week

Spatial sorting of skills and sectors