A Ricardian Model with a Continuum of Goods under Nonhomothetic Preferences: Demand Complementarities, Income Distribution, and North-South Trade

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This paper develops a Ricardian model of trade in which goods are indexed according to priority and higher-indexed goods are consumed only by richer households. South (North) has a comparative advantage in lower- (higher-) indexed goods and, hence, specializes in goods with lower (higher) income elasticities of demand. Product cycles and a southern terms-of-trade deterioration result from faster population growth and uniform productivity growth in South and a global productivity improvement. South's domestic income redistribution policy can improve its terms of trade so much that every household in South may be better off, at the expense of North.

I. Introduction

The Ricardian model takes cross-country differences in technology as a basis of trade; it is widely used as a building block in the recent literature on technology and trade (see Grossman and Helpman [1995] for a survey). Most studies in this literature assume that consumers have homothetic preferences for the analytical convenience. Not only is this assumption rejected empirically,[1] it is too restrictive for thinking about

A previous version of this paper is titled "The Dornbusch-Fischer-Samuelson Model without Homotheticity." I thank two anonymous referees and an editor for their comments.

[1] Empirical estimations of demand systems consistently show that there are large deviations of homotheticity and that certain commodity classes, such as food, account for a much larger budget share of low-income households (see, e.g., Deaton and Muellbauer 1983). A few studies have examined the empirical implications of nonhomothetic pref-

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many important issues in trade and development, where technological factors play central roles. For example, many policy makers and economists in developing countries, following Prebisch (1950) and Singer (1950), are concerned with the tendency that the terms of trade deteriorate continuously against poor countries. As the world economy grows, the argument goes, the relative demand shifts away from South, which specializes in goods with low income elasticities of demand, and toward industrialized North, which specializes in goods with higher income elasticities. Industrial policy advocates often suggest that a developing country should transform its industrial structure by targeting sectors with high income elasticities of demand so as to be able to enjoy the benefits of the growing global economy.[2] Furthermore, South would benefit little from productivity improvement in its export sectors since the increased purchasing power generated by the lower prices of southern goods would be spent mostly on northern goods. It has also been suggested, most notably by Linder (1961) and Vernon (1966), that new industries are born in North because only rich consumers can afford to purchase new, often luxury, products. The assumption of homothetic preferences, which implies that all the goods have the same unitary income elasticities and that the rich and the poor consume all the goods in the same proportion, is simply not appropriate for addressing these issues.

This paper develops an analytically tractable Ricardian model with nonhomothetic preferences, suitable for addressing these issues in trade and development. Following Dornbusch, Fischer, and Samuelson (1977), I assume that there is a continuum of traded goods, which helps to make the equilibrium depend smoothly on exogenous variables. In the present model, the goods are ordered in terms of priority. Goods at the lower end of the spectrum are consumed by all the households. As their income levels go up, the households expand their range of consumption by adding higher-indexed goods to their baskets. In the central case, there are two countries: North and South. Developed, high-income North has a comparative advantage in a higher spectrum of goods; underdeveloped, low-income South has a comparative advantage in a lower spectrum. This makes South (North) specialize in goods whose demand has lower (higher) income elasticities.

Because of the difference in income elasticities, a variety of exogenous changes have asymmetric effects on the terms of trade, patterns of specialization, and welfare. For example, the terms of trade move against

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erences on the trade flow. Hunter and Markusen (1988) and Hunter (1991) calculated that about 14 percent of world trade can be attributed to the difference in per capita income across countries.

[2] See Komiya, Okuno, and Suzumura (1984 [1988]) for a critical appraisal of the argument by industrial policy advocates.

South, and product cycles occur (i.e., new industries are born in North and old industries move from North to South), as a consequence of faster population growth in South, uniform productivity growth in South, and an improvement in global productivity. The welfare gain of productivity growth is also unevenly distributed. North can capture all the benefits of its own uniform productivity growth, whereas South may lose from its own uniform productivity growth. The reason for these effects is the asymmetry of *demand complementarities* between goods. When the prices of lower-indexed goods decline, demand for higher-indexed goods will increase. The reason is that the households respond to the higher real income, resulting from the reduction in prices of lower-indexed goods, by adding higher-indexed goods to their consumption baskets. In other words, the income effect makes higher-indexed goods complements to lower-indexed goods in demand. On the other hand, when the prices of higher-indexed goods decline, demand for lower-indexed goods will not increase.

Nonhomotheticity of preferences also implies that transfer payments, which affect *income distribution,* both within and across the two countries, have nontrivial effects. For example, South's domestic policy to redistribute income from the rich, who buy foreign imports, to the poor, who cannot afford to buy them, can lead to a large terms-of-trade improvement so that all the households in South may be better off at the expense of North.[3]

This paper is closely related to Flam and Helpman (1987) and Stokey (1991). Both studies presented Ricardian models with a continuum of goods, where consumers have nonhomothetic preferences, and applied them to North-South trade. There is a fundamental difference, however. In those papers, goods are indexed according to the product quality, and the assumed preferences imply that different goods are gross substitutes. That is, a reduction in the prices of lower-indexed goods induces the households to *switch* from a higher-indexed good to a lower-indexed good because of the substitution effect. This may be reasonable when the goods are vertically differentiated products within an industry, and the models are used to address the issues of intraindustry trade, which is indeed the interpretation offered by Flam and Helpman and Stokey. In the present model, on the other hand, goods are not gross substitutes; there are demand complementarities from southern goods to northern goods. That is, a reduction in the prices of lower-indexed goods induces the households to *expand* their consumption set toward higher-indexed

[3] Another implication of the present framework is that the effects of a population change are no longer isomorphic to those of a uniform change in technology. Even though both changes increase the effective supply of labor, they need to be treated separately. This point, however, is true for any nonhomothetic preferences, not unique to the present setting.

goods because of the income effect. The present model is more appropriate for addressing sectoral issues in trade and development in the presence of significant differences in the income elasticities of demand across sectors.[4]

The rest of the paper is organized as follows. Section II develops the basic model and highlights its key features in comparison with the Dornbusch et al. model. Section III considers the case in which each country has a homogeneous population. By abstracting from the effects of income distribution, this simplification helps to focus on the effects of demand complementarities. It conducts comparative statics under the assumption that the poor country has a comparative advantage in lower-indexed goods. That section also offers a more detailed comparison between the present model and those of Flam and Helpman and Stokey. Section IV looks at the case of heterogeneous populations and discusses the effect of income redistribution policies. Section V extends the model to a multicountry case. Section VI suggests the direction for future research.

II. The Model

This section develops the basic model, in which there are two countries, Home and Foreign. An extension to a multicountry case is considered in Section V.

[4] Some examples may be useful to illustrate the differences. In Flam and Helpman's paper, each household is restricted to choose only one good from the spectrum (the rich own a BMW but not a Yugo). Hence, North-South trade takes place only when there is nondenegerate income distribution within each country in their model. In Stokey's paper, each household may consume a range of goods from the spectrum (the rich may own both a bike and a car, whereas the poor own only a bike). Hence North-South trade takes place even if income distribution is degenerate within each country. (Indeed, her analysis is restricted to the case of homogeneous populations.) In the present model, the rich buy food and clothes from South and own a car made in North, and lower prices of food and clothes make a northern-made computer affordable to them (car production may move to South).

Wilson (1980) considered a variety of extensions to the Dornbusch et al. model, which include some nonhomothetic preferences, and examined the robustness of their results. He basically showed that many comparative statics results obtained by Dornbusch et al. carry over, as long as the difference in income elasticities is not significant and goods are gross substitutes. His analysis, however, offered little insights when there are significant differences in income elasticity or when goods are not gross substitutes. The main problem is that his model, by extending the Dornbusch et al. model, has lost the tractability of their model. This paper adopts a strategy different from his. Instead of presenting an *extension* of the Dornbusch et al. model, this paper presents an *alternative* to it that is capable of incorporating significant income effects in a tractable manner. The goal of this paper is not to examine the robustness of their model but to develop a Ricardian model applicable to many issues that are central in trade and development. (After this paper was accepted, Josef Zweimüller brought my attention to the diploma thesis written in German by a student of his [Aydemir 1998, chap. 2] in which a special case of the present model is developed and the same comparative statics are conducted.)

A. *Technology*

There is a continuum of competitive industries, indexed by $z \in [0, \infty)$, each producing a homogeneous good, also indexed by z. Labor is the only factor of production. Let $a(z)$ and $a^*(z)$ be the Home and Foreign unit labor requirements of sector z, that is, labor input required to produce one unit of output z in Home and Foreign. Following Dornbusch et al., I make the following assumption.

ASSUMPTION 1. $a^*(z)/a(z)$ is continuous and strictly decreasing in z.

Thus Home has a comparative advantage in a lower spectrum of goods and Foreign in a higher spectrum. Let us take Foreign labor as the *numeraire* and denote the price of Home labor by w. Then the price of good z is given by $p(z) = \min\{wa(z), a^*(z)\}$. Given assumption 1, there is a marginal good, m, the switching point in the chain of comparative advantage, so that Home produces only goods in $[0, m]$ and Foreign produces only goods in $[m, \infty)$, and the prices are determined by

$$\begin{aligned} p(z) &= wa(z), \quad z \in [0, m]; \\ p(z) &= a^*(z), \quad z \in [m, \infty). \end{aligned} \tag{1}$$

The marginal good is inversely related to w according to

$$w = \frac{a^*(m)}{a(m)}. \tag{2}$$

This relation is depicted by the downward-sloping curve in figure 1.

B. *Households*

There are N households in Home and N^* households in Foreign. There may be a nondegenerate income distribution due to skill differences, reflected in differences in the effective labor supply. Let $F(h)$ and $F^*(h^*)$ be the distributions of effective labor supply across the households in Home and Foreign, respectively. A Home household with h units of effective labor earns wh, and a Foreign household with h^* earns h^*.

All the households share the same preferences. The present model differs critically from the Dornbusch et al. model in the structure of preferences. A household with income I seeks to maximize $V = \int_0^\infty b(z)x(z)dz$, subject to the budget constraint $\int_0^\infty p(z)x(z)dz \leq I$, where $b(z) > 0$ is the utility weight attached to good z and $x(z)$ is an indicator function, with $x(z) = 1$ if good z is consumed and $x(z) = 0$ if it is not. The assumption that goods come in discrete units and that each household's desire for a particular good satiates after one unit, adopted from Murphy, Shleifer, and Vishny (1989), has a strong implication. An increase in the utility takes the form of increased diversity, not of increased

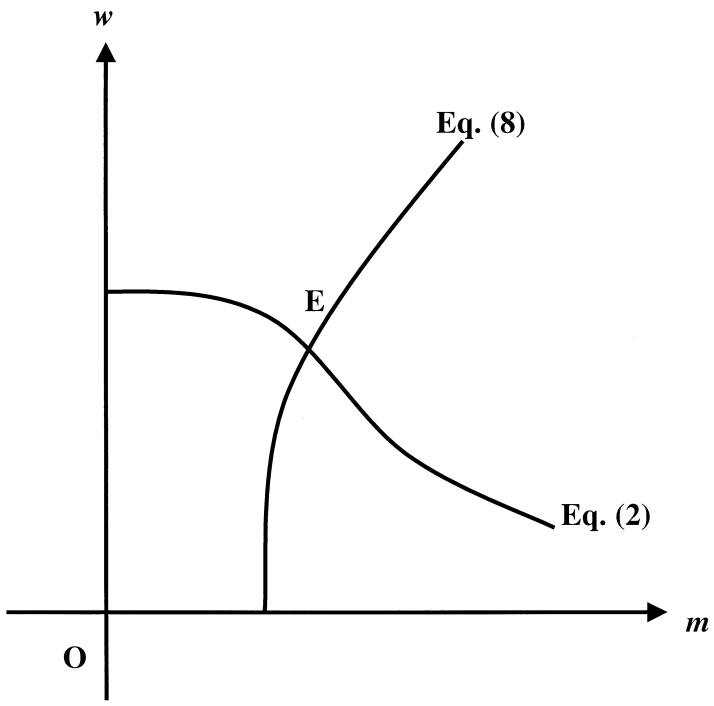

FIG. 1.—Equilibrium factor terms of trade and patterns of specialization

consumption of the same good, and wealthier households consume all the goods consumed by poorer households, plus some.[5] I make a further assumption.

ASSUMPTION 2. $b(z)/a(z)$ and $b(z)/a^*(z)$ are both strictly decreasing in z.

This assumption implies that

$$\frac{b(z)}{p(z)} = \frac{b(z)}{\min\{wa(z),\ a^*(z)\}}$$

is strictly decreasing in z for any w; hence the order in which the households purchase goods is the same as the order of goods given by assumption 1. Assumptions 1 and 2 jointly imply that Home has a comparative advantage in lower-indexed goods, which even poor households

[5] One may call lower-indexed goods "necessities" and higher-indexed goods "luxuries." However, none of the goods satisfies the standard definition of "a necessity" or "a luxury" on the basis of the property of demand function. Any particular good is a luxury for a sufficiently poor household and a necessity for a sufficiently rich household.

purchase, and that Foreign has a comparative advantage in higher-indexed goods, those purchased by wealthier households.[6] Let us define

$$E(z) \equiv \int_0^z p(s)ds = \int_0^z \min\{wa(s),\ a^*(s)\}ds. \tag{3}$$

Since the household purchases all the lower-indexed goods and expands its range of consumption upward as far as it can afford, a Home household with income wh chooses $x(z) = 1$ for $z \in [0,\ u(h)]$ and $x(z) = 0$ for $z \in (u(h),\ \infty)$, where $u(h)$ is given by

$$E(u(h)) = wh, \tag{4}$$

and attains the utility level $V(h) = B(u(h))$, where $B(z) \equiv \int_0^z b(s)ds$. Similarly, a Foreign household with income h^* chooses $x(z) = 1$ for $z \in [0,\ u^*(h^*)]$ and $x(z) = 0$ for $z \in (u^*(h^*),\ \infty)$, where

$$E(u^*(h^*)) = h^*, \tag{5}$$

and attains the utility level $V^*(h^*) = B(u^*(h^*))$. In this model, there is a one-to-one mapping between the level of utility attained by a household, $V(h)$ and $V^*(h^*)$, and the highest-indexed good it consumes, $u(h)$ and $u^*(h^*)$. For this reason, $u(h)$ and $u^*(h^*)$ may also be viewed as utility measures.

C. *The Labor Market Equilibriums and the Balanced Trade*

Since all the households whose income is greater than or equal to $E(z)$ consume one unit of good z, the total demand for good z is given by

$$Q(z) = N\left[1 - F\left(\frac{E(z)}{w}\right)\right] + N^*[1 - F^*(E(z))].$$

Since Home produces only the goods in $[0,\ m]$ and the goods in $[0,\ m)$ are produced only by Home, the Home labor market equilibrium condition is given by

[6] What is critical here is that there is some (either positive or negative) correlation between the ordering of goods based on the pattern of comparative advantage and the order in which the households purchase goods. In the Dornbusch et al. model, the correlation is assumed to be zero (i.e., homotheticity), which helps to make their model tractable. The present model assumes that the correlation is perfect in order to consider the implication that there is some correlation and, at the same time, to keep the model tractable. Note that no assumption has yet been made in terms of the direction of correlation (whether positive or negative). An additional assumption is needed to make this model one of North-South trade, where the poorer country produces goods with lower income elasticities of demand. This will be done later in Sec. III, in the form of assumption 3.

$$L = N\int_0^\infty hdF(h) = \int_0^m a(z)Q(z)dz.$$

Combining these expressions yields, after some algebra (see Matsuyama [1999*a*] for details),

$$\begin{aligned} wL &= wN\int_0^\infty hdF(h) \\ &= N\int_0^\infty \min\{wh,\ E(m)\}dF(h) + N^*\int_0^\infty \min\{h^*,\ E(m)\}dF^*(h^*). \end{aligned} \tag{6}$$

The left-hand side is the total labor income in Home, equal to the Home national income. The right-hand side is the Home gross national product, equal to the total spending on the Home goods; the first term is Home's expenditure on the Home goods, and the second term is Foreign's expenditure on the Home goods. Note that a Home household spends $\min\{wh, E(m)\}$ and a Foreign household spends $\min\{h^*, E(m)\}$ on the Home products. Similarly, the Foreign labor market equilibrium condition can be written as

$$\begin{aligned} L^* &= N^*\int_0^\infty h^*dF^*(h^*) \\ &= \int_m^\infty a^*(z)Q(z)dz \\ &= N\int_0^\infty \max\{wh - E(m),\ 0\}dF(h) \\ &\quad + N^*\int_0^\infty \max\{h^* - E(m),\ 0\}dF^*(h^*). \end{aligned} \tag{7}$$

The two labor market equilibrium conditions, (6) and (7), are indeed identical, because of Walras's law, and they can be rearranged to obtain

$$\begin{aligned} &N\int_0^\infty \max\left\{h - \int_0^m a(s)ds,\ 0\right\}dF(h) \\ &= N^*\int_0^\infty \min\left\{\frac{h^*}{w},\ \int_0^m a(s)ds\right\}dF^*(h^*), \end{aligned} \tag{8}$$

where use has been made of

$$E(m) = \int_0^m p(s)ds = w\int_0^m a(s)ds.$$

Equation (8) states that, given the static nature of the model, the trade is balanced. That is, the value of the Home imports, the left-hand side, must be equal to the value of the Home exports, the right-hand side. The balanced trade condition (8) is depicted in figure 1. It is upward-sloping as long as some Foreign households are poor enough to consume only the Home goods. An increase in the relative wage of Home labor, a rise in w, would force such poor Foreign households to cut their spending on the Home goods, thereby reducing indirect demand for Home labor. To restore the equilibrium, Home must expand the range of production, a rise in m. When w is sufficiently small that all the Foreign households are rich enough to consume some Foreign goods, a small change in w does not affect the demand for Home labor. In this case, the balanced trade condition (8) becomes $(N + N^*)\int_0^m a(s)ds = L$, and it is vertical, as depicted in figure 1.

Equations (2) and (8) jointly determine the equilibrium value of m and w. Then, from (4) and (5), one can determine the equilibrium range of goods consumed by different households, as well as their utility levels.

D. A Comparison with the Dornbusch et al. Model

Before we proceed, it is worth comparing the present model with the Dornbusch et al. model. These authors assumed that all the households have identical Cobb-Douglas preferences over a fixed range of goods, say $[0, 1]$, with $V = \int_0^1 \beta(z)\ln[x(z)]dz$, where $\beta(\cdot) > 0$ satisfies $\int_0^1 \beta(s)ds = 1$, and $x(z)$ can be any positive real number. This assumption implies that each household spends the fraction $\vartheta(z) \equiv \int_0^z \beta(s)ds$ of income on the goods in $[0, z]$, regardless of the income level. As a result, the labor market condition, or equivalently the balanced trade condition, becomes

$$w = \frac{\vartheta(m)}{1-\vartheta(m)}\frac{N^*\int_0^\infty h^* dF^*(h^*)}{N\int_0^\infty h dF(h)}$$

$$= \frac{\vartheta(m)}{1-\vartheta(m)}\frac{L^*}{L}, \tag{9}$$

which yields a positive relation between w and m. In the Dornbusch et al. model, the equilibrium levels of w and m are determined jointly by (2) and (9). Thus understanding the differences between (8) and (9) is the key for understanding the results below.

First, as seen in equation (9), the Dornbusch et al. model, because of the homothetic preferences, is independent of income distribution within each country, and hence so are the aggregate variables, such as m and w. On the other hand, as shown in equation (8), the equilibrium values of m and w depend on the distribution of h and h^* because of the nonhomothetic preferences. Second, equation (9) passes through the origin. That is, $w \to 0$ implies $m \to 0$. On the other hand, m approaches a positive number, satisfying $(N + N^*) \int_0^m a(s)ds = L$, according to equation (8). In the Dornbusch et al. model, as the Home wage rate and the price of Home goods become cheaper, all the households increase the amount of consumption of Home goods, through substitution effects, which increases demand for Home labor. To maintain the labor market equilibrium and the balanced trade, Home's production must keep shifting toward the bottom end of the goods spectrum. In the present model, the households do not increase the amount of consumption of lower-indexed goods when their prices go down. The aggregate demand for each good is bounded by $N + N^*$. For this reason, Home must continue to produce a certain range of lower-indexed goods to keep all the Home labor employed. Third, the unit labor requirements, $a(z)$ and $a^*(z)$, do not appear in equation (9). That is, at constant relative wages, neither a change in $a(z)$ nor a change in $a^*(z)$ affects the labor market equilibrium in the Dornbusch et al. model because of the Cobb-Douglas preferences.[7] In the present model, there is asymmetry between $a(z)$ and $a^*(z)$. Reducing $a(z)$ and hence the prices of the Home goods shifts the household spending away from Home goods toward Foreign goods, thereby increasing the relative demand for Foreign labor because of demand complementarities. To restore the balance, Home must expand its range of production. On the other hand, $a^*(z)$ does not appear in equation (8) because a reduction in $a^*(z)$ and the prices of Foreign goods induces the household to buy only other Foreign goods with higher indices and hence does not cause a spending shift between Home and Foreign goods. What matters for the following analysis is not so much that $a^*(z)$ does not affect equation (8), but rather that there is asymmetry in which $a(z)$ and $a^*(z)$ affect equation (8).

III. North-South Trade: The Case of Homogeneous Populations

In this section, it is assumed that households are homogeneous in each country. By abstracting the effect of income distribution, this assumption helps to focus on the implications of asymmetric demand complemen-

[7] As Wilson (1980) demonstrated, this feature of the Dornbusch et al. model does not hold even for general homothetic preferences.

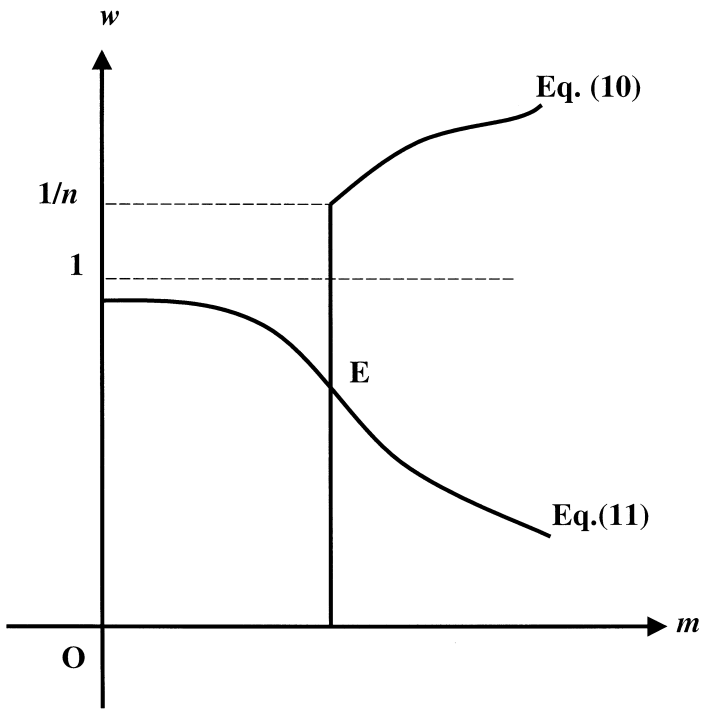

FIG. 2.—Equilibrium factor terms of trade and patterns of specialization between North and South: the case of homogeneous populations.

tarities. Let all the households be endowed with one unit of effective labor. Then, equation (8) becomes

$$\int_0^m a(s)ds = \frac{N}{N+N^*} \quad \text{if } w \le 1 + \frac{N^*}{N}$$

and

$$\int_0^m a(s)ds = 1 - \frac{N^*}{wN} \quad \text{if } w > 1 + \frac{N^*}{N},$$

as depicted in figure 2. I make a further assumption.

ASSUMPTION 3. $a(z) > a^*(z)$ for all z.

This assumption means that Foreign has an absolute advantage in all the industries.[8] This assumption ensures that the Foreign households are richer than the Home households, $w < 1$, in equilibrium. Combined

[8] The case in which assumption 3 is violated is discussed in Matsuyama (1999*a*, sec. 4).

with assumptions 1 and 2, assumption 3 implies that the poor (rich) country specializes in production of goods, with lower (higher) income elasticities of demand. It is the combination of assumptions 2 and 3 that makes this model one of North-South trade.[9] I hereafter identify Foreign as developed, high-income North and Home as less developed, low-income South.

The equilibrium conditions, equations (2) and (8), now become

$$\int_0^m a(s)ds = n \tag{10}$$

and

$$wa(m) = a^*(m), \tag{11}$$

where n is the share of South in the world population. Equation (11) is a reproduction of equation (2). These conditions are depicted in figure 2. Note that the downward-sloping curve intersects with the vertical section of the balanced trade/labor market equilibrium condition, which is why equation (10) is independent of w. This feature of equilibrium greatly simplifies the comparative statics exercises conducted below but is not essential for the central results.[10]

Equations (4) and (5) now become

$$w\int_0^m a(s)ds + \int_m^u a^*(s)ds = w \tag{12}$$

and

[9] The joint satisfaction of assumptions 1–3 can be justified as follows. Imagine that technology is subject to sector-specific, country-specific, learning by doing, as in Krugman (1987), and that, initially, the two countries are in autarky, with North having overall better technologies than South. Then, with the preference structure assumed here, technology gaps between North and South would become larger in higher-indexed sectors under autarky. (See Matsuyama [1999*b*] for such a model of a closed economy.) The model of this section can be interpreted as describing the situation in which these two countries start trading after these patterns of technologies have developed.

[10] What is essential is that, for a given w, a decline in $a(z)$ increases m more than a decline in $a^*(z)$ of equal magnitude decreases m along the balanced trade/labor equilibrium condition. The exact form of eq. (10), its independence of $a^*(z)$ and of w, is not essential. For example, one could extend the present model by putting leisure into the utility function, thereby endogenizing labor supply. (Or, equivalently, one could introduce a nontradable goods sector.) Such an extension makes the balanced trade/labor equilibrium condition dependent of $a^*(z)$ and of w but does not change the essential features of the model. Those who remain skeptical should also consult Sec. V, which shows the robustness of the results, even though nondegenerate income distribution makes the balanced trade/labor equilibrium condition dependent on w.

$$w\int_0^m a(s)ds + \int_m^{u^*} a^*(s)ds = 1, \tag{13}$$

where u and u^* are the highest-indexed goods consumed, hence the utility level attained, by southern and northern households. They satisfy $m < u < u^*$. Since North imports all the goods produced by South, South imports some northern goods in exchange. Hence, in this equilibrium, all the southern households consume all the basic goods produced in South, plus some goods produced in North ($m < u$). Northern households, which are richer than southern households, consume a wider range of their own goods and attain a higher level of utility ($u < u^*$).

The volume of trade per household is

$$2N^*\frac{\int_0^m a(s)ds}{N + N^*} = 2n(1 - n), \tag{14}$$

measured in the unit of Home labor. The volume of trade is thus independent of the terms of trade, w, and of the patterns of production, m.

A. *A Comparison with the Flam-Helpman and Stokey Models*

Before we proceed, it might be instructive to compare the present model with the North-South trade models of Flam and Helpman (1987) and Stokey (1991), which have apparent similarities. First, their models have a continuum of goods supplied competitively. Second, the preferences are nonhomothetic in such a way that only high-income households demand higher-indexed goods, and the set of goods produced in equilibrium is endogenous. Third, these authors make assumptions analogous to my assumptions 1–3. The country with a comparative advantage in higher-indexed goods is the developed North, having absolute advantage in all the indexed goods, and the less developed South has a comparative advantage in lower-indexed goods.

There is a fundamental difference, however, between the Flam-Helpman and Stokey models and the present model. In their models, goods are indexed according to the product quality, and the assumed preferences imply that different goods are gross substitutes. That is, a reduction in the prices of lower-indexed goods induces the households to *switch* from a higher-indexed good to a lower-indexed good because of the substitution effect. This may be reasonable when the goods are vertically differentiated products within an industry, and the models are used to address the issues of intraindustry trade, which is indeed the interpretation offered by these authors. In the present model, on the other hand, goods are not gross substitutes. A reduction in the prices

of lower-indexed goods induces the households to *expand* their consumption set toward higher-indexed goods because of the income effect. (Goods are not, however, Pareto-Edgeworth complements; there is no greater benefit of consuming the goods together than separately.) The present model should thus be interpreted as addressing intersectoral trade, where different sectors produce goods, whose demands have different income elasticities.

Because of the above-mentioned difference in the demand structures, the equilibrium in the Flam-Helpman and Stokey models has the following features. First, the goods at the bottom end of the spectrum are not produced. Second, there is a gap between the range of goods produced in the two countries. That is, there is a range of goods not produced in equilibrium, which are of higher quality than the highest-quality good produced by South and of lower quality than the lowest-quality good produced by North. In the present model, no gap exists in the range of goods produced. Third, a deterioration of South's terms of trade, which makes southern goods cheaper and causes a shift of production of some goods from North to South, also tends to discourage North from producing the upper end of the spectrum. In the present model, North introduces new goods at the upper end when South's terms of trade deteriorate, as will be seen below.

B. Population Size

This subsection discusses the effects of a change in the population sizes in the two countries. An increase in N means that the southern population and labor supply increase at the same rate. The vertical line, the balanced trade condition (10), shifts to the right in figure 2. The factor terms of trade move against South (a decline in w), and some industries move from North to South (an increase in m). A differentiation of (10) and (11) yields

$$a(m)dm = dn > 0 \tag{15}$$

and

$$dw = -\xi(m)dm < 0, \tag{16}$$

where $-\xi(m)$ is the slope of equation (11) at m, which measures the sensitivity of the factor terms of trade. Note that $\xi(m)$ can be any positive number. A total differentiation of (12) and (13), and use of (10) and (11), yields

$$a^*(u)du = (1 - n)dw < 0 \tag{17}$$

and

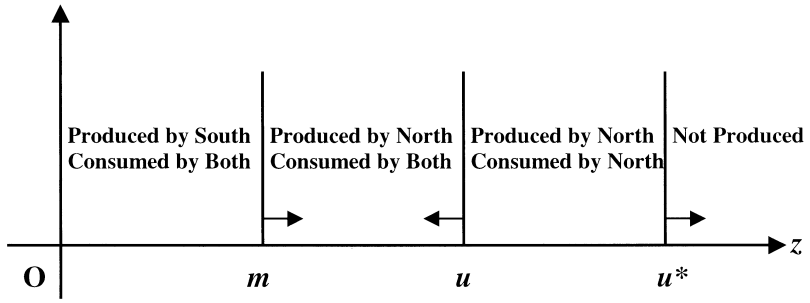

FIG. 3.—Effects of faster population growth in South: the case of homogeneous populations.

$$a^*(u^*)du^* = -ndw > 0. \tag{18}$$

The effects of a change in m cancel out because of (11). Note also that $a^*(u)ndu + a^*(u^*)(1 - n)du^* = 0$; hence the effect is purely distributional. An increase in N^* has exactly the opposite effects because the system (10)–(13) depends solely on the relative size of the two countries. Figure 3 summarizes these results, showing how the range of goods consumed and produced in each country would shift when the southern share of the population, n, changes. The volume of trade per household, from (14), reaches the highest level when $n = \frac{1}{2}$, the result previously obtained in models of trade in horizontal differentiated products (see Helpman and Krugman 1985, chap. 8).

In the Dornbusch et al. model, an increase in the Home country size, which shifts *down* the balanced trade curve at the same rate, reduces the Home relative wage proportionally less than the size increase. Hence, the share of Home income in the world rises. In the present model, an increase in the Home country size, which shifts the balanced trade line *to the right,* may reduce the Home relative wage more than the size increase because there is no restriction on the magnitude of $\xi(m)$, the slope of the downward-sloping curve, (11). Therefore, the share of Home income in the world may go down.

In contrast to the Dornbusch et al. model and as in the Flam-Helpman and Stokey models, the range of goods produced in the world economy changes in this model. Imagine that the population grows faster in South than in North over time. South experiences a secular decline in its terms of trade, and the lower-end industries in North move continuously to South. As the prices of imports from South decline, the northern households expand their range of consumption continuously toward higher-indexed goods, thereby giving birth to new industries in North. The

faster population growth in South can hence generate product cycle phenomena, similar to those discussed by Linder (1961) and Vernon (1966).

This result is in sharp contrast with those of Flam-Helpman and Stokey. First, in their models, new goods do not appear in North in response to the faster population growth in South. The upper end of the spectrum of goods produced does not change in the Flam-Helpman model. In Stokey's model, the highest-indexed goods are abandoned, as the northern households switch to cheaper goods produced in South. Second, although Flam-Helpman and Stokey also predict that North abandons the lower end of the goods in response, these goods are not immediately produced in South. Only after a time lag, South starts producing the goods previously produced in North.

C. *Productivity Changes*

This subsection examines the effects of productivity improvement, which means a reduction in $a(z)$ and $a^*(z)$. Let $g(z) \equiv -d\log[a(z)]$ and $g^*(z) \equiv -d\log[a^*(z)]$ be the rate of productivity growth in sector z in North and in South, respectively. Then, from (10)–(13),

$$a(m)dm = \int_0^m g(s)a(s)ds, \tag{19}$$

$$dw = -\xi(m)dm + w[g(m) - g^*(m)], \tag{20}$$

$$a^*(u)du = (1 - n)dw + w\int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds, \tag{21}$$

and

$$a^*(u^*)du^* = -ndw + w\int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds. \tag{22}$$

The first terms on the right-hand sides of (21) and (22) represent the (factor) terms-of-trade effect. Although an overall impact of productivity growth is positive,

$$a^*(u)ndu + a^*(u^*)(1 - n)du^* > 0,$$

the welfare gain can be unevenly distributed between North and South because of the terms-of-trade effect.

Northern Productivity Improvement: $g(z) = 0$ and $g^*(z) > 0$

First, let us consider the effect of a productivity improvement in North. In figure 2, this implies that the downward-sloping curve shifts down whereas the vertical line is unaffected. Hence, m remains unchanged and w declines at the rate equal to $g^*(m)$ (see eqq. [19] and [20]). Hence, from (22), $du^* > 0$; the northern households expand their range of consumption and their welfare improves. New industries are born in North. The effect on southern households is subtler. From (19) and (20), (21) becomes

$$a^*(u)du = -(1 - n)wg^*(m) + \int_m^u g^*(s)a^*(s)ds.$$

From (10) and (12), this can be further rewritten as

$$a^*(u)du = \int_m^u [g^*(s) - g^*(m)]a^*(s)ds.$$

If the productivity change is uniform across the export sectors, $g^*(z) = g^*$ for all $z \in [m, u]$, South's factor terms-of-trade deterioration (the decline in w) offsets exactly the productivity improvement in all the northern export sectors; hence, South's terms of trade measured in goods remain the same, and therefore the southern household's budget constraint remains intact. As a result, $du = 0$. This case thus serves as a useful benchmark. The result that North captures the entire gain of productivity improvement, without any spillover effect to South, offers a strong contrast with the effect of northern population growth, examined in the previous section, even though both changes imply an increase in the aggregate supply of effective labor in North. Population growth in North would lead to a proportional increase in northern demand for all the goods that they consume, so that some increase in demand goes to southern goods and South is better off. On the other hand, productivity improvement leads to an increase in income earned by each northern household and hence leads to an increase in demand only for northern products. This result also differs sharply from that in the Dornbusch et al. model, where a uniform improvement in northern labor productivity has the same effect as an increase in population, because of the homotheticity (see eq. [9]). The equivalence is lost when preferences are nonhomothetic, as in the present model.

The result that $du = 0$ critically depends on the uniformity of the change. If $g^*(z)$ is increasing over $[m, u]$, then $du > 0$, because the southern terms of trade improve. On the other hand, if $g^*(z)$ is decreasing in z, then $du < 0$ because the southern terms of trade deteriorate. In other words, South benefits when the change in North amplifies the

existing patterns of comparative advantage and loses otherwise. It is, however, wrong to interpret this result in terms of "export-biased" or "import-biased" growth, a common distinction in trade theory, because what matters here is the bias in northern productivity growth within the goods, $[m, u]$, all of which are exported to South.

Southern Productivity Improvement: $g(z) > 0$ and $g^*(z) = 0$

Let us now consider the effect of a productivity improvement in South. In figure 2, this shifts the vertical line to the right and the downward-sloping curve upward, and hence m increases unambiguously. That is, with improved technology in South, some industries migrate from North to South. The effect on w, u, and u^* is, on the other hand, ambiguous. To see what is involved, note that, from (19)–(21),

$$a(m)dm = \int_0^m g(s)a(s)ds > 0, \tag{23}$$

$$dw = -\xi(m)dm + wg(m), \tag{24}$$

and

$$a^*(u)du = (1 - n)dw + w\int_0^m g(s)a(s)ds. \tag{25}$$

From (23), (24), and (10), equation (22) becomes

$$a^*(u^*)du^* = \frac{n\xi(m)}{a(m)}\int_0^m g(s)a(s)ds + w\int_0^m [g(s) - g(m)]a(s)ds. \tag{26}$$

If the change is uniform across the export sectors, $g(z) = g$ for all $z \in [0, m]$, the expressions above are simplified to

$$a(m)dm = ng > 0,$$

$$\frac{dw}{w} = -\frac{\xi(m)}{w}dm + g = \left[1 - \frac{n\xi(m)}{wa(m)}\right]g < g,$$

$$a^*(u)du = (1 - n)dw + nwg = \left[w - \frac{n(1 - n)\xi(m)}{a(m)}\right]g,$$

and

$$a^*(u^*)du^* = \frac{n^2\xi(m)}{a(m)}g > 0.$$

With a uniform technological improvement in South, the terms of trade move in favor of North (since $dw/w < g$), and the cheaper southern goods allow the households in North to expand their consumption. The patterns of product cycles, the birth of new industries in North, $du^* > 0$, and the migration of some industries from North to South, $dm > 0$, thus emerge. Such product cycles do not appear in Flam-Helpman and Stokey's models. In Flam and Helpman's model, new goods are not introduced in North. In Stokey's model, the products at the upper end are dropped, as the northern households switch to cheaper southern products.

Even when the change is uniform across sectors, the effects on w and u are ambiguous. The reason is that the model imposes no restriction on $\xi(m)$, the slope of the downward-sloping curve in figure 2, and hence dw/w could take a value anywhere between $-\infty$ and g. If $\xi(m) > a(m)w/n = a^*(m)/n$, South's factor terms of trade deteriorate. If $\xi(m) > a^*(m)/n(1 - n)$, the deterioration is so large that the southern welfare declines and households are forced to cut back their consumption at the higher end, $du < 0$, generating the situation of immiserizing growth (see Bhagwati 1958).

Note the asymmetric effects of productivity improvements in North and in South. North cannot lose from its own productivity improvement, whereas South may lose from its own productivity improvement. Immiserizing growth is a possibility for South because it specializes in goods whose demand does not go up in response to a rising income. South's productivity improvement, without generating an increase in demand for its goods, reduces the demand for southern labor. In order to keep its workers fully employed, South must move into industries in which it has less comparative advantage, which could lead to a deterioration of the factor terms of trade. When South experiences an immiserizing growth, North captures more than 100 percent of the world's productivity gain. This cannot happen in the Flam-Helpman and Stokey models, where goods are gross substitutes and the lower prices, because of productivity growth, lead to an increase in demand. In their models, the southern household income and welfare rise after uniform productivity growth in South.

The result that North benefits from southern productivity growth depends on the uniformity of the change. As seen in equation (26), if $g(z)$ is sufficiently small over $[0, m]$ relative to $g(m)$, North could lose, $du^* < 0$, and South captures more than 100 percent of all the world's

productivity gain. In this case, North loses its industries at both ends of its spectrum. Such a situation may arise when the southern productivity growth is due to the technology transfer from North, because South has more to learn from North in sectors in which North has a greater absolute advantage, that is, a higher z. North could be worse off from such a technology transfer because there is less room left for taking advantage of the differences if South "narrows the technology gap" and becomes similar to North. (In the extreme case, if South succeeded in catching up to North completely and its technology became identical, the northern welfare would go down to the autarky level.) This point is general and holds true in any Ricardian model, including the Dornbusch et al. and Flam-Helpman and Stokey models. It should also be pointed out that what matters here is the bias across sectors within $[0, m]$, all of which produce goods that are exported to North. Hence, it is wrong to interpret in terms of export-biased or import-biased.

Global Productivity Improvement: $g(z) = g^*(z) > 0$

Finally, let us consider the global change, in which both North and South experience the same rate of productivity improvements in each sector, but the impact is not necessarily uniform across sectors. This can be analyzed by shifting the vertical line to the right, with the downward-sloping curve unperturbed in figure 2. Hence, $dw < 0$ and $dm > 0$. From (19)–(22),

$$a(m)dm = \int_0^m g(s)a(s)ds > 0,$$

$$dw = -\xi(m)dm < 0,$$

$$a^*(u)du = \left[w - (1 - n)\frac{\xi(m)}{a(m)}\right]\int_0^m g(s)a(s)ds + \int_m^u g^*(s)a^*(s)ds,$$

and

$$a^*(u^*)du^* = \left[w + n\frac{\xi(m)}{a(m)}\right]\int_0^m g(s)a(s)ds + \int_m^{u^*} g^*(s)a^*(s)ds > 0.$$

The effect on u is ambiguous because $\xi(m)$ and hence the effect on w can be arbitrarily large. On the other hand, $du^* > 0$, unambiguously. In spite of the fact that productivity improvement takes place worldwide, the asymmetry of demand response leads to a terms-of-trade movement against South, and the patterns of product cycles, where some industries

move from North to South and new industries are born in North, emerge ($dm,\ du^* > 0$).

D. Transfer Payments

Suppose that the transfer payments are made from North to South, financed by lump-sum taxes in North and distributed by lump-sum transfers in South. This has no effect on m and w. The new equilibrium would involve a trade deficit for South, equal to the transfer, and $du > 0$ and $du^* < 0$. The transfer has no effect on the prices because all the households, both in North and in South, spend their last income on northern goods. This would be different if there were poor households that could not consume northern-made goods, as will be seen in the following sections.

IV. North-South Trade: The Case of Heterogeneous Populations

Let us now examine the case in which there are nondegenerate distributions, $F(h)$ and $F^*(h^*)$, of incomes within each economy. Thus w and m are determined jointly by (2) and (8), and the consumption set and the utility level of each household are determined by (4) and (5). The case of heterogeneous populations is interesting when some households are so poor that, in equilibrium, they cannot afford to consume goods produced in North, that is, $wh < E(m)$ or $h^* < E(m)$ or, equivalently, $u(h) < m$ or $u^*(h^*) < m$. The existence of the poor households in North, those with $h^* < E(m)$, implies that in figure 1, the downward-sloping curve (2) intersects the upward-sloping part of the balanced trade curve (8). Let us also assume that the richest household in the world, whose consumption set determines the upward end of the goods produced in North, resides in North.

Population size.—An increase in m, or faster population growth in South, shifts the balanced trade curve to the right in figure 1, which leads to $dm > 0$ and $dw < 0$. North's terms of trade improve, all the northern households are better off, and new industries are born in North. Product cycle appears. The rich southern households, which consume Foreign imports, are worse off because of the terms-of-trade deterioration. On the other hand, the poor southern households, which cannot afford to buy Foreign imports, are unaffected because they essentially live in autarky, and their welfare is insulated from the terms-of-trade change.

Northern productivity improvement: $g(z) = 0$, $g^*(z) > 0$.—This shifts the downward-sloping curve down in figure 1, which leads to $dw < 0$. In contrast to the case of homogeneous populations, this also leads to $dm < 0$, and the rate of decline in w is less than $g^*(m)$. The reason is

that the poor northern households, whose marginal consumption good is a southern good, consume more southern goods when their income goes up. This increases the demand for labor in South. To keep South's labor market in balance, South specializes in a narrower spectrum of goods by abandoning the upper-end industries, which move from South to North. All the northern households are better off. In contrast to the case of homogeneous populations, the rich southern households gain if North's productivity growth is uniform across the sectors, which produce goods they import. The reason is that the decline in w is proportionately less than the productivity growth. If the productivity is biased, these households can be worse off. The poor southern households are unaffected.

Southern productivity improvement: $g(z) > 0$, $g^*(z) = 0$.—This shifts the downward-sloping curve up and the upward-sloping curve to the right, which leads to $dm > 0$; dw/w can be anywhere between $-\infty$ and $g(m)$. The poor southern households, insulated from the terms-of-trade change, are better off unambiguously. If productivity growth is uniform over $[0, m]$, then all the northern households are better off. In this case, new industries are born in North, and the patterns of product cycle emerge. If productivity growth is faster at m than at $[0, m)$, then North can be worse off. The effect on the rich southern households is ambiguous, even if the change is uniform. If the terms-of-trade deterioration is large, they can be worse off.

Global productivity improvement: $g(z) = g^*(z) > 0$.—This shifts the balanced trade curve to the right, whereas the downward-sloping curve is unaffected. Therefore, $dm > 0$ and $dw < 0$. All the northern households are better off, and new industries are born. Again, product cycle appears. The poor southern households are better off. The effect on the rich southern households is, however, ambiguous.

Income transfers.—Because of the coexistence of poor and rich households in each country—the poor who spend their additional income on southern goods and the rich on northern goods—aggregate variables, m and w, are generally affected when the transfer is made, whether it is across countries or within countries. Furthermore, such a transfer can have perverse welfare effects, in which donors may gain and recipients may lose. (For the literature of transfers and welfare, see Bhagwati, Brecher, and Hatta [1983] and the work cited therein.)

Let us consider South's domestic transfer policy, which redistributes income from the rich, whose marginal consumption good is an import from North, to the poor, whose marginal consumption good is a domestic good.[11] This policy shifts the upward-sloping curve up, which

[11] Such a domestic income distribution policy has been frequently proposed in many developing countries. The reason is not only that such a policy is believed to help to

leads to $dm < 0$ and $dw > 0$. The southern poor households are better off unambiguously because they are insulated from the terms-of-trade change. All the northern households are worse off because the terms of trade move against North. How about the rich households in South, whose incomes are taken away? Perhaps paradoxically, they may end up better off because of the improved terms of trade. To see this formally, suppose that northern households are homogeneous, with $h^* = 1$. There are two types of southern households: $N/2$ households with h_L and $N/2$ households with h_H, where $h_L < h_H$. Then if a transfer per household, equal to T measured in Home labor, is made from the rich to the poor in South, South's labor market equilibrium is

$$\frac{w(h_L + h_H)N}{2} = \frac{[w(h_L + T) + E(m)]N}{2} + N^*E(m)$$

or

$$\int_0^m a(s)ds = \frac{(h_H - T)(2 - n)}{n}.$$

Since the rich southern household's budget constraint is

$$w\int_0^m a(s)ds + \int_m^{u_H} a^*(s)ds = w(h_H - T),$$

the effect of an increase in T, evaluated at $T = 0$, is

$$dw = -\xi(m)dm = \frac{\xi(m)}{a(m)}\left(\frac{2 - n}{n}\right)dT > 0,$$

and

$$\begin{aligned} a^*(u_H)du_H &= -wdT + \left[\int_m^{u_H} a^*(s)ds\right]dw \\ &= \left\{\frac{\xi(m)}{a(m)}\left(\frac{2 - n}{n}\right)\left[\int_m^{u_H} a^*(s)ds\right] - w\right\}dT. \end{aligned}$$

Therefore, with a sufficiently large $\xi(m)$, the positive terms-of-trade effect offsets more than the primary effect of transfer. South's government can thus improve the welfare of all the southern households by adopting a "domestic" redistribution policy, which transfers income from the rich, who consume imports on the margin, to the poor, who consume domestic goods on the margin. By reversing the argument above, one can

alleviate the balance-of-payment problem by reducing the imports of foreign-made luxuries but also that a relatively large income disparity in some developing countries naturally generates political demands for redistribution.

also show that a redistribution from the poor to the rich in South can make all the southern households, including the rich, worse off.

Other types of transfer policies can be analyzed in a similar manner. For example, if North's government adopts a domestic policy of redistributing income from the rich to the poor, the resulting terms-of-trade deterioration can make all the households in North worse off, including the poor, who receive the transfer. South benefits from the terms-of-trade change. The effect of international aid, made from North to South, also depends critically on how the transfer is distributed within South. If it is distributed only to the rich households in South, an adverse terms-of-trade effect can eliminate much of the transfer's benefit to South.

V. A Multicountry World

One advantage of the present model over the Dornbusch et al. model is that it is relatively straightforward to extend the model to incorporate more than two countries. This section offers a sketch of how the analysis can proceed when there are many countries and points out some new issues that arise in a multilateral world.

Let J be the number of countries, with j being an index of country, $j = 1, 2, \ldots, J$, and $a_j(z)$ be country j's unit labor requirement in sector z, with the following property.

ASSUMPTION 4. For all $j = 1, 2, \ldots, J-1$, $a_{j+1}(z)/a_j(z)$ is continuous and strictly decreasing in $z \in [0, \infty)$.

Denote the wage rate in country j by w_j. Then country j produces only goods in $[m_{j-1}, m_j)$, where m_j is an increasing sequence, satisfying $m_0 = 0$,

$$\frac{w_j}{w_{j+1}} = \frac{a_{j+1}(m_j)}{a_j(m_j)}, \quad j = 1, 2, \ldots, J-1, \tag{27}$$

and $m_J = \infty$. The prices are given by $p(z) = w_j a_j(z)$ for $z \in [m_{j-1}, m_j)$. If there are N_j households in country j, with the distribution of skills given by $F^j(h_j)$, then the labor market equilibrium conditions in all the countries are given by

$$w_j N_j \int_0^\infty h_j dF^j(h_j) = \sum_k N_k \int_0^\infty \min\{w_k h_k - E(m_{j-1}), E(m_j) - E(m_{j-1})\} dF^k(h_k), \quad j = 1, 2, \ldots, J-1,$$

where $E(z) = \int_0^z p(s)ds$. Equations (27) and (28) jointly determine the equilibrium values of m_j and w_j ($j = 1, 2, \ldots, J-1$). The budget constraint of a household in country j with h_j units of labor endowment is

given by $E(u_j(h_j)) = w_j h_j$, which also determines the set of goods consumed by such a household, as well as the utility level achieved.

In what follows, the analysis is focused on the three-country case, $J = 3$, and each country is populated by homogeneous households with $h_j = 1$ $(j = 1, 2, 3)$. I make the following further assumption.

ASSUMPTION 5. $a_2(z)/a_1(z) < 1$ and $a_3(z)/a_2(z) < 1$ for all $z \in [0, \infty)$.

This assumption ensures that $w_1 < w_2 < w_3$. That is, a low-income country, 1, specializes in a lower spectrum of goods; a high-income country, 3, specializes in a higher spectrum; and a middle-income country, 2, specializes in an intermediate range. Let us choose country 3's labor as a numeraire, $w_3 = 1$. Then the initial task is to determine the equilibrium values of $m_1 < m_2$, $w_1 < w_2 < 1$, and u_1, u_2, and u_3.

From the budget constraint, $E(u_j) = w_j$, $w_1 < w_2 < 1$, and $u_1 < u_2 < u_3$. Since country 3 imports from country 1, country 1 must import some goods produced in country 2 $(m_1 < u_1)$, and country 2 must import from country 3 in equilibrium $(m_2 < u_2)$. Hence, there are two possible equilibrium configurations, depending on whether $u_1 > m_2$ or $u_1 < m_2$. In the first case, that is, $m_1 < m_2 < u_1 < u_2 < u_3$, all the households spend their marginal income on goods produced in country 3, and there is a two-way bilateral trade flow between each pair of countries. This case is similar in many ways to the case of two countries. On the other hand, in the second case, $m_1 < u_1 < m_2 < u_2 < u_3$, country 1 is not rich enough to be able to consume goods produced in country 3. Hence, country 1 runs a bilateral trade surplus vis-à-vis country 3, which in turn runs a bilateral trade surplus vis-à-vis country 2, which in turn runs a bilateral trade surplus vis-à-vis country 1. Furthermore, households in country 1 spend their marginal income on goods produced in country 2, and households in countries 2 and 3 spend their marginal income on goods produced in country 3.

The labor market equilibrium condition in country 1 is given by

$$w_1 N_1 = (N_1 + N_2 + N_3) E(m_1).$$

In the second case, the labor market equilibrium in country 2 is

$$w_2 N_2 = N_1[w_1 - E(m_1)] + (N_2 + N_3)[E(m_2) - E(m_1)].$$

From

$$E(m_1) = w_1 \int_0^{m_1} a_1(s)\,ds$$

and

$$E(m_2) - E(m_1) = w_2 \int_{m_1}^{m_2} a_2(s)ds,$$

the equilibrium is determined jointly by the following four conditions, which can be solved recursively:

$$\int_0^{m_1} a_1(s)ds = n_1,$$

$$\int_{m_1}^{m_2} a_2(s)ds + \frac{w_1}{w_2} n_1 = \frac{n_2}{n_2 + n_3},$$

$$\frac{w_1}{w_2} = \frac{a_2(m_1)}{a_1(m_1)},$$

and

$$w_2 = \frac{a_3(m_2)}{a_2(m_2)},$$

where n_j is the share of country j. From these conditions, $E(m_2) > E(u_1) = w_1$ can be shown to be equivalent to

$$\frac{n_2}{n_2 + n_3} > \frac{w_1}{w_2} = \frac{a_2(m_1)}{a_1(m_1)}.$$

Since m_1 is increasing in n_1, the right-hand side is decreasing in n_1. Hence, the second case, where the low-income country cannot afford to import from the high-income country and there are bilateral trade imbalances, is likely to occur if country 1 is larger, which lowers its income, and/or if country 2 is larger relative to country 3, which means that country 2 produces a wide range of goods in the middle spectrum. Fixing the country sizes, making country 1's technology worse or country 2's better, has the same effect, thereby making the second case more likely.

In the second case, transfers from country 2 or 3 to country 1 affect the terms of trade because country 1 spends the marginal income on country 2's goods and countries 2 and 3 spend the marginal income on country 3's goods. The detailed analysis is left as an exercise for the reader.

VI. Concluding Remarks

The Ricardian model of trade is a natural framework in which to investigate the roles of population size and technology in trade. Most existing studies in the literature assume that consumers have homothetic preferences for analytical convenience. The results obtained under the homotheticity assumption, which implies that all the goods have unitary income elasticities and that the rich and the poor consume all the goods in the same proportion, can be highly misleading when one is thinking about many issues in trade and development. This paper has developed an analytically tractable Ricardian model of trade under nonhomothetic preferences, which can be used to examine those issues that require a careful comparative statics analysis.

There are obviously many ways in which the analysis above can be extended. Only two will be mentioned here. First, the model assumed that each sector produces a homogeneous good. If each sector produces a continuum of goods, which are indexed in a manner similar to that used by Flam and Helpman (1987) and Stokey (1991), then there will be North-South trade both across and within sectors. This seems a most natural way of integrating the Flam-Helpman and Stokey models into the present framework. Second, one may endogenize productivity changes. Throughout this paper only the question of how technology affects trade is discussed. The exogeneity of technology makes it impossible to address the question of how trade affects technology. It is highly desirable to introduce learning by doing and research and development in the present model, along the lines pursued by the recent literature.[12] Such an extension is indeed essential to examine the validity of the argument made by industrial policy advocates, who believe that the income elasticity of demand is one of the key criteria for industrial targeting.

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[12] See Lucas (1993) and Grossman and Helpman (1995), which discuss various alternatives of endogenizing technologies. See also Matsuyama (1999*b*), which developed a closed economy model of sector-specific learning by doing, with the preference structure similar to that of the present model.

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