Chapter 10

Property Value Models

Economists have documented the relationship between the prices of housing units and quantities of environmental amenities since before this relationship was recognized as an application of the theory of hedonic prices (for example, Ridker and Henning 1967). Indeed, examples of the statistical analysis of the linkage between farmland prices and the characteristics of the land can be found as early as 1922. See Colwell and Dilmore (1999) for a review. The past 35 years have seen an explosion of both theoretical and empirical studies of the monetary values of nonmarket amenities and disamenities based on hedonic price theory. It is now well accepted that housing price differentials do reflect differences in the quantities of various characteristics of housing and that these differentials have significance for applied welfare analysis. For example, Smith and Huang (1995) conducted a meta-analysis of hedonic studies of air pollution and housing prices. They reported finding 37 studies and more than 160 separate estimates of the effects of air quality on housing prices.

At the same time, much has also been learned about the limitations of standard hedonic property analysis. As will be discussed later, the bulk of the empirical literature has focused on estimating the hedonic price function itself (that is, the equilibrium relationship between a home's prices and its characteristics). This "first stage" in the hedonic model can be used, under certain conditions, to infer the *marginal* willingness-to-pay (MWTP) for individual housing characteristics, including environmental amenities. While such information is valuable, most environmental policy scenarios envision discrete changes for which marginal analysis is no longer sufficient. Hedonic analysis can be extended to value discrete changes, but such "second stage" hedonic analyses are rare in practice, requiring complex econometrics and/or strong a priori restrictions on preferences (see Palmquist 2005).

In part, responding to this limitation of traditional hedonic price analysis, the "equilibrium sorting" literature has emerged in the past decade as a competing paradigm for characterizing how amenities are capitalized into the value of a property. Rather than focusing on a largely reduced form characterization of the hedonic price function, the sorting literature emphasizes the sorting process itself; that is, how the distribution of both individual characteristics and preferences interact with the distribution of housing characteristics in a market to determine market prices. Having characterized the sorting process, an analyst can, in theory, examine both the partial and general equilibrium implications of a discrete change in the distribution of property amenities. Smith et al. (2004), for example, estimated the impact of the projected ozone reductions in the Los Angeles air basin stemming from the 1990 Clean Air Act Amendments. The authors found that estimates of these impacts, and their distribution in the population, changed substantively depending upon whether or not individuals are allowed to re-sort (that is, move) and property values are allowed to change in response to the discrete air quality improvements. Such changes are indicative of the long run, general equilibrium impact of an environmental policy. The sorting literature, of course, is not without its limitations, requiring strong assumptions regarding the structure and distribution of preferences.

This chapter provides a summary of the current state of knowledge regarding these two approaches to property value modeling: hedonic price and equilibrium sorting. The first section focuses on hedonic pricing, beginning with a brief review of the evolution of economic thinking about property prices and environmental amenities, followed by a detailed exposition of the hedonic property value model. This presentation includes discussions of: (a) problems in estimating the hedonic price function; (b) approaches to recovering information on preferences and the demands for characteristics from the hedonic price function; and (c) the measurement of welfare change. The second section turns to the more recent equilibrium sorting literature. The distinction is made between what Kuminoff, Smith, and Timmins (2013) refer to as the "pure characteristics" sorting models of Epple and Sieg (1999) (among others) and the random utility sorting models pioneered by Bayer, McMillan, and Reuben (2004).

Hedonic Pricing

Historical Background

The theory of rents holds that the equilibrium price for a parcel of land will be the present value of the stream of rents produced by the land. Economic theory has long recognized that the productivity of land differs across sites. These productivity differentials will yield differential rents to land, and therefore differential land values. Where land is a producer's good, competition and free entry are sufficient to ensure that productivity differentials are fully reflected in the land rent structure. For any property where the land rent is less than the productivity, the activity occupying that land must be earning a profit. Some potential entrant will be willing to bid above the going rent in order to occupy that site and reap the rewards of a superior productivity. It is this competition that bids up land rents and eliminates the profit. Rent differentials will be equal to productivity differentials; and since the price at which a unit of land sells in the market is the present value of the stream of future rents, productivity differentials will also be reflected in land prices.

Some environmental characteristics such as air or water quality may affect the productivity of land as either a producer's good or a consumer's good. Where this is so, the structure of land rents and prices will reflect these environmentally determined productivity differentials. These results from classical rent theory aroused considerable interest among economists about the possibility of using data on land rent or land value for residential properties to measure the benefits to households brought about by improvements in environmental characteristics such as air or water quality. Ridker (1967) was the first economist to attempt to use residential property value data as the basis for estimating the benefits of changes in measures of environmental quality such as air pollution. He reasoned as follows:

> If the land market were to work perfectly, the price of a plot of land would equal the sum of the present discounted streams of benefits and costs derivable from it. If some of its costs rise (for example, if additional maintenance and cleaning costs are required) or if some of its benefits fall (for example, if one cannot see the mountains from the terrace), the property will be discounted in the market to reflect people's evaluation of these changes. Since air pollution is specific to locations and the supply of location is fixed, there is less likelihood that the negative effects of pollution can be significantly shifted on to other markets. We should therefore expect to find the majority of effects reflected in this market, and we can measure them by observing associated changes in property values.
>
> (Ridker 1967, 25)

The last sentence of the passage raises three questions. The first is whether environmental variables such as air pollution do systematically affect land prices. Assuming an affirmative answer to this question, the second is whether knowledge of this relationship is sufficient to predict changes in land prices when, say, air pollution levels change. The third question is whether changes in land prices accurately measure the underlying welfare changes.

Ridker (1967) and Ridker and Henning (1967) provided the first empirical evidence that air pollution affects property values by regressing median census tract property values in an urban area on a measure of sulfate air pollution. They then asserted positive answers to the second and third questions. Specifically, they argued that the coefficient on the air pollution variable in the regression equation could be used to predict the change in the price of any residence, conditioned on a change in its air pollution level. The sum of all such changes, they argued, could be taken as a measure of the benefit of improving air quality in an urban area (Ridker 1967, 136–137; Ridker and Henning 1967, 254).

This work stimulated a now large literature on the proper theoretical interpretation of the observed air pollution–property value relationship. Early contributions included those by Freeman (1971, 1974a, 1974b), and Anderson and Crocker (1972). Subsequent efforts to provide a sound theoretical basis for interpreting the air pollution–property value relationship have taken one of two paths. The first has been the development of models of the urban land market to determine whether and under what circumstances changes in aggregate land values accurately measure the benefits associated with environmental improvements. Early efforts in this direction included those by Strotz (1968), Lind (1973), Pines and Weiss (1976), Polinsky and Shavell (1976), and Kanemoto (1988). Although some of these models can be given an interpretation in the context of hedonic price theory, they do not lend themselves to empirical application, so they are not covered in this book. For further discussion of this branch of the literature, see Bartik and Smith (1987) and Palmquist (1991).

The second path drew on hedonic price theory, introduced in Chapter 4, to interpret the derivative of the cross-section regression equation with respect to air pollution as a marginal implicit price, and therefore, a marginal value for the air quality improvement (see Freeman 1974b and Rosen 1974). This section describes how hedonic price theory provides a basis for deriving welfare measures for public goods from observed differences in the prices of houses. The primary emphasis is on model specification and interpretation rather than econometric estimation. The goal of the section is to provide an overview of the methods of welfare measurement based on hedonic price theory and to identify the major conceptual issues. For other current treatments of these matters with greater emphasis on econometric and estimation issues, see Palmquist (2005) and Taylor (2003).

The Basic Hedonic Property Value Model

Assume that each individual's utility is a function of that person's consumption of a composite commodity z and a vector of amenities ($\boldsymbol{Q}$) associated with the house that the person occupies. These amenities, of course, include the structural characteristics of the house (such as size, number of rooms, age, and type of construction). They also include characteristics of the neighborhood in which the house is located (such as quality of local schools, accessibility to parks, stores, and work place, and crime rates), as well as location-specific environmental amenities (such as the local air and water quality).

Any large area has in it a wide variety of sizes and types of housing with different structural, neighborhood, and environmental characteristics. An important assumption of the hedonic technique is that the area as a whole can be treated as a single market for housing services. Individuals must have information on all alternatives and must be free to choose a house anywhere in the market. It is as if the area were one huge supermarket offering a wide selection of varieties. Indeed, Rosen's (1974) model assumed that the household can choose from a continuum of housing attributes, an assumption that is, at best, approximated in actual housing market settings.

Since the focus here is on the values of characteristics to buyers of houses, there is no need to model formally the supply side of this market. Instead, assume that the housing market is in equilibrium—that is, that all individuals have made their utility-maximizing residential choices given the prices of alternative housing locations, and that these prices just clear the market given the existing stock of housing and its characteristics. Under these assumptions, the rental price of the *j*th residential location can be taken to be a function of the structural, neighborhood, and environmental characteristics of that location. In other words,

$$R_j = R\left(\boldsymbol{Q}_j\right). \tag{10.1}$$

As explained in Chapter 4, this relationship can be linear in a characteristic if repackaging of that characteristic is possible. However, in general this need not be the case. Two living rooms with six-foot ceilings are not equal to one living room with a twelve-foot ceiling. Where repackaging is not possible, equation (10.1) will be nonlinear.

To model the problem more formally, consider an individual who occupies house *j*. Her utility is given by

$$u = u\left(z, \boldsymbol{Q}_j\right), \tag{10.2}$$

where z is a Hicksian composite good with a price of 1. This assumption makes the demands for characteristics independent of the prices of other goods, a convenient property for empirical work. The individual maximizes $u(\cdot)$ subject to the budget constraint:

$$M - R(\boldsymbol{Q}) - z = 0. \tag{10.3}$$

A typical first-order condition for the choice of amenity q (an element in $\boldsymbol{Q}$) is

$$\frac{\partial u/\partial q}{\partial u/\partial z} = \frac{\partial R(\boldsymbol{Q})}{\partial q}. \tag{10.4}$$

Assume now that the hedonic price function $R(\boldsymbol{Q})$ has been estimated for an urban area. Its partial derivative with respect to any of its arguments, for example q, gives the implicit marginal price of that characteristic—that is, the additional amount that must be paid by any household to move to a housing bundle with a higher level of that characteristic, other things being equal. If this function is nonlinear, the marginal implicit price of a characteristic is not constant, but depends on its level and perhaps the levels of other characteristics as well. If the individual is assumed to be a price taker in the housing market, that person can be viewed as facing an array of implicit marginal price schedules for various characteristics. An individual maximizes utility by simultaneously moving along each marginal price schedule until she reaches a point where her marginal willingness to pay for an additional unit of that characteristic just equals the marginal implicit price of that characteristic. If an individual is in equilibrium, the marginal implicit prices associated with the housing bundle actually chosen must be equal to the corresponding marginal willingness to pay for those characteristics.

Panel A in Figure 10.1 shows the partial relationship between q and $R(\boldsymbol{Q}) = R\left(q, \boldsymbol{Q}_{-q}\right)$ as estimated from equation (10.1), where $\boldsymbol{Q}_{-q}$ denotes all of

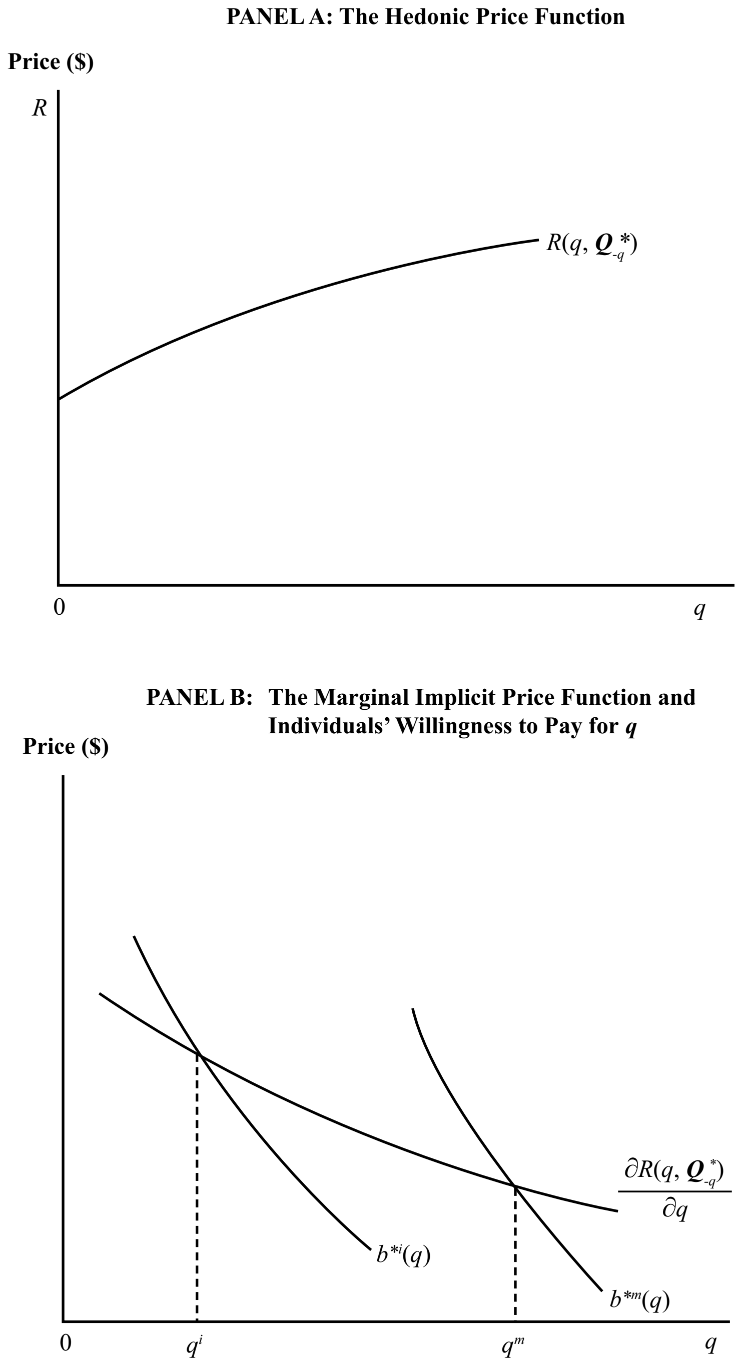

Figure 10.1 Property prices and environmental quality

the attributes except q. In the figure, $\boldsymbol{Q}_{-q}$ is fixed at some level $\boldsymbol{Q}^*_{-q}$. Panel B shows the marginal implicit price of q, $\partial R\left(q, \boldsymbol{Q}^*_{-q}\right) / \partial q$. It also shows the marginal willingness-to-pay curves for two individuals, i and m, who have chosen utility maximizing bundles of housing characteristics, labeled $b^{*i}(q)$ and $b^{*m}(q)$. These curves show each individual's marginal willingness to pay for changes in the characteristic, holding utility constant at the level achieved by maximizing equation (10.2) subject to (10.3). Let this level be u^*. Both individuals in Figure 10.1 have chosen locations where their marginal willingness to pay for q is equated with its marginal implicit price.

The analysis described here results in a measure of the price of, and the marginal willingness to pay for, q, but it does not directly reveal the marginal willingness-to-pay function. The fundamental problem is that, for any one individual i, we observe only a single point along their marginal willingness-to-pay function, namely the intersection between $b^{*i}(q)$ and $\partial R\left(q, \boldsymbol{Q}_{-q}\right) / \partial q$. The second stage of the hedonic technique seeks to combine the quantity and implicit price information, together with restrictions on the structure of preferences, in an effort to identify the marginal willingness-to-pay function for q. The individual's demand price or willingness to pay for q is a function of the level of q. Since there may be substitute and complementary relationships among characteristics, the willingness to pay for q may also depend on the levels (or marginal implicit prices) of other characteristics. It is convenient to assume that the utility function is weakly separable in housing so that prices of other goods can be omitted in the specification of the marginal willingness-to-pay function. Also, it is convenient to assume that each individual purchases only one housing bundle. Purchasing more than one would necessitate that the bundles be identical, or that the hedonic price function be linear in all characteristics. This is because there can be only one marginal implicit price recorded for each individual for each characteristic.

Given these assumptions, for the ith individual we can derive a marginal willingness-to-pay function for q by differentiating the expenditure function, as shown in Chapter 4. The result is

$$b^{*i} = b^{*i}\left(q, \boldsymbol{Q}^*_{-q}, u^*\right). \tag{10.5}$$

If equation (10.5) can be estimated, it can be used to estimate the welfare change of an individual associated with changes q, assuming that other things are held equal. Specifically, if the quantities of other characteristics and amenities do not change, the welfare change can be found by integrating b^{*i} over the relevant range of the change in q. However, a change in the quantity of one characteristic can result in changes in the quantities of other characteristics the individual chooses and in changes in the hedonic price function itself. The task of welfare measurement when individuals can fully adjust to the new supply of amenities and characteristics is discussed below in the section Measuring Welfare Changes.

Estimating the Hedonic Price Function

The Dependent Variable

In the discussion so far, the hedonic price has been the annual rental price of the property. What is observed usually is the purchase price of the house, which will be denoted as P_h for the rest of this chapter. P_h can be interpreted as the discounted present value of the stream of expected rental values; but this leads to two complications. First, when housing price differentials are used to estimate welfare changes that are usually expressed as annual flows, care must be taken to convert the house price measures into the appropriate temporal dimension. This topic is discussed in a later section. Second, it might be necessary to take account of expected changes in the characteristics of a house, especially environmental changes when estimating and interpreting the hedonic price function. For example, if air pollution at a given location is expected to improve over time, the present price of the house should be bid up to reflect not only the current conditions but the expected improvement as well.

One question to be asked is whether the dependent variable to be explained should be a pure site or land value or the full price of the house and land together. Since the environmental amenities of interest are location specific but not a part of the structure, the values of the environmental amenities should be reflected in the price of land alone. However, at least in the United States, land is not usually traded separately from the structures placed upon it, so the observed prices reflect the values of both the land and its structural improvements. This causes no problems at the theoretical level, but it does require that the hedonic price equation adequately control for structural characteristics.

Another question concerns the source of data on housing prices. Data on actual market transactions are preferable. For rental housing there is a regular monthly "market transaction," from which fairly accurate data on housing rents could be gathered. However, the majority of residential housing is owner-occupied, and only a small percentage of the total owner-occupied housing stock is exchanged through the market each year. The most preferred source of data is systematically collected information on actual sales prices of individual dwellings, along with relevant characteristics. Fortunately, in many parts of the country these data are collected by multiple-listing services and tax assessing agencies. Increasingly, these data are available electronically, either directly from the primary source (for example, online sites for individual county assessor offices) or from third-party data aggregators (for example, Dataquick). In recent years, virtually all published hedonic property value studies have used micro data on individual transaction prices.

An alternative source of property value data would be professional appraisals of individual properties constructed for taxation or other purposes. The appraisals themselves are typically based on one of two approaches: (a) a sales approach that assesses a home's value based on transaction prices for similar homes in the same area, and (b) a cost approach that assesses the cost of replacing the home. Some jurisdictions have developed computer-based systems of appraisals, made for tax purposes, which include data not only on appraised values but also on a variety of structural and site characteristics. For citations to studies examining the accuracy of appraisals and tax assessments, see Kiel and Zabel (1999, Table 1).

There are two fundamental problems with the use of appraisal data. First, to the extent that a sales approach is used, the data will mask underlying variability in property values, ignoring idiosyncratic factors influencing a home's value. Indeed, as the sales approach often involves a statistical analysis of recent transactions, the resulting assessments can be viewed as providing fitted values from a first stage hedonic regression. Subsequent analysis of these assessed values will be "successful" (that is, in the sense of fitting the data) only to the extent that the researcher happens to choose a functional form that mimics what was used by the assessor. Second, for assessed values based on a cost approach, the reported home prices reflect only the cost side of an equilibrium price function.

There are, of course, concerns with individual transaction data. For example, it is important to ensure that the transactions reflect so-called "arms-length" transactions, eliminating sales between individuals who are related or those in which the sales price has been altered for reasons unrelated to the home's underlying value. It is also good to keep in mind the underlying assumption in hedonic property analysis—that the housing market is in equilibrium (that is, that all opportunities for possible gains from further trade at the revealed set of prices have been exhausted). This is a heroic assumption, because buyers and sellers often operate with substantial ignorance about the true willingness to pay and willingness to accept offers of other potential buyers and sellers. Sellers typically state an asking price that effectively truncates potential offers at that price. Sellers must choose to accept or reject offers more or less at the time they are received, without knowing when (or even if) a higher offer might come along; and buyers lack information on possible prior bids made by others for a given property (Horowitz 1986). Horowitz developed an alternative model of the bidding and acceptance strategies of buyers and sellers, and estimated both a standard hedonic model and his alternative model with the same data set. He found that the statistical performance of his bidding model was substantially superior to the standard model in predicting sales prices. However, since he used principal components in his estimation procedure rather than actual attributes, it is not possible to analyze the impact of his alternative modeling strategy on marginal implicit prices for environmental attributes.

Explanatory Variables

In choosing the appropriate explanatory variables, the first question to be addressed is the way in which environmental amenities and location characteristics enter the hedonic price function. The typical practice has been to enter a simple scalar measure of an amenity—for example, parts per million (ppm) of an air pollutant, or distance to a park. However, Parsons (1990) showed that this practice is not consistent with a restriction imposed on the hedonic price function by profit maximizing behavior on the supply side of the housing market. The implication of profit maximization is that the effect of the environmental amenity can only be captured without bias by weighting the amenity by the area of the lot on which the house sits. The restriction is that if an area of land of given q is developed and sold in two or more different lot sizes, the prices of the lots must be such that the return per acre is independent of the sizes of the lots. For example, if two one-acre lots sell for $\$X$ apiece, one two-acre lot must sell for $\$2X$. This means that the premium on lots with higher levels of q must be twice as high for the two-acre lot compared with the one-acre lots. The higher premium on the larger lot is necessary to compensate the landowner for the forgone opportunity to capture two premiums with the smaller lots.

Although the argument is correct in principle, there is some question about its relevance in practice. As Parsons points out, once lots are developed, the cost of changing the size of lots on which houses sit may be too high to force amenity premiums to take the weighted form in the secondhand market for houses. Although Parsons showed that biased estimates of implicit prices for characteristics are possible, few empirical researchers have used his proposed weighted amenity values.

The levels of some environmental amenities are fixed by location, while the levels of others, especially those related to air quality, vary over time with changes in emissions and meteorological conditions. With time-varying amenities, there is the question of how best to represent the level of the amenity in the regression equation. The typical practice in air pollution–property value studies has been to use the annual mean as a summary statistic. However, Murdoch and Thayer (1988) have shown that in the case of visibility, using more information on the probability distribution of visual range improves the statistical performance of the hedonic price function.

A number of other conceptual and practical issues must be resolved in the course of selecting a set of explanatory variables for a hedonic price function. These include:

- Which measures of environmental quality should be used to characterize environmental amenities?
- Is it possible to separate the effects of different amenities on property values when measures of the amenities are correlated?
- What objective data best capture "neighborhood" characteristics?
- Does the spatial scale of the socioeconomic data often used in these studies correspond to peoples' perceptions of these characteristics? (For example, is there sufficient homogeneity within census tracts so that census tract means or medians adequately measure "neighborhood"?)
- Is there sufficiently close correspondence between peoples' perceptions of amenity levels (which presumably govern the choices reflected in property prices) and the objective measures of amenity levels that are available to the researcher?

Since the objective of the hedonic analysis is to determine the effect of one amenity on property values, other things being equal, a key issue is the control for structural, neighborhood, and other environmental variables. The issue is made more difficult by the likelihood of multicollinearity among housing characteristics. This raises the troublesome question of the tradeoff between increasing bias through the omission of variables that are correlated with the variable of concern and increasing the variance or imprecision of coefficient estimates when collinear variables are included. Theory does not provide any hard-and-fast answers to this question. Work by Atkinson and Crocker (1987) and Graves et al. (1988) suggested the value of approaching this question systematically, using Bayesian principles. These authors have also examined the effects of errors in the measurement of other explanatory variables on the estimates of the coefficients on the environmental variables of concern.

Finally, hedonic property analysis has often been employed to assess the impact of NIMBY ("not in my backyard") sites, such as toxic waste facilities (Kohlhase 1991), incinerators (Kiel and McClain 1995), Superfund sites (Kiel 1995), and animal confinement units (Palmquist, Roka, and Vukina 1997; Herriges, Secchi, and Babcock 2005). A difficulty in this setting is that a property may be exposed to multiple NIMBY sites, raising the question as to which site or sites to include in the analysis, and how to quantify their joint and marginal impacts. Most studies focus on the nearest site and assume that its marginal impact diminishes with distance. However, for some externalities, distance alone is a poor proxy for exposure. In the context of animal confinement units, for example, exposure to the odor generated by the various facilities can depend upon the prevailing winds. Cameron (2006) developed a more general representation of the hedonic price function that allows the marginal impact of a single site to depend upon its directional location relative to the property and to identify the direction from which the property value gradient is highest. The approach, however, still requires that the analyst focus on a single NIMBY site.

Functional Form

Functional forms for the hedonic price function that have been proposed or used in the literature include the linear, the quadratic, the log-log, the semi-log, the inverse semi-log, the exponential, and the Box–Cox transformation. The first step in choosing a functional form is to see what theory can tell us. According to theory, a hedonic price function is an equilibrium relationship derived from the interaction of individuals' preferences and suppliers' cost or profit functions. The only obvious general restriction on the form of the hedonic price function is that its first derivative with respect to an environmental characteristic be positive (negative) if the characteristic is a good (bad).

Rosen (1974) and Epple (1987) showed that it is possible to solve for the hedonic price function analytically after making specific assumptions about the form of individual utility functions and the distribution of suppliers' characteristics (Rosen) or the exogenous supply of housing characteristics (Epple). These analytical solutions are only possible for a very limited set of assumed forms of preferences and supply. For example, Rosen assumed that individuals' utility functions were linear. This is not an attractive assumption, especially if the ultimate objective of the analysis is to measure welfare values for changes in supplies of environmental characteristics.

Early researchers tried alternative functional forms for the hedonic price function (typically the semi-log, inverse semi-log, and log-linear) and selected one on the basis of goodness-of-fit criteria. Goodman (1978) was one of the first to experiment with a flexible functional form. He employed a Box–Cox transformation of the dependent variable:

$$P_h^{[\lambda]} = \frac{P_h^{\lambda} - 1}{\lambda}. \tag{10.6}$$

For $\lambda = 1$, this is a simple linear function. As λ approaches zero, this becomes the semi-log form. Some authors have found estimates of λ that were significantly different from both zero and one, indicating that this more complicated form fits the data better than either the linear or semi-log forms.

Transforming only the dependent variable still produces a very limited range of possibilities. Halvorsen and Pollakowski (1981) proposed estimating what they called a quadratic Box–Cox functional form. It would have the form

$$P_h^{[\lambda]} = \alpha_0 + \sum_{j=1}^{J} \alpha_j \cdot q_j^{[\gamma]} + \tfrac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{J} \beta_{jk} \cdot q_j^{[\gamma]} \cdot q_k^{[\gamma]}, \tag{10.7}$$

where j and k index the characteristics and λ and γ are estimated from the data.

Cassel and Mendelsohn (1985) pointed out that for welfare analysis, it is not the goodness-of-fit of the hedonic price function that matters; rather, it is the estimate of the marginal implicit price of the environmental attribute. In the regression equation for housing price, the environmental variable is likely to have relatively little influence in determining the estimated magnitude of γ. However, the estimate of γ would have a major impact on the estimated marginal implicit price of the environmental characteristic.

A more general flexible form would be an extension of the quadratic Box–Cox of equation (10.7) to

$$P_h^{[\lambda]} = \alpha_0 + \sum_{j=1}^{J} \alpha_j \cdot q_j^{[\gamma_j]} + \tfrac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{J} \beta_{jk} \cdot q_j^{[\gamma_j]} \cdot q_k^{[\gamma_k]}. \tag{10.8}$$

This functional form allows for different transformations for each independent variable.[1] This general form may not be estimable when there are a large number of characteristics. A compromise, proposed by Palmquist (1991), would be to set $\gamma_j = \gamma_k$ for all $j,k = 1, \ldots, n-1$, where n indexes the environmental attribute of

1 For dummy variables (for example, indicating whether or not a housing unit is in a particular neighborhood), γ_j cannot be identified, and is simply set equal to one.

interest. This would be responsive to the point raised by Cassel and Mendelsohn. Allowing for a separate transformation of the environmental amenity should give better results.

One question about functional form is whether the form chosen allows the marginal implicit price of the environmental characteristic to depend on the levels of the other attributes of houses. Of the commonly used functional forms, only the log and the Box–Cox transformation make the implicit prices of characteristics depend on the levels of other characteristics—the other forms impose independence. However, this is a question that should be answered by the data, not by assumption.

One of the early efforts to systematically consider the question of functional form was a study by Cropper, Deck, and McConnell (1988). The authors simulated the performance of a housing market using real data on buyer and housing characteristics drawn from the Baltimore, Maryland, area. After specifying the functional form and parameters of individuals' utility functions and the distribution of characteristics that reflect taste differences across individuals, they solved the assignment problem, producing a housing market equilibrium with each house being sold to the individual with the highest willingness to pay for its bundle of characteristics. With knowledge of the utility function parameters, it was then possible to calculate the true marginal implicit price for each individual and for the mean across all individuals. The authors used the equilibrium prices to estimate six alternative functional forms for the hedonic price function. They were then able to compare the mean true marginal bids with the bids calculated from each hedonic price function. They found that when all of the housing characteristics were included in the hedonic price function, the linear and quadratic versions of the Box–Cox transformation provided the most accurate estimates of marginal implicit prices. However, when they experimented with various forms of incorrectly specified hedonic price functions (by omitting variables or using proxy variables) they found that the linear version of the Box–Cox transformation was consistently superior in generating marginal implicit prices.

For over 20 years, this research has provided a rationale for relying on relatively simple functional forms when estimating hedonic price functions. More recently, however, Kuminoff, Parmeter, and Pope (2010) revisited this issue in light of newer econometric techniques, including spatial fixed effects and quasi-experimental methods, designed to control for omitted variables bias. The authors undertook an extensive Monte Carlo analysis evaluating over 540 different hedonic models, with a range of underlying utility functions and assumed functional forms for the price functions, as well differences in the nature of controls used by analyst to deal with omitted variables. Their overall conclusion was that "the more flexible specifications for the price function, such as the quadratic Box–Cox model, outperform the linear, log-linear, and log-log specifications that have dominated the empirical practice for the past two decades" (Kuminoff, Parmeter, and Pope 2010, 159).

The Hedonic Price Function as a Market Equilibrium

Interpreting the marginal implicit prices as measures of households' marginal willingness to pay requires the assumption that each household is in equilibrium with respect to a given vector of housing prices and that the vector of housing prices is the one that just clears the market for a given stock of housing. These conditions assure that the hedonic price function is the price vector that makes all participants in the market in aggregate just willing to hold the existing stock of housing. For these two aspects of equilibrium to be fully achieved, we require first that households have full information on all housing prices and attributes and that their transactions and moving costs be zero; and second, that the price vector adjust instantaneously to changes in either demand or supply. The market for housing can be viewed as a stock-flow model where the flow (change in stock) is a function of prices, but the prices at any point in time are determined only by the stock at that point in time.

This idealized model is clearly not an accurate representation of real-world housing markets. However, in evaluating the strength of this criticism of the hedonic price model, we must focus on several distinct issues. One issue concerns the speed of adjustment of the market to changed conditions of supply and demand. If adjustment is not complete, then observed marginal implicit prices will not accurately measure household marginal willingness to pay. A major question is whether imperfect adjustment will lead to systematic biases in estimates of willingness to pay.

Consider households' imperfect adjustment to changing prices. First, an increase in housing prices need not affect the marginal implicit prices of attributes, in which case no adjustment of the attribute bundles is necessary. Even if marginal implicit prices change, households will not move unless the potential utility gains to returning to full equilibrium exceed the information costs, transactions costs, and moving costs associated with the change. These costs help to define a band within which observed marginal implicit prices can diverge from household marginal willingness to pay for housing attributes. If housing prices change so that the marginal implicit price schedule for an attribute moves consistently in one direction, households will consistently lag in their adjustment to that change; and the marginal willingness to pay will be overstated or understated according to whether the marginal implicit price is rising or falling.

A second issue concerns expectations about future environmental amenity levels. Market prices for long-lived assets such as housing reflect the discounted present value of the stream of expected future services from that asset. A change in expectations about future environmental amenity levels can affect housing prices and marginal implicit prices independently of the present level of these amenities. For example, if there are widespread expectations of an improvement in air quality and the market adjusts reasonably quickly to these expectations, the price differential between presently dirty houses and clean houses should decrease. Correlating these prices with existing levels of air pollution would lead to an underestimate of the marginal implicit price of air quality.

Divergences from full equilibrium of the housing market in many circumstances will only introduce random errors into the estimates of marginal willingness to pay. However, where market forces are moving continuously in one direction or are expected to move in one direction, incomplete market adjustment, or full adjustment to changing expectations, or both, can introduce biases in both directions. We should be much more cautious about utilizing the cross-section hedonic price model in those cities and at those points in time during which market forces and environmental quality levels are changing rapidly (granted that "rapidly" is an imprecise term). However, it is also possible in these circumstances to determine the direction of bias. Thus, estimates of marginal willingness to pay derived from such studies can be labeled as an upper bound or a lower bound on the basis of that analysis.

Another issue concerns the possibility of corner solutions. If there is not a sufficiently wide variety of housing models available, corner solutions are likely. The hedonic price function defines an opportunity locus across attribute space. A household chooses a housing model such that its indifference surface is tangent to the given opportunity locus, provided that a model with that precise set of attributes is available. If the optimum model is not available, the household must pick the nearby housing model that gives the highest utility level; but then the first-order conditions for utility maximization are not satisfied as equalities (Mäler 1977, 361–362).

The hedonic model is based on an assumption that the implicit price function is differentiable and continuous. However, this is an artifact of the statistical and mathematical technique. If this assumption is not satisfied in practice, two sorts of problems can arise. The first problem is that the statistically fitted hedonic price function is a good approximation only when the number of housing units is large and there is more continuous variation in characteristics among units. A small number of distinctly different types of housing units might be better analyzed with one of the discrete choice models described later in this chapter. The second type of problem arises if there are no units available with particular combinations of attributes. If there are substantial gaps in the opportunity locus, some households will not be able to satisfy the first-order conditions as equalities. This could be a problem for certain subsets of the urban population.

Market Segmentation

Straszheim (1974) was the first to raise the question of market segmentation in the context of estimating hedonic price functions for housing within an urban area. He argued that the urban housing market really consisted of a series of separate, compartmentalized markets with different hedonic price functions in each. As evidence in support of the segmentation hypothesis, Straszheim showed that estimating separate hedonic price functions for different geographic areas of the San Francisco Bay area reduced the sum of squared errors for the sample as a whole.

For different hedonic price functions to exist in an urban area, two conditions must be met. The first is that the structure of demand, the structure of supply, or both must be different across segments—either buyers in separate submarkets must have different structures of demands, or the structure of characteristics of the housing stocks must be different. The second condition is that purchasers in one market segment must not participate significantly in other market segments. In other words, there must be some barrier to mobility of buyers among market segments that prevents arbitrage from occurring in response to differences in marginal implicit prices. Such barriers could be due to geography, discrimination, lack of information, or a desire for ethnically homogeneous neighborhoods. Even with buyer immobility, if demand and supply structures are the same they will produce similar structures of hedonic prices. Perfect mobility and information on the part of buyers will also eliminate differences in the implicit prices for any characteristic across market segments.

If market segmentation does exist, the hedonic price function estimated for the urban area as a whole will provide faulty estimates of the implicit prices facing subsets of buyers in different market segments. Thus, estimates of benefits and estimates of demand functions based on faulty price data will also be faulty. If market segmentation does exist, separate hedonic price functions must be estimated for each segment; and benefit and demand functions must be separately estimated for each segment with a different set of implicit prices.

It is not clear how significant the problem of market segmentation is for air pollution–property value studies within single urban areas—although there are enough positive results in the literature to suggest that it is not a problem that can be dismissed out of hand. Some authors have found evidence of different hedonic price functions for submarkets within larger urban areas, suggesting segmentation; however, this could be due to misspecification of the model, as others have not found evidence of segmentation in their data.

The existence of market segmentation does not render the hedonic price technique invalid; but rather, it makes application of the technique more difficult. If the appropriate basis for segmentation can be identified, it is conceptually possible to estimate separate implicit price functions for each submarket. Although these functions would be different across markets, they each would accurately reflect the outcome of the market processes in each submarket. Thus, the functions could be used to estimate equilibrium marginal willingness to pay.

Econometric Concerns

While the emphasis in this chapter has been on model specification and interpretation, a number of econometric developments in recent years are worth noting. Most of them stem from concerns regarding omitted variables. First, given the spatial nature of hedonic analysis, it is likely that the error terms in a hedonic price model are correlated over space, with the correlation being larger for housing units closer to each other, precisely because these units share common omitted variables. This so-called *spatial autocorrelation* is analogous to serial correlation in a time series setting, where in the latter case the correlation increases for observations closer to each other in time. Ignoring spatial autocorrelation will lead the researcher to overstate the precision with which individual parameters are estimated, potentially resulting in erroneous conclusions regarding statistical significance. A more problematic form of spatial dependence arises when the price of one unit depends upon the sales prices of other houses in the same region (known as spatial autoregression). Ignoring spatial autoregression can result in biased estimates of the price function parameters. Palmquist (2005) provided a helpful overview of the econometric techniques for handling both types of spatial dependence. Specific applications in the hedonic pricing literature include Bockstael (1996), Can (1992), and Bell and Bockstael (2000).

Second, as noted earlier, omitted variables can result in biased parameter estimates to the extent that the omitted factors are correlated with the regressors included in the hedonic price function. One approach to dealing with unobserved housing unit attributes (including locational amenities) is the use of repeat sales data (Palmquist 1982). Consider a housing price function given by

$$P_{hjt} = \alpha_0 + \sum_{k=1}^{K} \beta_k q_{kjt} + \xi_j + \varepsilon_{jt} = \alpha_0 + \sum_{k=1}^{K} \beta_k q_{kjt} + \eta_{jt}, \tag{10.9}$$

where P_{hjt} denotes the sales price of house j in time period t and q_{kjt} denotes the kth attribute for the same unit in time period t. The term ξ_j denotes unobserved attributes of the housing unit, which are subsumed into the error term $\eta_{jt} = \xi_j + \varepsilon_{jt}$. To the extent that ξ_j is correlated with the observable housing unit attributes (that is, the q_{kjt}), least squares regression will yield biased parameter estimates. However, if repeated sales data are available (say in time periods t and t'), then the two sales prices can be differenced, yielding a new regression equation:

$$\Delta P_{hj} \equiv P_{hjt'} - P_{hjt} = \sum_{k=1}^{K} \beta_k \left(q_{kjt'} - q_{kjt} \right) + \left(\varepsilon_{jt'} - \varepsilon_{jt} \right) = \sum_{k=1}^{K} \beta_k \Delta q_{kj} + \Delta \varepsilon_j. \tag{10.10}$$

In this case, by focusing on changes in housing prices over time for the same unit, any unobserved attributes of the house that are constant over time are differenced out.

The use of repeat sales data is not without its drawbacks. First, for any given market and time period, there will be fewer housing units that will have sold more than once, limiting the sample size available for the hedonic regression. Increasing the sample size by considering a larger market or a longer time period risks violating the assumption that a single hedonic price function applies for all the observations. Second, first differencing will also eliminate any observable site attributes that have remained constant over time. Third, if there are unobserved housing attributes that have changed over time, the potential for omitted variable bias remains, though it is likely to have been attenuated.[2]

In recent years, a variety of *quasi-experimental* techniques have been developed to eliminate or reduce the potential bias from omitted variables (see, for example, Angrist and Pischke 2008 and Imbens and Wooldridge 2009 for general treatments). Two prominent approaches are:

- *Difference-in-differences*: A natural measure of the impact that a locational amenity (for example, air quality) has had on property values is the difference (D_0) in housing prices before and after changes have occurred in that amenity. The risk, of course, is that other things have changed as well. In its simplest form, the difference-in-differences (DID) approach attempts to control for these other factors by also measuring the difference (D_0) in housing prices over the same time period for a control group that did not experience changes to the locational amenity of interest. The difference in these differences (that is, $D_1–D_0$) is assumed to eliminate the nuisance of other factors and isolate the impact of the locational amenity. Chay and Greenstone (2005) provided an example of the use of DID in measuring the impact of air quality on the housing market, while Kuminoff and Pope (2012) illustrated the risks in its use.
- *Spatial discontinuities*: The regression discontinuity approach provides a means of controlling for unobservables by examining outcomes (property values in the hedonic setting) on either side of an arbitrary threshold. For example, in considering the impact of school quality on the housing market, homes immediately on either side of a school district's boundary will share commonly unobserved locational attributes (such as crime rates, access to shopping, community "feel," etc.), with differences in property values attributable to differences in observable structural characteristics of the home and differences in the associated schools. Greenstone and Gallagher (2008) examined the impact of Superfund-sponsored cleanups on housing prices using a regression discontinuity approach, comparing housing prices in areas immediately above and immediately below the threshold to qualify for Superfund cleanup. Lee and Lemieux (2010) provided a general discussion of the technique.

Estimating Characteristics Demands

The attractiveness of the hedonic price model for applied welfare analysis lies in the potential for using estimates of individuals' marginal implicit prices for a characteristic to recover the uncompensated inverse demand function for q or information on the underlying structure of preferences. Rosen (1974) had argued

2 For a more extensive discussion of the repeat sales model, see Freeman (2003, 388–390).

that the inverse demand and marginal supply price functions could be estimated from the information contained in the hedonic price function in the following manner:

> … compute a set of implicit marginal prices … for each buyer and seller evaluated at the amounts of characteristics … actually bought or sold, as the case may be. Finally, use estimated marginal prices … as endogenous variables in the second-stage simultaneous estimation of [the inverse demand and supply price functions]. Estimation of marginal prices plays the same role here as do direct observations on prices in the standard theory and converts the second-stage estimation into a garden variety identification problem.
>
> (Rosen 1974, 50)

This suggestion has been the source of a large literature for more than three decades. Since the emphasis in this book is on models and basic economic method rather than on econometric issues, only an overview is provided here of the sources of problems in estimating and identifying demand functions for characteristics and alternative approaches to solving them.[3]

The difficulties in estimating and identifying the inverse demand functions from hedonic price data come in two forms. The first arises from the fact that the source of data for the dependent variable in the marginal willingness-to-pay function is not direct observation of the inverse demand prices; rather, it is the calculation of the marginal implicit price $\partial P_h / \partial q$ from the estimated hedonic price function. However, this variable is itself computed as a function of the same characteristics that are explanatory variables in the marginal willingness-to-pay function. Brown and Rosen (1982) and Mendelsohn (1987) showed that at least in some cases this procedure leads to parameter estimates for the marginal willingness-to-pay function that are identical to the estimated coefficients in the hedonic price function. As Brown and Rosen put it,

> Contrary to Rosen's original statement, we claim that marginal attribute prices constructed as above will not necessarily play the same role in estimation that direct observation on prices would play if they were available. Because such constructed prices are created only from observed sample quantities, any new information that they may provide (that is, any information beyond that already provided directly by observed sample quantities) can only come from a priori restrictions placed on the functional form of the price function $P_h(\cdot)$. In the absence of such additional restrictions, second-stage structural

3 Readers interested in more technical discussion, especially from an econometric perspective, should consult Brown and Rosen (1982), Epple (1987), Bartik (1987), and McConnell and Phipps (1987). Bartik and Smith (1987) and Palmquist (1991) also provided useful reviews.

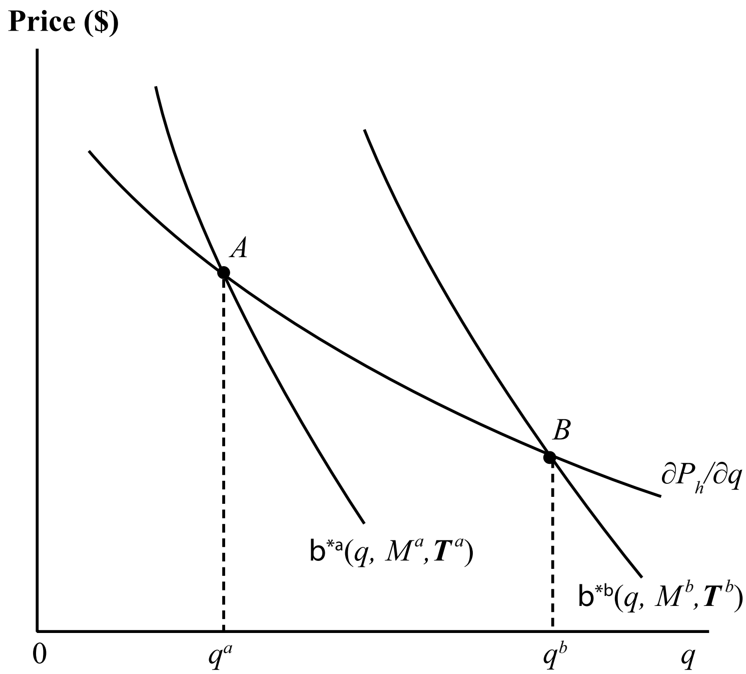

Figure 10.2 The identification problem when income and the quantity of the characteristic are correlated

> estimation of the sort suggested by Rosen may only reproduce the information already provided by the first-stage estimation of the $P_h(\cdot)$ function.
>
> (Brown and Rosen 1982, 176: notation changed by the authors)

In other words, since the second-stage estimation procedure utilizes no additional data beyond that already contained in the hedonic price function, it can do no more than reproduce the coefficients estimated from the hedonic price function.

The nature of the problem can be illustrated with Figure 10.2. This example is due to Bartik (1987, 84–85). Consider two individuals with the same income and uncompensated inverse demand functions of the form

$$b^* = b^*\left(q, M, \boldsymbol{T}\right), \tag{10.11}$$

where M is income, and $\boldsymbol{T}$ is a vector of unobserved determinants of tastes. For the first individual, we observe point A on her inverse demand function; but we have no information on the demand price for other levels of q. If the two individuals choose different levels of q, for example, q^a and q^b, it must be because of differences in tastes. This means that the demand-shifter, which is unobserved in the data of Figure 10.2, is correlated with the observed choices of q. As Palmquist put it, "the other marginal prices [on the individual's marginal willingness-to-pay function] are only observed for other individuals with other socioeconomic characteristics and provide no information on the original consumer's bid for different quantities of the characteristic" (Palmquist 1991, 96). This makes it very difficult to separate out the effects of demand-shifters from the price–quantity relationship itself.

One approach to solving the identification problem is to impose sufficient structure on the problem by assumption to assure that the conditions for identification of the inverse demand function are met. Quigley (1982), who assumed a functional form for preferences that included homotheticity as a property, provided an early example. See Chattopadhyay (1999) for another example of this approach. By specifying the relationship between income and demand, this assumption made it possible to separate the effects of income and quantity change on the marginal willingness to pay for characteristics. Note, however, that the assumptions about functional form are not testable.

Recently, Ekeland, Heckman, and Nesheim (2004) argued that the criticisms of Rosen's two-stage estimation procedure are misleading, based on "arbitrary linearizations that do not use all of the information in the model … Nonlinearities are generic features of equilibrium in hedonic models and a fundamental and economically motivated source of identification" (2004, S60). They went on to examine identification in the context of a normal-linear-quadratic version of a single market hedonic price model, and demonstrated that "[w]ith mild functional form assumptions, the model is completely identified" (2004, S96). While the approaches developed in Ekeland, Heckman, and Nesheim (2004) still rely upon structural assumptions, the line of research would seem to be promising in that the specifications considered are relatively general and authors draw instead on implications of underlying hedonic equilibrium to bolster identification.

An alternative approach to solving the identification problem is to find cases where the marginal implicit prices of characteristics vary independently of the other demand-shift variables. Specifically, this means finding cases where individuals with the same preferences, income, and other traits face different marginal implicit prices. This can only occur if similar individuals must choose in markets with different hedonic price functions, which in turn implies either segmented markets within a city or observations taken from several different housing markets (as, for example, in different cities).

The first step in implementing this approach is to estimate separate hedonic price functions for each housing market, using the same specification. The second step is to compute the marginal implicit price faced by each individual from the hedonic price function in that market. Then the computed marginal implicit prices can be regressed on the observed quantities of the characteristics and the exogenous demand-shifters to obtain the uncompensated bid function. Assuming sufficient independent variation across markets, and assuming that there are no unobserved differences in preferences across individuals, this approach will lead to reliable and properly identified bid functions. For examples of this approach, see Palmquist (1984), Bartik (1987), and Zabel and Kiel (2000).

The second source of difficulty in estimating inverse demands for attributes lies in the fact that both the quantity of the characteristic and its marginal implicit price are endogenous in the hedonic price model. Unlike the standard market model in which an individual faces an exogenously determined price and chooses a quantity, and unlike a quantity-rationed market in which an individual faces an exogenously determined quantity and reveals a marginal willingness to pay, the individual chooses both a point on the hedonic price schedule and its associated quantity. The choice of that point simultaneously determines the marginal willingness to pay and the quantity of the characteristic.

One approach to solving this problem is to find truly exogenous variables to be used as instruments. This appears to be a difficult task, however. For some of the suggested possibilities and their problems, see Mendelsohn (1984, 1985), Bartik (1987), Bartik and Smith (1987), and Palmquist (1991). Another possibility is to assume that there is a characteristic in the marginal implicit price function that is not an argument in the marginal willingness-to-pay function for another characteristic. This makes it possible to use the omitted characteristic as an instrument. Recently many authors have been critical of this approach, since the results are only as good as the assumptions imposed to obtain them—see Mendelsohn (1987), Bartik and Smith (1987), Horowitz (1987), and Palmquist (1984). Unfortunately, such assumptions are not testable. More recently, Ekeland, Heckman, and Nesheim (2004) suggested the use of nonlinear instrumental variables as an alternative approach to addressing the endogeneity problem.

Measuring Welfare Changes

It has been established that in a housing market in equilibrium, utility-maximizing individuals equate their marginal willingness to pay for housing characteristics with the marginal implicit prices of these characteristics, and that in some circumstances it may be possible to estimate inverse demand functions based on this information. The question then is how this information on prices and preferences extracted from the hedonic housing market can be used to calculate measures of welfare change for changes in environmental amenities. The basic concepts of welfare measurement at the level of the individual are straightforward and were introduced in Chapter 4. However, measurement of aggregate welfare changes based on hedonic prices is made difficult by the adjustments that people are likely to make in response to changes in environmental attributes and the possibility that the hedonic price function will change. Also, in principle, it is necessary to consider possible changes in the supply side of the hedonic property market.

In this subsection, the basic welfare measure is defined for marginal changes in a characteristic or environmental amenity, holding other things constant, in particular, individuals' choices of housing bundles. The benefits of a nonmarginal change in an amenity are then considered, assuming that individuals cannot adjust their housing bundles by moving. This measure looks only at benefits to purchasers of housing bundles. A fully general measure of welfare change is next examined, one that includes possible changes in profits on the supply side of the hedonic market as well as the consequences of individuals' adjustments on the demand side of the market. Since this conceptually correct measure is not implementable in practice, lower or upper bound approximations of the correct measure are considered. Finally, the subsection concludes with the consideration of localized changes in environmental quality that lead to benefits to some people without changing the hedonic price function.

Since a change in an environmental amenity in an urban area is nonexcludable and nondepletable, it is, in effect, a public good. The marginal value of the change, then, is simply the sum of the marginal willingness to pay of each of the N affected individuals evaluated at the existing housing equilibrium. In other words, for the amenity q:

$$w_q = \sum_{i=1}^{N} b^{*i} = \sum_{i=1}^{N} \left(\frac{\partial P_h}{\partial q_i} \right), \tag{10.12}$$

where w_q is the aggregate marginal welfare change and b^{*i} is the ith individual's marginal willingness to pay. Although most proposed environmental policy changes are nonmarginal in magnitude, the ease of calculating equation (10.12) may make it useful for indicating whether some improvement is desirable, by comparing this measure with an estimate of the marginal cost of the improvement.

Welfare Changes without Adjustments

Recall from Chapter 4 that the equilibrium hedonic price function is given by the double envelope of the bid and offer curves for all of the characteristics. A change in the level of q will place at least some individuals out of equilibrium, given the existing set of marginal implicit prices. Their efforts to restore their equilibria will result in changes in the hedonic price function and marginal implicit prices. Also, in principle, changes in the hedonic price function could trigger changes in the supplies of houses with different bundles of characteristics. In the next section, welfare measurement in the context of such changes will be discussed. But for now, assume that all individuals are constrained to stay at their original location, as might be the case with high transactions and moving costs or if a very short run perspective is taken; and also, assume that there is no supply response to the change in q.

Given these assumptions, the welfare value of the change in q from q^0 to q^1 is given by the sum of the areas under each individual's marginal willingness-to-pay curve over the change in q, or

$$W_q = \sum_{i=1}^{N} \int_{q^0}^{q^1} b^{*i} \left(q, \boldsymbol{Q}^*_{-q}, u_i^* \right) dq \ , \tag{10.13}$$

where W_q is the aggregate benefit. Notice that this measure requires knowledge of the marginal willingness-to-pay functions of individuals. If the uncompensated bid functions from the second stage of the hedonic price estimation are used, the welfare gain from an increase in q will be overestimated.

There is a method for calculating exact welfare measures for nonmarginal changes in a characteristic, holding all other things constant. This method is based on an adaptation by Horowitz (1984) of Hausman's technique for exact welfare measurement for price changes (see Chapter 3). Suppose that the ith individual's uncompensated inverse demand function for q,

$$b^i = b^i\left(q, \boldsymbol{Q}^*_{-q}, M - P_h\right), \tag{10.14}$$

has been identified. Using the indirect utility function, in equilibrium,

$$\frac{\partial v/\partial q}{\partial v/\partial M} = b^i(\cdot) \tag{10.15}$$

and for individual i

$$b^i = \left(\partial P_h/\partial q\right). \tag{10.16}$$

The left-hand side of equation (10.15) is the slope of the indifference curve between the numeraire, M, and q. So, in equilibrium

$$\frac{dM}{dq} = b^i(\cdot) = \left(\partial P_h/\partial q\right)_i. \tag{10.17}$$

This expression can be solved for

$$M = f\left(q, \boldsymbol{Q}^*_{-q}, C\right), \tag{10.18}$$

where C is a constant of integration. The benefit of an increase in q is

$$W_q = f\left(q^0, \boldsymbol{Q}^*_{-q}, C\right) - f\left(q^1, \boldsymbol{Q}^*_{-q}, C\right). \tag{10.19}$$

See Horowitz (1984) and Palmquist (2005) for examples of this approach with various forms of the utility function.

In those cases where neither the uncompensated or compensated inverse demand functions are available, welfare changes could be estimated by making some assumption as to the shape of the marginal willingness-to-pay function through the original equilibrium point. Three alternative assumptions can be used to establish bounds on the true measure. One is to assume that the marginal willingness to pay for each individual is constant—that is, that the marginal willingness-to-pay function for each individual is a horizontal line through the known point. In this case, each individual's benefit for the postulated improvement in the amenity is approximated by the product of the (assumed) constant marginal willingness to pay and the change in the amenity. The aggregate benefit is obtained by summing over all individuals. This assumption leads to an estimate of aggregate benefits that is biased upward.

A second convenient assumption would be that each individual's marginal willingness-to-pay curve decreases linearly from its observed point to the point of the highest attainable level of the amenity. Marginal willingness to pay would be zero at this point. It is not clear whether this approximation would lead to an overestimation or an underestimation of true benefits. The third assumption would be that all individuals' marginal willingness-to-pay functions are identical. Then, as discussed in Chapter 4, the marginal implicit price curve is identified as the marginal willingness-to-pay curve for the representative individual.

Welfare Changes with Full Adjustment

Bartik and Smith (1987, 1223) presented a welfare measure that takes account of all of the adjustments that individuals make in response to the nonmarginal change in q. At any location, the value of a nonmarginal change can be taken to be the integral of the values of a series of infinitesimal changes in the amenity. The value of each small change is taken to be the willingness to pay of the occupant of that site at that point in the sequence of changes. The measure for all sites together is the sum of the values for each site. It is given by

$$W_q = \sum_{j=1}^{J} \int_{q_j^0}^{q_j^1} \frac{\partial P_h\left(q, \boldsymbol{Q}_{-q}^*\right)}{\partial q} dq \, , \tag{10.20}$$

where $\boldsymbol{Q}_{-q}^*$ is the vector of all other site characteristics that are held constant by assumption, j indexes locations, and where the hedonic price function itself is changing in response to the adjustments that people are making.

In principle this measure allows individuals to relocate in response to changes in the quantity and price of the amenity, since, in effect, it sums individuals' marginal values as the amenity changes at each site. This is important, because a major limitation of some of the measures to be described below is their inability to account for individual relocation decisions. Furthermore, this measure does not require knowledge of either the marginal willingness to pay or the bid function. It relies on the fact that at each point in the sequence of changes, each individual's marginal bid is revealed by the marginal implicit price of the characteristic. However, since the hedonic price function is shifting as a consequence of the change in the amenity level, it is necessary to know how the hedonic price function and the marginal implicit prices at each location change as the levels of the amenities at each location change along the path of integration. As a practical matter, this is a major limitation of the measure.

This limitation has forced researchers to look for practical measures that can be interpreted as approximations or upper or lower bounds on the true welfare change. Following the analysis by Bartik (1988), suppose there are increases in several environmental amenities in an urban area. These increases need not be uniform across the area. Specifically, consider the case where the vector $\boldsymbol{Q}$ increases from $\boldsymbol{Q}^0$ to $\boldsymbol{Q}^1$.

First, assuming that individuals cannot move to new locations and that the hedonic price function does not change, the benefit to individuals is given by an expanded version of equation (10.11):

$$W_q = \sum_{i=1}^{N} \int_{Q_i^0}^{Q_i^1} \boldsymbol{B}^i \left(\boldsymbol{Q}, u_i^* \right) d\boldsymbol{Q}_i \,, \tag{10.21}$$

where each individual's welfare gain is computed from a path-independent line integral over the changes in the individual elements in $\boldsymbol{Q}$, and $\boldsymbol{B}^i(\cdot)$ is the vector of individual marginal willingness-to-pay functions for the characteristics.

Now, at the existing hedonic price function, some people may wish to choose different bundles of characteristics. If they do change, it must be because they perceive themselves to be better off after the adjustment. This welfare gain is in addition to that given by equation (10.21). Thus, (10.21) can be interpreted as a lower bound on the true measure; and it requires knowledge of only the bid or compensated inverse demand functions for the characteristics that change. Furthermore, the effort to adjust to different characteristics bundles is likely to affect the hedonic price function, unless the number of people wishing to do so is quite small relative to the market. It is also possible that the suppliers of housing will respond to changes in the hedonic price function by offering different bundles of housing characteristics. This could have further repercussions on the hedonic price function, and it will increase the profits of housing suppliers.

When all of these repercussions have worked themselves out, the aggregate benefit to individuals can be defined in terms of each individual's total willingness to pay for a housing unit with given characteristics, holding utility constant. Let this total willingness to pay be given by

$$WH_i \left(\boldsymbol{Q}_i^{*j}, u_i^* \right), \tag{10.22}$$

where $\boldsymbol{Q}_i^{*j}$ ($j = 0, 1$) indicates the vectors of environmental and other characteristics actually chosen by the individual in the original and new equilibrium. Each individual's total benefit is the increase in total willingness to pay for the characteristics actually chosen, holding utility constant, minus any increase in actual expenditure on housing. Summing across all individuals, we obtain

$$W_q = \sum_{i=1}^{N} \left[WH_i \left(\boldsymbol{Q}_i^{*1}, u_i^* \right) - WH_i \left(\boldsymbol{Q}_i^{*0}, u_i^* \right) \right] - \sum_{i=1}^{N} \left[P_h^1 \left(\boldsymbol{Q}_i^{*1} \right) - P_h^0 \left(\boldsymbol{Q}_i^{*0} \right) \right]. \tag{10.23}$$

Turning to the supply side of the market, producers, in aggregate, realize a change in aggregate profits given by the increase in expenditures on housing net of any change in their costs. This is given by

$$\Delta\text{Profit} = \sum_{i=1}^{N} \left[P_h^1 \left(\boldsymbol{Q}_i^{*1} \right) - P_h^1 \left(\boldsymbol{Q}_i^{*0} \right) \right] - \sum_{i=1}^{N} \left[C_h^1 \left(\boldsymbol{Q}_i^{*1} \right) - C_h^1 \left(\boldsymbol{Q}_i^{*0} \right) \right], \tag{10.24}$$

where $C_h^j(\cdot)$ is the cost function for producers.

The welfare change for society as a whole is the sum of equations (10.23) and (10.24). Notice that one component of this sum is simply a transfer of revenue from buyers to sellers, so it nets out. The total welfare change is the sum of the increase in total willingness to pay of individuals minus any cost increase on the part of producers of housing. Full implementation of this welfare measure would require enormous amounts of information. However, note that this measure reduces to equation (10.21) if the hedonic price function does not change and if the change in environmental amenities does not affect the costs of supplying housing for producers.

Even if this set of conditions is not satisfied, equation (10.21) can be interpreted as a lower bound on the true measure of benefits. This can be seen by decomposing the true benefit measure into a three-step sequence of changes and adjustments. Consider first the change in amenity levels without any adjustment on the part of individuals or suppliers. The welfare change associated with this step is given by equation (10.21) plus any reduction in the costs of supplying existing houses at the affected locations. Second, suppose hypothetically that the hedonic price function is shifted to its new equilibrium position but that no adjustments to the new price function by individuals or suppliers are permitted. At this stage, although some individuals and suppliers may gain while others lose, on net all of the price changes sum to zero. At this stage, there is no net change in welfare.

Finally, allow individuals and suppliers to respond to the new hedonic price function. Any adjustments that take place at this stage must represent welfare improvements for those responding. The total welfare change is the sum of equation (10.21), any costs reduction to suppliers, and the benefits of adjusting to the price change. The latter two components are either zero or positive. Thus, equation (10.21) represents a lower bound on the true measure of benefits; and the smaller the adjustment to the changes in the hedonic price function, the smaller is the error involved in using (10.21).

Localized Amenity Changes: A Special Case

If the hedonic price function does not shift, then exact welfare measurement may be a relatively easy task. Palmquist (1992a) identified one situation in which the hedonic price function could be assumed to be constant. That is when the number of sites at which there is a change in the amenity level is small relative to the total urban market. If this is the case, and if individuals can move without cost from one site to another in response to the change in environmental amenity levels, then exact welfare measurement is straightforward. The hedonic price function can be used to predict the changes in the prices of affected properties. Benefits are exactly measured by the increase in the values of the affected properties; and knowledge of the marginal bid functions is not required.

Consider the case of an improvement from q^0 to q^1 at just one site, as shown in Figure 10.3. Assume that moving costs for occupants who choose to relocate are zero (the impact of positive moving costs is discussed below). The change in the amenity level results in an increase in the price of this house from P_h^0 to P_h^1. The owner of the property is made better off by this increase in wealth. Even though the occupant of the property experiences the increase in amenity level, he or she is worse off because of the increase in the cost of occupying the property. The occupant is shifted from point A on the curve B^0 to point B on the curve B^1. However, with costless moving, the occupant can relocate to his or her

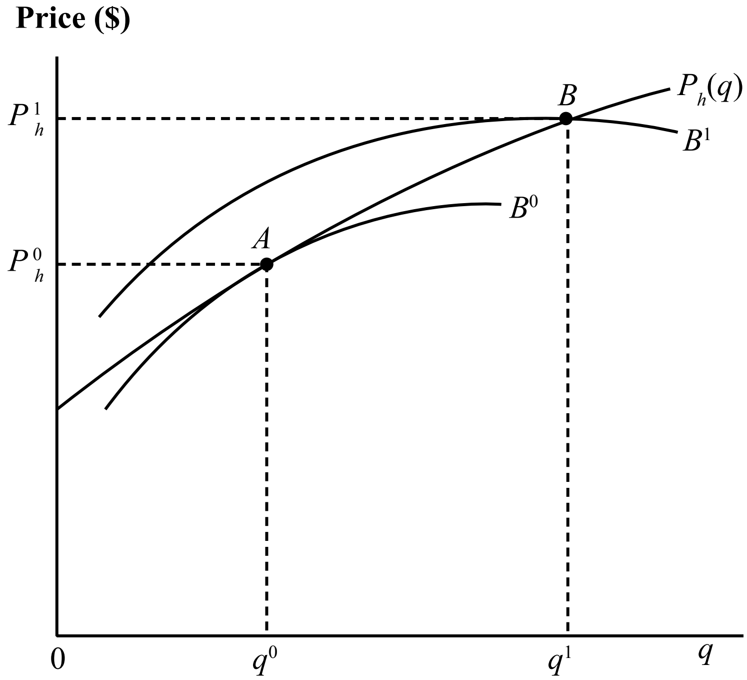

Figure 10.3 The benefit of an amenity improvement at one site when the hedonic price function is unchanged

original equilibrium position. So, the net welfare change is the increase in wealth to the owner. If the owner and occupant are the same person, the result is still the same. This individual might choose to move to a property with an amenity level somewhat greater than q^0 because of a wealth effect. However, the increase in wealth fully captures the benefit of the amenity improvement to this individual.

Now consider the case where the number of affected sites is small so that the hedonic price function does not change, but where moving costs are positive. The renter either loses the moving costs involved in adjusting his/her housing bundle or bears a loss of utility associated with staying at a less preferred location after the amenity change. In either case, the increase in property prices is an upper bound on the total benefit (Palmquist 1992b).

Rents, Taxes, and Property Prices

In the development and exposition of the theoretical model, the discussion has ignored the temporal dimension of housing prices and how welfare measures based on property prices might be converted to the annualized form usually used in welfare evaluation. It is typically the market price of a property that is observed. Inferences about the streams of rents and of benefits are drawn by converting observed present values to annual streams. The institutions of income and real property taxation affect the way in which the market capitalizes rents (and changes and differentials in rents) into market prices for properties. These effects must be properly understood if the process of retracing these steps to infer rents from property value observations is to be successful. This subsection develops and expands on some ideas first presented by Niskanen and Hanke (1977).

In the simplest case of a stream in perpetuity and with no taxes, the conversion of property value to rent is given by

$$R = P_h \cdot r, \tag{10.25}$$

where r is the appropriate discount rate. The discussion proceeds by examining first the effects of the two forms of taxation separately, then their combined effects. It is shown that the effects of these two kinds of taxation on the relationship between observed differences in property values and welfare measures depends of the specific features of the tax system and parameter values such as interest rates and tax rates.

Ad valorem taxation of property can be viewed as a device for capturing some of the rent of land for the government. Since taxation affects the net return to the property owner, it should affect the market value of property as an asset. An individual would purchase a property as an asset only if its market price, P_h, is equal to or less than the discounted present value of the rental stream net of property taxation. Market forces would establish the following relationship between property values and rents:

$$P_h = \frac{R - t \cdot P_h}{r}, \tag{10.26}$$

where t is the ad valorem tax rate.

If property values are known, the rental stream they represent can be computed by rearranging equation (10.24):

$$R = (r + t) P_h. \tag{10.27}$$

Assume that the property value–amenity relationship, $P_h(\cdot)$, has been estimated. The marginal benefit of a change in q at a site is

$$w_q = \frac{\partial R}{\partial q} = (r + t) \frac{\partial P_h}{\partial q}. \tag{10.28}$$

The present value of this stream of benefits is

$$\frac{w_q}{r} = \left(1 + \frac{t}{r}\right)\left(\frac{\partial P_h}{\partial q}\right). \tag{10.29}$$

In other words, when the hedonic price function is defined in terms of property value, ignoring the effect of property taxation on the capitalization of rents can lead to the underestimation of benefits. The term (t/r) is a measure of the percentage error resulting from omitting the tax term in the calculation of benefits. For an interest rate of 10 percent and an effective tax rate of 10–20 mils per dollar of market value (1–2 percent), the error is between 10 and 20 percent.

However, this is not the whole story. The income tax code treats the imputed rental income of homeowners differently than it does the rental income of landlords. The absence of a tax liability for imputed rent further complicates the task of inferring annual rents and benefits from observations of (capitalized) market prices for housing assets. This is because the market will place different values on two assets with the same rental stream if one is subject to income taxation while the other is not.

Assume two perpetual assets indexed as *a* and *b* with equal annual returns of *R* per year. The return to the first asset is taxable at the rate *g* percent, while the return to the second asset incurs no tax liability. If *r* represents the market rate of return on assets with taxable returns, the two assets will be priced so as to equalize the after-tax rate of return:

$$P_a = R_a / r \tag{10.30}$$

and

$$P_b = \frac{1}{(1-g)} \frac{R_a}{r} > P_a\,. \tag{10.31}$$

If P_b is observed, the tax-free rental stream can be computed as

$$R_b = r(1-g)P_b\,. \tag{10.32}$$

Taking account of this adjustment and using equations (10.26) and (10.27), the marginal annual benefits of amenity changes to homeowners and their present value are

$$w_q = r(1-g)\partial P_b / \partial q \tag{10.33}$$

$$\frac{w_q}{r} = (1-g)\partial P_b / \partial q\,. \tag{10.34}$$

Ignoring the effects of income taxation leads to an overestimation of benefits. The discount factor $r(1-g)$ is analogous to the municipal bond rate, and it arises for the same reason. However, where the marginal tax rate is itself a function of income, *g* varies across individuals, and equations (10.33) and (10.34) must be computed separately for each individual.

The tax code confers additional benefits on homeowners by exempting them from taxation on capital gains realized on the sale of a primary residence and by allowing them to deduct property tax payments in calculating taxable income. This latter provision lowers the real cost of the property tax by *g* percent. Combining these effects (ad valorem taxation, deductibility, and exemption of imputed rental income), we have

$$P_b = \left[R_b - (1-g)tP_b\right] / r(1-g)\,. \tag{10.35}$$

Solving for R_b gives

$$R_b = (r+t)(1-g)P_b. \tag{10.36}$$

Marginal benefits are calculated by

$$w_q = (r+t)(1-g)\left(\frac{\partial P_b}{\partial q}\right) \tag{10.37}$$

and

$$\frac{w_q}{r} = \left(1+\frac{t}{r}\right)(1-g)\left(\frac{\partial P_b}{\partial q}\right). \tag{10.38}$$

The effects of ignoring taxation in calculating benefits depend on the magnitudes of g and t/r. The higher the marginal income tax rate, the more likely benefits would be overstated if taxes were ignored. For an example, suppose the marginal income tax rate is 30 percent, the opportunity cost of capital is 10 percent, and the property tax rate is 2 percent. Then the terms in parentheses come to 0.84. Ignoring tax effects would lead to an overstatement of benefits by almost 20 percent. However, lower income tax and discount rates can reverse this conclusion. An alternative approach to dealing with taxation and discounting is to base the hedonic equation on measures of user cost (called gross rent by Sonstelie and Portney 1980). This variable captures the full cost of owning (and using) an asset such as a house. User cost would include property taxes and the opportunity cost of capital plus any change in the market price of the asset over the interval, say a year. It would be calculated as follows:

$$u = (r+t+m)P_h, \tag{10.39}$$

where m is the percentage rate of change in market value.

The user cost approach differs from that outlined above in two respects. The first is the inclusion of the change in market value over time. P_h could be changing because of physical depreciation of the house, general price inflation, changes in the price of housing relative to other goods, and changes in the variables determining P_h. Only the latter changes have relevance for benefit estimation, and they would be captured by modified versions of equations (10.37) and (10.38), which are generalized from the assumption of constant streams in perpetuity. However, the depreciation term might be useful in empirical work as an approximation of expected changes in these variables, provided that it were adjusted to net out general price level effects.

The second difference arises in considering the effects of some provisions of the income tax code on user cost. For one thing, the tax exemption for imputed rent does not affect the user cost of holding a house, since user cost is an opportunity cost. For another, the tax deductibility of mortgage interest does affect user cost, but it does not affect the market capitalization of streams of benefits. If user cost is used to compute benefits, the net result of these two effects is to overstate benefits in comparison with equations (10.37) and (10.38).

Discrete Choice Models

The hedonic price model is based on the assumption that each attribute of the housing bundle is a continuous variable and that an individual can choose any point on the continuous and differentiable hedonic price function in the *n*-dimensional attribute space. As noted above, this is clearly not a completely realistic assumption, and in some respects it may seriously misrepresent the problem of choosing a bundle of housing attributes. For example, the number of bedrooms in a house is not a continuous variable. There may be no one-bedroom houses on one-acre lots, or four-bedroom houses with swimming pool and attached garage on one-quarter-acre lots. Discrete choice models provide an alternative way of looking at housing choice and inferring values for housing attributes.

Some of the discrete choice models that have been used in the literature focus on the individual's bid function for housing bundles. Such models are based on the probability that an individual will be the highest bidder for a specified bundle of housing attributes. These are known as bid rent models or random bidding models. An alternative approach is to focus on the individual's utility function defined on housing attributes. These models investigate the probability that a specified bundle of housing attributes (including the price of the bundle) will be chosen by the individual, drawing on the Random Utility Maximization (RUM) models described in Chapter 3, and used extensively in the recreation demand literature.

Both types of models can be used to derive the marginal bid or marginal willingness-to-pay function for individual attributes from an estimate of the bid function or indirect utility function. Thus, in principle, both types of models allow for the calculation of the benefits of changes in an environmental attribute, at least assuming no relocation and no changes in the hedonic price function. Both types of models start with the assumption of utility maximization.

In the bid rent model developed by Ellickson (1981), the utility maximization problem is solved, subject to the standard budget constraint defined by income, prices of market goods, and the hedonic price function, to obtain the individual's bid function—that is, the bid as a function of the housing attributes and income, holding utility constant. As a practical matter, individuals are then grouped into broad "type" categories (based on, for example, income, household size, and other socio-demographic attributes), assuming that bid functions are homogenous within type. The bid function for individuals of type t for a particular housing unit j can be written as a function of the observable housing attributes ($\boldsymbol{Q}_j$) plus a random error term reflecting unobserved attributes of either the individual or the housing bundle:

$$B_{tj} = B_t\left(\boldsymbol{Q}_j\right) + \varepsilon_{tj}\,. \tag{10.40}$$

For examples of this type of model, see Ellickson (1981), Lerman and Kern (1983), and Gross (1988).

The probability that a household of type t will occupy housing unit j will be determined by whether or not they outbid any other household type; that is,

$$\begin{aligned}\Pr(t \mid j) &= \Pr\left(B_{tj} > B_{t'j}\ \forall t' \neq t\right) \\ &= \Pr\left[B_t\left(\boldsymbol{Q}_j\right) - B_{t'}\left(\boldsymbol{Q}_j\right) + \varepsilon_{tj} - \varepsilon_{t'j}\ \forall t' \neq t\right].\end{aligned} \tag{10.41}$$

If the random error terms are independently and identically distributed with a Type I Extreme Value distribution, this probability can be written in the logit form:

$$P\left[t \mid \boldsymbol{Q}_j\right] = \frac{\exp\left[B_t\left(\boldsymbol{Q}_j\right)\right]}{\sum_{s=1}^{T} \exp\left[B_s\left(\boldsymbol{Q}_j\right)\right]}\ , \tag{10.42}$$

where T denotes the number of household types. As Lerman and Kern (1983) and Gross (1988) pointed out, estimation of equation (10.42) fixes only the slope of the bid function, not its level. However, information on the bids actually paid can be used in the estimation process to fix these values and to make it possible to calculate the marginal bid functions for individual attributes.

Note that the focus in the bid-rent model is on which type of individual occupies a given housing unit. In contrast, in the RUM model, the emphasis is on modeling which housing unit is chosen by a given individual. Specifically, suppose that the conditional utility that individual i receives from choosing housing unit, j, is given by

$$V_{ij} = V\left(R_j, \boldsymbol{Q}_j, M_i, \boldsymbol{S}_i; \beta\right) + \varepsilon_{ij}\ , \tag{10.43}$$

where R_j denotes the rental cost of housing unit j, $\boldsymbol{S}_i$ is a vector of socio-demographic characteristics for individual i, β is a vector of parameters for the conditional utility function, and ε_{ij} is a random error term assumed to capture unobservable attributes of the individual/housing unit, known to the individual but unobserved by the analyst. The individual is presumed to choose the housing unit that maximizes their utility, so that

$$\begin{aligned}\Pr(j \mid i) &= \Pr\left(V_{ij} > V_{ij'}\ \forall j' \neq j\right) \\ &= \Pr\left[V\left(R_j, \boldsymbol{Q}_j M_i, \boldsymbol{S}_i; \beta\right) - V\left(R_{j'}, \boldsymbol{Q}_{j'} M_i, \boldsymbol{S}_i; \beta\right) + \varepsilon_{ij} - \varepsilon_{ij'}\ \forall j' \neq j\right].\end{aligned} \tag{10.44}$$

If, as in Cropper et al. (1993), the error terms are assumed to be independent and identically distributed with a Type I extreme value distribution, then the probability that individual i will choose housing unit j can be written in the standard logit form, with

$$\Pr(j \mid i) = \frac{\exp\left[V\left(R_j, \boldsymbol{Q}_j M_i, \boldsymbol{S}_i; \beta\right)\right]}{\sum_{j'=1}^{J} \exp\left[V\left(R_{j'}, \boldsymbol{Q}_{j'} M_i, \boldsymbol{S}_i; \beta\right)\right]}\ , \tag{10.45}$$

where J denotes the total number of housing units in the individual's choice set. Knowledge of the parameters of the indirect utility function makes it possible to compute welfare measures for changes in any of the housing attributes, including an environmental amenity. One practical issue with the RUM model in this setting is defining the choice set. An individual moving to a large metropolitan region may have literally thousands of housing units to choose from, but may actually only consider a small fraction of these options. In practice, analysts often focus on modeling the choice of a community or neighborhood, assuming that the housing units are largely homogeneous within a given neighborhood.

In an interesting simulation study, Cropper et al. (1993) compared estimates of welfare measures derived from a hedonic price model with those from the random utility model for given known household preferences. After simulating an equilibrium in an urban housing market, they used the resulting hedonic price and individual choice data to estimate both a hedonic price model with its marginal bid functions and a random utility model. They then calculated welfare measures for 25 percent and 100 percent changes in each of ten attributes, including both neighborhood attributes and attributes of individual houses (such as number of bathrooms, lot size, and age). They found that the random utility model provided more accurate estimates of the known welfare measure than the hedonic price model, and this was true even when they assumed that the researcher did not know the true form of individuals' utility functions. They suggest that the reason for this is the difficulty in identifying and obtaining accurate estimates of the marginal bid functions with the hedonic price model when data are generated by only a single market.

Other researchers (see for example, Bartik and Smith 1987, 1224–1225, and Palmquist 1991, 119) have suggested that while one of the strengths of the discrete choice model is its ability to generate welfare measures for nonmarginal changes relatively easily, the model only does so because it forces the researcher to make strong assumptions about the functional form of the utility function or bid function. If similar strong assumptions are made about the functional form of the inverse demand functions for attributes in the hedonic model, these functions can be identified too. Cropper et al. (1993) suggested that even when the functional form of preferences is known, the discrete choice model outperforms the hedonic model as a way of measuring welfare change. For other examples of empirically based efforts to compare welfare measures from standard hedonic models with random utility and random bidding models, see Chattopadhyay (1998, 2000) and Palmquist and Israngkura (1999).

Equilibrium Sorting Models

During the past decade, an alternative paradigm has emerged for modeling the supply and demand for differentiated commodities such as housing. Whereas the hedonic pricing literature focuses attention on the equilibrium outcome of the market, the equilibrium sorting literature seeks to characterize the sorting process itself (that is, how the distributions of individual preferences, production costs, and locational amenities interact to yield a market equilibrium in which the supplies and demands for housing amenities are equated, and both the distributions of housing types and their prices are determined).[4] With a model of the sorting process in hand, it is conceptually straightforward to evaluate both marginal and nonmarginal policy scenarios by simply simulating the new sorting equilibrium that would emerge. These capabilities, of course, come at a cost, as they invariably rely upon assumptions regarding the structure and/or distribution of preferences.

This section provides an overview of equilibrium sorting models. Much of the discussion draws on a recent and more comprehensive review of this literature by Kuminoff, Smith, and Timmins (2013). Attention here is focused on two particular sorting model frameworks: (a) the *pure characteristics sorting model* developed by Epple and Sieg (1999) and (b) the *random utility sorting model* developed by Bayer, McMillan, and Reuben (2004).[5] These are sometimes referred to in the literature as the *vertical* and *horizontal* sorting models, respectively. At the heart of both models is Tiebout's (1956) notion that households "vote with their feet," choosing among available communities both in terms of the private amenities that a specific house in the community would provide and the accompanying exposure to both positive and negative public goods. Specifically, let $\boldsymbol{G}_k$ denote the vector of public amenities provided by community $k = 1,\ldots, K$ and $\boldsymbol{H}_j$ denote the vector of private amenities provided by housing unit $j \in k$ in community k. In the notation of the previous section, the complete vector of amenities for housing unit j is then given by $\boldsymbol{Q}_j = \left(\boldsymbol{G}_k, \boldsymbol{H}_j\right)$. The individual is assumed to choose a community k, a housing unit $j \in k$ in that community, and a level of expenditures on a numeraire good z so as to maximize her utility subject to a budget constraint; that is,

$$\max_{k,j\in k,z} u\left(z,\boldsymbol{G}_k,\boldsymbol{H}_j;\alpha_i,\boldsymbol{S}_i\right) \qquad \text{s.t.} \quad z + R_{j\in k} = M_i, \tag{10.46}$$

where $R_{j\in k}$ denotes the annual rental expenditure for housing unit j in community k, α_i denotes unobserved individual characteristics influencing the individual's preferences and $\boldsymbol{S}_i$ denotes observable individual characteristics. In general, the individual's income might also depend upon the community chosen, but for ease of notation, this generalization is ignored here. The two modeling frameworks differ in the additional structure imposed on the consumer's optimization problem.

4 To be fair, of course, the hedonic pricing literature incorporates in theory many of these factors as well, as in Ekeland, Heckman, and Nesheim (2004), but has focused on the equilibrium price outcome as a practical matter.

5 Kuminoff, Smith, and Timmins (2013) also consider a third framework, which they refer to as the *Nechyba–Ferreyra model*, developed by Ferreyra (2007) and based on theoretical models of Nechyba (1997, 1999, 2000). This approach, however, has largely been applied to modeling housing and school choices, and the interaction of these choices with the production of education.

The Pure Characteristics Sorting Model

The vertical sorting model begins by assuming that the annual rental expenditures associated with living in housing unit *j* can be decomposed into a housing quantity index $h_j = h(\boldsymbol{H}_j)$, reflecting the private attributes of the unit, and a price index $p_k = p(\boldsymbol{G}_k)$, reflecting the cost of purchasing a home in community *k* (per unit of the housing index); that is, $R_{j\in k} = p_k h_j$. Thus, $h(\boldsymbol{H}_j)$ reflects the *amount* of housing one receives from a specific unit as it varies in physical attributes, such as square footage, number of bathrooms, etc., whereas $p(\boldsymbol{G}_k)$ captures how the price of the housing changes across communities with different levels of public amenities. In addition, it is traditionally assumed that the public good attributes of the community can be summarized by a one-dimensional index of the attribute vector $\boldsymbol{G}_k$ (i.e., $\tilde{g}_k = g(\boldsymbol{G}_k)$). Conditional on choosing community *k*, the individual agent's maximum utility becomes[6]

$$v_k = v(\tilde{g}_k, p_k, \alpha_i, M_i) \equiv \max_{h,z} u(z, \tilde{g}_k, h; \alpha_i) \qquad \text{s.t.} \quad z + p_k h = M_i . \tag{10.47}$$

In this context, α_i is treated as an indicator of the individual's preference for public goods; that is, all else equal, an individual with a high value of α_i will prefer to live in a community with more public goods.

The household is assumed to choose the community that maximizes its utility. Prices adjust to reflect the demand for housing in the community and the available stock of housing and public amenities. An equilibrium emerges when individuals are in their desired community and no longer wish to move. The problem is that, without further structure, it is not clear that such an equilibrium exists, or that it is unique.

The solution employed in the vertical sorting literature is the so-called *single crossing condition*, introduced by Ellickson (1971) and generalized by Epple and Platt (1998). The single crossing condition requires that indifference curves in $(\tilde{g}, p)$ space, implied by $v(\tilde{g}, p, \alpha, M)$ in equation (10.47), cross only once for different individuals. Formally, as Kuminoff, Smith, and Timmins (2013) note, the slope of these indifference curves is given by

$$F(\tilde{g}, p, \alpha, M) = -\frac{\partial v(\tilde{g}, p, \alpha, M)/\partial \tilde{g}}{\partial v(\tilde{g}, p, \alpha, M)/\partial p} . \tag{10.48}$$

The single crossing condition is satisfied if F is increasing in $(\alpha | M)$ and is increasing in $(M | \alpha)$. Figure 10.4 illustrates this condition for three individuals with the same preference parameter α, but different income levels. For any one individual, with their given level of income (M) and attitude towards public goods (α), utility is increasing as one moves down (due to lower housing prices) and to the right (due to higher levels of public goods). Individual *i* in the figure is

6 For simplicity, the observed individual attributes are dropped, subsumed for now into the preference parameter *i*.

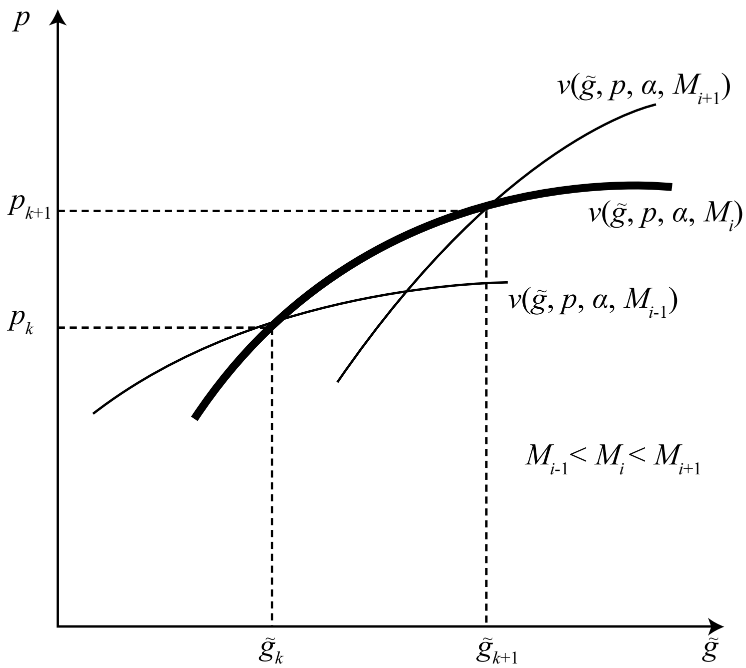

Figure 10.4 The single crossing condition

indifferent between communities k and $k + 1$, whereas individual $i - 1$ would prefer community k and individual $i + 1$ prefers community $k + 1$.

The single crossing condition yields three properties of the sorting equilibrium that are key to estimation: *boundary indifference, increasing bundles*, and *stratification* (Epple and Platt 1998). To define these properties, suppose that the K communities are ordered by the public good index $\tilde{g}_k$, with $\tilde{g}_1 < \tilde{g}_2 < \ldots < \tilde{g}_K$. Then the three properties are:

- Boundary indifference: There exists a "boundary" B_k between any two adjacent communities (k and $k + 1$) defined in (α, M)-space such that

  $$B_k = \left\{(\alpha, M) \middle| v(\tilde{g}_k, p_k, \alpha, M) = v(\tilde{g}_{k+1}, p_{k+1}, \alpha, M)\right\}. \tag{10.49}$$

  The term "adjacent" in this context refers to the communities' proximity to each other in the ordering from 1 to K, not necessarily their proximity geographically.
- Increasing bundles: The communities' rankings by the public good index match the rankings by price (i.e., $p_1 < p_2 < \cdots < p_K$).
- Stratification: For a given level of income, individuals sort themselves by preference parameter α; and individuals in communities $k - 1$, k, and $k + 1$ with the same income sort such that

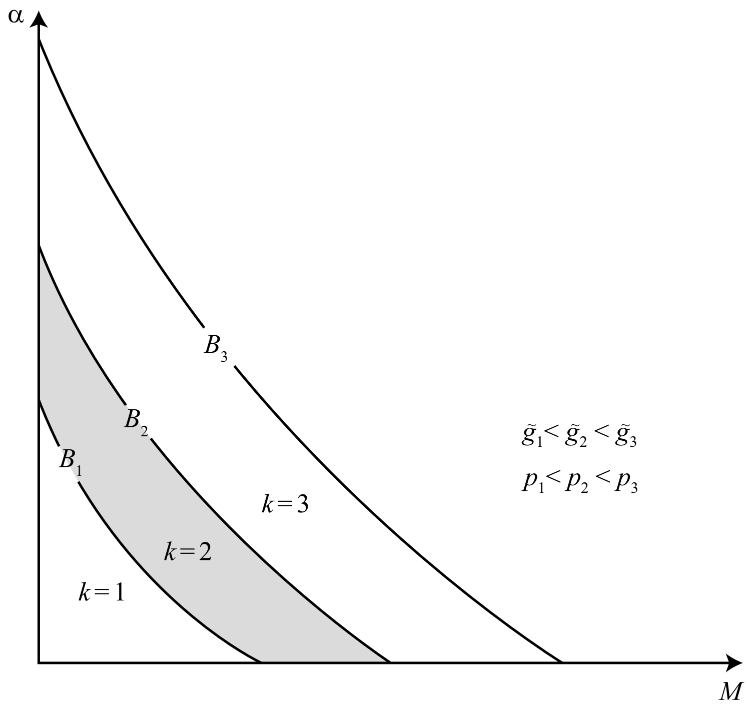

Figure 10.5 Market segmentation

$$\left(\alpha_{k-1} \middle| M\right) < \left(\alpha_k \middle| M\right) < \left(\alpha_{k+1} \middle| M\right). \tag{10.50}$$

Likewise, for a given level of the preference parameters, individuals sort themselves by income. In other words,

$$\left(M_{k-1} \middle| \alpha\right) < \left(M_k \middle| \alpha\right) < \left(M_{k+1} \middle| \alpha\right). \tag{10.51}$$

The implication of these three properties in terms of sorting is illustrated in Figure 10.5. The solid lines indicate the boundaries (B_k) between communities. The shaded region indicates the set of individuals (defined in terms of α and M) choosing to live in community k = 2. Note that the make-up of individuals in a given community is not defined solely in terms of either income (M) or solely in terms of preferences (i.e., α), but by their interaction. An individual with a relatively low income will choose to live in community 2 rather than the less expensive community 1 if they place a high enough value in the additional public goods that community 2 offers (that is, they have a high enough α). Conversely, an individual with a relatively high income may choose to live in community 2 rather than community 3 if they care relatively little for the available public goods (that is, they have a low enough α).

In order to close the model, additional structure is still needed. In particular, one needs a functional form for the indirect utility function in (10.47), an assumption regarding the joint distribution of income and preferences, and a functional form for the public goods index $\tilde{g}_k = g(\boldsymbol{G}_k)$. A commonly used specification for the indirect utility function is a constant elasticity of substitution (CES) form, with

$$v(\tilde{g}, p, \alpha, M) = \left\{ \alpha \cdot \tilde{g}^{\rho} + \left[\exp\left(\frac{M^{1-\upsilon} - 1}{1-\upsilon} \right) \exp\left(\frac{\beta \cdot p^{\eta+1}}{1+\eta} \right) \right]^{\rho} \right\}^{\frac{1}{\rho}}. \tag{10.52}$$

As Kuminoff, Smith, and Timmins (2013) noted, this structure implies that housing demand in the chosen community is given by

$$h = \beta p_k^{\eta} M^{\upsilon}, \tag{10.53}$$

so that η represents the price elasticity of housing demand and υ denotes the income elasticity of housing demand. The parameter ρ determines the substitutability between private and public goods.

The public good index is typically assumed to be a linear function of both the observed and unobserved public good attributes of a community; that is,

$$\tilde{g}_k = \gamma_1 g_{1,k} + \gamma_2 g_{2,k} + \cdots + \gamma_L g_{L,k} + \xi_k, \tag{10.54}$$

where $g_{\ell,k}$ denotes the ℓth observed public good for community k and ξ_k is a composite of all the unobserved attributes of community k. An important characteristic of this index is that it does not vary by individual. This implicitly assumes that all individuals weigh the component elements of the public good index the same. This is likely to be a strong assumption in some settings. For example, if one of the public goods is elementary education and another is public health care facilities, individuals with children may weigh these attributes differently than those who are single or elderly. In other settings, the assumption may be more innocuous, but it represents an important limitation of the vertical sorting model as it has been implemented to date and an area where additional research is needed.

The last element in specifying the model is to choose a joint distribution for the preference parameter and income pair (α, M). These are typically assumed to be from a joint lognormal distribution. Parameters of this distribution are inferred, in part, by comparing the observed income distributions in individual communities with those implied by the stratification boundaries (such as for community 2 in Figure 10.5).

Details of the econometric procedures required to estimate vertical sorting models are not described here, as they have varied somewhat across individual applications. Kuminoff, Smith, and Timmins (2013) provide a general discussion of some of the approaches used. To get a sense of at least part of the process, consider a stylized and simple version of Figure 10.5 depicted in Figure 10.6, in which it is assumed that there are only three communities, $k = 1, 2, 3$. Furthermore, suppose that *both* income (M) and preferences for public goods (α) are observable and known to be independently and uniformly distributed in the population; that is, $M \sim U[0, \overline{M}]$ and $\alpha \sim [0, \overline{\alpha}]$. Finally, suppose that the shaded region in Figure 10.6 depicts those individuals actually living in community 2. According to the

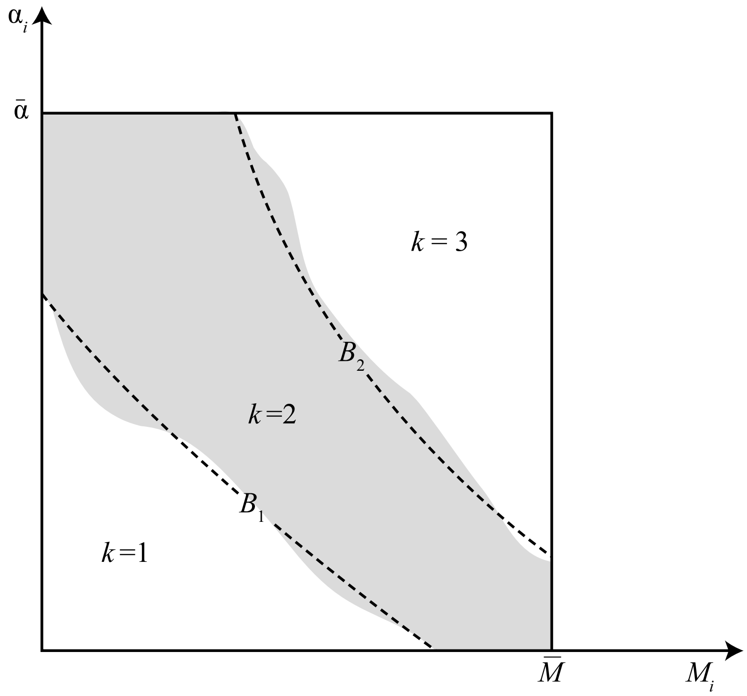

Figure 10.6 Estimating the parameters of a vertical sorting model

vertical sorting model outlined above, the proportion of the population living in community 2 is dictated by the region between the boundaries B_1 and B_2 defined in equation (10.10). Given observed housing prices in the three communities, these boundaries are in turn determined by the preference parameters in (10.13); that is, (β, η, ν, ρ). Estimation of these parameters involves adjusting them so that the resulting estimated boundaries (the dashed lines in Figure 10.6) imply as closely as possible the observed distribution of individuals in the three communities.

With the parameter estimates in hand, the general equilibrium implications of a policy changing the levels of public goods can be evaluated. Changes in the public goods will cause a shift in the community boundaries, which will in turn lead to a mismatch between existing housing supply and the new levels of demand in each community. The new equilibrium is constructed by finding the levels of housing prices that restore supply and demand in each community. Evaluation of the impact of the proposed policy can then include the overall impact of the policy and distribution of its impact on various subpopulations.

Individual applications include Epple and Sieg's (1999) study of school quality and public safety in the Boston metropolitan area, Smith et al.'s (2004) examination of general versus partial equilibrium implications of air quality improvements in the Los Angeles basin, and Walsh's (2007) analysis of open space as an endogenous public amenity.

The Random Utility Sorting Model

The second locational sorting model, initially developed by Bayer, McMillan, and Reuben (2004), builds on the discrete choice literature and McFadden's (1974) RUM model. Indeed, the starting point for this approach is essentially the same as the discrete choice RUM model discussed above at the end of the hedonic pricing model section. Specifically, an individual is assumed to face a discrete set of alternatives in their choice set, defined in terms of communities ($k = 1,\ldots,K$) and housing units (or housing types) available within each community (that is, $j \in k, j = 1,\ldots,J_k$). The conditional utility that individual i receives from choosing housing unit j in community k is assumed to take the form[7]

$$\begin{aligned} u^i_{j\in k} &= \boldsymbol{\alpha}^i_H \boldsymbol{H}_{j\in k} + \boldsymbol{\alpha}^i_G \boldsymbol{G}_k + \alpha_R R_{j\in k} + \xi_{j\in k} + \varepsilon^i_{j\in k} \\ &= v^i_{j\in k} + \varepsilon^i_{j\in k}\ , \end{aligned} \tag{10.55}$$

where $\boldsymbol{\alpha}^i_H$ and $\boldsymbol{\alpha}^i_G$ are vectors of individual-specific parameters associated with the respective vectors of housing and community characteristics, $\xi_{j\in k}$ represents unobserved attributes of the housing unit j (including potentially unobserved community characteristics), and $\varepsilon^i_{j\in k}$ denotes idiosyncratic unobservable factors influencing $u^i_{j\in k}$. The individual-specific parameters are typically modeled as functions of observable individual attributes (i.e., such as age, gender, etc. denoted by $\boldsymbol{S}_i$ in equation (10.46) above), with

$$\boldsymbol{\alpha}^i_a = \boldsymbol{\alpha}_{a0} + \sum_{\ell=1}^{L} \boldsymbol{\alpha}_{a\ell} s^i_\ell\ , \tag{10.56}$$

where s^i_ℓ denotes the ℓ th attribute in $\boldsymbol{S}_i$.

Substituting (10.56) into (10.55) and collecting terms, we can rewrite (10.55) as

$$u^i_{j\in k} = \alpha_{j\in k} + \tilde{\boldsymbol{\alpha}}^i_H \boldsymbol{H}_{j\in k} + \tilde{\boldsymbol{\alpha}}^i_G \boldsymbol{G}_k + \varepsilon^i_{j\in k}\ , \tag{10.57}$$

where

$$\alpha_{j\in k} = \boldsymbol{\alpha}_{H0} \boldsymbol{H}_{j\in k} + \boldsymbol{\alpha}_{G0} \boldsymbol{G}_k + \alpha_{R0} R_{j\in k} + \xi_{j\in k} \tag{10.58}$$

denotes a housing-type specific constant and

$$\tilde{\boldsymbol{\alpha}}^i_a = \sum_{\ell=1}^{L} \boldsymbol{\alpha}_{a\ell} s^i_\ell\ . \tag{10.59}$$

Individuals are assumed to choose a community (k) and housing unit ($j \in k$) that maximizes their utility.

Before getting into either the equilibrium sorting of households or the econometric issues associated with estimating the discrete choice model in (10.57) and (10.58), there are several of its features that are worth highlighting. First, unlike in the vertical sorting model, individuals are able to have differing preferences

7 One can also make the coefficient on rental costs (that is, αR) a function of individual observable attributes (that is, $\boldsymbol{S}i$), but this complication is not considered here in order to simplify the exposition.

regarding both the observable housing characteristics ($\boldsymbol{H}_{j\in k}$) and the community level public goods ($\boldsymbol{G}_k$). The baseline (or common) impact of each attribute is captured by either $\boldsymbol{\alpha}_{H0}$ or $\boldsymbol{\alpha}_{G0}$, but $\tilde{\boldsymbol{\alpha}}_H^i$ and $\tilde{\boldsymbol{\alpha}}_G^i$ allow for differences in the marginal utility of an attribute depending on socio-demographic characteristics. Thus, families with children can value local public schools and health care facilities differently than those who are single or elderly. Recall that in the vertical sorting model, everyone is assumed to weigh the community's public goods in the same manner (see equation 10.54). Second, the horizontal sorting model can readily handle discrete quality attributes for both communities and housing units. This is in contrast to the standard hedonic pricing model, wherein individuals are assumed to face a continuum of housing units in all quality dimensions. Third, the structure can also allow for friction in the sorting process in the form of moving costs, for example by incorporating a fixed cost associated with alternatives in either different communities or different cities (see, for example, Bayer, Koehane, and Timmins 2009). Finally, notice that the conditional indirect utilities in (10.57) include an alternative specific constant (ASC) $\alpha_{j\in k}$. These ASCs reflect the overall appeal of the alternative, which, as indicated by equation (10.58), depends upon the observed features of the housing unit ($\boldsymbol{H}_{j\in k}$), the public goods available in the community ($\boldsymbol{G}_k$), the cost of the unit ($R_{j\in k}$), and features of the unit not observed by the analyst ($\xi_{j\in k}$). The terms $\tilde{\boldsymbol{\alpha}}_H^i$ and $\tilde{\boldsymbol{\alpha}}_G^i$ capture how the appeal of the unit varies by observable individual attributes. The advantage of this structure is that it controls for a myriad of possible unobservable attributes for the housing unit. Unfortunately, it also means that there are as many ASCs to estimate as there are alternatives in the choice set. This creates a practical problem in terms of estimation, as the choice set can become large in individual applications. This has led to the use of a two-stage estimation procedure outlined below.

There are also econometric issues associated with insuring the consistency and asymptotic normality of the estimators used (see Berry, Linton, and Pakes 2004). This has led researchers to aggregate housing units into types or classes within each community. For example, Klaiber and Phaneuf (2010) organized housing units within communities by size (so that j references small, medium, and large housing types), whereas Tra (2010) organized housing units by ownership status, number of bedrooms, dwelling type, and when the housing unit was built.

Estimation of the model typically proceeds in two steps. If the $\varepsilon_{j\in k}^i$ are assumed to be independently and identically distributed according to a Type I extreme value distribution, then the probability that an individual i chooses housing unit (or housing type) j in community k is given by the usual logit structure, with

$$P_{j\in k}^i = \frac{\exp\left(v_{j\in k}^i\right)}{\sum_{m=1}^{K}\sum_{\ell=1}^{J_m}\exp\left(v_{\ell\in m}^i\right)}. \tag{10.60}$$

The alternative specific constants (i.e., the $\alpha_{j\in k}$) and the socio-demographic parameters $\tilde{\boldsymbol{\alpha}}_H^i$ and $\tilde{\boldsymbol{\alpha}}_G^i$ are recovered. One of the challenges at this stage of estimation is the potentially large number of alternative specific constants to be estimated. Researchers have typically drawn on a number of convenient features of the logit model to simplify estimation (see, for example, Murdock 2006 and Klaiber and Phaneuf 2010).

The second stage in the estimation process involves using the fitted alternative specific constants in order to estimate the parameters in (10.59). The key issue here is that the unobserved housing and community characteristics, represented by $\xi_{j\in k}$, are likely correlated with the observed housing and community characteristics, as well as the housing price itself (i.e., $R_{j\in k}$). In order to resolve this potential for omitted variables bias, the horizontal sorting literature has drawn on instrumental variables procedures developed in the industrial organization literature (for example, Berry, Levinsohn, and Pakes 1995).

Up to this point, the model is essentially no different from the discrete choice housing models discussed in the previous section. The difference arises when one recognizes that the choice probabilities themselves provide a measure of aggregate demand ($d_{j\in k}$) for housing by community and housing type. Specifically,

$$\begin{aligned} d_{j\in k} &= d_{j\in k}\left[\boldsymbol{H}_{j\in k}, \boldsymbol{G}_k, \alpha_{j\in k}\left(\boldsymbol{H}_{j\in k}, \boldsymbol{G}_k, R_{j\in k}\right)\right] \\ &= \sum_{i=1}^{N} P_{j\in k}^{i}\left[\boldsymbol{H}_{j\in k}, \boldsymbol{G}_k, \alpha_{j\in k}\left(\boldsymbol{H}_{j\in k}, \boldsymbol{G}_k, R_{j\in k}\right), \boldsymbol{S}_i\right]. \end{aligned} \tag{10.61}$$

Note that the price of housing ($R_{j\in k}$) has its impact in this specification entirely through the alternative specific constant. If $t_{j\in k}$ denotes the total available housing of type j in community k, then the equilibrium is characterized by

$$t_{j\in k} = d_{j\in k}\left[\boldsymbol{H}_{j\in k}, \boldsymbol{G}_k, \alpha_{j\in k}\left(\boldsymbol{H}_{j\in k}, \boldsymbol{G}_k, R_{j\in k}\right)\right]. \tag{10.62}$$

The general equilibrium impact of a change in the community public goods (for example, an improvement in air quality or public education, changing $\boldsymbol{G}_k$ to $\tilde{\boldsymbol{G}}_k$) is found by solving for the housing prices (say $\tilde{R}_{j\in k}$) that re-establish the equilibrium:

$$t_{j\in k} = d_{j\in k}\left[\boldsymbol{H}_{j\in k}, \tilde{\boldsymbol{G}}_k, \alpha_{j\in k}\left(\boldsymbol{H}_{j\in k}, \tilde{\boldsymbol{G}}_k, \tilde{R}_{j\in k}\right)\right]. \tag{10.63}$$

This involves solving for the new set of alternative specific constants

$$\tilde{\alpha}_{j\in k} = \alpha_{j\in k}\left(\boldsymbol{H}_{j\in k}, \tilde{\boldsymbol{G}}_k, \tilde{R}_{j\in k}\right), \tag{10.64}$$

that re-establishes the equilibrium in (10.63) and backs out the implied housing prices. Changes in welfare can then be inferred using standard log-sum formulas for evaluating changes in an RUM model. Changes in the composition of individuals living in a given community can be inferred by decomposing the housing demand in (10.61) by groups of interest (for example, the elderly, households with children, etc.).

There have been a number of applications of the horizontal sorting model to date, including Klaiber and Phaneuf's (2010) analysis valuing open space in the Twin Cities, Tra's (2010) examination of the impact in the Los Angeles basin of the 1990 Clean Air Act Amendments, and Bayer, Ferreira, and McMillan's (2007) consideration of how school quality impacts housing and community selection. As noted above, there are a number of appealing features of the horizontal sorting approach to valuing quality-differentiated commodities. There are, of course, limitations as well. First, much of this literature has relied on the standard logit structure in modeling choice probabilities. This specification makes the econometrics substantially easier, but requires the rather strong Independence of Irrelevant Alternatives (IIA) assumption (discussed in Chapter 3). In particular, it requires that the idiosyncratic error terms in (10.57) (that is, $\varepsilon^{i}_{j\in k}$) are uncorrelated across the choice options for a given individual. Since these terms capture unobserved individual specific factors influencing housing choice, this can be a strong assumption, depending upon how much information is observed about individual (and housing) characteristics. Relaxing this structure and using a nested logit or mixed logit model would seem worth consideration. Second, defining the choice set may be difficult, much like it is in the context of recreation demand. The housing units considered by individuals may span differing time periods (i.e., they may be looking for a house over a short or long time period) and communities. Banzhaf and Smith (2007) provided an excellent discussion of this issue, examining the impact of a wide range of possible choice sets. Their analysis, however, focused on the use of a single choice set across individuals, whereas the choice set itself may be individual specific. More research is needed into defining the choice set.

Summary

Hedonic price theory provides a coherent basis for explaining housing prices as a function of the levels of characteristics embedded in each house, including the environmental amenities and disamenities determined by the housing unit's location. Measures of value for marginal and nonmarginal changes in local public goods can be derived from a properly specified hedonic price model. Values for marginal changes in amenity levels are found simply by adding up the observed or computed marginal willingness to pay for all affected individuals. However, for nonmarginal amenity changes, when the hedonic price function itself might shift, welfare measurement requires knowledge of the inverse demand function or the income-compensated bid function for the amenity. These, in turn, require a solution to the daunting identification problem. As a result, much of the empirical literature has limited its attention to the so-called first stage of hedonic analysis—that is, estimating the hedonic price equation.

Beyond the difficulty in estimating both stages of the hedonic price model, and hence using it to assess nonmarginal shifts in amenities, there are some additional limitations to the hedonic property value model for use in estimating welfare effects.

First, the hedonic model assumes that consumers of housing can select their most preferred bundle of characteristics from a complete range of levels of all characteristics. In practice, the available housing units are finite both in numbers and in individual housing characteristics (for example, number of bathrooms and bedrooms, styles, etc.). This weakens the theoretical connection, depicted in equation (10.4), between the marginal cost of a housing attribute and the individual's marginal willingness to pay for the attribute that lies at the heart of using hedonic price analysis for welfare evaluation.

Second, since the property value models are based on the consequences of individuals' choices of residence, they do not capture willingness to pay for improvements in environmental amenities at other points in the urban area—for example, in the work place, shopping areas, or parks and recreational areas, or at second homes (see for example, Smith 2007).

Third, because the property value models are based on observing behavioral responses to differences in amenity levels across houses, they only capture willingness to pay for perceived differences in amenities and their consequences. For example, if there are subtle, long-term health effects associated with reduced environmental quality at some housing sites, but people are unaware of the causal link between these effects and the housing site, their willingness to pay to avoid the effects will not be reflected in housing price differences. Despite these limitations, the traditional hedonic price model provides a valuable tool in assessing or bounding the welfare implications of changing environmental amenities. It should also be noted that, while most applications of the hedonic model have used residential property values, the technique is also applicable to commercial and agricultural properties. For example, Mendelsohn, Nordhaus, and Shaw (1994) provided an analysis of the impact of climate on agricultural land values with a goal of assessing the effects of global warming on agriculture.

The equilibrium sorting literature has emerged in recent years as a means of addressing some of the issues associated with the hedonic pricing model. Both the vertical and horizontal sorting models allow for evaluation of nonmarginal shifts in public goods, including environmental amenities, by modeling the equilibrium process itself. This is particularly important in that many policy scenarios, such as those resulting from the 1990 Clean Air Act Amendments, involve significant changes in the environment that are likely to alter the housing market's equilibrium. Many of the applications to date suggest that ignoring general equilibrium effects of a policy (changing the hedonic pricing function and the resorting of households) can significantly bias the estimated welfare impacts of that policy. The horizontal sorting model also provides resolutions to two other issues in the hedonic price model by allowing for (a) a discrete, rather than continuous, stock of available housing units and (b) frictions in the housing market stemming from moving costs. The gains from the equilibrium sorting models, of course, come at a cost, including the need for additional assumptions regarding the structure of consumer preferences and the choice set available to consumers. At the same time, they represent a promising avenue for future research in efforts to understand how environmental amenities are capitalized into the housing market.

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