The General Validity of the Law of Comparative Advantage

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It is well known that the law of comparative advantage breaks down when applied to individual commodities or pairs of commodities in a many-commodity world. This paper shows that the law is nonetheless valid if restated in terms of averages across all commodities. Specifically, a theorem and several corollaries are derived which establish correlations between vectors of trade and vectors containing relative-autarky-price measures of comparative advantage. These results are proven in a general many-commodity model that allows for tariffs, transport costs, and other impediments to trade.

The purpose of this paper is to demonstrate, in a general model, the validity of a weak form of the Law of Comparative Advantage, that is, that the *pattern of international trade is determined by comparative advantage*. This is surely the oldest proposition in the pure theory of international trade and is common both to the Ricardian comparative-costs theory and the Heckscher-Ohlin factor-proportions theory, so long as comparative advantage is measured by relative autarky prices. As such, one might think that the proposition requires no further comment except in the basic textbooks whose job it is to explain important truths in simple terms.

Yet this proposition, like other more recent theorems of trade theory, has proven somewhat difficult to extend beyond the simple

models in which it was first formulated. Three examples should suffice to illustrate this difficulty. First, when Jones (1961) extended the doctrine of comparative advantage to a classical model with many goods and countries, he was forced to restate the concept of comparative costs in a form that lacked most of the simplicity and intuitive appeal of the original. Second, in the context of the Heckscher-Ohlin model, Melvin (1968) showed that if there are more goods than primary factors of production then the indeterminacy of the structure of production, that had been noted previously by Samuelson (1953), implies that any good may be exported by any country. This, it would seem, destroys altogether any determinate relationship between the pattern of trade and anything else. And third, Travis (1964, 1972) has argued that the introduction of impediments to trade, and particularly of tariffs, can alter the pattern of trade, causing goods that would have been exported to be imported and vice versa. Thus, it appears that if the two-commodity, two-country, free-trade model is extended or modified in plausible ways, it then ceases to be possible to explain the pattern of trade by simple comparisons of autarky prices. Most recently, this impossibility has been shown by Drabicki and Takayama (1979).

I will show in this paper, however, that a version of the comparative-advantage proposition *does* hold in a general model that allows for all of the complications just mentioned. This is not to say that the authors cited in the last paragraph were wrong. Instead, what is needed is to relax somewhat the rigidity of the proposition itself and require only that it hold in the sense of an appropriate average rather than for each commodity individually. While several forms of the proposition will be proved below, all may be summarized by the following statement: There must exist a negative correlation between any country's relative autarky prices and its pattern of net exports. Thus, on average, high autarky prices are associated with imports and low autarky prices are associated with exports.

This proposition will be demonstrated in a model that includes a variety of impediments to trade, as well as free trade, as special cases. I allow in a general way for transport costs, and I allow domestic and world prices to differ by additional amounts to reflect such artificial trade impediments as tariffs and quantitative restrictions. Unlimited interference with trade is not allowed, however, since it is clear that sufficient use of trade *subsidies* could lead to any pattern of trade and thus invalidate the law of comparative advantage. For ease of reference, and to distinguish it from the more restrictive case of free trade, I will refer to this combination of assumptions as defining "natural trade." Thus a natural trade equilibrium is one in which there are no trade subsidies or other artificial stimulants to trade, but in which

trade impediments of any sort may or may not be present. Free trade is then a special case of natural trade.[1]

My treatment of transport costs is somewhat unusual and should therefore also be mentioned in this introduction. Rather than postulate an explicit form for transport costs, I will distinguish goods on the basis of where they are delivered and incorporate transportation technology into a more general specification of the technology of production. For each country, a single production possibility set will define the constraints on its ability to produce for delivery at home and for delivery abroad. Thus any resources used up in transportation will be taken into account when the competitive producers and traders of the economy maximize the value of net delivered output. In the body of the paper I will simplify notation somewhat by assuming that all world trade passes through a single international port, though in an Appendix I show that most of my results carry over to a world of any number of such ports.

With this introduction the analysis can proceed. In Section I, I will state and discuss the assumptions of the model, which are broad enough to encompass a wide variety of models that have appeared in the literature. In Section II, I will first prove a basic theorem, which uses autarky prices to value the vector of goods that a country trades in a natural trade equilibrium. This result then leads readily to four corollaries which provide alternative statements of the law of comparative advantage in the average sense discussed above. In Section III, I discuss several ways that these results can be strengthened or modified. Finally, I return in Section IV to the particular issues raised above and show how my results contribute to an understanding of the various phenomena noted by other authors.

I. The Model

Consider a world of m countries, $i = 1, \ldots, m$, and n goods, $j = 1, \ldots, n$. The list of goods includes all final goods, intermediate goods, and services of primary factors of production. Each good may be delivered either on the country's home market or at the international port. Let Q^i be an n-vector of net supplies to country i's home market and T^i be an n-vector of net supplies by country i to the international port. Thus positive elements of Q^i represent goods available for consumption in country i, while negative elements represent net use of goods or factor services by production processes in country i. Similarly, the elements of the vector T^i represent the country's trade in each of the n goods: exports if positive and imports if negative. The country's total

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[1] Natural trade also includes autarky as another special case, though my results are, of course, of interest only in situations in which some trade actually does take place.

production, net of goods and resources used up in production and transport, is then $X^i = Q^i + T^i$.

Each country has its own net production possibility set, F^i, defined as the set of all feasible pairs of n-vectors, Q^i and T^i, given its technology and any constraints it may face on endowments of primary factors. Of the sets, F^i, I assume whatever is necessary to permit existence of the equilibria I will be studying. Thus, they must be closed, convex, and, in some weak sense, bounded from above.[2] In addition, I make the following assumption that says essentially that transport costs are nonnegative: If

$$(Q^i, T^i) \in F^i,$$

then   (1)

$$(Q^i + T^i, 0) \in F^i,$$

where 0 here represents an n-vector of zeros. This says that any total net output vector that is feasible with trade is also feasible without, since resources can only be used up, but never created, by transporting goods between the domestic market and the international port. Thus if it is feasible to produce a good and deliver it abroad, $T_j^i > 0$, then it is also feasible to produce it and deliver it at home. Likewise, if it is feasible to import a good, $T_j^i < 0$, then it is also feasible not to import it and to reduce deliveries on the home market by the same amount.[3]

To represent demand in each country, stronger assumptions will be used. I assume that preferences in each country can be represented by a family of n-dimensional community indifference curves, which it will be convenient to represent by a community utility function, U^i. These utility functions are assumed to have the property of local nonsatiation: For any Q^{i0} there exists a Q^{i1} arbitrarily close to Q^{i0} such that

$$U(Q^{i1}) > U(Q^{i0}).$$ (2)

This assumption will be used to rule out "thick" indifference curves.

Both producers and consumers are assumed to behave competitively, so that they maximize, respectively, the value of net output and the utility of consumption, subject to the prices that they face in each country.[4] In autarky equilibrium, production is for the domestic

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[2] It would be sufficient, though not necessary, to assume the existence of a vector $\bar{X}^i$ such that $Q + T \leq \bar{X}^i$ for all $(Q, T) \in F^i$.

[3] Note that this assumption includes, as a special case, the more explicit assumption made by Samuelson (1954), who specified that some fraction, α_j, of each good be used up in transport. If I let G be a more conventional production possibility set in which location of delivery is not specified, then my $F = \{(Q, T) \text{ such that } \bar{X} \in G \text{ where } \bar{X}_j = Q_j + T_j + \alpha_j |T_j|\}$. Assumption (1) then follows immediately.

[4] In order for producers to maximize the value of net output, I make the standard assumptions that domestic economies are competitive and that there are no externalities, production taxes, or other domestic distortions. Increasing returns to scale must also be ruled out.

market only, while with trade, producers maximize the sum of the value of output delivered at home and the value of output delivered abroad, the prices of which will in general be different due to transportation costs. The price of output delivered abroad will also in general be different from the world price, due to impediments to trade such as tariffs. Equilibrium in the home market requires that the vector of net supplies to the home market be consumed. Equilibrium in international trade requires in addition that the sum of all countries' net supplies to the international port be zero and that each country's trade be balanced at world prices.

I begin by characterizing autarky equilibrium. Let Q^{ai} be a vector of net outputs both supplied and demanded on the domestic market of country i under autarky, and let p^{ai} be a corresponding vector of autarky prices. Then the following three assumptions require that Q^{ai} be feasible, maximal, and preferred, given the prices p^{ai}:

$$(Q^{ai}, 0) \in F^i, \tag{3}$$

$$p^{ai}Q^{ai} \geqslant p^{ai}Q \text{ for all } (Q, 0) \in F^i, \tag{4}$$

$$U^i(Q^{ai}) \geqslant U^i(Q) \text{ for all } Q \text{ such that } p^{ai}Q \leqslant p^{ai}Q^{ni}. \tag{5}$$

Here, and throughout the paper, all products of vectors represent inner products.

To characterize a natural trade equilibrium, more notation is needed. Let Q^{ni} and T^{ni} be vectors of net supply by country i to domestic and foreign markets in a natural trade equilibrium. Let p^{qi} and p^{ti} be corresponding vectors of prices facing domestic producers, consumers, and traders in these markets, defined in terms of a single international numeraire. Thus the elements of p^{qi} are simply the domestic prices in country i, while those of p^{ti} are the prices paid or received by domestic traders at the international port and will be referred to as traders' prices. In particular, p^{ti} includes any tariffs that must be paid on domestic imports and is net of any export taxes paid on exports. The two vectors of prices will have to differ by enough to cover transport costs if trade is to take place, but this will be assured by the maximization assumption below. Assumptions analogous to those above require that production, trade, and consumption be feasible, maximal, and preferred given these prices:

$$(Q^{ni}, T^{ni}) \in F^i, \tag{6}$$

$$p^{qi}Q^{ni} + p^{ti}T^{ni} \geqslant p^{qi}Q + p^{ti}T \text{ for all } (Q, T) \in F^i, \tag{7}$$

$$U^i(Q^{ni}) \geqslant U^i(Q) \text{ for all } Q \text{ such that } p^{qi}Q \leqslant p^{qi}Q^{ni}. \tag{8}$$

Within the international port, there is also a vector of world prices, p^w, also measured in terms of the international numeraire. It represents the price at which international exchange actually takes place

and may differ from the national traders' prices, p^{ti}, to the extent that countries levy tariffs or export taxes. Each country's trade is assumed to be balanced at these world prices:

$$p^w T^{ni} = 0. \tag{9}$$

The relationship between world prices and national traders' prices can in general be complicated, depending both on the precise nature of trade impediments and on the direction of trade itself. However, all I need to characterize natural trade is to rule out trade subsidies, and this is done by the following simple assumption:

$$(p_j^w - p_j^{ti})T_j^{ni} \geq 0 \text{ for } \quad j = 1, \ldots, n. \tag{10}$$

What this says is that if a good is exported, $T_j^{ni} > 0$, then the world price must be at least as large as the price the exporter receives, any differences between the two representing an export tax levied by country i. And if a good is imported, $T_j^{ni} < 0$, then the world price must be no greater than the price the importer pays, any difference representing a tariff. Of course, if there were no policies interfering with trade, then (10) would be an equality.

Finally, I require that the world market for each good clear:

$$\sum_{i=1}^{m} T^{ni} = 0. \tag{11}$$

Assumptions (1) through (11) are sufficient for most of my results. However, later in the paper I will have occasion to make comparisons among autarky and world prices. For that purpose it will be convenient to normalize world prices and each country's autarky prices to lie on the unit simplex:

$$\sum_{j=1}^{n} p_j^w = \sum_{j=1}^{n} p_j^{ai} = 1 \quad i = 1, \ldots, m. \tag{12}$$

This is equivalent to taking, as numeraire, a bundle containing one unit of each good.

It may be well to note, before I proceed, that my list of assumptions does *not* include a number that are often made in trade theory. The utility functions have not been assumed to be differentiable and neither have the boundaries of the production possibility sets. The latter have not been assumed to be convex, nor the former to be homothetic. And neither have been assumed to be in any sense identical across countries. Thus the countries can differ arbitrarily in tastes, technologies, and factor endowments. None of this should be too surprising, however, since, while some of these assumptions may make certain modes of analysis more convenient, they are not really needed for establishing the role of comparative advantage.

More surprising, perhaps, is the limited amount I have had to say about the role of, and availability of, factor endowments in the model. Since the assumption of natural trade permits any of the "goods" to be nontraded, one can as well include the services of any or all factors of production as elements in the vector of n goods and even allow them to be in variable supply. One could even allow some or all of them to be traded internationally, so that the concept of comparative advantage is then extended to trade in the services of factors of production. Or, at the other extreme, one could identify as separate factors those which are employed in different industries and thus allow for various degrees of interindustry factor immobility. Finally, with some care one can allow for dynamic factor accumulation by interpreting the sets F as constraints on steady-state net output per capita.[5]

The point is that, while the list of assumptions earlier in this section was a long one, the assumptions themselves are much less restrictive than one often meets in models of international trade. With some care and ingenuity in interpreting the model, the results I am about to derive can shed light on the mechanism of international trade in quite a variety of contexts.

II. A Theorem and Its Corollaries

Now consider any one of the countries described in Section I. I will show first that if one uses autarky prices to evaluate its net trade vector in a natural trade equilibrium then that value must be less than or equal to zero. That is, the value of what a country gives up in trade is no greater, at autarky prices, than the value of what it acquires. While this may not seem to be a very surprising property, its proof uses most of the assumptions that were introduced in Section I. And once it is established, it leads fairly easily to a variety of results concerning comparative advantage.

The meaning of the theorem can be illustrated with the simple offer-curve diagram of figure 1. For a country which trades only two goods, the curve COC' shows the various possibilities for exports and imports of both goods with free trade. My theorem says that any trade which takes place will have a negative value when valued at autarky prices. But since autarky prices are given by the slope of the line tangent to the offer curve at the origin, POP', this means merely that

[5] Care is needed here in assuring that a competitive economy will still, in a dynamic context, maximize the steady-state value of net output per capita. If technology permits inputs and outputs at different times, then it is the discounted value of the input-output stream that is maximized by a competitive economy. This can be resolved for one's purposes either by assuming that the interest rate equals the natural rate of growth or, more generally, by making a distinction between rental markets for the services of stocks of goods and the markets for their flow as final sales.
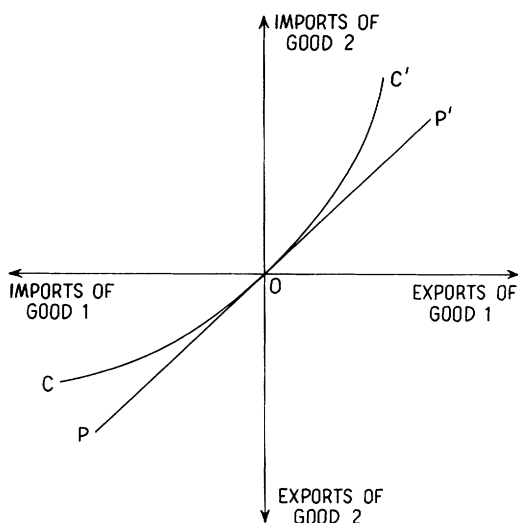
FIG. 1

COC' lies wholly to one side of this line. Well-behaved offer curves will have this property, and the reader may recognize a number of my assumptions as necessary to prevent the offer curve from "bending backward." Of course my model is far more general than figure 1, since it allows an arbitrary number of goods and does not require free trade. Still, one can interpret the theorem as saying that autarky prices provide a supporting hyperplane for the set of all possible trades.

Theorem.—If prices and quantities in autarky and trade satisfy assumptions (1)–(10) for a particular country, i, then

$$p^{ai}T^{ni} \leq 0. \tag{13}$$

Proof.—Since I am dealing with only a single country, I will omit the country superscript in what follows. I begin by adding up the inequalities in (10) to get

$$p^w T^n - p^t T^n \geq 0, \tag{14}$$

from which, using (9),

$$p^t T^n \leq p^w T^n = 0. \tag{15}$$

From (3) and (7) it then follows that

$$p^q Q^a \leq p^q Q^n + p^t T^n \leq p^q Q^n, \tag{16}$$

and this, from (8), implies

$$U(Q^n) \geq U(Q^a). \tag{17}$$

Thus the vector of goods consumed in natural trade is at least as preferred as that consumed in autarky. It follows from (2) and (5) that

$$p^a Q^n \geq p^a Q^a, \tag{18}$$

for if this were not the case, (2) would permit one to find another vector, $\bar{Q}$, in the neighborhood of Q^n, that would also cost less at autarky prices than Q^a but would be strictly preferred, violating (5).

Now (6) and (1) imply that $(Q^n + T^n, 0) \in F$, so that (4) implies

$$p^a Q^a \geq p^a (Q^n + T^n). \tag{19}$$

Rearranging and using (18) one gets the desired result:

$$p^a T^n \leq p^a Q^a - p^a Q^n \leq 0. \tag{20}$$

Q.E.D.

The theorem has been stated in the form of a weak inequality. Still it should be clear, from the chain of reasoning used in its proof, that there are many ways that the inequality can be strengthened. Some of these will be mentioned in Section III.

Consider, now, the issue of comparative advantage. My theorem is already very suggestive in this regard: For the autarky-price value of trade to be negative, it would have to be true that the autarky prices of exports are low compared with those of imports, and thus the country is exporting those goods with relatively low autarky prices as the principle of comparative advantage would suggest. To make this relationship more precise, I will now examine several correlations between particular vectors of relative autarky prices and net exports. By showing these correlations to be less than or equal to zero, I establish a *tendency* for high-autarky-priced items to be imported and for low-autarky-priced items to be exported. I do not attempt to say anything about the pattern of trade in any particular commodity, or pair of commodities, for it is known from the work of others that such statements are likely to be invalid in a model as general as mine. Still, if I can show a negative correlation such as just described, I will have demonstrated that comparative advantage is nonetheless valid as at least a partial determinant of the pattern of trade overall.

The sign of a correlation between two vectors is, of course, the same as the sign of their covariance. For any two n-vectors, x^1 and x^2, by definition

$$\text{cor}\,(x^1, x^2) = \frac{\text{cov}\,(x^1, x^2)}{\sqrt{\text{var}\,(x^1)\,\text{var}\,(x^2)}} \tag{21}$$

where

$$\text{cov}\,(x^1, x^2) = \sum_{j=1}^{n} (x_j^1 - \bar{x}^1)(x_j^2 - \bar{x}^2), \tag{22}$$

$$\text{var}\,(x^i) = \sum_{j=1}^{n} (x_j^i - \bar{x}^i)^2 \quad \text{for } i = 1, 2, \tag{23}$$

and

$$\bar{x}^i = \frac{1}{n} \sum_{j=1}^{n} x_j^i \quad \text{for } i = 1, 2. \tag{24}$$

Since the denominator of (21) is nonnegative (and nonzero if the correlation is defined), the correlation and covariance must have the same sign.[6] Furthermore, the covariance can be rewritten as follows:

$$\text{cov}\,(x^1, x^2) = x^1 x^2 - n\bar{x}^1\bar{x}^2. \tag{25}$$

Thus if either of the vectors sums to zero, so that it has zero mean, then the sign of their correlation is just the sign of their inner product. Since I will use this property several times, I state it formally for ease of reference: If

$$\sum_{j=1}^{n} x_j^i = 0 \quad \text{for } i = 1 \quad \text{or} \quad i = 2, \tag{26}$$

then

$$\text{cor}\,(x^1, x^2) \gtreqless 0 \qquad \text{as } x^1 x^2 \gtreqless 0.$$

In stating the role of comparative advantage, my first problem is to decide what is meant by "relative autarky prices." With many countries, there is no single set of foreign autarky prices with respect to which one country's can be compared. I will resolve this problem first by using world prices with trade as the basis for comparison. This gives corollaries 1 and 2 below. Then I show in corollary 3 that if in fact there are only two countries, so that a single vector of foreign autarky prices can be identified, then a comparison of the two vectors of autarky prices also yields the appropriate correlation with the vector of trade. Finally, in corollary 4 I establish a correlation between trade and autarky prices for a world of many countries without reference to world prices.

Corollary 1.—For any country let ρ be the vector of ratios of its autarky prices to the world prices that prevail with trade,

$$\rho_j = p_j^a / p_j^w \quad j = 1, \ldots, n, \tag{27}$$

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[6] I ignore, in what follows, the possibility of either vector having a zero variance so that the correlation is undefined. When that happens, as it will when there is no trade or when relative autarky prices are all identical, my conclusions about correlations can be restated as conclusions about the corresponding covariances only.

and let e be the vector of the country's net exports, valued at world prices,

$$e_j = p_j^w T_j^n \qquad j = 1, \ldots, n. \tag{28}$$

Then if assumptions (1)–(10) are satisfied for that country, it must be true that

$$\text{cor}\,(\rho, e) \leq 0. \tag{29}$$

   Proof.—From balanced trade (9),

$$\sum_{j=1}^{n} e_j = p^w T^n = 0 \tag{30}$$

so that (26) permits one to consider only the inner product, ρe. But

$$\rho e = \sum_{j=1}^{n} \frac{p_j^a}{p_j^w} p_j^w T_j^n = \sum_{j=1}^{n} p_j^a T_j^n = p^a T^n \leq 0 \tag{31}$$

by the theorem. Q.E.D.

   Here I have used ratios of autarky to world prices as the basis for comparison of industries. A slightly different comparison is also possible using the difference between autarky and world prices. For this difference to be meaningful, I assume that both price vectors are normalized to lie on the unit simplex, as stated in assumption (12).

   Corollary 2.—If assumptions (1)–(10) are satisfied for any country and if prices are normalized as in (12), then

$$\text{cor}\,(p^a - p^w, T^n) \leq 0. \tag{32}$$

   Proof.—For the inner product I have

$$(p^a - p^w)T^n = p^a T^n - p^w T^n = p^a T^n \leq 0 \tag{33}$$

by (9) and the theorem. Since, by (12),

$$\sum_{j=1}^{n} (p_j^a - p_j^w) = \sum_{j=1}^{n} p_j^a - \sum_{j=1}^{n} p_j^w = 0, \tag{34}$$

(26) permits one to deduce (32) from (33). Q.E.D.

   Notice that both of these results obtain for any particular country without any assumptions whatever about behavior in the rest of the world. Also, no use has been made of the requirement that world markets clear, assumption (11). This, of course, is because I have not yet tried to make comparisons across countries. I now make such a comparison for a two-country world.

   Corollary 3.—If the world contains only two countries, $i = 1, 2$, both satisfying (1)–(10), and if (11) is also satisfied, then

$$\text{cor}\,(p^{a1} - p^{a2}, T^{n1}) \leq 0, \tag{35}$$

where p^{a1} and p^{a2} are both normalized to the unit simplex (12).

 Proof.—From (11) with $m = 2$ note that

$$T^{n1} = -T^{n2}. \tag{36}$$

Thus

$$(p^{a1} - p^{a2})T^{n1} = p^{a1}T^{n1} + p^{a2}T^{n2}, \tag{37}$$

which is seen to be nonpositive by applying the theorem to each of the two terms on the right-hand side separately. The normalization (12), together with (26), then implies (35). Q.E.D.

This result, though derived only for a two-country world, has the advantage of placing a clear restriction on the possible pattern of natural trade. Given the two vectors of autarky prices, it says that, of all the conceivable patterns of trade that might emerge, only those which yield the indicated nonpositive correlation will be observed. Corollaries 1 and 2, on the other hand, do not embody such a clear restriction on the pattern of trade, since they contain the world-price vector which must be determined simultaneously with the pattern of trade.

To obtain a similar restriction for a many-country world I now prove a final corollary which deals with all countries and industries simultaneously. Let P^a be a vector of length mn containing the autarky prices of all countries and all industries, and let E be a vector of the same length containing the net exports of all countries and all industries, arranged in the same order as in P^a.

 Corollary 4.—If the world contains m countries, all satisfying assumptions (1)–(10), and if (11) is also satisfied, then

$$\text{cor}\,(P^a, E) \leq 0. \tag{38}$$

 Proof.—From the construction of E, it is clear that

$$\sum_{k=1}^{mn} E_k = \sum_{j=1}^{n}\left(\sum_{i=1}^{m} T_j^{ni}\right) = 0 \tag{39}$$

by (11). Thus (26) allows one to look only at the inner product of P^a and E. But

$$P^a E = \sum_{k=1}^{mn} P_k^a E_k = \sum_{i=1}^{m}\left(\sum_{j=1}^{n} p_j^{ai} T_j^{ni}\right)$$

$$= \sum_{i=1}^{m} p^{ai} T^{ni}. \tag{40}$$

From the theorem, each term in this summation is nonpositive, implying (38). Q.E.D.

To summarize, the first two corollaries provide alternative expla-

nations of a single country's trade, one in value terms, the other in terms of quantities, with comparative advantage measured by comparisons of autarky prices with world prices. The third corollary provides the general analogue to the traditional comparative-advantage proposition for two countries in terms of a good-by-good comparison of the two countries' autarky prices. Finally, the fourth and last corollary provides the most general statement of comparative advantage for the world as a whole.

III. Refinements

In this section I point out two ways that the proofs above can be modified, either to strengthen the results or to alter the assumptions needed for their validity.

Strong Inequalities

The theorem and its corollaries are stated in terms of weak inequalities. These can be strengthened by any of several additional assumptions which serve to contribute a strict inequality somewhere in the chain of reasoning used to prove the theorem. I leave it to the reader to verify that any of the following assumptions would serve this purpose. (1) Transport costs are strictly positive for nonzero trade. (2) Tariff or export tax revenues are strictly positive in the natural trade equilibrium. (3) The optimal consumption bundle given any positive price vector is unique and different in natural trade and autarky. (4) The production possibility set is strictly convex, and trade and autarky prices differ.

While the first of these assumptions, especially, is realistic, I have chosen not to use any of them throughout the paper since they would exclude one of the best-known models and results of trade theory. That is the classical Ricardian constant-costs model with two countries of different size. In that model a trade equilibrium can arise in which the world prices equal the autarky prices of the large country, which then alters production, but not consumption, when it moves from autarky to trade. In that case the value of its trade at its own autarky prices is of course zero, as are the correlations of corollaries 1 and 2 for the large country. However, aside from this and perhaps other equally special cases, one can expect strictly negative correlations to be the normal result.

Relation to the Gains from Trade

The reader will already have noted that part of the proof of the theorem bears a marked similarity to the proof of the gains from

trade (Samuelson 1939). I showed first that natural trade would be preferred to autarky (inequality [17]) and then that this implies the theorem. An alternative proof could have begun, then, with the assumption that trade is beneficial.

While this would not really be an improvement over the present analysis, since it would leave unanswered the question of whether there would indeed be gains from trade, this modification does suggest an alternative proof that does not rely on the fiction of community preferences.[7] Suppose that each country contains many individuals, each with his own preferences, and that a move from autarky to trade is accompanied by suitable redistribution of income so as to leave all individuals better off. Then an inequality like (18) can be obtained for each consumer individually, and (18) itself can be obtained by adding these up. The rest of the proof then follows.

IV. Discussion

At the beginning of the paper I noted that familiar simple statements about the role of comparative advantage become difficult or impossible when models are complicated in a variety of realistic ways. I have now developed an alternative way of representing the relationship between autarky prices and trade by looking at correlations between the two. This has enabled me to state a variety of simple propositions regarding comparative advantage, and I have proved these propositions in a very general model. I will now compare these results with others that have appeared in the literature.

Consider first the classical model of constant costs. Here it must be admitted that my contribution is limited, for the role of comparative advantage is already well understood in the classical model. The familiar proposition for a two-country world, that goods can be ranked in terms of comparative advantage with one end of the chain being exportable and the other importable, is considerably stronger than my own corollary 3 for the same case.[8] I note only what has sometimes been doubted: that while the classical theory predicts only the direction and not the magnitude of trade, it nonetheless permits one to infer a negative correlation between relative costs and net exports.[9]

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[7] This alternative proof was suggested by Ted Bergstrom.

[8] The explanation of trade in terms of a chain of comparative advantage was apparently first demonstrated by Haberler (1936).

[9] Doubt that this should be true has been expressed, e.g., by Bhagwati (1964, p. 11) in criticizing empirical tests of the comparative-costs theory. Similar doubts in the context of the factor-proportions theory also led Harkness and Kyle (1975) to employ logit analysis, rather than least-squares regression, to test that theory. While my results do not deal directly with factor endowments or factor intensities, they nonetheless suggest that tests for simple correlations may not, after all, be inappropriate.

With both many countries and many goods, the classical model has been examined in detail by Jones (1961). His results, again, allow a much more precise determination of the pattern of international specialization than does, say, my corollary 4. Still, my results do show that a straightforward comparison of costs does have something to say about the pattern of trade without going all the way to the solution of a mathematical programming problem as is done by Jones.[10]

Turning now to generalizations of the factor-proportions theory of trade, one sees that my model is consistent with the 3-good, 2-factor model in which Melvin (1968) found the pattern of trade to be indeterminate. But this indeterminacy has not prevented me from obtaining meaningful correlations. What I have done, in a sense, is to exploit those limitations that Melvin *was* able to place, implicitly, on the pattern of trade, as he did in his elegant figure 8, and to show that these limitations are enough to assure that the pattern of trade still, in a general sense, follows the dictates of comparative advantage. In his figure, Melvin showed that trade could be represented by any of an infinite number of lines, connecting two other parallel lines and passing through a single point representing demand. The clue to the validity of my correlations is that the two parallel lines must lie on opposite sides of the demand point in a manner that is prescribed by relative autarky prices. Thus in Melvin's model, and in more general multicommodity, multifactor models, the law of comparative advantage is weakened but not destroyed by the indeterminacy of the pattern of production.

Finally, a great advantage of my model is that it allows for a considerable amount of interference with the free flow of trade by such impediments to trade as transport costs and tariffs. It is true, as Travis (1964, 1972) has suggested, that such impediments can cause particular goods to be exported that would have been imported and vice versa.[11] But while this is possible for particular goods, my analysis shows that it cannot be true of so many goods as to reverse the average relationship that must hold between comparative advantage and trade. Only by subsidies could this average relationship be made not to hold—subsidies either of production, which would violate my assumptions (4) and (7), or of trade, which could violate my assumption (10).

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[10] In a further discussion of his results, Jones (1977) remarks that "alternative and equivalent criteria in two-by-two cases may not prove equivalent in more general settings. But knowledge of the general case can aid in recasting criteria in the simple model so that it can generalize." My results provide another example of this phenomenon, for my correlations, when applied to the two-by-two case, are equivalent to alternative statements of the comparative advantage criterion.

[11] An example of this is given in Deardorff (1979) and requires the presence of intermediate goods, as well as tariffs or transport costs, in a multicommodity model.

Appendix

Instead of the single international port assumed in the body of the paper, let there now be an arbitrary number of ports, l, indexed $h = 1, \ldots, l$. While it is not necessary, these could now be identified with the countries of the model, in which case one would have $l = m$. Instead of a single trade vector for each country, I must now distinguish trade vectors for each of the l destinations. Let ${}^h t^i$ represent country i's vector of net supplies to port h. Then the total trade vector, $T^i = \Sigma_{h=1}^{l} {}^h t^i$. In addition I must now distinguish separate world-price vectors at each port, ${}^h p^w$, and separate vectors of traders' prices facing country i's traders at each port, ${}^h p^{ti}$. A country's production possibilities set, F^i, now contains all feasible collections of the $l + 1$ vectors $(Q^i, {}^1 t^i, \ldots, {}^l t^i)$.

The following assumptions now replace those in Section I:

$$\text{If } (Q^i, {}^1 t^i, \ldots, {}^l t^i) \in F^i, \quad \text{then} \quad \left(Q^i + \sum_{h=1}^{l} {}^h t^i, 0, \ldots, 0 \right) \in F^i, \quad \text{(A1)}$$

$$(Q^{ai}, 0, \ldots, 0) \in F^i, \quad \text{(A3)}$$

$$p^{ai} Q^{ai} \geq p^{ai} Q \text{ for all } (Q, 0, \ldots, 0) \in F^i, \quad \text{(A4)}$$

$$(Q^{ni}, {}^1 t^{ni}, \ldots, {}^l t^{ni}) \in F^i, \quad \text{(A6)}$$

$$p^{ai} Q^{ni} + \sum_{h=1}^{l} {}^h p^{ih} t^{ni} \geq p^{ai} Q + \sum_{h=1}^{l} {}^h p^{ih} t \text{ for all } (Q, {}^1 t, \ldots, {}^l t) \in F^i, \quad \text{(A7)}$$

$$\sum_{h=1}^{l} {}^h p^{wh} t^{ni} = 0, \quad \text{(A9)}$$

$$({}^h p_j^w - {}^h p_j^{ti}) {}^h t_j^{ni} \geq 0 \quad \text{for } j = 1, \ldots, n; h = 1, \ldots, l, \quad \text{(A10)}$$

$$\sum_{i=1}^{m} {}^h t^{ni} = 0 \quad \text{for } h = 1, \ldots, l, \quad \text{(A11)}$$

$$\sum_{j=1}^{n} p_j^{ai} = 1 \qquad i = 1, \ldots, m. \quad \text{(A12)}$$

The remaining assumptions are unchanged.

The statement of the theorem is unchanged, but its proof must be modified. Inequality (14) is obtained for each port individually, and thus from (A9), I get

$$^h p^{th} t^n \leq 0 \quad \text{for } h = 1, \ldots, l. \quad \text{(A15)}$$

Inequality (16) then follows from (A7) as before; (17) and (18) are unchanged, while (19) follows from (A1), (A6), and (A4) as before; (20) follows, and the theorem is proved.

Corollaries 1 and 2 no longer make sense, as there are now many vectors of world prices. However, corollaries 3 and 4 can still be proved by using the theorem and (A11).

Thus most of the results of the paper continue to be valid in a world of many ports, and the fiction of a single port was needed only to simplify notation.

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