Problem Set 2
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Thank you to Nadia & Sunny for the helpful discussions.

1/ Fishing and Extinction

Setup

- Salmon fishery, single owner
- price taker: p constant, exo
- y_t extraction rate at t
- x_t stock at t
- $c(y)$ extract. cost : $c'(y_t) \in (0, p)$ $c''(y) > 0$
- $\dot{x}_t = f(x_t) - y_t$
  - ↳ $f(0) = 0$ $f'(0) > 0$ $f''(.) < 0$
  - ↳ $\exists\ \bar{x} > 0$ st $f(\bar{x}) = 0$
  - ↳ x_0 initial resource stock
- r = the owner's discount rate, $t \to \infty$

a) $$H = p y_t - c(y_t) + \lambda [f(x_t) - y_t]$$

↳ FOC $[y_t]$ $p - c'(y_t) - \lambda \leq 0$ $y_t \geq 0$ c.s

$[x_t]$ $-[\lambda f'(x_t)] = \dot{\lambda}_t - r\lambda_t$ $x_t \geq 0$

$\Leftrightarrow \dot{\lambda}_t = [r - f'(x_t)]\lambda_t$

TVC $\lim_{t\to\infty} \lambda_t e^{-rt} \geq 0$ $\lim_{t\to\infty} \lambda_t x_t e^{-rt} = 0$

b) "As t goes to ∞" $\Rightarrow$ steady state

$\dot{\lambda}_t = 0 \Rightarrow r = f'(x_t)$

$\dot{x}_t = 0 \Rightarrow f(x_t) = y_t$

Moreover, from the TVC, $\lim_{t\to\infty} x_t > 0 \Rightarrow \lim_{t\to\infty} \lambda_t e^{-rt} = 0 \Rightarrow$

$\dot{\lambda}_t \leq 0 \Rightarrow [r - f'(x_t)] \leq 0$

$r \leq f'(x_t) \Rightarrow r = f'(x_{ss})$

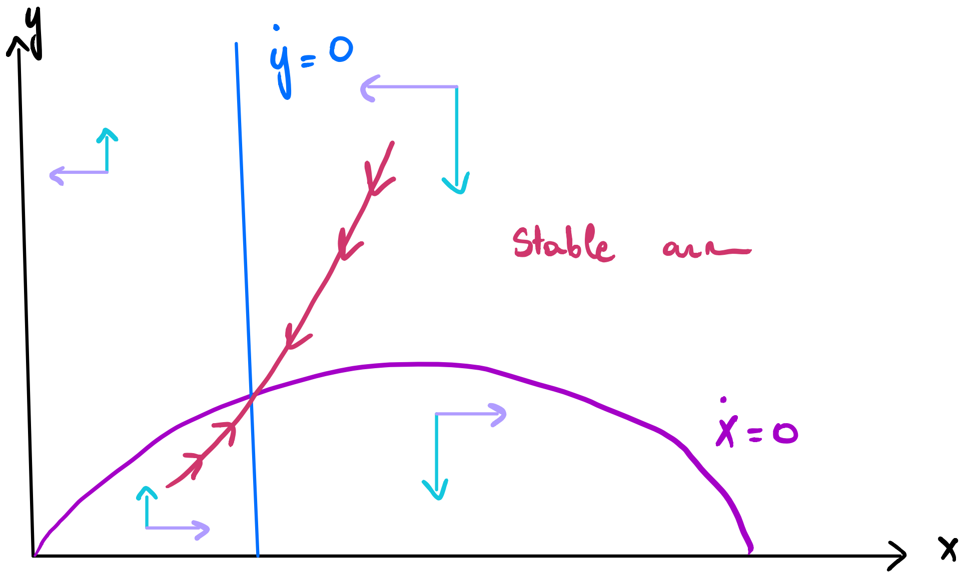
With $x_{ss} > 0$ and $f'(.) \searrow$ as $f''(.) < 0$ it must be that $x_{ss} > 0$

$\Rightarrow f'(x_{ss}) < f'(0)$

$$\underbrace{r - f'(x_{ss})}_{=0} > r - f'(0)$$

$$r < f'(0)$$

c)



$\dot x = 0 \Rightarrow f(x_t) = y_t$

$\dot\lambda_t = [r - f'(x_t)]\,\lambda_t$ + FOC $\quad \lambda_t = p - c'(y_t)$

$\longleftrightarrow \dot\lambda_t = -c''(y_t)\,\dot y_t$

$$\Rightarrow -c''(y_t)\,\dot y_t = [r - f'(x_t)][p - c'(y_t)]$$

$$\Rightarrow \dot y_t = \frac{[f'(x_t) - r][p - c'(y_t)]}{c''(y_t)}$$

$\rightarrow c''(y_t) > 0$

$p - c'(y_t) > 0$

$\Rightarrow \dot y_t = 0 \Rightarrow f'(\bar x) = r$

$\bar x = f'^{-1}(r)$

<u>Stable arm</u>

$$\dot y_t = \frac{[f'(x_t) - r][p - c'(y_t)]}{c''(y_t)}$$

↳ $x > x_{ss} \Rightarrow f'(x) < f'(x_{ss})$

$\rightarrow \boxed{\dot y < 0}$

$$\dot{X}_t = f(X_t)\ y_t$$

↳ $y > y_{ss} \Rightarrow \dot{X}_t < 0$

d/ At T : The fishery will close. Still assume that $f'(0) > r$

↳ We are not in an infinite time problem anymore, so the TVC conditions become

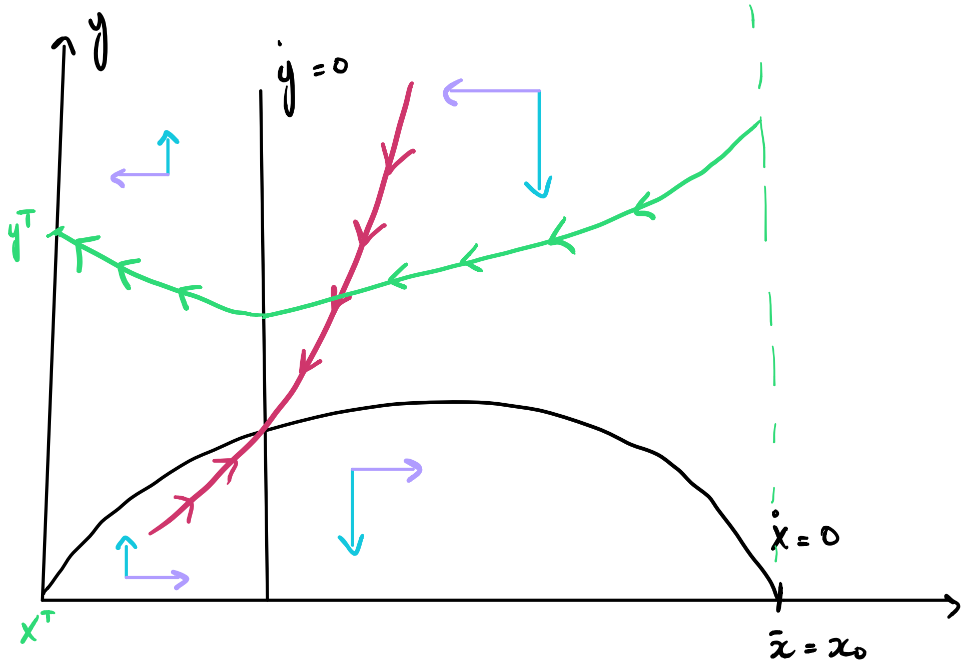
$$\lambda^T e^{-rT} \geq 0 \qquad X_T \geq 0 \qquad \lambda_T e^{-rT} X_T = 0$$

↳ the stock has no more value at T as the fishery closes so $\lambda^T > 0$

However, we also have $\lambda_t = p - c'(y_t) > 0$

$\Rightarrow$ it must be that $X_T = 0$ for $\lambda_T e^{-rT} X_T = 0$ with $T < \infty$

e/



× path

When we pass to the left of $\dot{y}_t = 0$, and have $\dot{y}_t > 0$, what happens is that $f'(X_t) > r$ so the extract rate increases as future stock is worth less on the margin.

From the FOC

$$\underbrace{p - c'(\overset{\uparrow}{y_t})}_{\downarrow} = \underset{\downarrow}{\lambda_t}$$

2) Common pool with an exhaustible Resource

Setup :
S = initial oil reserve
N = number of firms
Resource = common pool
MC strictly ↗ in rate of extract° but no stock effect

a) In continuous time, the firm's problem is

$$\max \int_0^{\infty} \left(p\, y_{it} - c(y_{it})\right) e^{-\rho t}\, dt \quad \text{st} \quad \dot{X}_t = g(X_t) - y_{it} - \Sigma_{j\neq i}\, y_j(X_t)$$

$$y_{it} \geq 0 \qquad X_0 > 0$$

$$X_t \geq 0$$

Write as a CV Hamiltonian:

$$H = p\, y_{it} - C(y_{it}) + \lambda_t \left[g(X_t) - y_{it} - \Sigma_{j\neq i}\, y_j(X_{it}) \right]$$

↳ FOC $[y_{it}]$: $p - c'(y_{it}) - \lambda_t \leq 0 \qquad y_{it} \geq 0 \qquad$ CS

$[X_t]$: $\dot{\lambda}_t = \rho \lambda_t - \lambda_t\, g'(X_t) + \lambda_t \Sigma_{j\neq i}\, y_{it}'(X_t)$

TVC $\lim\limits_{t\to\infty} \lambda_t e^{-rt} \geq 0 \qquad \lim\limits_{t\to\infty} \lambda_t X_t e^{-rt} = 0 \qquad X_t \geq 0$

Using symmetry as all firms play a Markov Perfect Eq^M:

$$\dot{\lambda}_t = \rho\lambda_t - \lambda_t\, g'(X_t) + \lambda_t (N-1) \frac{dy_{it}}{dX_t}$$

b) FOC on y_{it} : $p = \lambda_t + c'(y_{it})$ for interior solutions where the LHS is the marginal return from extracting an additional unit (and selling it at p) and the RHS is the marginal cost. The latter has two parts
- the MC of extraction $c'(y_{it})$
- the foregone MV (shadow) of an additional unit of stock.

FOC on $[X_t]$ $\Rightarrow \frac{\dot{\lambda}_t}{\lambda_t} = \left[\rho - g'(X_t) + (N-1) \frac{dy_{it}}{dX_t} \right]$

the growth rate of the marginal shadow value of an additional unit of stock X_t is

- increasing in the discount factor ρ: the more patient the agent, the more valuable future stock
- increasing in the growth rate of the stock ($g'(X_t) < 0 \Rightarrow -g'(X_t) > 0$)

the common pool problem

- accounts for other firm's reactions to changes in X_t (envelope theorem): the larger $\frac{dy_{it}}{dX_t}$, the more positive (less negative) $\frac{\dot{\lambda}_t}{\lambda_t}$ because the initial extraction rate is larger.

We can plug FOCs into each other: $p = \lambda_t + c'(y_{it})$

$\Rightarrow \lambda_t = p - c'(y_{it})$

$$\Rightarrow \dot{\lambda}_t = \left[\rho - g'(X_t) + (N-1) \frac{dy_{it}}{dX_t} \right] \left[p - c'(y_{it}) \right]$$

at the ss $\dot{\lambda}_t = 0$ $\Rightarrow g'(X_t) = \rho + (N-1) \frac{dy_{it}}{dX_t}$

This compares to the social optimum ss from the class notes

$$g'(X_t) = \rho \quad \text{(no stock effect)}$$

$$\Rightarrow g'(X_{ss,\,social}) < g'(X_{ss,\,CE})$$

$$\Leftrightarrow X_{ss,\,social} > X_{ss,\,CE}$$

3) Highway Congestion Externalities

Set up : N drivers commute daily to Manhattan across Hudson River

GW bridge direct, crowded: $z = \alpha X$

alternative: $z = 1$

z = daily commute time GW

X = nb drivers using GW

w = value of time

a) $\max \{ -w(\alpha X^{NE}) \; ; \; -w \}$

↳ take GW as long as $\alpha X^{NE} \leq 1$

$X^{NE} \leq \frac{1}{\alpha}$

$\Rightarrow$ NE : $X^{NE} = \frac{1}{\alpha}$ drivers take GW

b) Pareto efficient : MB of GW = MC of GW

↳ MB of GW $= (1 - \alpha X_p) w$

↳ MC of GW $= (X_p \alpha) w$ → impose α additional minutes on all X_p drivers

$$\rightarrow 1 - \alpha X = \alpha X_p$$

$$1 = 2\alpha X_p$$

$$X_p = \frac{1}{2\alpha} = 0.5 \frac{1}{\alpha} = 0.5 \; X^{NE}$$

c) At X_p, the externality is $\frac{d(\alpha X)}{dX} \cdot X_p \cdot w = \alpha \cdot \frac{1}{2\alpha} w = \frac{w}{2}$

αw = net welfare taking the bridge

d) Imposing a "Pigouvian toll" → $T = \alpha X_p$

↳ the driver maximizes $\max \{ -w\alpha X - T \; ; -w \}$

↳ the marginal driver on GW has

$$w = w\alpha X + T$$

$$w(1 - \alpha X) = T \quad \Rightarrow \quad 1 - \alpha X = \frac{T}{w}$$

$$\Rightarrow \alpha X = 1 - \frac{T}{W}$$

$$T = \frac{W}{2}$$

$$X = \frac{1}{\alpha} - \frac{T}{\alpha W} = \frac{1}{\alpha} - \frac{W}{2}\frac{1}{W\alpha}$$

$$= \frac{1}{2\alpha} = X_p$$

→ at the equilibrium, only the gov captures the efficiency gains.

Drivers are not better off since $-W = -$ cost drive GW still holds.

h) Targeting Pigouvian Taxes

Good produced with polluting technology.

$U(q) = aq - \left(\frac{b}{2}\right)q^2$ $\quad a, b > 0$ $\quad$ private benef

τe $\quad \tau \geq 0$ $\quad$ social damage

$f(k,e)$ production funct. $\Rightarrow$ CRS $\quad f_k > 0 \quad f_{kk} < 0$

k_i input $\Rightarrow$ price r_i $\quad f_e > 0 \quad f_{ee} < 0$

a) Optimal tax on emissions t^* is such that the social cost of pollut. τe is internalized by the firm.

Planner's problem: $\max_{k,e} \; a f(k,e) - \frac{b}{2} f(k,e)^2 - rk - \tau e$

↳ FOC wrt k_i $(\forall i)$ and e

$*$ $\quad [k_i] \quad a f_i(k,e) - b f(k,e) f_i(k,e) - r_i = 0$

$*$ $\quad [e] \quad a f_e(k,e) - b f(k,e) f_e(k,e) - \tau = 0$

Firm's problem $\max_{k,e} \; a f(k,e) - \frac{b}{2} f(k,e)^2 - rk - te$

↳ we see that is the same problem and the same FOCs if $\tau = t^*$

b) The planner's pb doesn't change, but the firm problem becomes

$\max_{k,e} \; (a-t) f(k,e) - \frac{b}{2} f(k,e)^2 - rk$

↳ FOC $\quad [k_i] \quad (a-t) f_i(k,e) - b f(k,e) f_i(k,e) - r_i = 0 \quad *'$

$[e] \quad (a-t) f_e(k,e) - b f(k,e) f_e(k,e) = 0 \quad **''$

Rearranging $\quad *' \quad a f_i(k,e) - b f(k,e) f_i(k,e) - r_i = t f_i(k,e)$

$**' \quad a f_e(k,e) - b f(k,e) f_e(k,e) - t f_e(k,e)$

In order for $\begin{cases} *' \\ **' \end{cases}$ to match $\begin{cases} * \\ ** \end{cases}$ it must be the case that

$$\begin{cases} t f_e(k,e) = \tau \\ t f_i(k,e) = 0 \end{cases} \Rightarrow t = 0 \Rightarrow \tau = 0$$

Because taxing e implies taxing k too, any tax will distort the firm's decision and the optimal outcome. Therefore, the only non-distortionary eq^M implies that $t = 0 \Rightarrow \tau = 0$.

c) The firm's problem becomes

$$\max_{k,e} \; a f(k,e) - \frac{b}{2} f(k,e)^2 - (r + t_i) k$$

$*''$ ↳ FOC $[k_i]$: $a f_i(k,e) - b f(k,e) f_i(k,e) - r - t_i = 0$

$**''$ $[e]$: $a f_e(k,e) - b f(k,e) f_e(k,e) = 0$

↳ For $\begin{cases} *'' \\ **'' \end{cases}$ to equal $\begin{cases} * \\ ** \end{cases}$ it must be the case that

$t_i = 0$; $\tau = 0$.

d) The firm's problem

$$\max_{k,e} \; (a - t_0) f(k,e) - \frac{b}{2} f(k,e)^2 - (r + t_i) k$$

$*'''$ ↳ FOC $[k_i]$: $a f_i(k,e) - b f(k,e) f_i(k,e) - r - t_i - t_0 f_i(k,e) = 0$

$**'''$ $[e]$ $a f_i(k,e) - b f(k,e) f_e(k,e) - t_0 f_e(k,e) = 0$

For $\begin{cases} *''' \\ **''' \end{cases}$ to equal $\begin{cases} * \\ ** \end{cases}$ it must be the case that

$$\begin{cases} t_0 f_e(k,e) = \tau \\ t_i + t_0 f_i(k,e) = 0 \quad \forall i \end{cases} \Rightarrow \begin{matrix} i+1 \text{ unknowns,} \\ i+1 \text{ equations} \end{matrix}$$

$\Rightarrow t_0 = \tau / f_e(k,e)$ tax $\Rightarrow t_i = -\tau f_i(k,e)$

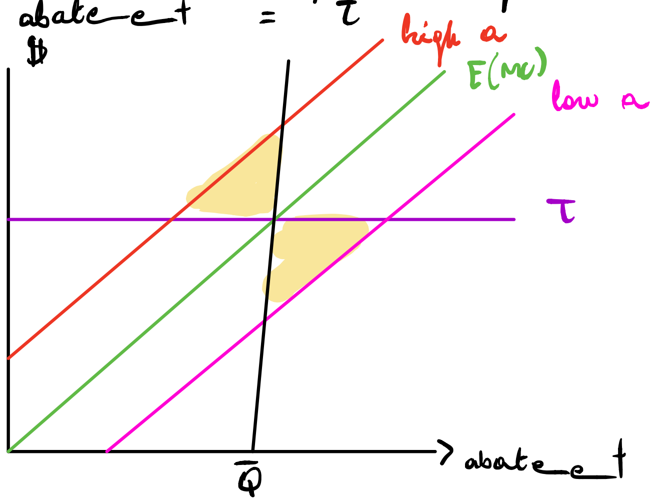
e) a is unknown

$\nearrow p = 0.5 \Rightarrow$ low a

$\searrow p = 0.5 \Rightarrow$ high a

We can think about a tax on emissions as abatement were

- MC abatement = firm's private benef
- MB abatement = τ



Here because MB of abatement, τ is constant, we always get it right by using a tax on emissions $= \tau$ while the expected DWL From a quantity standard $\bar{Q}$ is positive

5) Permit Market Dynamics

Setup
- E_0 aggregate annual firm pollut°
- $\bar{S} < 10 E_0$ → using cap and trade
- at each t, firms com
  - abate pollut° at cost
  - permits $p(t)$

→ $D(p_t)$ demand for permits

$D(0) = E_0$ $\quad D(p) > 0 \quad \forall p \geq 0$
$D'(\cdot) \leq 0$

$C'(E_0 - D(p))$ marginal abatement cost $\overset{\text{no arbitrage}}{\Longrightarrow}$ $p_t = C'(E_0 - D(p_t))$

a) $p(0)$ such that $\sum_{t=0}^{10} D[P_0 e^{rt}] = \bar{S}$

↳ Note that $P_t = P_0 e^{rt}$ $\forall t$ because of no arbitrage.

b) Concerned that $P_{10} = P_0 e^{10r}$ too high ⇒ at $10-\tau$ the price reaches P_c. However, the no-arbitrage condition can't hold anymore at $P_t = P_c$ $t \geq 10-\tau$. So firms won't hold any permit after $10-\tau$

$$\Rightarrow \sum_{t=0}^{10-\tau} D(P_0 e^{rt}) = \bar{S}$$

↳ Demand each period ↑ ⇒ price each period decreases.

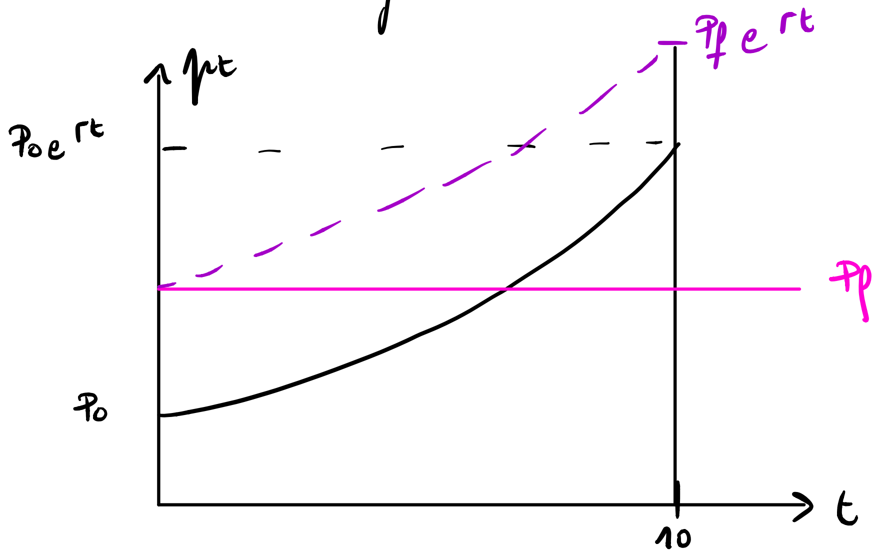
After $10-\tau$ ⇒ $\Delta = \tau D(p_c)$

c) There will be a speculative attack if $\tau D(p_c) = \Delta > G$

↳ if firms anticipate that after $10-\tau$ there will be more demand for permits than supply, then all firms would buy at $10-\tau$ and hold on the permits for later

d) We need not only $\tau D(p_c) = \Delta > G$ (speculative attack)
but also
$$G \leq \sum_{t=10-\tau}^{10} D(p_c e^{rt})\, dt$$

e) Concern $\Rightarrow P_0 e^{10r}$ too low. Gov willing to purchase at any time at $p_f \in (p_0, p_0 e^{10r})$



$$\Rightarrow \quad \bar{S} - B = \sum_{t=0}^{10} D(p_f e^{rt})$$

6/ Clean Back stop Technology

Setup: p = MU of consumpt° → demand for transportat° fuel perfectly elastic

↳ 2 potential sources of fuel:
- crude oil
- biofuel

Crude oil: stock $S_0 > 0$
product° rate y_t
cost of extraction $c_y(y_t) > 0$, $c_y'(y_t) > 0$, $c_y''(y_t) > 0$
$c_y'(0) \in (0, p)$

Biofuel rate of product° b_t
cost of product° $c_b(b_t) > 0$ $c_b'(b_t) > 0$ $c_b''(b_t) > 0$
$c_b'(0) \in (0, p)$

r consumers' discount rate
$t = \infty$

a) $$\max_{\{y_t, b_t\}_{t=0}^{\infty}} \int_{t=0}^{\infty} \left[p(y_t + b_t) - c_y(y_t) - c_b(b_t) \right] e^{-rt} dt$$ $\dot{S}_t = -y_t$

$\int_{t=0}^{\infty} y_t \leq S_0$ give >0

$$H = p \cdot (y_t + b_t) - c_y(y_t) - c_b(b_t) - \mu_t y_t$$

↳ FOC $[y_t]$ $p - c_y'(y_t) - \mu_t \leq 0$ $y_t \geq 0$ CS

$[b_t]$ $p - c_b'(b_t) \leq 0$ $b_t \geq 0$ CS

$[S_t]$ $\dot{\mu}_t = r \mu_t$

TVC $\lim_{t \to \infty} \mu_t e^{-rt} \geq 0$ $\lim_{t \to \infty} \mu_t S_t e^{-rt} = 0$

b) Note that here the price is constant and doesn't depend on $y_t + b_t$. Assuming interior solut. for b_t

FOC on b_t : $p = C_b'(b_t) \Rightarrow b_t = C_b'^{-1}(p)$ → constant

y_t : $p \leq C_y'(y_t) + \mu_t$

↳ $\mu_t = \mu_0 e^{rt}$

$p - \mu_0 e^{rt} \leq C_y'(y_t)$

$y_t \geq C_y'^{-1}(p - \mu_0 e^{rt})$

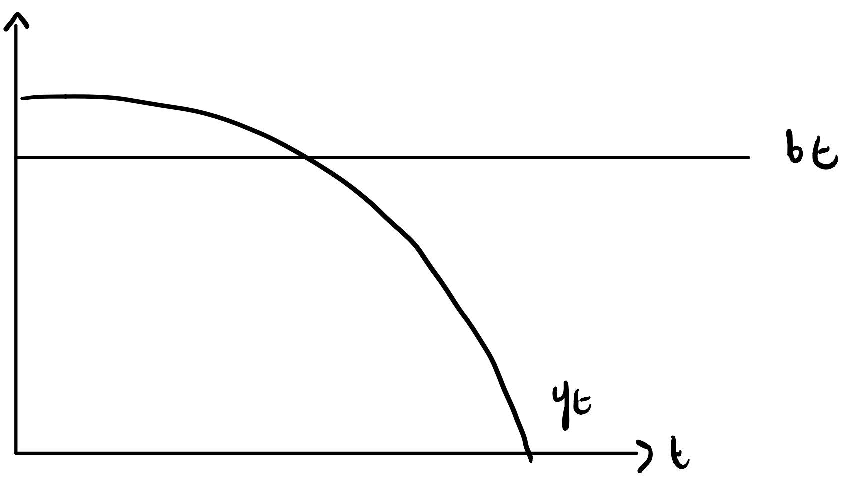
↳ $y_t = \max\{ C_y'^{-1}(p - \mu_0 e^{rt}) ; 0 \}$

↳ C_y' is increasing $\Rightarrow C_y'^{-1}$ increasing

↳ $p - \mu_0 e^{rt}$ decreasing exponentially

$\Rightarrow C_y'^{-1}(p - \mu_0 e^{rt})$ decreasing exponentially

⇒ concave



Note that $b(0) > y(0)$ depends on the cost function

c) All cars banned at T imposes that $\mu_T = 0 \Rightarrow \mu_t = 0 \ \forall t$ because μ_t is positive.

Thus, from the FOC on y_t

$p = C_y'(y_t)$

$y_t = C_y'^{-1}(p)$ constant

T needs to be such that

$$\int_0^T y(t)\, dt < S_0$$ where S_0 is the initial stock of y_t.

$$T \; c_y'^{-1}(p) < S_0$$

d/ The externality!

The planner's problem becomes:

$$H = p(y_t + b_t) - c_y(y_t) - \tau e_y y_t - c_b(b_t) - \tau e_b b_t - \mu_t y_t$$

↳ FOC (assuming interior) : $[y_t]$ $$p - c_y'(y_t) - \mu_t - \tau e_y = 0$$

$[b_t]$ $$p - c_b'(b_t) - \tau e_b = 0$$

The firm's problem with t_b, t_y taxes on y_t, b_t

$$H = (p - t_b) b_t + (p - t_y) y_t - c_y(y_t) - c_b(b_t) - \mu_t y_t$$

↳ FOC ...

⇒ setting t_b and t_y at the FOC of the planner's match the FOC of the firm:

$$t_b = \tau e_b$$
$$t_y = \tau e_y$$

e/ The planner's pb doesn't change. The firm's pb becomes

$$H = p y_t + p b_t - c_y(y_t) - c_b(b_t) - \mu_t y_t - \lambda_t \left(e_y y_t + e_b b_t - (y_t + b_t)\sigma\right)$$

↳ FOC : $$p - c_y'(y_t) - \mu_t - \lambda_t(e_y - \sigma) \leq 0 \qquad y_t \geq 0 \qquad CS$$

: $$p - c_b'(b_t) - \lambda_t(e_b - \sigma) \leq 0 \qquad b_t \geq 0 \qquad CS$$

$$\sigma \in [e_b, e_y]$$

Matching the planner and firm's FOC we see that the FB implies

$$\tau e_b = \lambda(\sigma - e_b)$$

$\Rightarrow e_b = 0$ (otherwise subsidy) $\Rightarrow \sigma = 0$

$\Rightarrow y_t = 0$ bc we need $\dfrac{e_y\, y_t}{b_t + y_t} \leq 0$

$\Rightarrow$ we need $y_t = 0$ to be optimal (corner solution). Thus from the FOC

$$p - c_y'(0) - \mu_t + \tau e_y \leq 0 \quad \forall t$$

Besides, from the TVC $\Rightarrow$ we need $\mu_t = 0 \;\; \forall t$

$$\Rightarrow \quad p + \tau e_y < c_y'(0)$$

7/ Mustang and Civics

2 types of car drivers: economic faculty F $\Rightarrow$ mustang φ_F grams NO_x per gasoline $e_F > 0$ per mile

economics grad student S $\Rightarrow$ civics $\varphi_S > \varphi_F > 0$ grams NO_x per gasoline $e_S = e_F$ per mile

τ = env damage per gram NO_x emitted

m_F, m_S miles driven $m(p+t)$

a) Assume $m_F(p+t) = m_S(p+t)$ same demand

$m_F'(p+t) = m_S'(p+t)$

FB welfare outcome $\Leftrightarrow$ pigou $\Leftrightarrow$ tax = Marginal damage at optimum

We want to achieve the "FB" $\Rightarrow$ we maximize the planner's pb

$$\max_{M_F, M_S} U_F(M_F) + U_S(M_S) - (p + \tau\varphi_F)\, e_F M_F - (p + \tau \varphi_S)\, e_S M_S$$

$\hookrightarrow$ FOC

$$[M_F]: \quad U_F'(M_F) - (p + \tau\varphi_F)\, e_F = 0$$

$$[M_S]: \quad U_S'(M_S) - (p - \tau\varphi_S)\, e_S = 0$$

However, a simple tax $t = \tau\varphi_F$ or $t = \tau\varphi_S$ or any combination of the two would not allow the 2 FOC to hold simultaneously.

Let's now move to the "second best" ie the tax that minimizes the loss in welfare, taking into account how Professors and students optimize (ie the planner doesn't choose m_F, m_S directly, but instead chooses t with m_S, m_F being functions of t)!

$$\max_t U_F(m_F(t)) + U_S(m_S(t)) - (p + \tau\varphi_F)\, e_F\, m_F(t) - (p + \tau\varphi_S)\, e_S\, m_S(t)$$

$\Rightarrow$ with $m_F = m_S$ same demand:

$$U_F(m(t)) + U_S(m(t)) - (p + \tau\varphi_F)\, e_F\, m(t) - (p + \tau\varphi_S)\, e_S\, m(t)$$

FOC $[t]$: $U_F'\, m' + U_S'\, m' - \left[(p + \tau\varphi_F)\, e_F + (p + \tau\varphi_S)\, e_S\right] m' = 0$

* The prof problem $\max_{M} U_F(m(t)) - (p+t)\, e_F\, m(t)$

↳ FOC [M] : $U_F' - (p+t)\, e_F = 0$

Same for the student

FOC [M] $U_S' - (p+t)\, e_S = 0$

plugging in for U_F', U_S' into the planner's FOC

$$\left[(p+t)\, e_F + (p+t)\, e_S\right] m' = \left[(p+\tau \varphi_F) e_F + (p+\tau \varphi_S)\, e_S\right] m'$$

$$\Rightarrow t = \tau \left[\frac{\varphi_F\, e_F + \varphi_S\, e_S}{e_F + e_S}\right]$$

$$t = \tau\left[\frac{\varphi_F + \varphi_S}{2}\right] \quad \text{as } e_F = e_S \text{ by assumpt.}$$

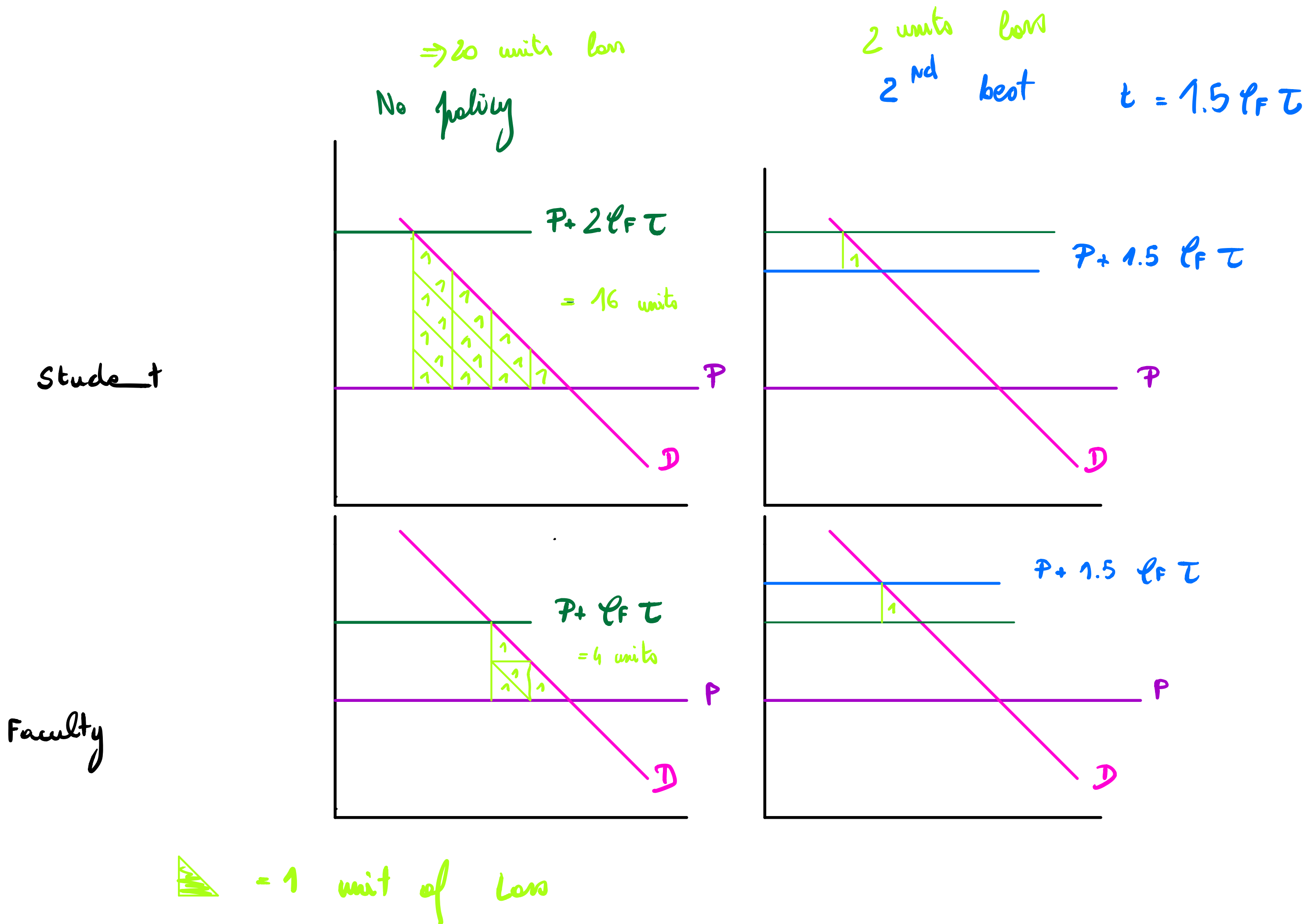
⇒ because both students and faculty have the same demand functions (same elasticity) and same starting point of consumpt. $e_F = e_S$, the tax should equally spread the burden on the two.

b/ Assume $m' < 0$ $\quad \varphi_S = 2\varphi_F$

Let's compare the DWL from
- No policy vs 1st best
- 2nd best vs 1st best

1st best : tax on student $= \varphi_S \tau = 2\tau \varphi_F$
Faculty $= \varphi_F \tau$

2nd best (from qA) $\quad t = \tau\left[\frac{\varphi_F + \varphi_S}{2}\right] = \frac{3}{2}\tau \varphi_F$



$$\Rightarrow \frac{\text{DWL 2}^{nd}\text{ best}}{\text{DWL No policy}} = \frac{2}{20} = \frac{1}{10} \Rightarrow \text{Recovered} = 1 - \frac{1}{10} = 90\%$$

Nb : we have the same nb of students and faculty

⇒ Thus we have recovered 90% of welfare through the 2nd best policy.

(I guess there was a more formal way to go but I can't figure it out...)

c) Now $m_F' = 0$
$m_S' < 0$

⇒ Faculty don't respond to the tax ⇒ the planner won't be able to change MF. Instead the planner can now focus on student.

Setting the planner's FOC to the student's FOC ⇒ $t = \tau \varphi_S$