Matching and Inequality in the World Economy

Arnaud Costinot

Massachusetts Institute of Technology and National Bureau of Economic Research

Jonathan Vogel

Columbia University and National Bureau of Economic Research

This paper develops tools and techniques to analyze the determinants of factor allocation and factor prices in economies with a large number of goods and factors. The main results of our paper characterize sufficient conditions for robust monotone comparative statics predictions in a Roy-like assignment model. These general results are then used to generate new insights about the consequences of globalization.

I. Introduction

This paper develops tools and techniques to analyze the determinants of factor allocation and factor prices in economies with a large number of goods and factors. We then illustrate how these tools and techniques can be applied to generate new insights about the consequences of globalization.

Empirically, understanding the determinants of factor allocation and factor prices in economies with a large number of goods and factors is important for at least two reasons. First, large changes in factor allo-

We thank Daron Acemoglu, David Autor, Ariel Burstein, Pierre-André Chiappori, Gene Grossman, Gordon Hanson, Samuel Kortum, two anonymous referees, seminar participants at many institutions, and especially Esteban Rossi-Hansberg for helpful comments and discussions. We have also benefited from stimulating discussions by Pol Antras, Bob Staiger, and Dan Trefler at the NBER Summer Institute and the Princeton International Economics Section Summer Workshop. Costinot also thanks Princeton IES for its hospitality.

[*Journal of Political Economy,* 2010, vol. 118, no. 4]

cation and factor prices occur at high levels of disaggregation. For instance, a number of authors in the labor and public finance literatures have documented (i) large changes in inequality at the top of the income distribution, (ii) divergent trends in inequality in the top and in the bottom halves of the income distribution, (iii) divergent trends in employment growth of high- and low-wage occupations, and (iv) changes in both between- and within-group inequality (see, respectively, Piketty and Saez 2003; Autor, Katz, and Kearney 2008; Goos and Manning 2007; Juhn, Murphy, and Pierce 1993). Second, even changes occurring at low levels of disaggregation, such as variations in the relative wage of college to non–college graduates, often reflect average changes taken over a large number of imperfectly substitutable factors.

Theoretically, however, the analysis of neoclassical general equilibrium models with a large number of goods and factors has proven elusive. Comparative static predictions derived in such environments tend to be either weak or unintuitive, limiting the scope of these models in addressing the empirical phenomena mentioned above. For example, the famous "Friends and Enemies" result by Jones and Scheinkman (1977) states that a rise in the price of some good causes an even larger proportional increase in the price of some factor; but depending on the number of goods and factors, it may or may not lead to an even larger proportional decrease in the price of some other factor. A common theme in the existing literature (see, e.g., Ethier 1984) is that comparative static results in high-dimensional environments heavily hinge on the answer to one fairly abstract question: Are there more goods than factors in the world?

In order to move away from such discomforting considerations and make progress on the analysis of factor allocation and factor prices in economies with more than two goods and two factors, we focus on a Roy-like assignment model in which a continuum of factors, which we refer to as "workers," are used to produce a continuum of goods, which we refer to as "tasks."[1] Markets are perfectly competitive, and tasks are combined into a unique final good using a Dixit-Stiglitz production function. In equilibrium, workers sort across tasks on the basis of their comparative advantage.[2] A crucial feature of this model is that the marginal product of any worker in any task is independent of the set of workers employed in that particular task. While this assumption is obviously stronger than those imposed in standard neoclassical models, it will allow us to transform the analysis of any competitive equilibrium

[1] Though our approach is more elegant in the continuum-by-continuum case, none of our results hinge on the dimensionality of our economy. We come back to this issue in Sec. VII.

[2] See Gibbons et al. (2005) for empirical evidence demonstrating the importance of comparative advantage for the allocation of workers across industries and occupations.

into the analysis of a matching problem and in turn to generate both sharp and intuitive predictions in economies with many goods and factors.

The first part of our paper describes our theoretical framework and develops tools and techniques to derive robust monotone comparative static predictions in this environment. We first introduce definitions of skill abundance and skill diversity to conceptualize changes in factor supply and definitions of skill-biased and extreme-biased technologies to conceptualize changes in factor demand. These definitions do not rely on any functional form assumptions but rely instead on standard concepts in information economics (see, e.g., Milgrom 1981). They naturally extend standard notions of relative factor supply and demand from two-good, two-factor models to models with a large number of goods and factors. Using these concepts, we then derive sufficient conditions for various patterns of changes in factor allocation—for example, job polarization—and factor prices—for example, pervasive changes in inequality[3]—to occur in a closed economy.

The second part of our paper uses these general results to offer a unifying perspective on the effects of North-South trade, North-North trade, global technological change, and offshoring. We consider a world economy comprising two countries. Since factor price equalization always holds in our framework, we can easily build on our closed economy results to analyze the impact of North-South trade, which we model as trade between countries differing in either skill abundance or the skill bias of their technologies. When North-South trade is driven by differences in factor endowments, we obtain high-dimensional counterparts to the classic two-by-two Heckscher-Ohlin results. In particular, trade integration induces a pervasive rise in inequality in the North and a pervasive fall in inequality in the South. Perhaps surprisingly, when North-South trade is driven by differences in technological biases, we show that the exact same logic leads to the exact opposite conclusion.

This observation, which arises naturally in the context of our model, has important empirical implications. According to our model, predictions regarding the impact of trade integration crucially depend on the correlation between factor endowment and technological differences. In a series of influential papers, Acemoglu (1998, 2003) argues that skill-abundant countries tend to use skill-biased technologies. With this correlation in mind, we should not be surprised if (i) similar countries have different globalization experiences depending on which of these two forces, supply or demand, dominates, and (ii) the overall effect of

[3] Throughout this paper, whenever we say "changes in inequality," we formally mean "changes in the return to skill." In practice, changes in the income distribution may, of course, reflect changes in both the distribution of skills and their returns. We briefly return to this issue in Sec. IV.

trade liberalization on factor allocation and factor prices tends to be small in practice. This observation provides a simple rationale for empirical findings summarized in Goldberg and Pavcnik (2007).

Using the same theoretical apparatus, we can also study the implications of North-North trade. While trade between countries with similar average skill levels accounts for the vast majority of world trade, the two-by-two Heckscher-Ohlin model has nothing to say about its implications for inequality. The same is true of "new" trade models (see, e.g., Helpman and Krugman 1985). By contrast, the high-dimensionality of our framework allows us to model North-North trade as trade between a more and a less diverse country. This view of North-North trade, introduced by Grossman and Maggi (2000), is consistent with the large differences in the dispersion of skills across rich countries (see, e.g., Bombardini, Gallipoli, and Pupato 2009).

Compared to North-South trade, we find that North-North trade may either increase or decrease the relative wage between high- and low-skill workers as well as the relative price of the goods they produce. This observation provides a simple rationale for the empirical findings of Lawrence and Slaughter (1993). According to our model, the consequences of North-North trade are to be found at a higher level of disaggregation. When trading partners vary in terms of skill diversity, changes in inequality occur within low- and high-skill workers, respectively. Similarly, North-North trade does not yield a decrease (or increase) in the employment shares of the skill-intensive tasks; instead, it leads to a U-shape (or inverted U-shape) relationship between tasks' employment growth and their skill intensity.

Our last set of comparative static exercises illustrates how our general results may also shed light on the consequences of global technological change and offshoring. Among other things, we show that global skill-biased technological change increases inequality between countries and that offshoring, in contrast to North-South trade, induces skill downgrading and a pervasive rise in inequality in both countries.[4] Taken together, the previous results demonstrate the richness and flexibility of our theoretical framework for analyzing the consequences of globalization and technological change around the world.

The rest of our paper is organized as follows. Section II discusses the related literature. Section III presents our theoretical framework. Section IV derives comparative static results in the closed economy. Section V contrasts the effects of North-South and North-North trade integration, whereas Section VI describes the consequences of global techno-

[4] A related mechanism was first studied by Feenstra and Hanson (1996) and, subsequently, Zhu and Trefler (2005) in an economy with two types of workers, skilled and unskilled. We come back to the relationship between their results and ours in Sec. VI.

logical change and offshoring. Section VII discusses extensions of our basic framework. Section VIII offers concluding remarks.

II. Related Literature

Assignment literature.—Our paper is related to two distinct literatures. The first one is the assignment literature (see Sattinger [1993] for an overview). Typical results in this field fall into two broad categories. On the one hand, authors including Becker (1973), Heckman and Honore (1990), Shimer and Smith (2000), Legros and Newman (2002, 2007), and Eeckhout and Kircher (2010) offer general results focusing on cross-sectional predictions, such as sufficient conditions for positive assortative matching to arise. On the other hand, authors including Teulings (1995, 2005), Garicano and Rossi-Hansberg (2006), Gabaix and Landier (2008), and Tervio (2008) offer specific comparative static predictions under strong functional form restrictions on the distribution of skills, worker productivity, and the pattern of substitution across goods. Among the previous papers, our paper is most closely related to those by Teulings (1995, 2005), who analyzes the determination of wages in a Roy-like assignment model similar to ours.[5] Our contribution to this literature is to offer sufficient conditions for robust monotone comparative static predictions in this environment. These general results are useful because they deepen our understanding of an important class of models in the labor and trade literatures, clarifying how relative factor supply and relative factor demand affect factor allocation and factor prices in such environments.

International trade literature.—Our paper also contributes to the theory of international trade. The analysis of the effects of trade integration on factor allocation and factor prices is at the core of neoclassical trade theory (see, e.g., Stolper and Samuelson 1941; Jones 1965; Melvin 1968; Dornbusch, Fischer, and Samuelson 1980). In high-dimensional environments, however, this analysis has been plagued by technical difficulties.[6] A central contribution of our paper is to show that by restricting the supply side of neoclassical trade models—namely, by focusing on a Roy-like assignment model—one can derive strong and intuitive predictions on the consequences of globalization in economies with an

[5] Using the terminology of Sattinger (1993), one can think of Garicano and Rossi-Hansberg (2006), Gabaix and Landier (2008), and Tervio (2008) as variations or extensions of the "differential rents model," which our analysis does not cover. Compared to Roy-like assignment models, the equilibrium sorting of workers in these models may not necessarily reflect comparative advantage.

[6] In a recent paper, Anderson (2009) develops a neoclassical trade model with a continuum of goods and factors. In his model, however, the allocation of the continuum of factors is exogenously given, and therefore, results are restricted to changes in factor prices.

arbitrarily large number of both goods and factors. This "assignment approach" enables us to discuss, within a unified framework, phenomena that would otherwise fall outside the scope of standard trade theory, such as pervasive changes in inequality and wage and job polarization.

Our paper is part of a small but rapidly growing literature using assignment models in an international context; see, for example, Grossman and Maggi (2000), Grossman (2004), Yeaple (2005), Ohnsorge and Trefler (2007), Blanchard and Willmann (2008), Costinot (2009), Monte (2009), and Sly (2009) for applications to trade and Kremer and Maskin (2003), Antras, Garicano, and Rossi-Hansberg (2006), and Nocke and Yeaple (2008) for applications to offshoring. Among the previous papers, our analysis is most closely related to that of Costinot (2009), who develops a generalized version of the supply side of our theoretical framework in which both productivity and factor endowment differences are sources of comparative advantage. Compared to the present paper, however, his analysis is restricted to cross-sectional predictions on the pattern of trade rather than comparative static predictions on factor allocation and factor prices.

Our paper is also related, though less closely, to the recent literature on trade and inequality in monopolistically competitive environments with ex ante homogeneous workers, heterogeneous firms, and labor market imperfections (see, e.g., Davis and Harrigan 2007; Amiti and Davis 2008; Sethupathy 2008; Egger and Kreickemeier 2009; Helpman, Itskhoki, and Redding, forthcoming). The central idea in these papers is that, because of labor market imperfections, firms with different productivities pay different wages. As a result, changes in trade barriers that affect the distribution of firm productivity also affect the distribution of wages. Compared to any neoclassical trade model (including ours), such models provide a rich set of predictions at the firm level. They are, however, abstracting from ex ante worker heterogeneity, which is our main focus.[7]

III. The Closed Economy

A. Basic Environment

Endowments.—We consider an economy populated by a continuum of workers with skill $s \in \mathbb{R}$. We denote by $V(s) \geq 0$ the inelastic supply of workers with skill s and by $S \equiv \{s \in \mathbb{R} | V(s) > 0\}$ the set of skills available

[7] Helpman et al. (forthcoming) develop an interesting extension of their model with ex ante worker heterogeneity. With a large number of factors, this model leads to the same type of ambiguous predictions as standard neoclassical trade models. Davidson, Matusz, and Shevchenko (2008) offer a simple model with firm heterogeneity and two types of workers.

in the economy. Throughout this paper, we restrict ourselves to skill distributions such that $S = [\underline{s}, \bar{s}]$, though different V's may have different supports.

Technology.—There is one final good, which we use as the numeraire. Producing the final good requires a continuum of intermediate goods or tasks indexed by their skill intensity $\sigma \in \mathbb{R}$.[8] Output of the final good is given by the following Dixit-Stiglitz production function:

$$Y = \left\{ \int_{\sigma\in\Sigma} B(\sigma)[Y(\sigma)]^{(\varepsilon-1)/\varepsilon} d\sigma \right\}^{\varepsilon/(\varepsilon-1)}, \tag{1}$$

where $Y(\sigma) \geq 0$ is the endogenous output of task σ, $0 < \varepsilon < \infty$ is the constant elasticity of substitution across tasks,[9] $B(\sigma) \geq 0$ is an exogenous technological parameter, and $\Sigma \equiv \{\sigma \in \mathbb{R} | B(\sigma) > 0\}$ corresponds to the set of tasks available in the economy. As before, we restrict ourselves to technologies such that $\Sigma = [\underline{\sigma}, \bar{\sigma}]$, though different B's may have different supports.

Producing tasks requires only workers. Workers are perfect substitutes in the production of each task but vary in their productivity, $A(s, \sigma) > 0$. Output of task σ is given by

$$Y(\sigma) = \int_{s\in S} A(s, \sigma) L(s, \sigma) ds, \tag{2}$$

where $L(s, \sigma) \geq 0$ is the endogenous number of workers with skill s performing task σ. We assume that $A(s, \sigma)$ is twice differentiable and strictly log-supermodular:

$$A(s', \sigma')A(s, \sigma) > A(s, \sigma')A(s', \sigma) \quad \text{for all } s' > s \text{ and } \sigma' > \sigma. \tag{3}$$

Since $A(s, \sigma) > 0$, property (3) can be rearranged as $A(s', \sigma')/A(s', \sigma) > A(s, \sigma')/A(s, \sigma)$. In other words, high-skill workers have a comparative advantage in tasks with high skill intensities.

Market structure.—All markets are perfectly competitive, with all goods being produced by a large number of identical price-taking firms. Total profits for the final good are given by

$$\Pi = \left\{ \int_{\sigma\in\Sigma} B(\sigma)[Y(\sigma)]^{(\varepsilon-1)/\varepsilon} d\sigma \right\}^{\varepsilon/(\varepsilon-1)} - \int_{\sigma\in\Sigma} p(\sigma) Y(\sigma) d\sigma, \tag{4}$$

[8] One could, of course, reinterpret intermediate goods or tasks as final goods and Y as a utility function. We prefer the task interpretation for two reasons. First, it will allow us to talk about "skill-biased technical change" rather than "skill-biased shifts in consumer tastes." Second, it will allow us to discuss, albeit briefly, environments in which the final good is tradable.

[9] For expositional purposes, we assume that $\varepsilon > 0$. Our results continue to hold when $\varepsilon = 0$.

where $p(\sigma) > 0$ is the price of task σ. Similarly, total profits for intermediate good σ are given by

$$\Pi(\sigma) = \int_{s \in S} [p(\sigma)A(s,\sigma) - w(s)]L(s,\sigma)ds, \tag{5}$$

where $w(s) > 0$ is the wage of a worker with skill s. For technical reasons, we also assume that B and V are continuous functions.

B. *Definition of a Competitive Equilibrium*

In a competitive equilibrium, all firms maximize their profits and all markets clear. Profit maximization by final good producers requires

$$Y(\sigma) = I \times [p(\sigma)/B(\sigma)]^{-\varepsilon} \quad \text{for all } \sigma \in \Sigma, \tag{6}$$

where $I \equiv \int_{s \in S} w(s)V(s)ds$ denotes total income. Since there are constant returns to scale, profit maximization by intermediate good producers requires

$$\begin{aligned} p(\sigma)A(s,\sigma) - w(s) &\le 0 \quad \text{for all } s \in S; \\ p(\sigma)A(s,\sigma) - w(s) &= 0 \quad \text{for all } s \in S \text{ such that } L(s,\sigma) > 0. \end{aligned} \tag{7}$$

Finally, good and labor market clearing require

$$Y(\sigma) = \int_{s \in S} A(s,\sigma)L(s,\sigma)ds \quad \text{for all } \sigma \in \Sigma; \tag{8}$$

$$V(s) = \int_{\sigma \in \Sigma} L(s,\sigma)d\sigma \quad \text{for all } s \in S. \tag{9}$$

In the rest of this paper, we formally define a competitive equilibrium as follows.

Definition 1. A competitive equilibrium is a set of functions $Y: \Sigma \to \mathbb{R}^+$, $L: S \times \Sigma \to \mathbb{R}^+$, $p: \Sigma \to \mathbb{R}^+$, and $w: S \to \mathbb{R}^+$ such that conditions (6)–(9) hold.

C. *Properties of a Competitive Equilibrium*

Given our assumptions on worker productivity, $A(s,\sigma)$, profit-maximization condition (7) imposes strong restrictions on competitive equilibria.

Lemma 1. In a competitive equilibrium, there exists a continuous and strictly increasing matching function $M: S \to \Sigma$ such that (i) $L(s,\sigma) > 0$ if and only if $M(s) = \sigma$, and (ii) $M(\underline{s}) = \underline{\sigma}$ and $M(\bar{s}) = \bar{\sigma}$.

Lemma 1 can be understood as follows. First, because markets are perfectly competitive and factors of production are perfect substitutes within each task, equation (2), the value of output goes entirely to the worker, condition (7). This implies that comparative advantage determines factor allocation. Second, because A is strictly log-supermodular, property (3), high-skill workers have a comparative advantage in tasks with high skill intensity. This implies the monotonicity of this matching function.[10]

The rest of our analysis crucially relies on the following lemma.

LEMMA 2. In a competitive equilibrium, the matching function and wage schedule satisfy

$$\frac{dM}{ds} = \frac{A[s, M(s)]V(s)}{I \times \{p[M(s)]/B[M(s)]\}^{-\varepsilon}}, \tag{10}$$

$$\frac{d \ln w(s)}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s}, \tag{11}$$

with $M(\underline{s}) = \underline{\sigma}$, $M(\bar{s}) = \bar{\sigma}$, and $p[M(s)] = w(s)/A[s, M(s)]$.

According to lemma 2, the two key endogenous variables of our model, the matching function, M, and the wage schedule, w, are given by the solution of a system of ordinary differential equations. Equation (10) captures how market clearing determines the matching function. Formally, the slope of the matching function, dM/ds, equates the supply of any factor, $V(s)$, with its demand

$$I\frac{dM}{ds} \times \frac{\{p[M(s)]/B[M(s)]\}^{-\varepsilon}}{A[s, M(s)]}.$$

Equation (11) captures how profit maximization determines the wage schedule. Intuitively, differences in relative productivity, $\partial \ln A\,[s, M(s)]\,/\partial s$, must be reflected in differences in relative wages, $d \ln w(s)/ds$. Once w and M have been computed, Y and p can be computed by simple substitutions using equations (6) and (7).

IV. Comparative Statics in the Closed Economy

Armed with the knowledge that a competitive equilibrium is characterized by equations (10) and (11), we now investigate how exogenous changes in factor supply and demand, captured by changes in V and B, affect factor allocation and factor prices. In each case, we first determine how exogenous changes in V and B affect the matching function, M. We then consult equation (11) to draw conclusions about its implications for the wage schedule, w.

[10] Formally, the log-supermodularity of A is necessary and sufficient for $p(\sigma)A(s,\sigma)$ to satisfy the single crossing property in (s,σ) for all $p(\sigma)$ and, therefore, for positive assortative matching to arise for any price schedule.

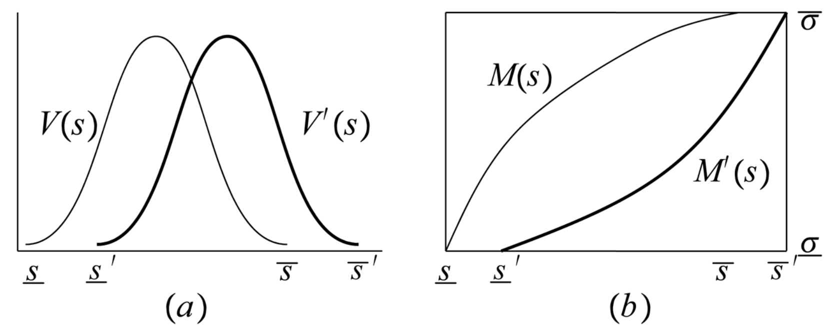

FIG. 1.—Changes in skill abundance and matching

A. *Changes in Factor Supply*

For our two first comparative static exercises, we focus on exogenous changes in factor supply. In a closed economy, such changes may capture, for instance, the effects of immigration or educational reforms. As we will see in Section V, these comparative static exercises will be important for analyzing the consequences of North-South and North-North trade integration.

1. Skill Abundance

We first consider a change in factor supply from V to V' such that

$$V'(s')V(s) \geq V(s')V'(s) \quad \text{for all } s' \geq s. \tag{12}$$

Property (12) corresponds to the monotone likelihood ratio property (see Milgrom 1981).[11] It captures the idea that there are relatively more high-skill workers under V' than under V. If $s,\ s' \in S \cap S'$, property (12) simply implies $V'(s')/V'(s) \geq V(s')/V(s)$. This is the natural generalization of the notion of skill abundance in a two-factor model. Property (12), in addition, allows us to consider situations in which different sets of skills are available under V and V'. If $s, s' \notin S \cap S'$, then property (12) implies that $s \in S$ and $s' \in S'$ or, equivalently, that S' is greater than S in the strong set order: $\underline{s}' \geq \underline{s}$ and $\bar{s}' \geq \bar{s}$. In other words, the highest-skill workers must be in the economy characterized by V' and the lowest-skill workers in the economy characterized by V. Property (12) is illustrated in figure 1*a*.

In the rest of this paper, we use the following definition.

[11] There exists a close mathematical connection between log-supermodularity and the monotone likelihood ratio property. Formally, if we let $V(s) \equiv \tilde{V}(s,\gamma)$ and $V'(s) \equiv \tilde{V}(s,\gamma')$ with $\gamma' \geq \gamma$, then V and V' satisfy property (12) if and only if $\tilde{V}$ is log-supermodular.

DEFINITION 2. V' is skill abundant relative to V, denoted $V' \succcurlyeq_a V$ if property (12) holds.

It is worth pointing out that the monotone likelihood ratio property, on which our notion of skill abundance builds, is a stronger notion of dominance than first-order stochastic dominance. If V' is skill abundant relative to V, then for any skill level s^c, the proportion of workers with skill above s^c is higher under V' than under V, and therefore, the proportion of workers with skill levels below s^c is lower under V' than under V.

We first analyze the impact of a change in skill abundance on matching. Let M and M' be the matching functions associated with V and V', respectively. Our first result can be stated as follows.

LEMMA 3. Suppose $V' \succcurlyeq_a V$. Then $M'(s) \leq M(s)$ for all $s \in S \cap S'$.

From a worker standpoint, moving from V to V' implies *task downgrading*: each type of worker performs a task with lower skill intensity under V'. From a task standpoint, this means *skill upgrading*: each task is performed by workers with higher skills under V'. This change in the matching function is illustrated in figure 1*b*.[12] At a broad level, the intuition behind lemma 3 is very simple. As the relative supply of the high-skill workers rises, market-clearing conditions require more tasks to be performed by high-skill workers. So the M schedule should shift down.

At a technical level, the proof of lemma 3 also illustrates the important but limited role of the Dixit-Stiglitz production function in our analysis. Given the generality of the assumptions imposed on V, A, and B, Dixit-Stiglitz production functions do not lead to closed-form solutions. In our paper, the functional form restriction imposed in equation (1) serves only one purpose: it implies that, as in a two-by-two model, the relative demand for two tasks depends only on the relative price of these tasks.

Now let us consider the associated impact of a change in skill abundance on wages. Let w and w' be the wage schedules associated with V and V', respectively, where $V' \succcurlyeq_a V$. Combining lemma 3, equation (11), and the log-supermodularity of A, we obtain

$$\frac{d \ln w}{ds} = \frac{\partial \ln A[s, M(s)]}{\partial s} \geq \frac{\partial \ln A[s, M'(s)]}{\partial s} = \frac{d \ln w'}{ds}.$$

Integrating the above inequality implies

$$\frac{w(s')}{w(s)} \geq \frac{w'(s')}{w'(s)} \quad \text{for all } s' > s \text{ in } S \cap S'. \tag{13}$$

[12] For expositional purposes, we have chosen to state all our definitions and predictions using weak inequalities. It should be clear, however, that both our definitions and predictions have natural, though slightly more involved, counterparts with strict inequalities.

Moving from V to V' leads to a *pervasive fall in inequality*: for any pair of workers, the relative wage of the worker with a higher skill level—who is relatively more abundant under V'—goes down.[13] In our model, an increase in the relative supply of the high-skill workers triggers a reallocation of all workers away from tasks with high skill intensity. Since high-skill workers have a comparative advantage in these tasks, their relative wage decreases.

2. Skill Diversity

We now consider the case in which V and V' satisfy

$$\begin{aligned} &\text{(i) } V \succcurlyeq_a V' \quad \text{for all } s < \hat{s}, \text{ and} \\ &\text{(ii) } V' \succcurlyeq_a V \quad \text{for all } s \geq \hat{s}, \text{ with } \hat{s} \in S. \end{aligned} \tag{14}$$

Property (14) captures the idea that there are relatively more workers with extreme skill levels (either high or low) under V' than under V. If different sets of skills are available under V and V', then property (14) implies $S \subseteq S'$. Moreover, for any pair of distinct skill levels $s \leq s' < \hat{s}$ with $s, s' \in S$, there are relatively more high-skill workers in the economy characterized by V, $V(s')/V(s) \geq V'(s')/V'(s)$; and for any pair of distinct skill levels $s' \geq s \geq \hat{s}$ with $s, s' \in S$, there are relatively more high-skill workers in the economy characterized by V', $V'(s')/V'(s) \geq V(s')/V(s)$. Property (14) is illustrated in figure 2*a*.[14]

In the rest of this paper, we say that the following definition holds.

Definition 3. V' is more diverse than V, denoted $V' \succcurlyeq_d V$, if property (14) holds.

Definition 3 is a stronger notion of diversity than in Grossman and Maggi (2000) in the sense that we impose likelihood ratio dominance on either side of $\hat{s}$ whereas they impose only first-order stochastic dominance. It also is a weaker notion, however, in the sense that Grossman and Maggi impose symmetry on V and V' whereas we do not.

As before, let M and M' be the matching functions associated with V and V', respectively. Our second result can be stated as follows.

Lemma 4. Suppose $V' \succcurlyeq_d V$. Then there exists a skill level $s^* \in S$

[13] As mentioned in the introduction, "changes in inequality" in our paper refer to "changes in the return to skill." Since both the distribution of skills V and the wage schedule w are changing simultaneously, inequality (13) has no direct implications for changes in the income distribution. Note, however, that when we turn to the impact of globalization, the distribution of skills V is fixed in all countries. In this situation, changes in the return to skill also have direct implications for changes in the income distribution.

[14] Mexican migration toward the United States can be thought of as a real-world counterpart to this stylized comparative static exercise. According to Chiquiar and Hanson (2005), migrants are from the middle of the Mexican skill distribution. Thus migration tends to increase the relative supply of workers with extreme skill levels in Mexico.

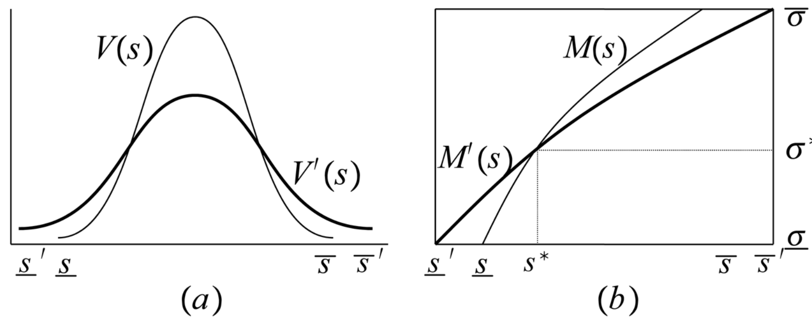

FIG. 2.—Changes in skill diversity and matching

such that $M'(s) \geq M(s)$ for all $s \in [\underline{s}, s^*]$, and $M'(s) \leq M(s)$ for all $s \in [s^*, \bar{s}]$.

Moving from V to V' implies *job convergence*: skill downgrading for low–skill intensity tasks, $\sigma < \sigma^*$, and skill upgrading for high–skill intensity tasks, $\sigma^* < \sigma$, where $\sigma^* \equiv M(s^*) = M'(s^*)$. This change in the matching function is illustrated in figure 2*b*.[15]

As in the case of a change in skill abundance, the broad intuition behind these two results relies on our market-clearing conditions. For workers with skill $s \in (\underline{s}, \hat{s})$, a change in the distribution of skills from V to V' decreases the relative supply of high-skill workers. The converse is true for workers with skill $s \in (\hat{s}, \bar{s})$. These two observations imply that among the least skill-intensive tasks, fewer tasks should employ high-skill workers (i.e., the M schedule should shift up), whereas the converse should be true among the most skill-intensive tasks (i.e., the M schedule should shift down). With the same strategy used in lemma 3, the rest of our proof simply establishes that M and M' cannot cross more than once.

Now let us turn to the associated wage schedules, w and w', under the restriction that $V' \succcurlyeq_d V$. Combining lemma 4, equation (11), and the log-supermodularity of A, we obtain

$$\frac{d \ln w'}{ds} \geq \frac{d \ln w}{ds} \quad \text{for all } \underline{s} < s < s^*;$$

$$\frac{d \ln w'}{ds} \leq \frac{d \ln w}{ds} \quad \text{for all } s^* < s < \bar{s}.$$

[15] It is worth pointing out that lemma 4 implies neither (i) $s^* = \hat{s}$ nor (ii) $s^* \in (\underline{s}, \bar{s})$. Regarding point i, one can find simple examples such that either $s^* > \hat{s}$ or $s^* < \hat{s}$ (details available on request). Regarding point ii, examples of sufficient conditions that guarantee $s^* \in (\underline{s}, \bar{s})$ are $V' \succcurlyeq_d V$ and $S \subset S'$.

Integrating this series of inequalities gives

$$\frac{w'(s')}{w'(s)} \geq \frac{w(s')}{w(s)} \quad \text{for all } \underline{s} \leq s < s' \leq s^*;$$
$$\frac{w'(s')}{w'(s)} \leq \frac{w(s')}{w(s)} \quad \text{for all } s^* \leq s < s' \leq \bar{s}. \tag{15}$$

Hence, moving from V to V' implies *wage convergence*: an increase in inequality among low-skill workers, $s < s^*$, and a decrease in inequality among high-skill workers, $s > s^*$.

B. Changes in Factor Demand

In the previous subsection, we focused on exogenous changes in factor supply. We now briefly demonstrate how our concepts and techniques can be extended to analyze exogenous changes in factor demand, which we think of as technological changes in practice.

1. Skill-Biased Technological Change

We now consider a shift in the B schedule, from B to B', such that

$$B'(\sigma')B(\sigma) \geq B'(\sigma)B(\sigma') \quad \text{for all } \sigma' \geq \sigma. \tag{16}$$

Property (16) captures changes in relative factor demand that are biased toward high-skill workers. With prices held constant, equation (6) and property (16) imply $Y'(\sigma')/Y'(\sigma) \geq Y(\sigma')/Y(\sigma)$ for any pair of tasks $\sigma' \geq \sigma$ in $\Sigma \cap \Sigma'$. In other words, a shift from B to B' increases the relative demand for tasks performed by high-skill workers. Property (16), in addition, allows us to consider situations in which the technologies characterized by B and B' use different sets of tasks. If $\Sigma \neq \Sigma'$, then property (16) implies that Σ' is greater than Σ in the strong set order: $\underline{\sigma}' \geq \underline{\sigma}$ and $\bar{\sigma}' \geq \bar{\sigma}$. Put simply, the most skill-intensive tasks must be used under B' and the least skill-intensive tasks under B.

In the rest of this paper, we say that the following definition holds.

DEFINITION 4. B' is skill biased relative to B, denoted $B' \succcurlyeq_s B$, if property (16) holds.

Let M and M' denote the matching functions associated with B and B', respectively. The demand version of the results on changes in factor supply derived in lemma 3 can be stated as follows.

LEMMA 5. Suppose $B' \succcurlyeq_s B$. Then $M(s) \leq M'(s)$ for all $s \in S$.

Broadly speaking, if the relative demand for the skill-intensive goods rises, then market-clearing conditions require workers to move toward tasks with higher skill intensities in order to maintain equilibrium. This implies skill downgrading at the task level and task upgrading at the

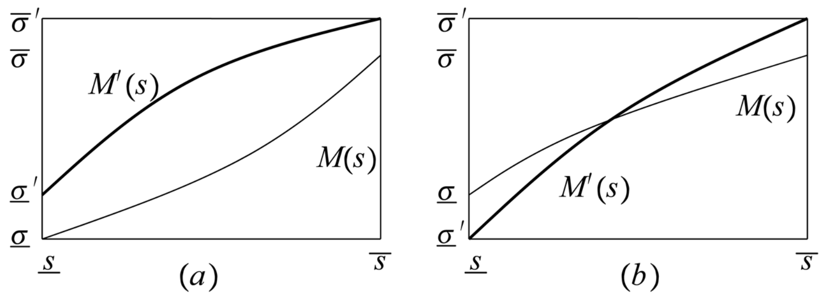

FIG. 3.—Skill- and extreme-biased technological change and matching

worker level. This change in the matching function is illustrated in figure 3*a*.

Finally, let w and w' be the wage schedules associated with B and B', respectively, where $B' \succcurlyeq_s B$. Combining lemma 5, equation (11), and the log-supermodularity of A, we now obtain, after integration,

$$\frac{w'(s')}{w'(s)} \geq \frac{w(s')}{w(s)} \quad \text{for all } s' \geq s. \tag{17}$$

Moving from B to B' leads to a *pervasive rise in inequality*: for any pair of workers, the relative wage of the more skilled worker increases. The mechanism linking the matching function to the wage schedule is the same as in Section IV.A.1. By lemma 5, an increase in the relative demand for goods with high skill intensities triggers a reallocation of workers toward such tasks. Since high-skill workers have a comparative advantage in these tasks, their relative wage increases.

2. Extreme-Biased Technological Change

Finally, we consider a shift in the B schedule, from B to B', such that

$$\begin{aligned} &\text{(i) } B \succcurlyeq_s B' \quad \text{for all } \sigma < \hat{\sigma} \text{ and} \\ &\text{(ii) } B' \succcurlyeq_s B \quad \text{for all } \sigma \geq \hat{\sigma}, \text{with } \hat{\sigma} \in \Sigma. \end{aligned} \tag{18}$$

A shift from B to B' increases the relative demand for tasks with low skill intensities over the range $\sigma < \hat{\sigma}$ and increases the relative demand for tasks with high skill intensities over the range $\sigma \geq \hat{\sigma}$. Property (18) is reminiscent, for instance, of the impact of computerization, as modeled by Autor et al. (2006). As in our previous comparative static exercise, the change in relative factor demand captured by property (18) may result, among other things, from the introduction of a new set of tasks in the economy, that is, $\Sigma \subset \Sigma'$.

In the rest of this paper, we say that the following definition holds.

Definition 5. B' is extreme-biased relative to B, denoted $B' \succcurlyeq_e B$, if property (18) holds.

Let M and M' denote the matching functions associated with B and B', respectively. The demand version of the results on changes in factor supply derived in lemma 4 can be stated as follows.

Lemma 6. Suppose $B' \succcurlyeq_e B$. Then there exists a skill level $s^* \in S$ such that $M(s) \geq M'(s)$ for all $s \in [\underline{s}, s^*]$, and $M(s) \leq M'(s)$ for all $s \in [s^*, \bar{s}]$.

Moving from B to B' induces workers to reallocate out of intermediate-σ tasks and toward extreme-σ tasks. We refer to this reallocation as *job polarization*. This change in the matching function is illustrated in figure 3*b*. Relative wages are given by

$$\frac{w'(s')}{w'(s)} \leq \frac{w(s')}{w(s)} \quad \text{for all } \underline{s} \leq s < s' \leq s^*;$$

$$\frac{w'(s')}{w'(s)} \geq \frac{w(s')}{w(s)} \quad \text{for all } s^* \leq s < s' \leq \bar{s}.$$

Hence, extreme-biased technological change implies *wage polarization*: a decrease in inequality among low-skill workers, $s < s^*$, and an increase in inequality among high-skill workers, $s > s^*$.

V. The World Economy

In the remainder of this paper we consider a world economy comprising two countries, Home (H) and Foreign (F). Workers are internationally immobile, the unique final good is not traded, and all intermediate goods are freely traded at price $p_T(\sigma)$.[16] Without loss of generality, we use the price of the final good in the Home country as the numeraire and denote by P_F the price of the final good in the Foreign country. In each country, we assume that production is as described in Section III.A and that factor productivity differences across countries are Hicks-neutral, $A_i(s, \sigma) \equiv \gamma_i A(s, \sigma)$ for $i = H, F$, with $\gamma_i > 0$. Hence, cross-country differences in factor endowments, V_H and V_F, and technological biases, B_H and B_F, are the only rationale for trade.[17] Throughout this section, we denote by $S_W \equiv S_H \cup S_F$ and $\Sigma_W \equiv \Sigma_H \cup \Sigma_F$ the set of skills and tasks available in the world economy, respectively.

[16] We come back to the case in which the final good is freely traded at the end of Sec. V.B.

[17] It should be clear that differences in technological biases are not Ricardian technological differences. In our model, differences in technological biases play very much the same role as differences in preferences in a standard Heckscher-Ohlin model.

A. Free-Trade Equilibrium

Before analyzing the consequences of globalization, we characterize a free-trade equilibrium. Given our work in Section III.B, this is a straightforward exercise. A competitive equilibrium in the world economy under free trade is a set of functions $(Y_H, L_H, w_H, Y_F, L_F, w_F, p_T, P_F)$ such that producers maximize profits, trade is balanced, and factor and good markets clear. Compared to the closed economy, the equilibrium condition for intermediate goods is now given by

$$Y_H(\sigma) + Y_F(\sigma) = \int_{s \in S_W} [A_H(s,\sigma)L_H(s,\sigma) + A_F(s,\sigma)L_F(s,\sigma)]ds$$

$$\text{for all } \sigma \in \Sigma_W.$$

Since technological differences at the task level are Hicks-neutral, our model is isomorphic to a model in which tasks are produced using the same technology around the world, but countries' factor supplies are given by $\tilde{V}_i \equiv \gamma_i V_i$. Moreover, since factors of production are perfect substitutes within each task, factor price equalization necessarily holds in efficiency units; see condition (7). This is a significant advantage of our assignment approach. It allows us to focus on the free-trade equilibrium that replicates the integrated equilibrium.[18]

Let M_T, w_T, and p_T denote the matching function, the wage expressed in Home units, and the price schedule in the integrated equilibrium, respectively. Using the same logic as in lemma 2, one can easily check that

$$\frac{dM_T}{ds} = \frac{A[s, M_T(s)]V_W(s)}{I_W \times \{p_T[M_T(s)]/B_W[M_T(s)]\}^{-\varepsilon}},$$

$$\frac{d\ln w_T(s)}{ds} = \frac{\partial \ln A[s, M_T(s)]}{\partial s},$$

where $M_T(\underline{s}_W) = \underline{\sigma}_W$ and $M_T(\bar{s}_W) = \bar{\sigma}_W$ are the boundary conditions for the matching function; $V_W \equiv \gamma_H V_H + \gamma_F V_F$ is the world distribution of skills;

$$p_T[M_T(s)] = w_T(s)/\{\gamma_H A[s, M_T(s)]\}$$

is the price schedule;

$$B_W[M_T(s)] \equiv \{(I_H/I_W)B_H[M_T(s)]^{\varepsilon} + (I_F/I_W)P_F^{\varepsilon-1}B_F[M_T(s)]^{\varepsilon}\}^{1/\varepsilon}$$

[18] Since factor price equalization holds in efficiency units, our model is perfectly consistent with large observed differences in income per capita across countries; see, e.g., Trefler (1993, 1995).

characterizes the skill bias of the "world's technology"; and

$$I_W \equiv \int_{s \in S_W} w_T(s)[V_H(s) + (\gamma_F/\gamma_H)V_F(s)]ds$$

is world income.

B. Consequences of North-South Trade

We conceptualize North-South trade as situations in which countries differ in either (i) their skill abundance, $V_H \succcurlyeq_a V_F$, or (ii) the skill bias of their technologies, $B_H \succcurlyeq_s B_F$.

1. The Role of Cross-Country Differences in Factor Endowments

To isolate the role of factor supply considerations, we first assume that Home is skill abundant relative to Foreign, $V_H \succcurlyeq_a V_F$, but that the final good is produced using the same technology around the world, $B_H = B_F$. In a two-by-two Heckscher-Ohlin model, when the skill-abundant country opens up to trade, (i) the skill intensity of both tasks decreases, (ii) the skill-intensive task expands, and (iii) the skill premium rises. Conversely, when the non-skill-abundant country opens up to trade, (i) the skill intensity of both tasks increases, (ii) the non-skill-intensive task expands, and (iii) the skill premium falls. We now use our assignment model to offer high-dimensional counterparts to these classic results.[19] Our analysis builds on the following lemma.

LEMMA 7. Suppose $V_H \succcurlyeq_a V_F$. Then V_W satisfies $V_H \succcurlyeq_a V_W \succcurlyeq_a V_F$.

As in the two-factor model, if Home is skill abundant relative to Foreign, then Home is skill abundant relative to the world and the world is skill abundant relative to Foreign.

We first consider the implications of trade integration on the matching of workers to tasks. Let M_H and M_F be the matching functions at home and abroad, respectively, under autarky. By lemmas 3 and 7, trade integration induces skill downgrading for all tasks at home and skill upgrading for all tasks abroad:

$$M_H^{-1}(\sigma) \geq M_T^{-1}(\sigma) \geq M_F^{-1}(\sigma) \quad \text{for all } \sigma \in \Sigma_W. \tag{19}$$

[19] We omit the counterparts to the Heckscher-Ohlin theorem because both Ohnsorge and Trefler (2007) and Costinot (2009) prove this result with arbitrarily many factors and tasks.

This is the counterpart to effect i in the two-by-two Heckscher-Ohlin model. A direct corollary of inequality (19) is that for any $\sigma \in \Sigma_W$,

$$\begin{aligned} \int_{M_T^{-1}(\sigma)}^{M_T^{-1}(\bar{\sigma}_W)} V_H(s)ds &\geq \int_{M_H^{-1}(\sigma)}^{M_H^{-1}(\bar{\sigma}_W)} V_H(s)ds; \\ \int_{M_T^{-1}(\underline{\sigma}_W)}^{M_T^{-1}(\sigma)} V_F(s)ds &\geq \int_{M_F^{-1}(\underline{\sigma}_W)}^{M_F^{-1}(\sigma)} V_F(s)ds. \end{aligned} \tag{20}$$

According to inequality (20), the employment share in tasks with high skill intensities, from σ to $\bar{\sigma}_W$, increases at home, whereas the employment share in tasks with low skill intensities, from $\underline{\sigma}_W$ to σ, increases abroad. This is the counterpart to effect ii in the two-by-two Heckscher-Ohlin model.

We now turn to the implications of trade integration on inequality. Let w_H and w_F be the wage schedules at home and abroad, respectively, in autarky. As in Section IV.A.1, changes in the matching function, inequality (22), together with the comparative advantage of high-skill workers in tasks with high skill intensity, property (3), imply a pervasive rise in inequality at home and a pervasive fall in inequality abroad:

$$\begin{aligned} \frac{w_H(s')}{w_H(s)} &\leq \frac{w_T(s')}{w_T(s)} \quad \text{for all } \underline{s}_H \leq s \leq s' \leq \bar{s}_H; \\ \frac{w_T(s')}{w_T(s)} &\leq \frac{w_F(s')}{w_F(s)} \quad \text{for all } \underline{s}_F \leq s \leq s' \leq \bar{s}_F. \end{aligned} \tag{21}$$

Inequality (21) is the counterpart to effect iii. It captures a strong Stolper-Samuelson effect: anywhere in the skill distribution, workers with higher skills get relatively richer in the skill-abundant country under free trade, whereas they get relatively poorer in the other country. As in a two-by-two Heckscher-Ohlin model, one can further show that free trade raises the real wage of high-skill workers and lowers that of low-skill workers in the skill-abundant country if $S_H = S_F$; see Appendix B for details. The converse occurs in the other country.

To get a better sense of the previous effect, denote by s_i^q the skill of the worker at the qth percentile of the skill distribution in country i and by

$$I_i^A(q) \equiv \int_{s_i^{100-q}}^{\bar{s}_i} w_i(s)V_i(s)ds$$

and

$$I_i^T(q) \equiv \int_{s_i^{100-q}}^{\bar{s}_i} w_T(s)(\gamma_i/\gamma_H)V_i(s)ds$$

the total earnings of the top q percent of the skill distribution in country $i = H, F$ under autarky and free trade, respectively. For any $1 > q' \geq q \geq 0$, inequality (21) implies

$$\frac{I_H^T(q')}{I_H^T(q)} \geq \frac{I_H^A(q')}{I_H^A(q)}.$$

In other words, changes in inequality are *fractal* in nature: within any truncation of the skill distribution, free trade makes high-skill workers richer at home. Similarly, in the Foreign country, we have

$$\frac{I_F^T(q')}{I_F^T(q)} \leq \frac{I_F^A(q')}{I_F^A(q)}.$$

In spite of the large number of goods and factors in this economy, the fundamental forces linking trade integration and inequality remain simple. Because of changes in the relative supply of skills, trade integration induces skill downgrading for all tasks in the skill-abundant country. Thus, workers move into tasks with higher skill intensities, which increases the marginal return to skill and, in turn, inequality. Proposition 1 summarizes our results on the consequences of North-South trade when driven by factor endowment differences.

Proposition 1. If Home is skill abundant relative to Foreign, then, all else equal, trade integration induces (i) skill downgrading for all tasks at home and skill upgrading for all tasks abroad, (ii) an increase in the employment share of tasks with high skill intensities at home and low skill intensities abroad, and (iii) a pervasive rise in inequality at home and a pervasive fall in inequality abroad.

The simple two-by-two Stolper-Samuelson effect, which part iii of proposition 1 extends to the case of a high-dimensional assignment model, is one of the most tested implications of trade theory. Empirical results, however, are mixed. For example, O'Rourke and Williamson (1999), Wei and Wu (2001), Menezes-Filho and Muendler (2007), Michaels (2008), and Broda and Romalis (2009) find either direct or indirect support; for an extensive list of papers finding violations, see Goldberg and Pavcnik (2007). Goldberg and Pavcnik provide the following summary of the state of this empirical literature: "Overall, it appears that the particular mechanisms through which globalization affected inequality are country, time, and case specific; that the effects of trade liberalization need to be examined in conjunction with other concurrent policy reforms" (78). Seen through the lens of our theory, the previous empirical results can be interpreted as follows. For a given country's globalization experience, cross-country differences in relative factor supply may or may not be the main determinant of changes in inequality. With this in mind, we now turn to the implications of cross-country differences in relative factor demand.

2. The Role of Cross-Country Differences in Technological Biases

To isolate the role of factor demand considerations, we now assume that countries differ in terms of their final good production functions, $B_H \succcurlyeq_s B_F$, but have identical factor supply, $V_H = V_F$. As in the case of differences in factor supply, our analysis builds on the following lemma.

LEMMA 8. Suppose $B_H \succcurlyeq_s B_F$. Then B_W satisfies $B_H \succcurlyeq_s B_W \succcurlyeq_s B_F$.

If Home's technology is skill biased relative to Foreign's, then Home's technology is skill biased relative to the world's and the world's technology is skill biased relative to Foreign's.

We first consider the impact of trade integration on the matching of workers to tasks. Let M_H and M_F be the matching functions at home and abroad, respectively, under autarky. By lemmas 5 and 8, trade integration induces skill upgrading for all tasks at home and skill downgrading for all tasks abroad:

$$M_H(s) \ \geq \ M_T(s) \ \geq \ M_F(s) \quad \text{for all } s \in S_W. \tag{22}$$

Note that if Home and Foreign use different sets of tasks under autarky, $\Sigma_H \neq \Sigma_F$, then trade integration induces workers to move into the production of new tasks. In the Foreign country, the most skilled workers become employed in tasks whose skill intensity is higher than the intensity of any tasks performed under autarky. The converse is true in the Home country, where the least skilled workers become employed in tasks whose skill intensity is lower than the intensity of any tasks performed under autarky.

What happens to the distribution of wages? As in Section IV.B, changes in the matching function, inequality (22), together with the comparative advantage of high-skill workers in tasks with high skill intensity, property (3), imply a pervasive fall in inequality at home and a pervasive rise in inequality abroad:[20]

$$\frac{w_H(s')}{w_H(s)} \geq \frac{w_T(s')}{w_T(s)} \geq \frac{w_F(s')}{w_F(s)} \quad \text{for all } s' \ \geq \ s. \tag{23}$$

To sum up, the consequences of North-South trade driven by demand considerations are the exact opposite of the consequences of North-South trade driven by supply considerations.

PROPOSITION 2. If Home's technology is skill biased relative to Foreign's, then, all else equal, trade integration induces (i) skill upgrading for all tasks at home and skill downgrading for all tasks abroad, (ii) an increase in the employment share of tasks with low skill intensities at home and high skill intensities abroad, and (iii) a pervasive fall in inequality at home and a pervasive rise in inequality abroad.

[20] Verhoogen (2008) provides a partial equilibrium framework yielding similar predictions, at the firm level, and empirically finds supportive evidence in Mexico.

Propositions 1 and 2 together imply that predictions regarding the impact of globalization crucially depend on the correlation between supply and demand considerations. In a series of influential papers, Acemoglu (1998, 2003) argues that skill-abundant countries tend to use skill-biased technologies. With our notation, this means that if $V_H \succcurlyeq_a V_F$, then $B_H \succcurlyeq_s B_F$. Combining the insights of propositions 1 and 2, we should therefore not be surprised if (i) similar countries have different globalization experiences depending on which of these two forces, supply or demand, dominates; and (ii) the overall effect of trade liberalization on factor allocation and factor prices tends to be small in practice.

To conclude this section on the consequences of North-South trade, we briefly discuss the case in which both final and intermediate goods are freely traded. This extension provides a simple rationale for why, as sometimes argued, trade may increase inequality in both the North and the South. To see this, consider the case in which Home is skill abundant, $V_H \succcurlyeq_a V_F$; it uses a skill-biased technology, $B_H \succcurlyeq_s B_F$; and its producers have an absolute advantage in the production of the final good, for example, $B_H(\sigma) > B_F(\sigma)$ for all σ. In this situation, the final good is produced at home under free trade. In this country, opening up to trade is therefore equivalent to a change in relative factor supply, which increases inequality. By contrast, opening up to trade has two effects abroad: it increases the relative supply of high-skill workers but also the relative demand for tasks with high skill intensity. While the first force tends to lower inequality, the second tends to raise it. This opens up the possibility of a simultaneous rise in inequality in both countries if the second force dominates, for example, if (i) there are large differences in technological biases, (ii) there are small differences in factor endowments, or (iii) Foreign is large is relative to Home.[21]

C. *Consequences of North-North Trade*

To avoid a taxonomic exercise, we focus on the case in which countries differ only in factor supply and conceptualize North-North trade as a situation in which Home is more diverse than Foreign, $V_H \succcurlyeq_d V_F$. Under this assumption, we demonstrate that the familiar mechanisms at work in North-South trade apply equally well to North-North trade, which allows us, in turn, to generate new results on the consequences of international trade.

Our analysis of North-North trade builds on the following lemma.

[21] Formally, a sufficient (though of course not necessary) condition for a simultaneous increase in inequality in both countries is that Foreign is infinitely large compared to Home.

LEMMA 9. Suppose $V_H \succcurlyeq_d V_F$. Then V_W satisfies $V_H \succcurlyeq_d V_W \succcurlyeq_d V_F$.

Consider the Foreign country. By lemmas 4 and 9, trade integration induces task upgrading for low-skill workers and task downgrading for high-skill workers. Formally, there exists $s_F^* \in [\underline{s}_F, \bar{s}_F]$ such that

$$\begin{aligned} M_F(s) &\leq M_T(s) \quad \text{for all } s \in [\underline{s}_F, s_F^*]; \\ M_F(s) &\geq M_T(s) \quad \text{for all } s \in [s_F^*, \bar{s}_F]. \end{aligned} \tag{24}$$

This means skill downgrading for low–skill intensity tasks and skill upgrading for high–skill intensity tasks. The converse is true in the Home country. Namely, there exists $s_H^* \in [\underline{s}_H, \bar{s}_H]$ such that

$$\begin{aligned} M_H(s) &\geq M_T(s) \quad \text{for all } s \in [\underline{s}_H, s_H^*]; \\ M_H(s) &\leq M_T(s) \quad \text{for all } s \in [s_H^*, \bar{s}_H]. \end{aligned} \tag{25}$$

The differential impact of North-North trade integration on the tasks performed by high- and low-skill workers has stark implications on inequality in the two countries. At Home, inequality (24) and the log-supermodularity of A imply

$$\begin{aligned} \frac{w_H(s')}{w_H(s)} &\geq \frac{w_T(s')}{w_T(s)} \quad \text{for all } \underline{s}_H \leq s \leq s' \leq s_H^*; \\ \frac{w_H(s')}{w_H(s)} &\leq \frac{w_T(s')}{w_T(s)} \quad \text{for all } s_H^* \leq s \leq s' \leq \bar{s}_H. \end{aligned} \tag{26}$$

Moving from autarky to free trade leads to a polarization of the wage distribution in the more diverse country. Among the least skilled workers, those with lower skills get relatively richer, whereas the converse is true among the most skilled workers. Similarly, in the less diverse country we have

$$\begin{aligned} \frac{w_F(s')}{w_F(s)} &\leq \frac{w_T(s')}{w_T(s)} \quad \text{for all } \underline{s}_F \leq s \leq s' \leq s_F^*; \\ \frac{w_F(s')}{w_F(s)} &\geq \frac{w_T(s')}{w_T(s)} \quad \text{for all } s_F^* \leq s \leq s' \leq \bar{s}_F. \end{aligned} \tag{27}$$

Inequality (27) implies convergence abroad, as the "middle class" benefits relatively more from free trade. Proposition 3 summarizes our results on the consequences of North-North trade.

PROPOSITION 3. If Home is more diverse than Foreign, then, all else equal, trade integration induces (i) skill upgrading in tasks with low skill intensities at home and high skill intensities abroad, (ii) skill downgrading in tasks with high skill intensities at home and low skill intensities abroad, and (iii) wage polarization at home and convergence abroad.

It is worth emphasizing that, unlike propositions 1 and 2, proposition

3 has no clear implications for the overall level of inequality. Under North-North trade, the relative wage between high- and low-skill workers—as well as the relative price of the goods they produce—may either increase or decrease. This observation provides a simple rationale for the empirical findings of Lawrence and Slaughter (1993). The consequences of North-North trade are to be found at a higher level of disaggregation. When trading partners vary in terms of skill diversity, changes in inequality occur within low- and high-skill workers, respectively. Similarly, proposition 3 does not predict a decrease (or increase) in the employment shares of the skill-intensive tasks. According to our theory, North-North trade leads to a U-shape (or inverted U-shape) relationship between tasks' employment growth and their skill intensity.

VI. Technological Change in the World Economy

In this section we consider the impact of skill-biased technological change and offshoring in the world economy. For expositional purposes, we restrict ourselves to the North-South case in which $V_H \succcurlyeq_a V_F$ and assume that $\gamma_H \geq \gamma_F$. In other words, the skill-abundant country also is (weakly) more productive in all tasks.

A. *Global Skill-Biased Technological Change*

We first analyze the impact of global skill-biased technological change (SBTC), modeled as a shift from B_W to B'_W such that $B'_W \succcurlyeq_s B_W$. Among other things, this analysis will help us illustrate how our previous predictions—which were concerned with the entire wage distribution—may easily be aggregated to shed light on cross-country income differences.

We denote M_T and M'_T the matching functions in the integrated equilibrium under B_W and B'_W, respectively, and w_T and w'_T the associated wage schedules. From our previous work in a closed economy, we already know that global SBTC induces skill downgrading for all tasks in both countries:

$$M_T(s) \leq M'_T(s) \quad \text{for all } s \in S_W.$$

We also know that this change in matching implies

$$\frac{w'_T(s')}{w'_T(s)} \geq \frac{w_T(s')}{w_T(s)} \quad \text{for all } s' \geq s,$$

which leads to a pervasive rise in inequality within each country. Compared to a closed economy, however, we can further ask how global SBTC affects inequality between countries. Let

$$I_i^{(\prime)} \equiv \int_{s \in S_i} w_T^{(\prime)}(s)(\gamma_i/\gamma_H)V_i(s)ds$$

denote total income in country $i = H, F$. Our predictions about the impact of global SBTC on cross-country inequality can be stated as follows.

LEMMA 10. Suppose $V_H \geqslant_a V_F$ and $B_W' \geqslant_s B_W$. Then total income satisfies $I_H'/I_F' \geq I_H/I_F$.

According to lemma 10, an increase in the relative labor demand for skill-intensive tasks worldwide increases inequality between Home and Foreign. The formal argument relies on the fact that log-supermodularity is preserved by multiplication and integration, but the basic intuition is simple: high-skill workers gain relatively more from such a change, and Home has relatively more of them. In our model, within- and between-country inequality tend to go hand in hand: ceteris paribus, changes in matching that increase inequality in both countries also increase inequality across countries. Proposition 4 summarizes our results on the consequences of global SBTC.

PROPOSITION 4. Global SBTC induces (i) skill downgrading for all tasks in each country, (ii) a pervasive rise in inequality in each country, and (iii) an increase in inequality between countries.

Finally, it is worth pointing out that proposition 4 also has interesting implications for the consequences of trade liberalization when our Dixit-Stiglitz production function is reinterpreted as a utility function. Suppose, for example, that a country's preferences are a function of its aggregate income I and that wealthier countries have a relative preference for skill-intensive goods: $B^{I'} \geqslant_s B^I$ for all $I' \geq I$. Then, by increasing income in all countries, trade liberalization would tend to lead to a pervasive rise in inequality around the world.

B. *Offshoring Tasks*

For our final comparative static exercise, we analyze the impact of an increase in Foreign workers' productivity from $\gamma_F A(s, \sigma)$ to $\gamma_F' A(s, \sigma)$, where $\gamma_F' > \gamma_F$. A natural way to think about such a technological change is *offshoring*, that is, the ability of Home firms to hire Foreign workers using Home's superior technology.[22] This is the interpretation we adopt in the rest of this subsection.

Our analysis of task offshoring builds on two simple observations.

[22] This way of modeling offshoring is in the spirit of Grossman and Rossi-Hansberg (2008).

First, as far as the integrated equilibrium is concerned, increasing the productivity of all Foreign workers from γ_F to $\gamma_F' > \gamma_F$ is similar to increasing their supply by γ_F'/γ_F. Second, since Home is skill abundant relative to Foreign, an increase in effective units of Foreign factor supply, from $\gamma_F V_F$ to $\gamma_F' V_F$, makes the world relatively less skill abundant, as we show in the following lemma.

Lemma 11. Suppose $V_H \succcurlyeq_a V_F$ and $\gamma_F' > \gamma_F$. Then $V_W \equiv \gamma_H V_H + \gamma_F V_F$ and $V_W' \equiv \gamma_H V_H + \gamma_F' V_F$ satisfy $V_W \succcurlyeq_a V_W'$.

To sum up, if domestic firms offshore their production, it is as if the world distribution becomes relatively less skill abundant. Therefore, the results of Section IV.A.1 directly imply that

$$M_T'(s) \geq M_T(s) \quad \text{for all } s \in S_W,$$

where M_T and M_T' are the matching functions in the integrated equilibrium before and after offshoring, respectively. By lemmas 3 and 11, offshoring induces task upgrading for all workers, as the world's matching function moves closer toward Foreign's matching function under autarky. This implies a pervasive rise in inequality in both countries:

$$\frac{w_T'(s')}{w_T'(s)} \geq \frac{w_T(s')}{w_T(s)} \quad \text{for all } s' \geq s.$$

For any pair of workers in either country, the relative wage of the more skilled worker increases as a result of offshoring. In the integrated equilibrium, offshoring is similar to an increase in the relative size of the Foreign country. As Foreign grows relative to Home, world prices converge to those that hold in Foreign under autarky. Since the wage schedule is steeper abroad than at home under autarky, offshoring increases inequality in both countries. Proposition 5 summarizes our results on the consequences of offshoring.

Proposition 5. Offshoring in the world economy induces (i) skill downgrading for all tasks in both countries and (ii) a pervasive rise in inequality in both countries.

The previous results are reminiscent of those in Feenstra and Hanson (1996) and, subsequently, Zhu and Trefler (2005). In addition to the fact that they apply to the full distribution of earnings rather than just the skill premium, these results also demonstrate that neither Ricardian technological differences nor a lack of factor price equalization is necessary to yield these predictions. The key mechanism is that offshoring leads to task upgrading for workers around the world, thereby increasing the marginal return to skill in all countries.

VII. Extensions

A. *Number of Goods and Factors*

The results derived so far all relied on the assumption of a continuum of tasks and a continuum of workers. As mentioned in the introduction, comparative static predictions on factor allocation and factor prices in neoclassical trade models typically are very sensitive to assumptions made on the number of goods and factors. The objective of this section is to demonstrate that, by contrast, our results generalize to the case of an arbitrary but discrete number of goods and factors. In order to avoid a taxonomic exercise, we focus on a move from V to $V' \succcurlyeq_a V$ in the closed economy.

Throughout this section, we assume that there is a discrete number of factors, $s_1 < \cdots < s_M$, and a discrete number of tasks, $\sigma_1 < \cdots < \sigma_N$. The rest of our model is unchanged. In terms of notation, we let $\Sigma(s) \equiv \{\sigma \in \Sigma | L(s, \sigma) > 0\}$ denote the set of tasks employing workers with skills s and $S(\sigma) \equiv \{s \in S | L(s, \sigma) > 0\}$ denote the set of skills employed in task σ, where $L(s, \sigma)$ is the allocation of workers to tasks in a competitive equilibrium. We use similar notation for the allocation under V'.

In this environment, we can derive the following counterparts to lemmas 1 and 3.

Lemma 1 (Discrete). In a competitive equilibrium, $\Sigma(s') \geq \Sigma(s)$ in the strong set order for any $s' \geq s$.

Lemma 3 (Discrete). Suppose $V' \succcurlyeq_a V$. Then $\Sigma'(s) \leq \Sigma(s)$ in the strong set order for all $s \in S \cap S'$.

This last lemma further implies that if $V' \succcurlyeq_a V$, then $w'(s')/w'(s) \leq w(s')/w(s)$ for all $s' \geq s$ in $S \cap S'$. In other words, a move from V to $V' \succcurlyeq_a V$ leads to a pervasive fall in inequality, as previously shown in the continuum-by-continuum case. The formal proofs can be found in Costinot and Vogel (2009).

B. *Observable versus Unobservable Skills*

Although our theory assumes a continuum of skills, an econometrician is unlikely to observe a continuum of skills in practice. To bring our theory one step closer to data, we now introduce explicitly the distinction between observable and unobservable skills.[23] The objective of this section is to demonstrate how, under reasonable assumptions, our results about change inequality over a continuum of unobservable skills can easily be mapped into observable measures of inequality such as (i)

[23] Of course, the analysis of this section has similar implications in the case in which the econometrician observes only a coarse measure of task skill intensity, such as “occupation” or “sector” of employment.

between-group inequality (e.g., the skill premium) and (ii) within-group inequality (e.g., the 90–10 log hourly wage differential among college graduates). For pedagogical purposes, we restrict ourselves to the case of North-South trade integration with factor endowment differences: $V_H \succcurlyeq_a V_F$ and $B_H = B_F$.

Throughout this section, we assume that workers are partitioned into n groups on the basis of some socioeconomic characteristic $e_1 < \cdots < e_n$, such as years of education or experience. While firms and workers perfectly observe s, we assume that the econometrician observes only e but knows the inelastic supply of workers with skill s in group e in country i: $V_i(s, e) \geq 0$. In particular, the econometrician knows that $V_i(s, e)$ is log-supermodular:

$$V_i(s', e')V_i(s, e) \geq V_i(s, e')V_i(s', e) \quad \text{for all } s' \geq s \text{ and } e' \geq e. \tag{28}$$

Property (28) captures the idea that, in both countries, high-skill workers are relatively more likely in groups with high levels of education or experience.

Armed with a link between observable and unobservable skills, we may now discuss between- and within-group inequality. For any pair of groups e' and e in country i, we define between-group inequality as the relative average wage between two groups:

$$\bar{w}_i(e')/\bar{w}_i(e) = \int_{s \in S_i} w_i(s)V_i(s, e')ds \bigg/ \int_{s \in S_i} w_i(s)V_i(s, e)ds.$$

For any group e in country i, we define within-group inequality as $w_i[s_i^{q'}(e)]/w_i[s_i^q(e)]$, where $100 \geq q' \geq q \geq 0$ are arbitrary percentiles of the skill distribution and $s_i^{q'}(e)$ and $s_i^q(e)$ denote the skills of the workers at these points.

With the previous notation in hand, we are ready to state the implications of North-South trade integration for between- and within-group inequality. If $V_H \succcurlyeq_a V_F$ and $B_H = B_F$, then

$$\begin{aligned} \frac{\bar{w}_H(e')}{\bar{w}_H(e)} &\leq \frac{\bar{w}_T(e')}{\bar{w}_T(e)} \quad \text{for all } e' \geq e; \\ \frac{\bar{w}_T(e')}{\bar{w}_T(e)} &\leq \frac{\bar{w}_F(e')}{\bar{w}_F(e)} \quad \text{for all } e' \geq e. \end{aligned} \tag{29}$$

Inequality (29) states that North-South trade integration, when driven by factor endowment differences, leads to an increase in between-group inequality at home and a decrease in between-group inequality abroad. From inequality (21), it is also easy to check that

$$\begin{aligned}\frac{w_H[s_H^{q'}(e)]}{w_H[s_H^{q}(e)]} &\le \frac{w_T[s_H^{q'}(e)]}{w_T[s_H^{q}(e)]} \quad \text{for all } e \text{ and } q' \ge q;\\ \frac{w_T[s_F^{q'}(e)]}{w_T[s_F^{q}(e)]} &\le \frac{w_F[s_F^{q'}(e)]}{w_F[s_F^{q}(e)]} \quad \text{for all } e \text{ and } q' \ge q.\end{aligned} \tag{30}$$

Thus North-South trade integration, when driven by factor endowment differences, also leads to an increase in within-group inequality at home and a decrease in within-group inequality abroad.[24] In each country, between- and within-group inequality tend to go hand in hand. Proposition 6 summarizes our results on between- and within-group inequality.

PROPOSITION 6. If Home is skill abundant relative to Foreign, then, all else equal, trade integration induces an increase in between- and within-group inequality at home and a decrease in between- and within-group inequality abroad.

Although we have chosen to focus on the consequences of North-South trade integration, similar results can be derived in the case of our other comparative static exercises. For instance, it is easy to check that (global) skill-biased technological change would increase between- and within-group inequality. This final observation may provide a simple rationale for the increase in residual wage inequality documented by Juhn et al. (1993).

VIII. Concluding Remarks

In the assignment literature, comparative static predictions are typically derived under strong functional form restrictions. The first contribution of our paper is to offer sufficient conditions for robust monotone comparative static predictions in a Roy-like assignment model. These general results are useful because they deepen our understanding of an important class of models in the labor and trade literatures, clarifying how relative factor supply and relative factor demand affect factor allocation and factor prices in such environments.

The second contribution of our paper is to show how these general results can be used to derive sharp predictions about the consequences of globalization in economies with an arbitrarily large number of both goods and factors. This new approach enables us to discuss, within a unified framework, phenomena that have been recently documented in the labor and public finance literatures but would otherwise fall

[24] As in Sec. V.B.2, North-South trade integration, when driven by technological differences, would lead to the exact opposite results.

outside the scope of standard trade theory, such as pervasive changes in inequality and wage and job polarization.[25]

Finally, while we have emphasized the consequences of globalization, we believe that our general results also have useful applications outside of international trade. As Heckman and Honore (1990) note, "The analysis of choice of geographical location . . . , schooling levels . . ., occupational choice with endogenous specific human capital . . . , choice of industrial sectors . . . , and the consequences of these choices for earnings inequality all fall within the general framework of the Roy model" (1121). Accordingly, our tools and techniques can also potentially shed light on each of these choices and their consequences for inequality.

Appendix A

Proofs I: The Closed Economy

Proof of Lemma 1

Throughout the proof, we denote $S(\sigma) \equiv \{s \in S | L(s,\sigma) > 0\}$ and $\Sigma(s) \equiv \{\sigma \in \Sigma | L(s,\sigma) > 0\}$. Clearly $s \in S(\sigma) \Leftrightarrow \sigma \in \Sigma(s)$. We proceed in five steps.

Step 1. $S(\sigma) \neq \emptyset$ for all $\sigma \in \Sigma$ and $\Sigma(s) \neq \emptyset$ for all $s \in S$.

Condition (9) and $V(s) > 0$ for all s directly imply $\Sigma(s) \neq \emptyset$ for all $s \in S$. To show that $S(\sigma) \neq \emptyset$ for all σ, we proceed by contradiction. Suppose that there exists σ' such that $S(\sigma') = \emptyset$. Since $\Sigma(s) \neq \emptyset$ for all $s \in S$, we know that there exists σ such that $S(\sigma) \neq \emptyset$. Thus we have $Y(\sigma') = 0$ and $Y(\sigma) > 0$. By condition (6), this further implies $p(\sigma)/p(\sigma') = 0$. Since there exists $s \in S(\sigma)$, we know by condition (7) that we must have

$$\frac{p(\sigma)}{p(\sigma')} \geq \frac{A(s,\sigma')}{A(s,\sigma)} > 0,$$

a contradiction.

Step 2. $S(\cdot)$ satisfies the two following properties: (i) for any $\sigma \in \Sigma$, $S(\sigma)$ is a nonempty interval of $[\underline{s}, \bar{s}]$; and (ii) for any $\sigma' > \sigma$, if $s' \in S(\sigma')$ and $s \in S(\sigma)$, then $s' \geq s$.

We first demonstrate property i. In step 1, we have already shown that $S(\sigma)$ was nonempty. To show that $S(\sigma)$ is an interval, we proceed by contradiction. Suppose that there exists a task σ and three workers $s_1 < s_2 < s_3$ such that $s_1, s_3 \in S(\sigma)$ but $s_2 \notin S(\sigma)$. Since $\Sigma(s_2) \neq \emptyset$ by step 1, we know that there must be $\sigma' \neq \sigma$ such that $s_2 \in S(\sigma')$. Now suppose that $\sigma > \sigma'$. The argument in the case $\sigma < \sigma'$ is similar. Condition (7) implies

$$p(\sigma)A(s_1,\sigma) - w(s_1) = 0; \tag{A1}$$

[25] Of course, whether or not globalization actually caused such changes is an empirical matter. But to assess empirically whether or not this is the case, we first need a trade model that can "speak" to these phenomena, which our paper provides.

$$p(\sigma')A(s_2,\sigma') - w(s_2) = 0; \tag{A2}$$

$$p(\sigma')A(s_1,\sigma') - w(s_1) \leq 0; \tag{A3}$$

$$p(\sigma)A(s_2,\sigma) - w(s_2) \leq 0. \tag{A4}$$

By equation (A1) and inequality (A3), we have

$$p(\sigma')A(s_1,\sigma') \leq p(\sigma)A(s_1,\sigma). \tag{A5}$$

By equation (A2) and inequality (A4), we have

$$p(\sigma)A(s_2,\sigma) \leq p(\sigma')A(s_2,\sigma'). \tag{A6}$$

Combining inequalities (A5) and (A6), we obtain $A(s_1,\sigma')A(s_2,\sigma) \leq A(s_1,\sigma)A(s_2,\sigma')$, which contradicts $A(s,\sigma)$ strictly log-supermodular. Property i follows.

Let us turn to property ii. We again proceed by contradiction. Suppose that there exists $\sigma' > \sigma$ and $s' > s$ such that $s' \in S(\sigma)$ and $s \in S(\sigma')$. Using condition (7) in the exact same way as before, it is easy to check that $A(s',\sigma')A(s,\sigma) \leq A(s,\sigma')A(s',\sigma)$, which contradicts $A(s,\sigma)$ strictly log-supermodular. Property ii follows.

STEP 3. $S(\sigma)$ is a singleton for all but a countable subset of Σ.

By step 2's property i, we know that $S(\sigma)$ is measurable for any σ. Let Σ_0 be the subset of tasks σ such that $\mu[S(\sigma)] > 0$, where μ is the Lebesgue measure over $\mathbb{R}$. We first show that Σ_0 is a countable set. Choose an arbitrary task $\sigma \in \Sigma_0$ and let $\underline{s}(\sigma) \equiv \inf S(\sigma)$ and $\bar{s}(\sigma) \equiv \sup S(\sigma)$. The fact that $\mu[S(\sigma)] > 0$ has strictly positive measure implies $\underline{s}(\sigma) < \bar{s}(\sigma)$. So for any $\sigma \in \Sigma_0$, there must exist $j \in \mathbb{N}$ such that $\bar{s}(\sigma) - \underline{s}(\sigma) \geq (\bar{s} - \underline{s})/j$. By step 2's property ii, we also know that for any $\sigma \neq \sigma'$, $\mu[S(\sigma) \cap S(\sigma')] = 0$. Thus for any $j \in \mathbb{N}$, there can be at most j elements $\{\sigma_1, \ldots, \sigma_j\} \equiv \Sigma_0^j \subset \Sigma_0$ for which $\bar{s}(\sigma_i) - \underline{s}(\sigma_i) \geq (\bar{s} - \underline{s})/j$ for $i = 1, \ldots, j$. By construction, we have $\Sigma_0 = \cup_{j\in\mathbb{N}} \Sigma_0^j$, where Σ_0^j is a countable set. Since the union of countable sets is countable, Σ_0 must be a countable set. The fact that $S(\sigma)$ is a singleton for all but a countable subset of Σ directly derives from this observation and the fact that the only nonempty intervals of $[\underline{s}, \bar{s}]$ with measure zero are singletons.

STEP 4. $\Sigma(s)$ is a singleton for all but a countable subset of S.

This follows from the same arguments as in steps 2 and 3.

STEP 5. $S(\sigma)$ is a singleton for all $\sigma \in \Sigma$.

We proceed by contradiction. Suppose that there exists $\sigma \in \Sigma$ for which $S(\sigma)$ is not a singleton. By step 2's property i, we must have $\mu[S(\sigma)] > 0$. By step 4, we know that $\Sigma(s) = \{\sigma\}$ for μ-almost all $s \in S(\sigma)$. Hence, condition (9) implies

$$L(s,\sigma) = V(s)\delta[1 - \mathbf{1}_{S(\sigma)}] \quad \text{for } \mu\text{-almost all } s \in S(\sigma), \tag{A7}$$

where δ is a Dirac delta function. By step 3, we must also have $\sigma' \in \Sigma$ for which $S(\sigma') = \{s'\}$. Thus condition (9) implies

$$L(s',\sigma') \leq V(s')\delta[1 - \mathbf{1}_{S(\sigma')}]. \tag{A8}$$

Combining equations (8), (A7), and (A8) with $\mu[S(\sigma)] > 0$, we obtain $Y(\sigma) =$

$+\infty$ and $Y(\sigma') < +\infty$. By condition (6), this further implies $p(\sigma)/p(\sigma') = 0$. Since there exists $s \in S(\sigma)$, we know by condition (7) that we must have

$$\frac{p(\sigma)}{p(\sigma')} \geq \frac{A(s,\sigma')}{A(s,\sigma)} > 0,$$

a contradiction.

Step 5 implies the existence of a function $H: \Sigma \rightarrow S$ such that $L(s,\sigma) > 0$ if and only if $H(\sigma) = s$. By step 2's property ii, H must be weakly increasing. Since $\Sigma(s) \neq \emptyset$ for all $s \in S$ by step 1, H must also be continuous and satisfy $H(\underline{\sigma}) = \underline{s}$ and $H(\bar{\sigma}) = \bar{s}$. Finally, by step 4, H must be strictly increasing. Therefore, there exists a continuous and strictly increasing function $H: \Sigma \rightarrow S$ such that (i) $L(s,\sigma) > 0$ if and only if $H(\sigma) = s$ and (ii) $H(\underline{\sigma}) = \underline{s}$ and $H(\bar{\sigma}) = \bar{s}$. To conclude the proof of lemma 1, we set $M \equiv H^{-1}$. QED

Proof of Lemma 2

We first consider equation (11). Condition (7) and lemma 1 imply

$$p[M(s)]A[s,M(s)] - w(s) \geq p[M(s)]A[s+ds,M(s)] - w(s+ds),$$

$$p[M(s+ds)]A[s+ds,M(s+ds)] - w(s+ds) \geq p[M(s+ds)]A[s,M(s+ds)] - w(s).$$

Combining the two previous inequalities, we obtain

$$\begin{aligned} &\frac{p[M(s)]\{A[s+ds,M(s)] - A[s,M(s)]\}}{ds} \\ &\leq \frac{w(s+ds) - w(s)}{ds} \\ &\leq \frac{p[M(s+ds)]\{A[s+ds,M(s+ds)] - A[s,M(s+ds)]\}}{ds}. \end{aligned}$$

Since A is continuous, profit maximization, condition (7), and factor market clearing, equation (9), require w to be continuous. Since $p(\sigma) = w[M^{-1}(\sigma)]/A[M^{-1}(\sigma),\sigma]$, by condition (7) and lemma 1, and M^{-1} is continuous, by lemma 1, p is continuous as well. Taking the limit of the previous chain of inequalities as ds goes to zero, we therefore get

$$w_s(s) = p[M(s)]A_s[s,M(s)]. \tag{A9}$$

Since $p[M(s)] = w(s)/A[s,M(s)]$, we can rearrange equation (A9) as

$$\frac{d\ln w(s)}{ds} = \frac{\partial \ln A[s,M(s)]}{\partial s}.$$

This completes the first part of our proof. We now turn to equation (10). Lemma 1 and condition (9) imply that, for all $s \in S$,

$$L(s,\sigma) = V(s)\delta[\sigma - M(s)], \tag{A10}$$

where δ is a Dirac delta function. Now consider condition (8). At $\sigma = M(s)$, we have

$$Y[M(s)] = \int_{s\in S} A[s',M(s)]L[s',M(s)]ds'.$$

Using equation (A10), we can rearrange the previous expression as

$$Y[M(s)] = \int_{s'\in S} A[s', M(s)]V(s')\delta[M(s) - M(s')]ds'.$$

By lemma 1, we know that there exists $\sigma' = M(s')$, which implies

$$Y[M(s)] = \int_{\sigma'\in \Sigma} A[M^{-1}(\sigma'), M(s)]V[M^{-1}(\sigma')]\delta[M(s) - \sigma']\frac{1}{M_s(M^{-1}(\sigma'))}d\sigma'.$$

By definition of the Dirac delta function, this simplifies into

$$M_s(s) = \frac{A[s, M(s)]V(s)}{Y[M(s)]}. \tag{A11}$$

Combining equations (A11) and (6), we obtain equation (10). This completes the second part of our proof. The equalities $M(\underline{s}) = \underline{\sigma}$, $M(\bar{s}) = \bar{\sigma}$, and $p[M(s)] = w(s)/A[s, M(s)]$ derive from condition (7) and lemma 1, as previously mentioned. QED

Proof of Lemma 3

We proceed by contradiction. Suppose that there exists $s \in S \cap S'$ at which $M'(s) > M(s)$. Since $V' \succcurlyeq_a V$, we know that $S \cap S' = [\underline{s}', \bar{s}]$. By lemma 1, we also know that M and M' are continuous functions such that $M'(\underline{s}') = \underline{\sigma} \leq M(\underline{s}')$ and $M(\bar{s}) = \bar{\sigma} \geq M'(\bar{s})$. So, there must exist $\underline{s}' \leq s_1 < s_2 \leq \bar{s}$ and $\underline{\sigma} \leq \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that (i) $M(s_1) = M'(s_1) = \sigma_1$ and $M(s_2) = M'(s_2) = \sigma_2$, (ii) $M_s(s_1) \geq M'_s(s_1)$ and $M_s(s_2) \geq M'_s(s_2)$, and (iii) $M'(s) > M(s)$ for all $s \in (s_1, s_2)$. Condition ii implies

$$M_s(s_1)/M_s(s_2) \leq M'_s(s_1)/M'_s(s_2). \tag{A12}$$

Combining condition i with inequality (A12) and equation (10), we obtain

$$\frac{V(s_2)}{V(s_1)}\left[\frac{p(\sigma_2)}{p(\sigma_1)}\right]^{\varepsilon} \geq \frac{V'(s_2)}{V'(s_1)}\left[\frac{p'(\sigma_2)}{p'(\sigma_1)}\right]^{\varepsilon}.$$

With the zero-profit condition, this can be rearranged as

$$\frac{V(s_2)}{V(s_1)}\left[\frac{w(s_2)}{w(s_1)}\right]^{\varepsilon} \geq \frac{V'(s_2)}{V'(s_1)}\left[\frac{w'(s_2)}{w'(s_1)}\right]^{\varepsilon}. \tag{A13}$$

Inequality (A13) and $V' \succcurlyeq_a V$ require $w(s_2)/w(s_1) \geq w'(s_2)/w'(s_1)$. This inequality cannot hold because of equation (11), the strict log-supermodularity of A, and condition iii. QED

Proof of Lemma 4

We proceed by contradiction. Suppose that there does not exist $s^* \in S \cap S'$ such that $M'(s) \geq M(s)$, for all $s \in [\underline{s}, s^*]$, and $M'(s) \leq M(s)$, for all $s \in [s^*, \bar{s}]$. Since $V' \succcurlyeq_d V$, we know that $S \cap S' = [\underline{s}, \bar{s}]$. By lemma 1, we also know that M and M' are continuous functions such that $M(\underline{s}) = \underline{\sigma} \leq M'(\underline{s})$ and $M(\bar{s}) = \bar{\sigma} \geq M'(\bar{s})$. So, there must exist $\underline{s} \leq s_0 < s_1 < s_2 \leq \bar{s}$ and $\underline{\sigma} \leq \sigma_0 < \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that (i) $M(s_0) = M'(s_0) = \sigma_0$, $M(s_1) = M'(s_1) = \sigma_1$, and $M(s_2) = M'(s_2) = \sigma_2$; (ii) $M'_s(s_0) \leq M_s(s_0)$, $M'_s(s_1) \geq M_s(s_1)$, and $M'_s(s_2) \leq M_s(s_2)$; and (iii) $M'(s) < M(s)$ for all $s \in (s_0, s_1)$ and $M'(s) > M(s)$ for all $s \in (s_1, s_2)$. At this point, there are two possible cases: $s_1 < \hat{s}$ and $s_1 \geq \hat{s}$. If $s_1 < \hat{s}$, we can follow the same steps as in the proof of lemma 3 using s_0 and s_1. Formally, condition ii implies

$$M'_s(s_0)/M'_s(s_1) \leq M_s(s_0)/M_s(s_1). \tag{A14}$$

Combining condition i with inequality (A14), equation (10), and the zero-profit condition, we obtain

$$\frac{V'(s_1)}{V'(s_0)}\left[\frac{w'(s_1)}{w'(s_0)}\right]^{\varepsilon} \geq \frac{V(s_1)}{V(s_0)}\left[\frac{w(s_1)}{w(s_0)}\right]^{\varepsilon}. \tag{A15}$$

Inequality (A15) and $V \succcurlyeq_a V'$ for all $s < \hat{s}$ require $w'(s_1)/w'(s_0) \geq w(s_1)/w(s_0)$. This inequality cannot hold because of equation (11), the log-supermodularity of A, and condition iii. This completes our proof in the case $s_1 < \hat{s}$. If $s_1 \geq \hat{s}$, we can again follow the same steps as in the proof of lemma 3 but using s_1 and s_2. The formal argument is identical and is omitted. QED

Proof of Lemma 5

We proceed by contradiction. Suppose that there exists $s \in [\underline{s}, \bar{s}]$ at which $M(s) > M'(s)$. Since $B' \succcurlyeq_s B$, we know that $\underline{\sigma} \leq \underline{\sigma}'$ and $\bar{\sigma} \leq \bar{\sigma}'$. By lemma 1, we also know that M and M' are continuous functions such that $M(\underline{s}) = \underline{\sigma} \leq \underline{\sigma}' = M'(\underline{s})$ and $M(\bar{s}) = \bar{\sigma} \leq \bar{\sigma}' = M'(\bar{s})$. So there must exist $\underline{s} \leq s_1 < s_2 \leq \bar{s}$ and $\underline{\sigma}' \leq \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that (i) $M'(s_1) = M(s_1) = \sigma_1$ and $M'(s_2) = M(s_2) = \sigma_2$, (ii) $M'_s(s_1) \leq M_s(s_1)$ and $M'_s(s_2) \geq M_s(s_2)$, and (iii) $M(s) > M'(s)$ for all $s \in (s_1, s_2)$. Condition ii implies

$$M'_s(s_1)/M'_s(s_2) \leq M_s(s_1)/M_s(s_2). \tag{A16}$$

Combining condition i with inequality (A16) and equation (10), we obtain

$$\frac{B'(\sigma_2)}{B'(\sigma_1)}\frac{p'(\sigma_1)}{p'(\sigma_2)} \leq \frac{B(\sigma_2)}{B(\sigma_1)}\frac{p(\sigma_1)}{p(\sigma_2)}.$$

With the zero-profit condition, this can be rearranged as

$$\frac{B'(\sigma_2)}{B'(\sigma_1)}\frac{w'(s_1)}{w'(s_2)} \leq \frac{B(\sigma_2)}{B(\sigma_1)}\frac{w(s_1)}{w(s_2)}. \tag{A17}$$

Inequality (A13) and $B' \succcurlyeq_s B$ require $w'(s_1)/w'(s_2) \leq w(s_1)/w(s_2)$. This inequality cannot hold because of equation (11), the log-supermodularity of A, and condition iii. QED

Proof of Lemma 6

We proceed by contradiction. Suppose that there does not exist $s^* \in S$ such that $M(s) \geq M'(s)$, for all $s \in [\underline{s}, s^*]$, and $M(s) \leq M'(s)$, for all $s \in [s^*, \bar{s}]$. Since $B' \succcurlyeq_e B$, we know that $\underline{\sigma}' \leq \underline{\sigma}$ and $\bar{\sigma} \leq \bar{\sigma}'$. By lemma 1, we also know that M and M' are continuous functions such that $M'(\underline{s}) = \underline{\sigma}' \leq \underline{\sigma} = M(\underline{s})$ and $M(\bar{s}) = \bar{\sigma} \leq \bar{\sigma}' = M'(\bar{s})$. So, there must exist $\underline{s} \leq s_0 < s_1 < s_2 \leq \bar{s}$ and $\underline{\sigma} \leq \sigma_0 < \sigma_1 < \sigma_2 \leq \bar{\sigma}$ such that (i) $M'(s_0) = M(s_0) = \sigma_0$, $M'(s_1) = M(s_1) = \sigma_1$, and $M'(s_2) = M(s_2) = \sigma_2$; (ii) $M_s(s_0) \leq M'_s(s_0)$, $M_s(s_1) \geq M'_s(s_1)$, and $M_s(s_2) \leq M'_s(s_2)$; and (iii) $M(s) < M'(s)$ for all $s \in (s_0, s_1)$ and $M(s) > M'(s)$ for all $s \in (s_1, s_2)$. At this point, there are two possible cases: $\sigma_1 < \hat{\sigma}$ and $\sigma_1 \geq \hat{\sigma}$. If $\sigma_1 < \hat{\sigma}$, we can follow the same steps as in the proof of lemma 5 using σ_0 and σ_1. Formally, condition ii implies

$$M_s(s_0)/M_s(s_1) \leq M_s'(s_0)/M_s'(s_1). \tag{A18}$$

Combining condition i with inequality (A18), equation (10), and the zero-profit condition, we obtain

$$\frac{B'(\sigma_1)}{B'(\sigma_0)}\frac{w'(s_0)}{w'(s_1)} \geq \frac{B(\sigma_1)}{B(\sigma_0)}\frac{w(s_0)}{w(s_1)}. \tag{A19}$$

Inequality (A19) and $B \succcurlyeq_s B'$ for all $\sigma < \hat{\sigma}$ require $w'(s_0)/w'(s_1) > w(s_0)/w(s_1)$. This inequality cannot hold because of equation (11), the log-supermodularity of A, and condition iii. This completes our proof in the case $\sigma_1 < \hat{\sigma}$. If $\sigma_1 \geq \hat{\sigma}$, we can again follow the same steps as in the proof of lemma 5 but using σ_1 and σ_2. The formal argument is identical and is omitted. QED

Appendix B

Proofs II: The World Economy

Proof of Lemma 7

We formally show that $V_H \succcurlyeq_a V_F \Rightarrow V_H \succcurlyeq_a V_W$. The proof that $V_H \succcurlyeq_a V_F \Rightarrow V_W \succcurlyeq_a V_F$ is similar and is omitted. For any s' and s,

$$\frac{V_H(s')}{V_H(s)} \geq \frac{V_F(s')}{V_F(s)}$$

implies

$$\frac{V_H(s')}{V_H(s)} \geq \frac{\gamma_H V_H(s') + \gamma_F V_F(s')}{\gamma_H V_H(s) + \gamma_F V_F(s)}.$$

By definition of V_W, this further implies

$$\frac{V_H(s')}{V_H(s)} \geq \frac{V_W(s')}{V_W(s)}.$$

QED

Proof of Lemma 8

We formally show that $B_H \succcurlyeq_s B_F \Rightarrow B_H \succcurlyeq_s B_W$. The proof that $B_H \succcurlyeq_s B_F \Rightarrow B_W \succcurlyeq_s B_F$ is similar and is omitted. For any σ' and σ,

$$\frac{B_H(\sigma')}{B_H(\sigma)} \geq \frac{B_F(\sigma')}{B_F(\sigma)}$$

implies

$$\frac{B_H(\sigma')}{B_H(\sigma)} \geq \left\{\frac{I_H[B_H(\sigma')]^{\varepsilon} + I_F P_F^{\varepsilon-1}[B_F(\sigma')]^{\varepsilon}}{I_H[B_H(\sigma)]^{\varepsilon} + I_F P_F^{\varepsilon-1}[B_F(\sigma)]^{\varepsilon}}\right\}^{1/\varepsilon}.$$

By definition of B_W, this further implies

$$\frac{B_H(\sigma')}{B_H(\sigma)} \geq \frac{B_W(\sigma')}{B_W(\sigma)}.$$

QED.

Stolper-Samuelson Effect

In the main text, we have argued that North-South trade integration driven by factor endowments raises the real wage of high-skill workers and lowers that of low-skill workers at home, whereas the converse occurs abroad. We now demonstrate this result formally for Home if $S_H = S_F$. The argument for the Foreign country is similar. Throughout this proof, M_H, w_H, and p_H denote the matching function and wage and price schedules for Home under autarky, respectively. We proceed in two steps.

STEP 1. $p_H(\sigma')/p_H(\sigma) \leq p_T(\sigma')/p_T(\sigma)$ for all $\sigma \leq \sigma'$.

Profit maximization, condition (7), implies

$$\ln p_i(\sigma) = \ln w_i[M_i^{-1}(\sigma)] - \ln A[M_i^{-1}(\sigma), \sigma].$$

Differentiating with respect to σ and using the envelope theorem, we get

$$\frac{d \ln p_i(\sigma)}{d\sigma} = -\frac{\partial \ln A[M_i^{-1}(\sigma), \sigma]}{\partial \sigma}. \tag{B1}$$

Combining equation (B1) with inequality (19) and A log-supermodular, we obtain

$$\frac{d \ln p_H(\sigma)}{d\sigma} \leq \frac{d \ln p_T(\sigma)}{d\sigma}.$$

Integrating the previous expression, we finally get

$$\frac{p_H(\sigma')}{p_H(\sigma)} \leq \frac{p_T(\sigma')}{p_T(\sigma)}$$

for all $\sigma \leq \sigma'$.

STEP 2. $w_H(\underline{s}) \geq w_T(\underline{s})$ and $w_H(\bar{s}) \leq w_T(\bar{s})$.

We start by showing that $w_H(\underline{s}) \geq w_T(\underline{s})$. To do so, we first establish that

$$\frac{w_H(\underline{s})}{p_H(\sigma)} \geq \frac{w_T(\underline{s})}{p_T(\sigma)} \quad \text{for all } \sigma. \tag{B2}$$

Since $M_H(\underline{s}) = M_T(\underline{s}) = \underline{\sigma}$, the zero-profit condition for the producer of task $\underline{\sigma}$ implies

$$\frac{w_H(\underline{s})}{w_T(\underline{s})} = \frac{p_H(\underline{\sigma})}{p_T(\underline{\sigma})}.$$

Hence, inequality (B2) can be rearranged as

$$\frac{p_H(\sigma)}{p_H(\underline{\sigma})} \leq \frac{p_T(\sigma)}{p_T(\underline{\sigma})}$$

for all σ, which is true by step 1. In order to complete our proof, we then use the fact that the price of the final good at home is equal to one in both the autarky and free-trade equilibria. Inequality (B2) therefore immediately implies that $w_H(\underline{s}) \geq w_T(\underline{s})$. The proof that $w_H(\bar{s}) \leq w_T(\bar{s})$ is similar and is omitted.

The fact that North-South trade integration driven by factor endowments raises the real wage of high-skill workers and lowers that of low-skill workers at home follows from step 2 and inequality (21), according to which $w_T(s)/w_H(s)$ is weakly increasing in s. QED

Proof of Lemma 9

By definition, $V_H \succcurlyeq_d V_F$ implies $V_F \succcurlyeq_a V_H$, for all $s < \hat{s}$, and $V_H \succcurlyeq_a V_F$, for all $s \geq \hat{s}$. Thus, the result follows from lemma 7 applied separately to $s < \hat{s}$ and $s \geq \hat{s}$. QED

Proof of Lemma 10

Define $I(i, j) \equiv \gamma_i \int_{\underline{s}}^{\bar{s}} w(s, j)V(s, i)ds$, where $i = 1$ for Foreign and $i = 2$ for Home; $j = 1$ under B_W and $j = 2$ under B'_W; $w(s, j)$ is the world wage function for $j = 1, 2$; and $V(s, i) = V_i(s)$. The fact that $V_H \succcurlyeq_a V_F$ implies that $V(s, i)$ is log-supermodular. According to inequality (7), $w(s, j)$ is also log-supermodular. Since log-supermodularity is preserved by multiplication and integration, $I(i, j)$ is log-supermodular (see Karlin and Rinott 1980). This can be rearranged as $I(2, 2)/I(1, 2) \geq I(2, 1)/I(1, 1)$, which is equivalent to $I'_H/I'_F \geq I_H/I_F$. QED

Proof of Lemma 11

For any s' and s, we know that $V_H(s')/V_H(s) \geq V_F(s')/V_F(s)$. Since $\gamma'_F > \gamma_F$, this implies

$$\frac{\gamma_H V_H(s') + \gamma_F V_F(s')}{\gamma_H V_H(s) + \gamma_F V_F(s)} \geq \frac{\gamma_H V_H(s') + \gamma'_F V_F(s')}{\gamma_H V_H(s) + \gamma'_F V_F(s)}.$$

By definition of V_W and V'_W, this further implies $V_W(s')/V_W(s) \geq V'_W(s')/V'_W(s)$. QED

Appendix C

Proofs III: Observable versus Unobservable Skills

Proof of Proposition 6

We first demonstrate inequality (29) for the Home country. Let $w(s, x) \equiv w_H(s)$ if $x = 1$ and $w(s, x) \equiv w_T(s)$ if $x = 2$. By inequality (21), $w(s, x)$ is log-supermodular. By assumption, $V_H(s, e)$ is log-supermodular. We know that log-supermodularity is preserved by multiplication and integration (see Karlin and Rinott 1980). Therefore, $\bar{w}(e, x) \equiv \int_{s \in S} w(s, x)V_H(s, e)ds$ must be log-supermodular. This directly implies $\bar{w}_H(e)/\bar{w}_H(e') \leq \bar{w}_T(e)/\bar{w}_T(e')$. The argument for the Foreign country is similar. Inequality (30) directly derives from the fact that inequality (21) holds for all $s \geq s'$. QED

References

Acemoglu, D. 1998. "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *Q.J.E.* 113 (4): 1055–89.

———. 2003. "Patterns of Skill Premia." *Rev. Econ. Studies* 70 (2): 199–230.

Amiti, M., and D. R. Davis. 2008. "Trade, Firms, and Wages: Theory and Evidence." Working Paper no. 14106, NBER, Cambridge, MA.

Anderson, J. E. 2009. "Globalization and Income Distribution: A Specific Factors Continuum Approach." Manuscript, Boston Coll.

Antras, P., L. Garicano, and E. Rossi-Hansberg. 2006. "Offshoring in a Knowledge Economy." *Q.J.E.* 121 (1): 31–77.

Autor, D. H., L. F. Katz, and M. S. Kearney. 2006. "The Polarization of the U.S. Labor Market." Working Paper no. 11986, NBER, Cambridge, MA.

———. 2008. "Trends in U.S. Wage Inequality: Revising the Revisionists." *Rev. Econ. and Statis.* 90 (2): 300–323.

Becker, G. S. 1973. "A Theory of Marriage: Part I." *J.P.E.* 81 (4): 813–46.

Blanchard, E., and G. Willmann. 2008. "Trade, Education, and the Shrinking Middle Class." Manuscript, Univ. Virginia.

Bombardini, M., G. Gallipoli, and G. Pupato. 2009. "Skill Dispersion and Trade Flows." Working Paper no. 15097, NBER, Cambridge, MA.

Broda, C., and J. Romalis. 2009. "The Welfare Implications of Rising Price Dispersion." Manuscript, Univ. Chicago, Booth School Bus.

Chiquiar, D., and G. H. Hanson. 2005. "International Migration, Self-Selection, and the Distribution of Wages: Evidence from Mexico and the United States." *J.P.E.* 113 (2): 239–81.

Costinot, A. 2009. "An Elementary Theory of Comparative Advantage." *Econometrica* 77 (4): 1165–92.

Costinot, A., and J. Vogel. 2009. "Matching and Inequality in the World Economy." Working Paper no. 14106, NBER, Cambridge, MA.

Davidson, C., S. Matusz, and A. Shevchenko. 2008. "Globalization and Firm Level Adjustment with Imperfect Labor Markets." *J. Internat. Econ.* 75 (2): 295–309.

Davis, D. R., and J. Harrigan. 2007. "Good Jobs, Bad Jobs, and Trade Liberalization." Working Paper no. 13139, NBER, Cambridge, MA.

Dornbusch, R., S. Fischer, and P. A. Samuelson. 1980. "Heckscher-Ohlin Trade Theory with a Continuum of Goods." *Q.J.E.* 95 (2): 203–24.

Eeckhout, J., and P. Kircher. 2010. "Sorting and Decentralized Price Competition." *Econometrica* 78 (2): 539–74.

Egger, H., and U. Kreickemeier. 2009. "Firm Heterogeneity and the Labor Market Effects of Trade Liberalization." *Internat. Econ. Rev.* 50 (1): 187–216.

Ethier, W. J. 1984. "Higher Dimensional Issues in Trade Theory." In *Handbook of International Economics*, edited by R. W. Jones, and P. B. Kenen, 131–84. Amsterdam: Elsevier.

Feenstra, R. C., and G. H. Hanson. 1996. "Foreign Investment, Outsourcing and Relative Wages." In *Political Economy of Trade Policy: Essays in Honor of Jagdish Bhagwati*, edited by R. C. Feenstra, G. M. Grossman, and D. A. Irwin, 89–127. Cambridge, MA: MIT Press.

Gabaix, X., and A. Landier. 2008. "Why Has CEO Pay Increased So Much?" *Q.J.E.* 123 (1): 49–100.

Garicano, L., and E. Rossi-Hansberg. 2006. "Organization and Inequality in a Knowledge Economy." *Q.J.E.* 121 (4): 1383–1435.

Gibbons, R., L. F. Katz, T. Lemieux, and D. Parent. 2005. "Comparative Advantage, Learning, and Sectoral Wage Determination."*J. Labor Econ.* 23 (4): 681–724.

Goldberg, P. K., and N. Pavcnik. 2007. "Distributional Effects of Globalization in Developing Countries." *J. Econ. Literature* 45:39–82.

Goos, M., and A. Manning. 2007. "Lousy and Lovely Jobs: The Rising Polarization of Work in Britain." *Rev. Econ. and Statis.* 89 (1): 118–33.

Grossman, G. M. 2004. "The Distribution of Talent and the Pattern and Consequences of International Trade." *J.P.E.* 112 (1): 209–39.

Grossman, G. M., and G. Maggi. 2000. "Diversity and Trade." *A.E.R.* 90 (5): 1255–75.

Grossman, G. M., and E. Rossi-Hansberg. 2008. "Trading Tasks: A Simple Theory of Offshoring." *A.E.R.* 98 (5): 1978–97.

Heckman, J. J., and B. E. Honore. 1990. "The Empirical Content of the Roy Model." *Econometrica* 58 (5): 1121–49.

Helpman, E., O. Itskhoki, and S. Redding. Forthcoming. "Inequality and Unemployment in a Global Economy." *Econometrica.*

Helpman, E., and P. Krugman. 1985. *Market Structure and Foreign Trade: Increasing Returns, Imperfect Competition, and the International Economy.* Cambridge, MA: MIT Press.

Jones, R. W. 1965. "The Structure of Simple General Equilibrium Models." *J.P.E.* 73 (6): 557–72.

Jones, R. W., and J. A. Scheinkman. 1977. "The Relevance of the Two-Sector Production Model in Trade Theory." *J.P.E.* 85 (5): 909–35.

Juhn, C., K. M. Murphy, and B. Pierce. 1993. "Wage Inequality and the Rise in Returns to Skill." *J.P.E.* 101 (3): 410–42.

Karlin, S., and Y. Rinott. 1980. "Classes of Orderings of Measures and Related Correlation Inequalities. I. Multivariate Totally Positive Distributions." *J. Multivariate Analysis* 10:467–98.

Kremer, M., and E. Maskin. 2003. "Globalization and Inequality." Manuscript, Harvard Univ.

Lawrence, R. Z., and M. J. Slaughter. 1993. "International Trade and American Wages in the 1980s: Giant Sucking Sound or Small Hiccup?" *Brookings Papers Econ. Activity* (2): 161–226.

Legros, P., and A. F. Newman. 2002. "Monotone Matching in Perfect and Imperfect Worlds." *Rev. Econ. Studies* 69 (4): 925–42.

———. 2007. "Beauty Is a Beast, Frog Is a Prince: Assortative Matching with Nontransferabilities." *Econometrica* 75 (4): 1073–1102.

Melvin, J. 1968. "Production and Trade with Two Factors and Three Goods." *Rev. Econ. and Statis.* 58:1249–68.

Menezes-Filho, N. A., and M.-A. Muendler. 2007. "Labor Reallocation in Response to Trade Reform." Manuscript, Univ. California, San Diego.

Michaels, G. 2008. "The Effect of Trade on the Demand for Skill—Evidence from the Interstate Highway System." *Rev. Econ. and Statis.* 90 (4): 683–701.

Milgrom, P. R. 1981. "Good News and Bad News: Representation Theorems and Applications." *Bell J. Econ.* 12 (2): 380–91.

Monte, F. 2009. "Skill Bias, Trade and Wage Dispersion." Manuscript, Univ. Chicago.

Nocke, V., and S. Yeaple. 2008. "An Assignment Theory of Foreign Direct Investment." *Rev. Econ. Studies* 75 (2): 529–57.

Ohnsorge, F., and D. Trefler. 2007. "Sorting It Out: International Trade with Heterogeneous Workers." *J.P.E.* 115 (5): 868–92.

O'Rourke, K. H., and J. G. Williamson. 1999. *Globalization and History: The Evolution of a Nineteenth-Century Atlantic Economy.* Cambridge, MA: MIT Press.

Piketty, T., and E. Saez. 2003. "Income Inequality in the United States, 1913–1998." *Q.J.E.* 118 (1): 1–39.

Sattinger, M. 1993. "Assignment Models of the Distribution of Earnings." *J. Econ. Literature* 31:831–80.

Sethupathy, G. 2008. "Offshoring, Wages, and Employment: Theory and Evidence." Manuscript, Columbia Univ.

Shimer, R., and L. Smith. 2000. "Assortative Matching and Search." *Econometrica* 68:343–70.

Sly, N. 2009. "Labor Matching Behavior in Open Economies and Trade Adjustment." Manuscript, Univ. Oregon.

Stolper, W. F., and P. A. Samuelson. 1941. "Protection and Real Wages." *Rev. Econ. Studies* 9:58–73.

Tervio, M. 2008. "The Difference That CEOs Make: An Assignment Model Approach." *A.E.R.* 98 (3): 642–68.

Teulings, C. N. 1995. "The Wage Distribution in a Model of the Assignment of Skills to Jobs." *J.P.E.* 103:280–315.

———. 2005. "Comparative Advantage, Relative Wages, and the Accumulation of Human Capital." *J.P.E.* 113:425–54.

Trefler, D. 1993. "International Factor Price Differences: Leontief Was Right!" *J.P.E.* 101 (6): 961–87.

———. 1995. "The Case of the Missing Trade and Other Mysteries." *A.E.R.* 85 (5): 1029–46.

Verhoogen, E. A. 2008. "Trade, Quality Upgrading, and Wage Inequality in the Mexican Manufacturing Sector." *Q.J.E.* 123 (2): 489–530.

Wei, S.-J., and Y. Wu. 2001. "Globalization and Inequality: Evidence from within China." Working Paper no. 8611, NBER, Cambridge, MA.

Yeaple, S. R. 2005. "A Simple Model of Firm Heterogeneity, International Trade, and Wages." *J. Internat. Econ.* 65 (1): 1–20.

Zhu, S. C., and D. Trefler. 2005. "Trade and Inequality in Developing Countries: A General Equilibrium Analysis." *J. Internat. Econ.* 65 (1): 21–48.