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Comparative Advantage, Trade, and Payments in a Ricardian Model with a Continuum of Goods

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This paper discusses Ricardian trade and payments theory in the case of a continuum of goods. The analysis thus extends the development of many-commodity, two-country comparative advantage analysis as presented, for example, in Gottfried Haberler (1937), Frank Graham (1923), Paul Samuelson (1964), and Frank W. Taussig (1927). The literature is historically reviewed by John Chipman (1965). Perhaps surprisingly, the continuum assumption simplifies the analysis neatly in comparison with the discrete many-commodity case. The distinguishing feature of the Ricardian approach emphasized in this paper is the determination of the competitive margin in production between imported and exported goods. The analysis advances the existing literature by formally showing precisely how tariffs and transport costs establish a range of commodities that are not traded, and how the price-specie flow mechanism does or does not give rise to movements in relative cost and price levels.

The formal *real* model is introduced in Section I. Its equilibrium determines the *relative* wage and price structure and the efficient international specialization pattern. Section II considers standard comparative static questions of growth, demand shifts, technological change, and transfers. Extensions of the model to nontraded goods, tariffs, and transport costs are then studied in Section III. Monetary considerations are introduced in Section IV, which examines the price-specie mechanism under stable parities, floating exchange rate regimes, and also questions of unemployment under sticky money wages.

I. The Real Model

In this section we develop the basic real model and determine the equilibrium relative wage and price structure along with the efficient geographic pattern of specialization. Assumptions about technology are specified in Section IA. Section IB deals with demand. In Section IC the equilibrium is constructed and some of its properties are explored. Throughout this section we assume zero transport costs and no other impediments to trade.

A. Technology and Efficient Geographic Specialization

The many-commodity Ricardian model assumes constant unit labor requirements $(a_1, \ldots, a_n)$ and $(a_1^*, \ldots, a_n^*)$ for the n commodities that can be produced in the home and foreign countries, respectively. The commodities are conveniently indexed so that relative unit labor requirements are ranked in order of diminishing home country comparative advantage,

$$a_1^*/a_1 > \ldots > \ldots > a_i^*/a_i > \ldots > a_n^*/a_n$$

where an asterisk denotes the foreign country.

In working with a continuum of goods, we similarly index commodities on an interval, say [0, 1], in accordance with diminishing home country comparative advantage. A commodity z is associated with each point on the interval, and for each commodity there are unit labor requirements in the two countries, $a(z)$ and $a^*(z)$, with relative unit labor requirement given by

$$(1) \qquad A(z) \equiv \frac{a^*(z)}{a(z)} \qquad A'(z) < 0$$

The relative unit labor requirement function in (1) is by strong assumption continuous

and by construction (ranking or indexing of goods), decreasing in z. The function $A(z)$ is shown in Figure 1 as the downward sloping schedule.

Consider now the range of commodities produced domestically and those produced abroad, as well as the relative price structure associated with given wages. For that purpose we define as w and w^* the domestic and foreign wages measured in *any* (common!) unit. The home country will efficiently produce all those commodities for which domestic unit labor costs are less than or equal to foreign unit labor costs. Accordingly, any commodity z will be produced at home if

$$(2) \qquad a(z)w \leq a^*(z)w^*$$

Thus

$$(2') \qquad \omega \leq A(z)$$

where (3) defines the parameter ω, fundamental to Ricardian analysis,

$$(3) \qquad \omega \equiv w/w^*$$

This is the ratio of our real wage to theirs (our "double-factoral terms of trade"). It follows that for a given relative wage ω the home country will efficiently produce the range of commodities

$$(4) \qquad 0 \leq z \leq \tilde{z}(\omega)$$

where taking (2') with equality defines the borderline commodity z, for which

$$(5) \qquad \tilde{z} = A^{-1}(\omega)$$

$A^{-1}(\ )$ being the inverse function of $A(\ )$. By the same argument the foreign country will specialize in the production of commodities in the range

$$(4') \qquad \tilde{z}(\omega) \leq z \leq 1$$

The minimum cost condition determines the structure of relative prices. The relative price of a commodity z in terms of any other commodity z', when both goods are produced in the home country, is equal to the ratio of home unit labor costs:

$$(6) \quad P(z)/P(z') = wa(z)/wa(z')$$
$$= a(z)/a(z');$$
$$z \leq \tilde{z}, z' \leq \tilde{z}$$

The relative price of home produced z in terms of a commodity z'' produced abroad is by contrast

$$(7) \quad P(z)/P(z'') = wa(z)/w^*a^*(z'')$$
$$= \omega a(z)/a^*(z'');$$
$$z < \tilde{z} < z''$$

In summarizing the supply part of the model we note that any specified relative real wage is associated with an efficient geographic specialization pattern characterized by the borderline commodity $\tilde{z}(\omega)$ as well as by a relative price structure. (The pattern is "efficient" in the sense that the world is out on, and not inside, its production-possibility frontier.)

B. Demand

On the demand side, the simplest Mill-Ricardo analysis imposes a strong homothetic structure in the form of J. S. Mill or Cobb-Douglas demand functions that associate with each ith commodity a *constant expenditure share*, b_i. It further assumes *identical* tastes for the two countries or *uniform* homothetic demand.

By analogy with the many-commodity case, which involves budget shares

$$b_i = P_i C_i / Y \qquad b_i = b_i^*$$
$$\sum_1^n b_i = 1$$

We therefore prescribe for the continuum case a given $b(z)$ profile:

$$(8) \qquad b(z) = P(z)C(z)/Y > 0$$
$$b(z) = b^*(z)$$
$$\int_0^1 b(z)\,dz = 1$$

where Y denotes total income, C demand for and P the price of commodity z.

Next we define the fraction of income spent (anywhere) on those goods in which the home country has a comparative advantage:

$$(9) \qquad \vartheta(\tilde{z}) \equiv \int_0^{\tilde{z}} b(z)\,dz > 0$$
$$\vartheta'(\tilde{z}) = b(\tilde{z}) > 0$$

where again $(0, \tilde{z})$ denotes the range of commodities for which the home country enjoys a comparative advantage. With a fraction ϑ of each country's income, and therefore of world income, spent on home produced goods, it follows that the fraction of income spent on foreign produced commodities is

$$(9') \quad 1 - \vartheta(\tilde{z}) \equiv \int_{\tilde{z}}^{1} b(z)\, dz$$

$$0 \leq \vartheta(z) \leq 1$$

C. Equilibrium Relative Wages and Specialization

To derive the equilibrium relative wage and price structure and the associated pattern of efficient geographic specialization, we turn next to the condition of market equilibrium. Consider the home country's labor market, or equivalently the market for domestically produced commodities. With $\tilde{z}$ denoting the *hypothetical* dividing line between domestically and foreign produced commodities, equilibrium in the market for home produced goods requires that domestic labor income wL equals world spending on domestically produced goods:

$$(10) \quad wL = \vartheta(\tilde{z})(wL + w^*L^*)$$

Equation (10) associates with each $\tilde{z}$ a value of the relative wage w/w^* such that market equilibrium obtains. This schedule is drawn in Figure 1 as the upward sloping locus and is obtained from (10) by rewriting the equation in the form:

$$(10') \quad \omega = \frac{\vartheta(\tilde{z})}{1 - \vartheta(\tilde{z})} (L^*/L) = B(\tilde{z}; L^*/L)$$

where it is apparent from (9) that the schedule starts at zero and approaches infinity as $\tilde{z}$ approaches unity.

To interpret the $B(\ )$ schedule we note that it is entirely a representation of the demand side; and in that respect it shows that if the range of domestically produced goods were increased at constant relative wages, demand for domestic labor (goods) would increase as the dividing line is shifted —at the same time that demand for foreign
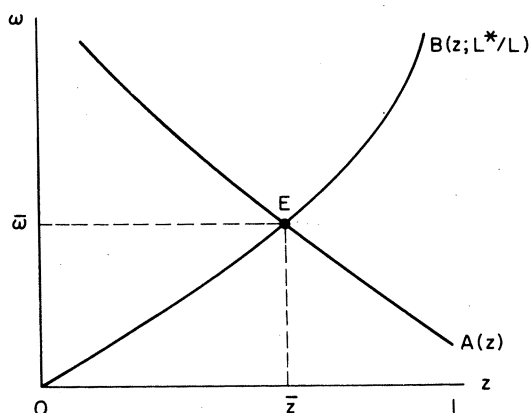

FIGURE 1

labor (goods) would decline.[1] A rise in the domestic relative wage would then be required to equate the demand for domestic labor to the existing supply.

An alternative interpretation of the $B(\ )$ schedule as the locus of trade balance equilibria uses the fact that (10) can be written in the balance-of-trade form:

$$(10'') \quad [1 - \vartheta(\tilde{z})]wL = \vartheta(\tilde{z})w^*L^*$$

This states that equilibrium in the trade balance means imports are equal in value to exports. On this interpretation, the $B(\ )$ schedule is upward sloping because an increase in the range of commodities hypothetically produced at home at constant relative wages lowers our imports and raises our exports. The resulting trade imbalance would have to be corrected by an increase in our relative wage that would raise our import demand for goods and reduce our exports, and thus restore balance.

The next step is to combine the demand side of the economy with the condition of efficient specialization as represented in equation (5), which specifies the competitive margin as a function of the relative wage. Substituting (5) in (10') yields as a solution the unique relative wage $\bar{\omega}$, at which the world is efficiently specialized, is in bal-

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[1]Throughout this paper we refer to "domestic" goods as commodities produced in the home country rather than to commodities that are nontraded. The latter we call "nontraded" goods.

anced trade, and is at full employment with
all markets clearing:

$$(11) \qquad \bar{\omega} = A(\bar{z}) = B(\bar{z}; L^*/L)$$

The equilibrium relative wage defined in
(11) is represented in Figure 1 at the inter-
section of the $A(\ )$ and $B(\ )$ schedules.[2]
Commodity $\bar{z}$ denotes the equilibrium bor-
derline of comparative advantage between
commodities produced and exported by the
home country $(0 \leq z \leq \bar{z})$, and those com-
modities produced and exported by the for-
eign country $(\bar{z} \leq z \leq 1)$.

Among the characteristics of the equilib-
rium we note that the equilibrium relative
wages and specialization pattern are deter-
mined by technology, tastes, and relative
size (as measured by the relative labor
force).[3] The relative price structure asso-
ciated with the equilibrium at point E is
defined by equations (6) and (7) once (11)
has defined the relative wage $\bar{\omega}$ and the
equilibrium specialization pattern $\bar{z}(\bar{\omega})$.

The equilibrium levels of production
$Q(z)$ and $Q^*(z)$, and employment in each
industry $L(z)$ and $L^*(z)$, can be recovered
from the demand structure and unit labor
requirements once the comparative advan-
tage pattern has been determined.

We note that with identical homothetic
tastes across countries and no distortions,
the relative wage $\bar{\omega}$ is a measure of the well-

being of the representative person-laborer
at home relative to the well-being of the
representative foreign laborer.

II. Comparative Statics

The unique real equilibrium in Figure 1
is determined jointly by tastes, technology,
and relative size, L^*/L. We can now exploit
Figure 1 to examine simple comparative
static questions.

A. Relative Size

Consider first the effect of an increase in
the relative size of the rest of the world. An
increase in L^*/L by (10) shifts the $B(\ )$
trade balance equilibrium schedule upward
in proportion to the change in relative size
and must, therefore, raise the equilibrium
relative wage at home and reduce the range
of commodities produced domestically. It
is apparent from Figure 2 that the domestic
relative wage increases *proportionally less*
than the decline in domestic relative size.

The rise in equilibrium relative wages due
to a change in relative size can be thought
of in the following manner. At the initial
equilibrium, the increase in the foreign rela-
tive labor force would create an excess sup-
ply of labor abroad and an excess demand
for labor at home—or, correspondingly, a
trade surplus for the home country. The re-
sulting increase in domestic relative wages
serves to eliminate the trade surplus while

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[2] See the Appendix for the relation of the diagram
to previous analyses.

[3] The construction of the $B(\ )$ schedule relies heavily
on the Cobb-Douglas demand structure. If, instead,
demand functions were identical across countries and
homothetic, an analogous schedule could be con-
structed. In the general homothetic case, however, a set
of relative prices is required at each z to calculate the
equivalent of the $B(\ )$ schedule; the relative prices are
those that apply on the $A(\bar{z})$ schedule for that value
of z. In this case the independence of the $A(\ )$ and
$B(\ )$ schedules is obviously lost. In the general homo-
thetic case there is still a unique intersection of the
$A(\ )$ and $B(\ )$ schedules. For more general nonhomo-
thetic demand structures, it is known that an equilib-
rium exists; but even in the case of two Ricardian
goods there may be *no* unique equilibrium even though
there will almost always be a finite number of equi-
libria. See Gerard Debreu and Stephen Smale. Exten-
sions of our analysis with respect to the demand struc-
ture and the number of countries are developed in un-
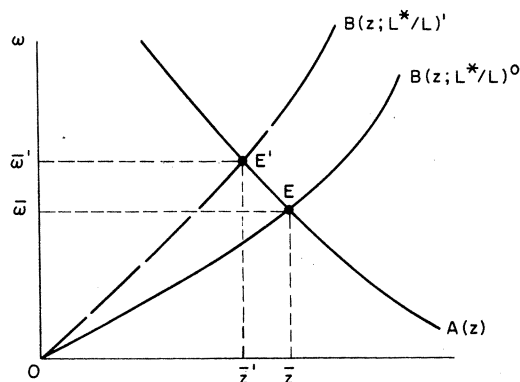published work by Charles Wilson.

FIGURE 2

at the same time raising relative unit labor costs at home. The increase in domestic relative unit labor costs in turn implies a loss of comparative advantage in marginal industries and thus a needed reduction in the range of commodities produced domestically.

The welfare implications of the change in relative size take the form of an unambiguous improvement in the home country's real income and (under Cobb-Douglas demand) a reduction in real income per head abroad. We observe, too, that from the definition of the home country's share in world income and (10), we have

$$(12) \qquad wL/(wL + w^*L^*) = \vartheta(\bar{z})$$

It is apparent, as noted above, that a reduction in domestic relative size in raising the domestic relative wage (thereby reducing the range of commodities produced domestically) must under our Cobb-Douglas demand assumptions lower the home country's share in total world income and spending—even though our per capita income rises.

B. Technical Progress

To begin with, we are concerned with the effects of uniform technical progress. By equation (1), a uniform proportional reduction in foreign unit labor requirements implies a reduction in $a^*(z)$ and therefore a proportional downward shift of the $A(z)$ schedule in Figure 1. At the initial relative wage $\bar{\omega}$, the loss of our comparative advantage due to a reduction in foreign unit labor costs will imply a loss of some industries in the home country and a corresponding trade deficit. The resulting induced decline in the equilibrium relative wage serves to restore trade balance equilibrium, and to offset in part our decline in comparative advantage.

The net effect is therefore a reduction in domestic relative wages, which must fall proportionally short of the decline in relative unit labor requirements abroad. The home country's terms of trade therefore improve as can be noted by using (7) for any two commodities z and z'', respectively, produced at home and abroad:

$$(13) \quad \hat{P}(z) - \hat{P}(z'') = \hat{\bar{\omega}} - \hat{a}^*(z'') > 0$$

where a "hat" denotes a proportional change. Domestic real income increases, as does foreign real income.[4] The range of goods produced domestically declines since domestic labor, in efficiency units, is now relatively more scarce.

An alternative form of technical progress that can be studied is the international transfer of the least cost technology. Such transfers reduce the discrepancies in relative unit labor requirements—by lowering them for each z in the relatively less efficient country—and therefore flatten the $A(z)$ schedule in Figure 1. It can be shown that such harmonization of technology must benefit the innovating low-wage country, and that it may reduce real income in the high-wage country whose technology comes to be adopted. In fact, the high-wage country must lose if harmonization is complete so that relative unit labor requirements now become identical across countries and all our consumer's surplus from international trade vanishes.[5]

C. Demand Shifts

The case with a continuum of commodities requires a careful definition of a demand shift. For our purposes it is sufficient to ask: What is the effect of a shift from high z commodities toward low z commodities? It is apparent from Figure 2 that such a shift will cause the trade balance equilibrium schedule $B(\ )$ to shift up and to the left. It follows that the equilibrium domestic rela-

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[4] The purchasing power of foreign labor income in terms of domestically produced goods is $w^*L^*/wa(z) = L^*/a(z)\bar{\omega}$ and in terms of foreign goods $L^*/a^*(z)$. The fact that foreigners' real income per head rises is guaranteed by our Cobb-Douglas demand assumption. In the general homothetic case, a balanced reduction in $a^*(z)$ can be immiserizing abroad if the real wage falls strongly in terms of all previously imported goods; however, the balanced drop in $a^*(z)$ in the general homothetic case *always* increases our real wage.

[5] Complete equilization of unit labor requirements implies that the $A(\ )$ schedule is horizontal at the level $\omega = A(z) = 1$. In this case geographic specialization becomes indeterminate and inessential.

tive wage will rise while the range of com-
modities produced by the home country de-
clines. Domestic labor is allocated to a
narrower range of commodities that are
consumed with higher density while foreign
labor is spread more thinly across a larger
range of goods.

Welfare changes cannot be identified in
this instance because tastes themselves have
changed. It is true that domestic relative
income rises along with the relative wage.
Further we note that since $\bar{\omega}$ rises, the rela-
tive well-being of home labor to foreign
labor (reckoned at the new tastes) is greater
than was our laborers' relative well-being
(reckoned at the old tastes).

D. Unilateral Transfers

Suppose foreigners make a continual uni-
lateral transfer to us. With uniform homo-
thetic tastes and no impediments to trade,
neither curve is shifted by the transfer since
we spend the transfer *exactly* as foreigners
would have spent it but for the transfer. The
new equilibrium involves a recurring trade
deficit for us, equal to the transfer, but there
is no change in the terms of trade. As Bertil
Ohlin argued against John Maynard
Keynes, here is a case where full equilibra-
tion takes place solely as a result of the
spending transfers. When we introduce non-
traded goods below, Ohlin's presumption
will be found to require detailed qualifica-
tions, as it also would if tastes differed geo-
graphically.

III. Extensions of the Real Model

Extensions of the real model taken up in
this section concern nontraded goods,
tariffs, and transport costs. The purpose of
this section is twofold. First we establish
how the exogenous introduction of non-
traded goods qualifies the preceding analy-
sis. Next we turn to a particular specifica-
tion of tariffs and transport costs to establish
an equilibrium range of endogenously de-
termined nontraded goods as part of the
equilibrium solution of the model. Trans-
fers are then shown to affect the equilibrium

relative price structure and the range of
goods traded.

A. Nontraded Goods

To introduce nontraded goods into the
analysis we assume that a fraction k of in-
come is everywhere spent on internationally
traded goods, and a fraction $(1 - k)$ is
spent in each country on nontraded com-
modities. With $b(z)$ continuing to denote
expenditure densities for traded goods, we
have accordingly

$$(14) \qquad k \equiv \int_0^1 b(z)\,dz < 1$$

where z denotes traded goods.[6] As before
the fraction of income spent on domesti-
cally exportable commodities is $\vartheta(z)$, except
that ϑ now reaches a maximum value of
$\vartheta(1) = k$.

Equation (1) remains valid for traded
goods, but the trade balance equilibrium
condition in (10″) must now be modified to:

$$(15) \quad [1 - \vartheta(\bar{z}) - (1 - k)]wL$$
$$= \vartheta(\bar{z})w^*L^*$$

since domestic spending on imports is equal
to income less spending on *all* domestically
produced goods including nontraded com-
modities. Equation (15) can be rewritten as

$$(15') \qquad \omega = \frac{\vartheta(\bar{z})}{k - \vartheta(\bar{z})}\,(L^*/L)$$

where k is a constant and therefore inde-
pendent of the relative wage structure.

We note that (15′) together with (5) de-
termines the equilibrium relative wage and
efficient geographic specialization, $(\bar{\omega}, \bar{z})$.
Further it is apparent that (15′) has exactly
the same properties as (10′) and that ac-
cordingly a construction of equilibrium like
that in Figure 1 remains appropriate. The
equilibrium relative wage again depends on

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[6]We can think of the range of nontraded goods as
another [0, 1] interval with commodities denoted by x
and expenditure fractions on those goods given by
$c(x)$. With these definitions we have $\int_0^1 c(x)\,dx \equiv$
$1 - k$, a positive fraction.

relative size, technology, and demand conditions. In this case demand conditions explicitly include the fraction of income spent on traded goods:

$$(11')\qquad \bar{\omega} = \frac{\vartheta(\bar{z})}{k - \vartheta(\bar{z})}\,\frac{L^*}{L} = A(\bar{z})$$

This nicely generalizes our previous equilibrium of (11) to handle exogenously given nontraded goods.[7]

Two applications of the extended model highlight the special aspects newly introduced by nontraded goods. First consider a shift in demand (in each country) toward *nontraded* goods. To determine the effects on the equilibrium relative wage we have to establish whether this shift is at the expense of high or low z commodities. In the former case the home country's relative wage increases while in the latter case it declines. If the shift in demand in each country is uniform so that $b(z)$ is reduced in the same proportion for all z in both countries, then the relative wage remains unchanged.

Consider next a transfer received by the home country in the amount T measured in terms of foreign labor. As is well known, and already shown, with identical homothetic tastes and *no* nontraded goods, a transfer leaves the terms of trade unaffected. In the present case, however, the condition for balanced trade, inclusive of transfers, becomes:

$$(16)\quad T = (k - \vartheta)[\omega L + T]$$
$$- \vartheta[L^* - T]$$

or, in equilibrium,

$$(16')$$
$$\bar{\omega} = \frac{1 - k}{k - \vartheta(\bar{z})}\,(T/L) + \frac{\vartheta(\bar{z})}{k - \vartheta(\bar{z})}\,(L^*/L)$$

It is apparent from (16') that a transfer receipt by the home country causes the trade balance equilibrium schedule in Figure 1 to shift upward at each level of z. Accordingly, the equilibrium domestic rela-

tive wage increases and the range of commodities produced domestically is reduced. The steps in achieving this result are, first, that at the initial relative wage only a fraction of the transfer is spent on imports in the home country, while foreign demand for domestic goods similarly declines only by a fraction of their reduced income. The resulting surplus for the home country has to be eliminated by, second, an increase in the domestic relative wage and a corresponding improvement in the home country's terms of trade.[8]

The analysis of nontraded goods therefore confirms in a Ricardian model the "orthodox" presumption with respect to the terms of trade effects of transfers.[9]

B. *Transport Costs: Endogenous Equilibrium for Nontraded Goods*

The notion that transport costs give rise to a range of commodities that are nontraded is established in the literature and is particularly well stated by Haberler (1937). In contrast with the previous section we shall now endogenously determine the range of nontraded commodities as part of the equilibrium. We assume, following the "iceberg" model of Samuelson (1954), that transport costs take the form of "shrinkage" in transit so that a fraction $g(z)$ of commodity z shipped actually arrives. We further impose the assumption that $g = g(z)$ is identical for all commodities and the same for shipments in either direction.

The home country will produce commodities for which domestic unit labor cost falls short of foreign unit labor costs adjusted for shrinkage, and we modify (2') accordingly:

$$(17)\qquad wa(z) \le (1/g)w^*a^*(z)$$
$$\text{or}\qquad\qquad \omega \le A(z)/g$$

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[7]Diagrams much like Figures 1 and 2 again apply: the descending $A(z)$ schedule is as before; and now the new rising schedule looks much as before. As before, a rise in L^*/L and a balanced drop in $a^*(z)$ will raise $\bar{\omega}$ and lower $\bar{z}$.

[8]At constant relative wages the current account worsens by $[(1 - k - \vartheta) + \vartheta]dT = (1 - k)dT$ which is less than the transfer, since it is equal to the fraction of income spent on nontraded goods.

[9]The pre-Ohlin orthodox view of Keynes, Taussig, Jacob Viner and other writers is discussed in Viner (1937) and Samuelson (1952, 1954). A recent treatment with nontraded goods is Ronald Jones (1975).

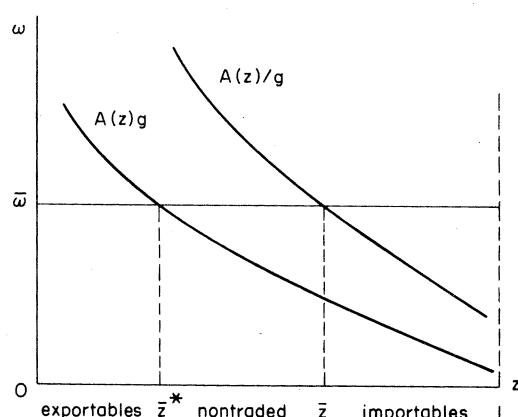exportables $\bar{z}^*$ nontraded $\bar{z}$ importables

FIGURE 3

Similarly the foreign country produces commodities for which foreign unit labor cost falls short of adjusted unit labor costs of delivered imports:

(18) $w^*a^*(z) \leq (1/g)wa(z)$

or $A(z)g \leq \omega$

In Figure 3 we show the adjusted relative unit labor requirement schedules $A(z)/g$ and $A(z)g$. It is apparent from (17) and (18) that for any given relative wage the home country produces and exports commodities to the left of the $A(z)g$ schedule, both countries produce as nontraded goods commodities in the intermediate range, and the foreign country produces and exports commodities in the range to the right of $A(z)/g$.

To determine the equilibrium relative wage we turn to the trade balance equilibrium condition in (19)—together with (20) and (21)—which is modified to take account of the endogenous range of nontraded goods:

(19) $(1 - \lambda)wL = (1 - \lambda^*)w^*L^*$

The variable λ is the fraction of home country income spent on our domestically (or home) produced goods—exportables and nontraded—and λ^* is the share of foreigners' income spent on goods they produce. Both λ and λ^* are endogenously determined because the range of goods produced in each country depends on the relative wages.

(20) $\lambda(g\omega) \equiv \int_0^{\bar{z}} b(z)dz \quad \lambda'(g\omega) < 0$

        $\lambda^*(\omega/g) \equiv \int_{\bar{z}}^1 b(z)dz \quad \lambda^{*\prime}(\omega/g) > 0$

The dependence of $\lambda(\ )$ and $\lambda^*(\ )$ on the variables specified in (20) and the respective derivatives follow from (21) below.

The limits of integration $\bar{z}$ and $\bar{z}^*$ are derived from the conditions for efficient production in (17) and (18) by imposing equalities and so defining the borderline commodities. Thus, in Figure 3, $\bar{z}$ is the borderline between domestic nontraded goods and imports for the home country, and $\bar{z}^*$ denotes the borderline between foreign nontraded goods and the home country's exports:

(21) $\bar{z}^* = A^{-1}(\omega/g) \quad d\bar{z}^*/d(\omega/g) < 0$
        $\bar{z} = A^{-1}(g\omega) \qquad d\bar{z}/d(g\omega) < 0$

Of course, equilibrium $\bar{z}$ and $\bar{z}^*$ are yet to be determined by the interaction of technology and demand conditions.

From (21) an increase in the relative wage reduces the range of commodities domestically produced and therefore raises the fraction of income spent on imports. Abroad the converse holds. An increase in the domestic relative wage increases the range of goods produced abroad and therefore reduces the fraction of income spent on imports. It follows that we can solve:

(19') $\bar{\omega} = \dfrac{1 - \lambda^*(\bar{\omega}/g)}{1 - \lambda(g\bar{\omega})} (L^*/L)$

          $\equiv \varphi(\bar{\omega}; L^*/L, g) \quad \partial\varphi/\partial\bar{\omega} < 0$

for the unique equilibrium relative wage as a function of relative size and transport costs:

(22) $\bar{\omega} = \bar{\omega}(L^*/L, g)$

Because (19')'s right-hand side declines as $\bar{\omega}$ rises, a rise in L^*/L must still raise $\bar{\omega}$; a rise in g can shift $\bar{\omega}$ in either direction, depending on the $B(z)$ and $A(z)$ profiles.

The equilibrium relative wage in (22), taken in conjunction with (21), determines the equilibrium geographic production pattern, $\bar{z}$ and $\bar{z}^*$. Since the range of nontraded

goods $\bar{z}^* \leq z \leq \bar{z}$ depends in this formulation on the equilibrium relative wage, it is obvious that shifts in given parameters will shift the range of nontraded commodities. Thus, a transfer that raises the equilibrium relative wage at home causes previously exported commodities to become nontraded, and previously nontraded commodities to become importables.

C. Tariffs

We consider next the case of zero transport cost but where each country levies a uniform tariff on imports at respective rates t and t^*, with proceeds rebated in lump sum form. This case, too, leads to cost barriers to importing, and to a range of commodities that are not traded, with the boundaries defined by:

$$(23) \qquad \bar{z} = A^{-1}\left(\frac{\omega}{1+t}\right)$$

and $\qquad \bar{z}^* = A^{-1}(\omega(1+t^*))$

From (23) it is apparent that the presence of tariffs in either or both countries must give rise to nontraded goods because in this case $\bar{z} \neq \bar{z}^*$.

The trade balance equilibrium condition at international prices becomes, in place of (19),

$$(24) \quad (1-\lambda)Y/(1+t) =$$
$$(1-\lambda^*)Y^*/(1+t^*)$$

where Y and Y^* denote incomes inclusive of lump sum tariff rebates. Using the fact that rebates are equal to the tariff rate times the fraction of income spent on imports, we arrive at the trade balance equilibrium condition in the form:[10]

$$(25) \quad \omega = \left(\frac{1-\lambda^*}{1-\lambda}\right)\frac{1+t\lambda}{1+t^*\lambda^*}(L^*/L)$$

where λ and λ^* are functions of (ω, t, t^*).

[10]Tariff rebates in the home country are equal to $R = (1-\lambda)Yt/(1+t)$. With $Y \equiv WL + R$ we therefore have $Y = WL(1+t)/(1+\lambda t)$ as an expression for income inclusive of transfers. From equations (20) and (23) we have $\lambda = \lambda[\omega/(1+t)]$ and $\lambda^* = \lambda^*[\omega(1+t^*)]$, having substituted the tariff instead of transport costs as the obstacle to trade.

The implicit relations (25) can be solved for the equilibrium relative wage as a function of relative size and the tariff structure:

$$(26) \qquad \bar{\omega} = \bar{\omega}(L^*/L, t, t^*)$$

From (26) and (23) it is apparent now that the range of nontraded goods will be a function of both tariff rates. It is readily shown that an increase in the tariff improves the imposing country's relative wage and terms of trade. Furthermore, as is well known, when all countries but one are free traders, then one country can always improve its own welfare by imposing a tariff that is not too large.

A further question suggested by (26) concerns the effect of a uniform increase in world tariffs. Starting from zero, a small uniform increase in tariffs raises the relative wage of the country whose commodities command the larger share in world spending. This result occurs for two reasons. First, at the initial relative wage a larger share of spending out of tariff rebates falls on the goods of the country commanding a larger share in world demand. Second, the tariff induces new nontraded goods and therefore increases net demand for the borderline commodity of the country whose residents have the larger income, or equivalently, the larger share in world income.

If countries are of equal size as measured by the share in world income, such a uniform tariff increase has zero effect on relative wages, but of course reduces well-being in both places. Multilateral tariff increases, in this case, unnecessarily create some nontraded goods, and artificially raise the relative price of importables in terms of domestically produced commodities in each country exactly in proportion to the tariff.

IV. Money, Wages, and Exchange Rates

In this section we extend the discussion of the Ricardian model to deal with monetary aspects of trade. Specifically we shall be interested in the determination of exchange rates in a flexible rate system, in the process of adjustment to trade imbalance under fixed rates, and in the role of wage sticki-

ness. The purpose of the extension is to in-
tegrate real and monetary aspects of trade.

A. Flexible Exchange Rates

The barter analysis of the preceding sec-
tions is readily extended to a world of
flexible exchange rates and flexible money
wages. Assume a given nominal quantity of
money in each country, M and M^*, re-
spectively. Further, in accordance with the
classical Quantity Theory, assume constant
expenditure velocities V and V^*.[11] A flexible
exchange rate, and our stipulating the ab-
sence of nonmonetary international asset
flows, will assure trade balance equilibrium
and therefore the equality of income and
spending in each country. The nominal
money supplies and velocities determine
nominal income in each country:

$$(27) \quad WL = MV \quad \text{and} \quad W^*L^* = M^*V^*$$

where W and W^* (now in capital letters)
denote domestic and foreign money wages
in terms of the respective currencies. Fur-
ther, defining the exchange rate e as the
domestic currency price of foreign ex-
change, the foreign wage measured in terms
of domestic currency is eW^*, and the rela-
tive wage therefore is $\omega \equiv W/eW^*$.

From the determination of the equilib-
rium real wage ratio $\bar{\omega}$ by our earlier "real"
relations, we can now find an expression for
the equilibrium exchange rates:

$$(28) \quad \bar{e} = (1/\bar{\omega})(\overline{W}/\overline{W}^*) =$$
$$(1/\bar{\omega})(MV/M^*V^*)(L^*/L)$$

where (27′) defines equilibrium money
wages:

$$(27') \qquad \overline{W} = MV/L$$

and

$$\overline{W}^* = M^*V^*/L^*$$

In this simple structure and with wage
flexibility, we can keep separate the de-
terminants of all equilibrium real variables
from all monetary considerations. Mone-

[11]This is a strong assumption since it makes spend-
ing independent of income and nonliquid assets even in
the short run.

tary changes or velocity changes in one
country will be reflected in equiproportion-
ate changes in prices in that country and in
the exchange rate in the fashion of the
neutral-money Quantity Theory. However,
a real disturbance, as (28) shows, definitely
does have repercussions on the nominal ex-
change rate as well as on the real equilib-
rium.

Using the results of Section II, we see
that an increase in the foreign relative labor
force causes, under flexible exchange rates
and given $\overline{M}$ and $\overline{M}^*$, a depreciation in the
home country's exchange rate as does uni-
form technical progress abroad. A shift in
real demand toward foreign goods likewise
leads to a depreciation of the exchange rate
as well as to a reduction in real $\bar{\omega}$. A rise in
foreign tariffs will also cause our currency
to depreciate. Each of these real shifts is
assumed to take place while (M, M^*) are
unchanged and on the simplifying proviso
that real income changes leave V and V^*
unchanged.

B. Fixed Exchange Rates

In the fixed exchange rates case we as-
sume currencies are fully convertible at a
parity pegged by the monetary authorities.
In the absence of capital flows and steriliza-
tion policy, a trade imbalance is reflected in
monetary flows. In the simplest metal
money model, the world money supply is
redistributed toward the surplus country at
precisely the rate of the trade surplus. We
assume that the world money supply is
given and equal to $\overline{G}$, measured in terms of
domestic currency. The rate of increase of
the domestic quantity of money is therefore
equal to the reduction in foreign money,
valued at the fixed exchange rate $\bar{e}$:

$$(29) \qquad \dot{M} = -\bar{e}\dot{M}^*$$

where $\dot{M} \equiv dM/dt$.

For a fixed rate world we have to de-
termine in addition to the real variables $\bar{\omega}$
and $\bar{z}$, the levels of money wages W and W^*
as well as the equilibrium balance of pay-
ments associated with each short-run equi-
librium. In the long run the balance of pay-
ments will be zero as money ends up

redistributed internationally to the point where income equals spending in each country. In the short run an initial misallocation of money balances implies a discrepancy between income and spending and an associated trade imbalance. To characterize the preferred rate of adjustment of cash balances in the simplest and most manageable way, we assume that spending by each country is proportional to money holdings.[12] On the further simplifying assumption that velocities are equal in each country, $V = V^*$,[13] world spending is equal to

(30) $VM + eV^*M^* \equiv V\overline{G}$

For the tastes and technology specified in Section I, world spending on domestically produced goods is given by

(31) $V\overline{G} \int_0^{\tilde{z}} b(z)dz \equiv \vartheta(\omega) V\overline{G}$

$$\tilde{z} = A^{-1}(\omega)$$

In equilibrium, world spending on our goods must equal the value of our full-employment income WL:

(32) $WL = \vartheta(\omega) V\overline{G}$

Equilibrium requires, too, that world spending on foreign goods equals the value of foreign full-employment income:

(33) $\overline{e}W^*L^* = [1 - \vartheta(\omega)] V\overline{G}$

Equations (32) and (33) express what would seem to be the *joint* determination of real and monetary variables. But, in fact, we could have taken the shortcut of recognizing that the real equilibrium is precisely that of the barter analysis developed in Section I. Dividing (32) by (33) and substitut-

---

[12]The assumption that spending is proportional to cash balances is only one of a number of possible specifications. Conditions for this expenditure function to be optimal are derived in Dornbusch and Michael Mussa. In general, expenditure will depend on both income and cash balances.

[13]In the long-run equilibrium, higher V than V^* leaves us with a smaller share of the world money stock than foreigners, but with nominal and real income *shares* in the two countries the same as when $V = V^*$.
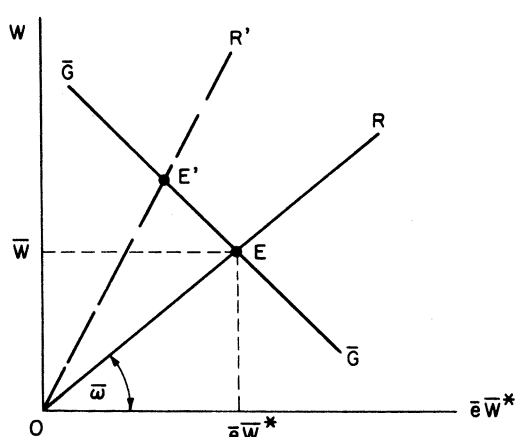

FIGURE 4

ing from (11) for the equilibrium relative wage $\overline{\omega}$, we can employ equations (32) and (33) to determine money wage levels.

The equilibrium determined by equations (32) and (33) can be analyzed in terms of Figure 4. The figure emphasizes the separation of real and monetary aspects of the equilibrium under our assumptions of traded goods only, and no distribution effects. From the ratio of (32) and (33) we obtain the equilibrium relative wage $\overline{\omega}$ as a function of tastes and technology solely from the barter model. This equilibrium relative wage is plotted as the ray OR in Figure 4.

The equality of world income and spending

(30') $WL + \overline{e}W^*L^* = V\overline{G}$

is shown as the downward sloping straight line $\overline{G}\overline{G}$, which is drawn for given velocity, world quantity of money, and labor forces. Point E is the equilibrium where relative prices and the level of wages and prices are such that all markets clear. At a level of wages and prices higher than point E, there would be a world excess supply of goods, and conversely at points below E.

Figure 4 immediately shows some comparative static results. Thus a doubling of both countries' labor forces, from the analysis of the barter model, will leave the relative wage unaffected but will double

world output. Given unchanged nominal
spending $V\bar{G}$, wages and prices will have to
halve. This would be shown by a parallel
shift of the $\bar{G}G$ schedule halfway toward the
origin. A shift in demand toward the home
country's output by contrast would rotate
the OR ray to a position like OR' since it
raises our relative wage. The ensuing mone-
tary adjustment is then an increase in our
money wage and money income and a de-
cline in foreign wages, prices, and incomes
(point E').

The real and nominal equilibrium at
point E in Figure 4 is independent of the
short- and long-run distribution of the
world quantity of money. The independence
of the real equilibrium derives from the uni-
form homothetic tastes. The independence
of the nominal equilibrium is implied by
identical velocities. What does, however,
depend on the short-run distribution of
world money is the transition periods' bal-
ance of payments. As in the absorption ap-
proach of Sidney Alexander (1952), we
know this: when goods markets clear, the
trade surplus or balance of payments $\dot{M}$ of
the home country is equal to the excess of
income over spending, or:

$$(34) \qquad \dot{M} = \bar{W}L - VM$$

With the nominal wage independent of the
distribution of world money, equation (34)
therefore implies that the trade balance
monotonically converges to equilibrium at
a rate proportional to the discrepancy from
long-run equilibrium:[14]

$$(34') \qquad \dot{M} = V(\bar{M} - M); \; \bar{M} = \vartheta(\bar{\omega})\bar{G}$$

The assumptions of this section were de-
signed to render inoperative most of the tra-
ditional mechanisms discussed as part of

[14]Suppose $V > V^*$ and our share of the world
money supply is initially larger than our equilibrium
share. Then, as we lose M, total world nominal income
and nominal GNP falls. Always our share of nominal
world GNP stays the same under the strong demand
assumptions. Total world real output never changes
during the transition; only regional consumption
shares change. Therefore, both countries' nominal
price and wage levels fall in the transition, but such
balanced changes have no real effects on either the
transient or the final real equilibrium.

the adjustment process: changes in the
terms of trade, in home and/or foreign
price levels, in relative prices of traded and
nontraded goods (there being none of the
latter), in double factoral terms of trade;
and any discrepancies in the price of the
same commodity between countries. The
features of the adjustment process of this
section rely on 1) identical, constant expen-
diture velocities, 2) uniform-homothetic de-
mand, and 3) the absence of trade impedi-
ments. If velocities were constant but
differed between countries, the absolute
levels of money wages and prices, though
not *relative* wages or prices, would depend
on the world distribution of money. Relaxa-
tion of the uniform-homothetic taste as-
sumption would make equilibrium relative
prices a function of the distributions of
spending. Finally, the presence of non-
traded goods would, together with Ricardo's
technology, provide valid justification for
some of the behavior of relative prices and
price levels frequently asserted in the litera-
ture; this behavior is studied in more detail
in the next section.

C. The Price-Specie Flow Mechanism under More General Conditions

We now discuss the adjustment process
to monetary disequilibrium and enquire
into the price effects associated with a redis-
tribution of the world money supply when
there are nontraded goods. Common ver-
sions of the Hume price-specie flow mech-
anism usually involve the argument that in
the adjustment process, prices decline along
with the money stock in the deficit country,
while both rise in the surplus country. There
is usually, too, an implication that the defi-
cit country's terms of trade will necessarily
worsen in the adjustment process and in-
deed have to do so if the adjustment is to be
successful.

Section IVB demonstrated that the redis-
tribution of money associated with mone-
tary imbalance need have no effects on real
variables (production, terms of trade, etc.)
and on nominal variables other than the
money stock and spending. While this is

clearly a very special case, it does serve as a benchmark since it establishes that the monetary adjustment process would be effective even in a one-commodity world.

To approach the traditional view of the adjustment process more clearly and provide formal support for that view, we consider an extension to the monetary realm of our previous model involving nontraded goods. We return to the assumption that a fraction $(1 - k)$ of spending in each country falls on nontraded goods, and accordingly equations (32) and (33) become:

$$(32') \qquad WL = \vartheta(\omega)V\overline{G} + (1 - k)\gamma V\overline{G};$$
$$\gamma \equiv M/\overline{G}$$
$$(33') \quad \overline{e}W^*L^* = [k - \vartheta(\omega)]V\overline{G}$$
$$+ (1 - \gamma)(1 - k)V\overline{G}$$

These hold both in final equilibrium, and in transient equilibrium where specie is flowing. Equations $(32')$ and $(33')$ imply that the equilibrium relative wage does depend on the distribution of the world money supply. Solving these equations for the equilibrium relative wage we have:

$$(35) \qquad \overline{\omega} = \overline{\omega}(\gamma) \qquad \frac{\partial \overline{\omega}}{\partial \gamma} > 0$$

An increase in the home country's initial share in the world money supply γ raises our relative wage.

Using this extended framework, we can draw on the analysis of the transfer problem in Section II to examine the adjustment that follows an initial distribution of world money between the two countries that differs from the long-run equilibrium distribution.

Suppose our M is initially excessive, say from a gold discovery here. Assume also that the gold discovery occurred when the world was in long-run equilibrium with the previous world money stock. As a result of our excess M, we spend more than our earnings, incurring a balance-of-payments deficit equal to the rate at which our M is flowing out. In effect, the foreign economy is making us a real transfer to offset our deficit. As seen earlier, we, the deficit country, are devoting some of our excess spend-

ing to nontraded goods, shifting some of our resources to their production at the expense of our previous exports. We not only export fewer types of goods, but also import more types, and import more of each ($\overline{\omega}$ rises and $\overline{z}$ falls).

During the transition, while the real transfer corresponding to our deficit is taking place, our terms of trade are more favorable than in the long-run state. The new gold raises both their W^* and our W, but in addition, our W is up relative to their W^*. Therefore the price level of goods we continue to produce is up relative to the price level of goods they continue to produce. This is true both for our nontraded goods and for our exportables. The prices of goods we produce rise relative to the prices of goods they produce in proportion to the change in relative wages.

Thus the price levels in the two countries have been changed differentially by the specie flow and implied real transfer. But that does not mean that any traded good ever sells for different prices in two places. In fact the divergence in weighted average (consumer) price levels is due to nontraded goods. The price level will rise in the gold-discovering country relative to the other country the greater is the share of nontraded goods in expenditure, $1 - k$. It is a bit meaningless to say, "What accomplished the adjustment is the relative movements of price levels for nontraded goods in the two countries," since we have seen that the adjustment can and will be made even when there are no such nontraded goods. It is meaningful to say, "The fact that people want to direct some of their expenditure to nontraded goods makes it necessary for resources to shift in and out of them as a result of a real transfer, and such resource shifts take place only because the terms of trade (double-factoral and for traded goods) do shift in the indicated way."

The adjustment process to a monetary disturbance is stable in the sense that the system converges to a long-run equilibrium distribution of money with balanced trade. To appreciate that point, we supplement equations $(32')$ and $(33')$ with (34) that con-

tinues to describe the monetary adjustment process. We note, however, that now W and W^* are endogenous variables whose levels in the short run do depend on the distribution of the world money supply. A redistribution of money toward the home country would raise our spending and demand for goods, and reduce foreign spending and demand. As before, spending changes for traded goods offset each other precisely so that the net effect is an increase in demand for nontraded goods at home and a decline abroad. As a consequence our wages will rise and foreign wages decline. Therefore, starting from full equilibrium, a redistribution of money toward the home country will create a deficit equal to

$$(36) \quad d\dot{M}/dM = -V(1 - \delta) \quad 0 \leq \delta < 1$$

where δ is the elasticity of our nominal wages with respect to the quantity of money and is less than unity.[15] Equation (36) implies that the price-specie flow mechanism is stable.

It is interesting to observe in this context that the presence of nontraded goods in fact *slows down* the adjustment process by comparison with a world of only traded goods (contrary to J. Laurence Laughlin's turn of the century worries). As we saw before, with all goods freely tradeable, wages are independent of the distribution of money, and accordingly $\delta = 0$. Further we observe that the speed of adjustment depends on the relative size of countries. Thus the more equal countries are in terms of size, the slower tends to be the adjustment process.

In concluding this section we note that nontraded goods (and/or localized demand) are essential to the correctness of traditional insistence that the adjustment process necessarily entails absolute and relative price, wage, and income movements. They are, of course, in no way essential to the existence of a stable adjustment process, nor is there at any time a need for a discrepancy of prices of the same commodity across countries in either case.[16]

A final remark concerns the adjustment to real disturbances such as demand shifts or technical progress. It is certainly true that whether the exchange rate is fixed or flexible, real adjustment will have to take place and cannot be avoided by choice of an exchange rate regime. So long as wages and prices are flexible, it is quite false to think that fixed parities "put the whole economy through the wringer of adjustment" while in floating rate regimes "only the export and import industries have to make the real adjustment." It is true, however, that once we depart from flexible wages and prices there may well be a preference for one exchange rate regime over another. The next section is devoted to that question.

D. Sticky Money Wages

The last question we address in this section concerns the implications of sticky money wages. For a given world money supply, *downward* stickiness of money wages implies the possibility of unemployment. We assume upward flexibility in wages, once full employment is attained.

We start with a fixed exchange rate $\bar{e}$. The relation between wages and the world quantity of money is brought out in Figure 5. Denote employment levels in each country, as opposed to the labor force, by the new symbols $\bar{L}$ and $\bar{L}^*$, respectively; denote nominal incomes by Y and Y^*. The equality of world income and spending is again shown by the $\bar{G}\bar{G}$ schedule, the equation of which now is

$$(37) \quad V\bar{G} = Y + \bar{e}Y^* \equiv \overline{W}\bar{L} + \bar{e}\overline{W}^*\bar{L}^*$$

---

[15]The value of δ can be calculated from equations (32') and (33') to be

$$\delta \equiv (1 - k)\frac{\gamma(1 - \gamma)}{\gamma(1 - \gamma) + \vartheta\epsilon}$$

where ϵ is the elasticity of the share of our traded goods in world spending, $\epsilon \equiv -\vartheta'\omega/\vartheta > 0$. The elasticity δ is evaluated at the long-run equilibrium where $\gamma \equiv \vartheta/k$. If $A'(z)$ falls slowly, ϵ will be large.

[16]The continuum Ricardian technology is special in that there can be no range of goods both imported and produced at home. Therefore, the cross elasticity of supply between nontraded goods and exports must be greater than the zero cross elasticity between nontraded goods and imports. Consequently, a transfer must shift the terms of trade (for goods and factors) in the stated orthodox way, favorably for the receiver.

FIGURE 5

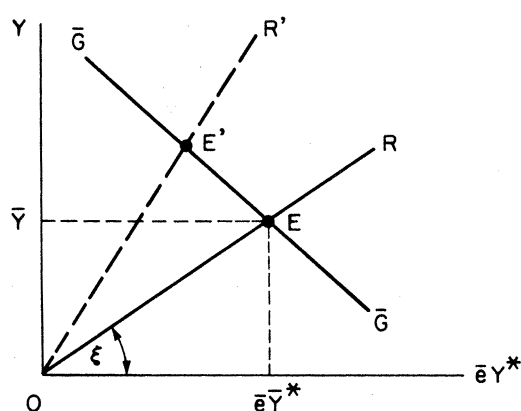where $\overline{W}$ and $\overline{W}^*$ are the fixed money wages set at too high sticky levels. The schedule is drawn for given money wages, a given world quantity of money, and a pegged parity for $\overline{e}$. The ray OR now is predetermined by the given sticky relative wage $\overline{\omega} = \overline{W}/\overline{e}\,\overline{W}^*$. From equations (32) and (33) the ratio of money incomes $Y/\overline{e}Y^*$ is just a function of the relative wage now given exogenously by rigid money wages and the exchange rate:

$$(38) \quad Y/\overline{e}Y^* = \frac{\vartheta(\omega)}{1 - \vartheta(\omega)} \equiv \xi(\overline{W}/\overline{e}\,\overline{W}^*)$$

$$\xi'(\omega) < 0$$

Point E is the nominal equilibrium where by assumption the world quantity of money is insufficient relative to wage rates to ensure full employment. Although that equilibrium is one with unemployed labor, it is efficient in other respects. Specifically, geographic specialization follows comparative advantage as laid out above, but now labor employed adjusts to sticky wage patterns of specialization.

Employment levels $\overline{L}$ and $\overline{L}^*$ now are determined by (39)

$$(39) \quad \overline{L} = \overline{Y}/\overline{W}, \quad \overline{L}^* = \overline{e}\,\overline{Y}^*/\overline{W}^*$$

where $\overline{Y}$ and $\overline{Y}^*$ are the equilibrium levels of nominal income determined by equations (37) and (38) or by point E in Figure 5.

Consider now the impact of a foreign increase in money wages. The effect of the im-

plied reduction in our relative wages and the resulting increase in our relative income are shown in Figure 5 by the rotation from OR to OR'.

The new equilibrium is at E' where our money income and employment have risen while income and employment decline abroad. Thus an increase in the foreign wage rate, by moving the terms of trade against us, shifts comparative advantage and employment toward the home country. The extent to which the home country benefits from the adverse terms of trade shift in terms of employment will depend on both the substitutability in demand and the elasticity of the $A(z)$ schedule in Figure 1. We observe, too, that the move from E to E' will bring about a transitory balance-of-payments surplus. Given the initial distribution of money and hence of spending, the foreign decline in income and the increase at home implies that we will spend less than our income and therefore have a trade surplus. This surplus persists until money is redistributed to match the new levels of income at E'.

Next we move to flexible exchange rates. Under flexible rates an increase in the foreign money wage $\overline{W}^*$, given money supplies in each country, will similarly have real repercussion effects on relative prices and employment at home. Now employment in each country is determined by money supplies and prevailing wages:

$$(39') \quad \overline{L} = V\overline{M}/\overline{W}; \quad \overline{L}^* = V\overline{M}^*/\overline{W}^*$$

Given the employment levels thus determined, we know from the analysis of the earlier barter model that there is a unique relative wage at which the trade balance achieves equilibrium. The higher is $\overline{M}^*/\overline{W}^*$, the higher will be employment abroad—and, therefore, the higher will be our relative wage $\overline{\omega}$. It is thus apparent that an increase in the foreign money wage, $\overline{W}^*$, will reduce employment abroad. Employment declines only in proportion to the increase in wages and thus declines by less than it would under fixed exchange rates when specie is lost abroad.

We saw in the barter model that a reduc-

tion in effective foreign labor causes a de-
cline in our relative wage, but that the
decline in our relative wage falls propor-
tionately short of the foreign reduction in
labor. Now, at the initial exchange rate, the
increase in foreign wages reduces our rela-
tive wage and their employment in the same
proportion. The decline in our relative wage
is therefore excessive. Domestic goods are
underpriced and the exchange rate appre-
ciates to *partly* offset the gain in cost com-
petitiveness. The net effect is therefore a
decline in our relative wage and an appre-
ciation of our exchange rate (a decline in e)
that falls short of the foreign increase in
wages. Since our terms of trade unambig-
uously deteriorate without any compensat-
ing gain in employment, it must be true that
welfare declines at home. Abroad, the loss
in employment is offset by a gain in the
terms of trade, but there too the net effect is
a loss in welfare under our strong Mill-
Ricardo assumption.

The adjustment to money wage distur-
bances under fixed and flexible rates differs
in several respects. Under fixed rates em-
ployment effects are transmitted, while
under flexible rates they are bottled up in
the country initiating the disturbance.
Under fixed rates the terms of trade move
one for one with money wage, while under
flexible rates exchange rate movements
partly offset increases in the foreign money
wage rate.

The difference between fixed and flexible
rates in relation to the adjustment process is
further brought out by an example of a real
disturbance. Consider a shift in world de-
mand toward our goods. Under fixed rates
the resulting increase in our relative income
will, from (38), move us in Figure 5 from
E to E'. Employment rises at home and falls
abroad. Demand shifts are fully reflected in
employment changes. Under flexible rates,
by contrast, with given wages and money, a
demand shift has no impact on employ-
ment—as we observe from (39). At the ini-
tial exchange rate the demand shift would
give rise to an excess demand for our goods
and to an excess supply abroad. Domestic
income and employment would tend to rise
while falling abroad. The resulting trade

surplus causes our exchange rate to appre-
ciate until the initial employment levels and
therefore trade balance equilibrium are re-
stored. The demand shift is fully absorbed
by a change in the terms of trade and a shift
in competitive advantage that restores de-
mand for foreign goods and labor.

Real and nominal equilibria are thus seen
to be uniquely definable in our continuum
model with constant-velocity spending de-
terminants. The difference between sticky
and flexible wage rates under fixed exchange
rates is understandable as the difference be-
tween (a) having the crucial relative wage $\bar{\omega}$
be imposed in the sticky wage case with em-
ployments having then to adjust; or (b) hav-
ing the full employments be imposed and $\bar{\omega}$
having to adjust. Under floating exchange
rates, sticky nominal wages impose employ-
ment levels in each country and the crucial
relative wage $\bar{\omega}$ then adjusts to those em-
ployment levels.

Appendix

Historical Remark

Figure 1 seems to be new. G. A. Elliot
(1950) gives a somewhat different diagram,
one that makes explicit the meaning of
Marshall's 1879 "bales" (which, by the way,
happen to work only in the two-country
constant labor costs case). In terms of the
present notations, Elliott plots for the *U.S.*
offer curve the following successive points
traced out for all ω on the range [0, ∞]: on
the vertical axis is plotted our total real im-
ports valued in foreign labor units ("our
demand for bales of their labor," so to
speak), namely,

$$\int_z^1 [P^*(z)/w^*]\,C(z)\,dz = \int_z^1 a^*(z)\,C(z)\,dz$$

and on the horizontal axis, our total real
exports valued in home labor units ("our
supply of bales of labor to them"), namely,

$$\int_0^z [P(z)/w][Q(z) - C(z)]\,dz =$$
$$L - \int_0^z a(z)\,C(z)\,dz$$

It is to be understood that $\bar{z}$ is a function of ω, namely the inverse function $A^{-1}(\omega)$; also that $C(z)$ are the amounts demanded as a function of our real income L and of the $P(z)/W$ function defined for each, namely min $[\omega a(z), a^*(z)]$. Because we have a continuum of goods, we avoid Elliott's branches of the offer curve that are segments of various rays through the origin. The reader will discern by symmetry considerations how the foreign offer curve is plotted in the same (L, L^*) quadrant, by varying ω to generate the respective coordinates

$$\left[\int_0^{\bar{z}} a(z) C^*(z) dz, \right.$$

$$\left. L^* - \int_{\bar{z}}^1 a^*(z) C^*(z) dz \right]$$

Our model forces the Elliott-Marshall diagram to generate a *unique* solution under uniform-homothetic demand. Unlike our Figure 1, the Elliott diagram can handle the general case of nonhomothetic demands in the two countries; but then, as is well known, multiple solutions are possible, some locally stable and some unstable. The price one pays for this generality is that, as Edgeworth observed, the Marshallian curves are the end products of much implicit theorizing, with much that is interesting having taken place offstage.

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