**Ph.D. Core Examination**

**PRICE THEORY (MICROECONOMICS)**

Friday, July 12, 2013

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WRITE in **black ink** and write only on **one side** of each page.

DO NOT WRITE in the margins.

**Be sure the "random number" stamped on your paper is the correct number** (the number you were given by the department office). Be sure to put your **random number** in the upper left hand corner of **every page** you write on.

On the **first page** of your examination write:

- the name of the examination
- the date of the examination

Put the **page number** in the upper right hand corner of **every page** of your examination.

When you have completed your examination, write the **total number** of pages on the **back** of the last page.

Put your examination in the envelope to be turned in.

Results of the examination will be posted by your random number outside the Office of Graduate Student Affairs and sent to you by letter.

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**TIME: THREE HOURS**

1. **You must answer ALL questions and justify ALL answers.**

2. **The exam is out of 180 points. Part I is worth a total of 90 points. Each of Parts II, III, and IV are worth 30 points.**

Note: The envelope you will be given will contain a colored writing pad to be used for notes and a white writing pad with your random number to be used for writing your final answers.

## I. BASIC PRICE THEORY QUESTIONS (90 points – 10 points each)

Answer each question as TRUE, FALSE or UNCERTAIN. Justify your answer. Your score will depend entirely on your justification. If the question makes several claims address each of those claims in your answer. Penalties will be assessed for irrelevant material.

1. Assume that consumers only have wage income. A 10% increase in the wage rate due to technological progress in the market sector combined with a 10% increase in the productivity of production for all household commodities would not change hours worked by these consumers.
2. Suppose a marriage market where men and women both differ only in education, and where there is transferable utility between men and women in a marriage. With a marriage market competitive equilibrium, an increase in the number of men with low education would necessarily lower the utilities from marriage of both low and high education men, and raise the utilities from marriage of both low and high education women.
3. The United States has greatly increased its production of natural gas because of fracking techniques. If US regulations prevent this increased production of gas from being exported, the US domestic price of gas will fall. Market forces will also make the international price of gas fall to the same level as the US domestic price.
4. A law that prevents the advertising of cigarettes will raise the profits of cigarette companies, and raise the price of cigarettes.
5. When consumers overestimate the value received from a good and as a result purchase more of that good than they otherwise would the loss suffered by consumers will be smaller when supply of that good is less elastic since inelastic supply limits the degree of over consumption.
6. A new law which mandates that employers provide health insurance to all workers will help workers that do not currently have employer provided health insurance but will make those that currently have employer provided health insurance worse off.
7. A carbon tax (that taxes each ton of $CO_2$ emitted) would likely raise the profits of some fossil fuel producers if those fuels (coal, oil, gas) differ in their $CO_2$ emissions per unit of energy produced and the demand for energy is relatively inelastic.
8. A reduction in the cost of producing a capital good will reduce the price of that good more in the long run than the short-run but will increase production more in the short run than the long run.
9. Assume drivers have the choice of two roads for making a given commute and that those roads are subject to congestion. The introduction of a toll on one of the two roads will make some drivers worse off.

II. GENERAL EQUILIBRIUM AND CHOICE THEORY(30 points)

(a) (10 points) Consider an exchange economy with two consumers and two goods. Consumer 1's continuous utility function is $u(x_1^1) + u(x_2^1)$ and consumer 2's continuous utility function is $v(x_1^2) + v(x_2^2)$, where $x_k^i \geq 0$ is the amount of good $k$ consumed by consumer $i$. Assume that $u', v' > 0$, and $u'', v'' < 0$. Consumer 1's endowment vector is $(\alpha, 0)$ and consumer 2's endowment vector is $(0, 1)$, where $\alpha > 0$. Thus, consumer 1 is endowed with $\alpha$ units of good 1 and zero units of good 2, and consumer 2 is endowed with zero units of good 1 and one unit of good 2.

(i) (5 points) State and verify the conditions of a theorem that guarantees the existence of a Walrasian equilibrium and prove that there is a unique Walrasian equilibrium when $\alpha = 1$.

(ii) (5 points) Suppose that consumer 1's endowment of good 1 increases to $\alpha > 1$. Prove that in every Walrasian equilibrium the price of good 2 exceeds the price of good 1. Prove also that consumer 2 is now strictly better off than in part (a). Is it clear whether consumer 1 is now better off than in part (a)? Why or why not?

(b) (10 points)

(i) (5 points) State the Gibbard Satterthwaite theorem (you need **not** define the terms used in the statement of the theorem).

(ii) (5 points) Show that the conclusion of the Gibbard Satterthwaite theorem does not hold when there are just two alternatives. (Hint: You may assume an odd number of individuals if this is helpful.)

(c) (10 points) Suppose that $u : \mathbb{R}_+ \to \mathbb{R}$ is a consumer's von Neumann-Morgenstern utility function over wealth. Suppose also that $u$ is twice differentiable and that $u' > 0$. Suppose that whenever this consumer prefers one wealth gamble over another, she also prefers the one gamble over the other when all outcomes of the two gambles are multiplied by any positive constant $\lambda > 0$. Prove that this consumer must display constant relative aversion, i.e., prove that the ratio $\frac{wu''(w)}{u'(w)}$ must be independent of $w$. (Hint: Consider, for any $\lambda > 0$, the two vNM utility functions $u(\lambda w)$ and $u(w)$ over wealth, $w$. How are they related?) Can you describe her utility function?

III. GAME-THEORY QUESTION (30 points)

Consider a game where player 1 chooses T or B, player 2 independently chooses L or M or R, and their utility payoffs ($u_1$,$u_2$) depend on their choices as follows:

| Player 1 \ Player 2: | L | M | R |
|---|---|---|---|
| T | 2, 9 | 6, 7 | 4, 3 |
| B | 5, 0 | 8, 5 | x, 6 |

The payoff $x = u_1(B,R)$ has been left unspecified in the above table, so that we can consider some different assumptions about it in parts (a) and (b) below:

(a) Suppose first that $x = 1$ (and this fact is common knowledge among the players).
Find a Nash equilibrium of this game and compute each player's expected payoff.

(b) Now suppose that x is a random variable drawn from a Uniform distribution on the interval from 1 to 2. Player 1 knows the actual value of x when he chooses his action in this game, but player 2 knows only the distribution from which x is drawn. (So x may be called 1's "type" here.)
Find a Bayesian equilibrium of this game.

*Numerical answers may be expressed as ratios of whole numbers, without reducing.*

IV. INFORMATION ECONOMICS (30 points)

There are two types of doctors, "skilled" and "unskilled." Each doctor knows what type he is, but a doctor's type is her private information. There are two types of patients, "difficult" (i.e., difficult to treat) and "easy" (i.e., easy to treat). Everyone can observe whether a patient is difficult or easy to treat, and doctors can choose which patients to treat. Any doctor, no matter what her type, is more likely to cure an easy patient than a difficult one, and for any given patient type, a skilled doctor is more likely to cure that patient than an unskilled doctor. The following table gives the probabilities that a particular doctor type (row) cures a particular patient type (column).

| | difficult | easy |
|---|---|---|
| skilled | $\frac{1}{2}$ | $\frac{3}{4}$ |
| unskilled | $\frac{1}{4}$ | $\frac{1}{2}$ |

So for example, an unskilled doctor cures a difficult patient with probability 1/4. Both types of doctors are risk neutral in wealth and have von Neumann-Morgenstern utility function $u_d(w) = w - 1$ if they treat a difficult patient and $u_e(w) = w$ if they treat an easy patient. Suppose that you run a hospital that sees 200 patients per day, 100 easy patients and 100 difficult patients. Each doctor can treat at most one patient per day. Each day you wish to hire 100 skilled doctors to treat the difficult patients and 100 unskilled doctors to treat the easy patients. (Of course you cannot distinguish between skilled and unskilled doctors.) Doctors get to choose which type of patient to treat so you cannot, for example, force any particular doctor to treat a difficult patient. On any day, any doctor can obtain utility $u_0 = 1$ by playing golf instead of working. Suppose that you want to minimize the expected total wages paid to your doctors so that all 200 patients are treated each day and so that the unskilled doctors you hire treat only easy patients and the skilled doctors you hire treat only difficult patients. You can observe whether the patient treated is difficult or easy. You may assume an unlimited supply of skilled and unskilled doctors. Negative wages are not permitted.

(a) (15 points) Suppose there is just one day. Let $w_d$ denote the wage paid to a doctor if she cures a difficult patient that day and let $w_e$ denote the wage paid to a doctor if she cures an easy patient that day. Find the optimal contract $(w_d, w_e)$ for this one-day model.

(b) (15 points) Suppose there are two days and there is no discounting. What is the expected total cost to the hospital of simply repeating the contract from part (a) on each of the two days? Can you strictly improve on this by "backloading" the contract, i.e., by paying any doctor hired on the first day only if she treats and cures two patients, one on each day. Find the optimal such backloaded contract, keeping in mind that doctors can quit and play golf anytime and that new doctors can be hired on the second day if necessary (what contract will they be offered?). (Hint: Remember, the hospital can observe whether patients are difficult or easy. What should the hospital pay a doctor who treats an easy patient and then a difficult patient, or treats an easy patient and then plays golf, etc.?)