Price Theory Summary, Autumn 2012

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1 Consumer Theory

1.1 Utility Maximization

First order conditions:

$$MRS = \frac{\frac{\partial U}{\partial x_i}}{\frac{\partial U}{\partial x_i}} = \frac{p_i}{p_j}$$

$$\lambda = \frac{\frac{dU}{dx_i}}{p_i} = \frac{\frac{dU}{dx_j}}{p_j}$$

where i,j = 1, ..., N. From these equations, we can obtain the generalized demand function (or Marshallian demand function): $x_1(p_1, \ldots, p_N, M)$

- Hold $p_1, \ldots, p_N$ fixed, we will obtain the Engel curve: if $\frac{\partial x_i}{\partial M} > 0$, it is normal good, and definitions of luxury and necessity goods.
- Hold $p_2, \ldots, p_N, M$ fixed, we will obtain the ordinary demand curve.
- Hold $p_1, p_3, \ldots, p_N, M$ fixed, we will get the cross price demand function (if the slope is positive, it is a substitute good, if the slope is negative, it is a complement good).

1.2 Useful Equations for Marshallian and Hicksian Demand Functions

These equations are obtained by differentiating the budget constraint with different arguments for the Marshallian Demand curve:

- An increase in income is spread over different goods:

$$\sum_{i=1}^{N} s_i \eta_i = 1$$

- Adding up constraint:

$$\sum_{i=1}^{N} s_i \epsilon_{ij} + s_j = 0 \quad \forall j$$

- Homogeneity:

$$\sum_{j=1}^{N} \epsilon_{ij} + \eta_i = 0 \quad \forall i$$

For the Hicksian Demand curve:

- Adding up version

$$\sum_{i=1}^{N} s_i \epsilon_{ij}^H + s_j = \quad \forall j$$

- Homogeneity version?

$$\sum_{j=1}^{N} \epsilon_{ij}^H = 0 \quad \forall i$$

Symmetry: $C_{ij} = \frac{\partial x_i^H}{\partial p_j} = \frac{\partial x_j^H}{\partial p_i} = C_{ji}$

If $\eta_i = \eta_j$, then $\frac{\partial x_i}{\partial p_j} = \frac{\partial x_j}{\partial p_i}$

Also from symmetry: $s_i \epsilon_{ij}^H = \epsilon_{ji}^H s_j$, if $s_i = s_j \implies \epsilon_{ij}^H = \epsilon_{ij}^H$

1.3 Slutsky 's Equation

- $\epsilon_{ij} = \epsilon_{ij}^H - s_j\eta_i \implies \epsilon_{ii} = \epsilon_{ii}^H - s_i\eta_i$
- $\frac{\partial x_i}{\partial p_j} = \frac{\partial x_i^H}{\partial p_j} - \frac{\partial x_i}{\partial M}x_j \implies \frac{\partial x_i}{\partial p_i} = \frac{\partial x_i^H}{\partial p_i} - \frac{\partial x_i}{\partial M}x_i$

1.4 Social Multiplier

$X_1 = \sum_{j=1}^N x_1^j \implies \frac{\partial X_1}{\partial p_1} = \sum_{j=1}^N \frac{\partial x_1^j}{\partial p_1} + \sum_{j=1}^N \frac{\partial x_1^j}{\partial X_1}\frac{\partial X_1}{\partial p_1}$
So, $\frac{\partial X_1}{\partial p_1} = \frac{\sum_{j=1}^N \frac{\partial x_1^j}{\partial p_1}}{1-\sum_{j=1}^N \frac{\partial x_1^j}{\partial X_1}}$

1.5 Law of Demand

First Law of Demand: Long run demand curve is downward sloping ($\epsilon_{ii} < 0$)
More generally, let $(x_1^0, \ldots, x_n^0)$ be the cost minimizing bundle at $\vec{P}^0$ and $(x_1^1, \ldots, x_n^1)$ be the cost minimizing bundle at $\vec{P}^1$. Then, $\sum_i x_i^1 P_i^1 \leq \sum_i x_i^0 P_i^1$ and $\sum_i x_i^0 P_i^0 \leq \sum_i x_i^1 P_i^0 \implies \sum_i (x_i^1 - x_i^0)(P_i^1 - P_i^0) \leq 0$.

Second Law of Demand: The reaction of quantity demanded to price change is larger in the long run than in the short run.
Example: $Gas = f(P_{GAS}, Q_{CAR}, OTHERS)$

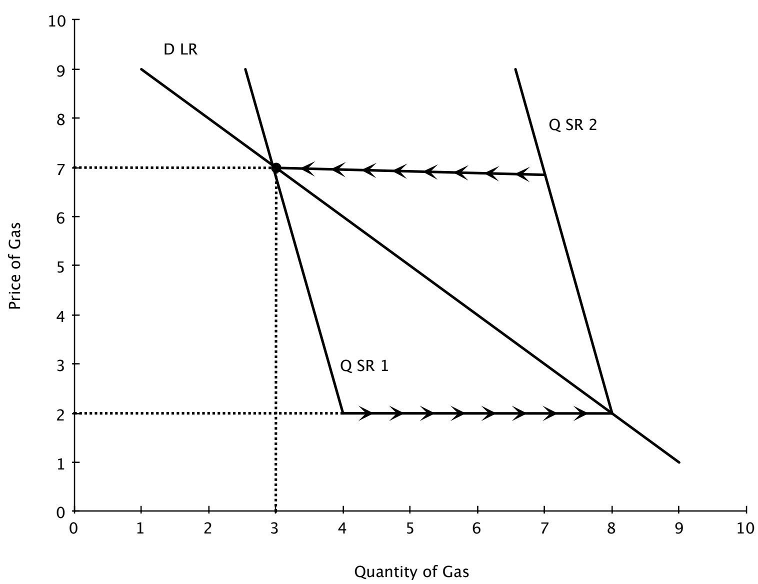


Figure 1: In the short-run, people would response to an increase in price of oil by driving less, take shorter vacations and such, but in the long run, they might change to more efficient cars.

1.6 Household production theory

Given $U\left(z_1, \ldots, z_m\right)$ and the home production functions $z_i = f_i\left(x_{1i}, \ldots, x_{ni}, h_{1i}, \ldots, h_{ki}, E\right)$, where x are the goods in the maket and h are the time spent in household production. We can solve the household utility maximization problem by diving it into 2 stages:

First Stage: $min\, C\left(\pi_1, \ldots, \pi_m, U\right)$ s. t. $U = \bar{U}$

Second Stage: $max\, U\left(z_1, \ldots, z_m\right)$ given $C\left(\pi_1, \ldots, \pi_m, U\right)$

The first order conditions are:

$$\frac{\partial U}{\partial z_i} \cdot \frac{\partial z_i}{\partial x_i} = \lambda p_i \quad and \quad \frac{\partial U}{\partial z_i} \cdot \frac{\partial z_i}{\partial h_i} = \lambda w$$

$$\frac{\frac{\partial z_i}{\partial x_i}}{\frac{\partial z_i}{\partial h_i}} = \frac{p_i}{w}$$

If the individual is not working, we can obtain the shadow wage as $\frac{\mu}{\lambda}$, where μ is the multiplier associated with the first stage problem and λ is the multiplier associated with the second stage problem:

$$\frac{\frac{\partial z_i}{\partial x_i}}{\frac{\partial z_i}{\partial h_i}} = \frac{\lambda p_i}{\mu}$$

1.7 Uncertainty

Under assumptions of separability and probability effect is linear, we get: $V = \sum_{i=1}^{N} p_i U_i\left(I_i\right)$ If no-state dependence, $V = \sum_{i=1}^{N} p_i U\left(I_i\right)$.

$$max\, V = \sum_{i=1}^{N} p_i U\left(I_i\right) \text{s. t.} \sum_{i=1}^{N} \pi_i I_i = \bar{I}$$

$$p_i U'(I_i) = \lambda \pi_i$$

$$p_i U'(I_j) = \lambda \pi_j$$

$$\implies \frac{\pi_i}{\pi_j} = \frac{p_i U'(I_i)}{p_j U'(I_j)}$$

If the bet is actuarially fair, then $p_1 + p_2\left(-\frac{\pi_1}{\pi_2}\right) = 0$, implies, $\frac{p_1}{p_2} = \frac{\pi_1}{\pi_2}$.
Assume $\pi_i = p_i$, , then first order condition gives us: $U'(I_i) = U'(I_j) \implies I_i = I_j$ (full insurance). But we don't observe full insurance, due to:

- Moral hazard: once an individual obtained the insurance, he/she is more likely to take risky behaviors or over use healthcare treatments
- Systemic or aggregate risk: for example, financial risk that the whole market will collapse
- Adverse selection: more risky people are more likely to take insurance, but the insurance company can't distinguish the individuals.

- State-dependence: utility functions are different. In this case, first order condition is: $U_i'(I_i) = U_j'(I_j)$, if $\pi_i = p_i$. If not, then the first order condition is $\frac{\pi_i}{\pi_j} = \frac{p_i U_i'(I_i)}{p_j U_j'(I_j)}$.
- Risk-loving: convex region of utility function

1.8 Rent and distance from the city model

$$\max_{C,L,t} U(C,L) \text{ s.t. } L = 1 - h - t \quad and \quad A + wh = C + R(t) \tag{1}$$
$$\mathscr{L} = U(C,L) + \lambda(A + w(1 - L - t) - C - R(t)) \tag{2}$$
$$[C] : \frac{\partial U}{\partial C} = \lambda \tag{3}$$
$$[L] : \frac{\partial U}{\partial L} = \lambda w \tag{4}$$
$$[t] : 0 = \lambda(-w - R'(t)) \implies w = -R'(t) \tag{5}$$

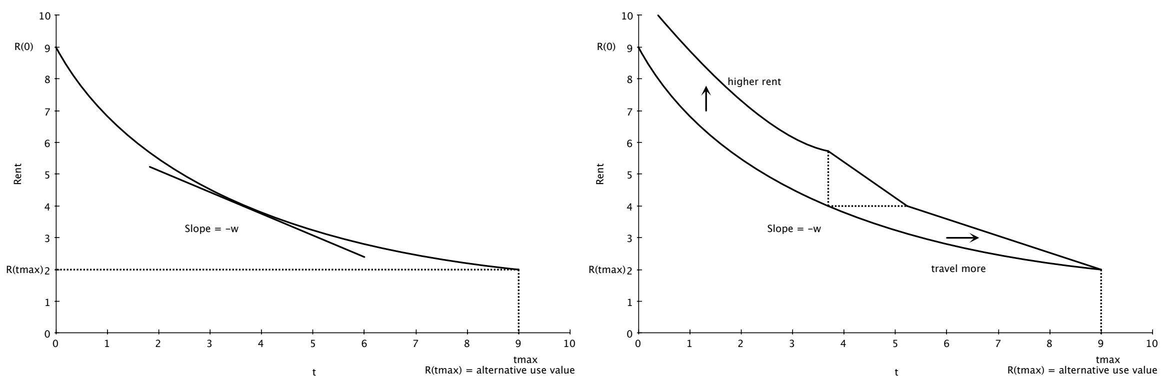


Figure 2: Insert people in the middle with the appropriate wages

1.9 Life-Cycle Budget Constraint

$$max \int_0^T \exp(-\rho t) U(c(t), L(t)) dt \tag{6}$$
$$\text{s.t. } A(0) + \int_0^T \exp(-\int_0^t r(\tau) d\tau) y(t) dt = \int_0^T \exp(-\int_0^t r(\tau) d\tau) c(t) dt \tag{7}$$
$$FOC : U'(c(t)) = \lambda exp(-\int_0^t (r(\tau) - \rho) d\tau) \tag{8}$$

If $r < \rho, \dot{c} < 0$. If $r = \rho, \dot{c} = 0$. If $r > \rho, \dot{c} > 0$.

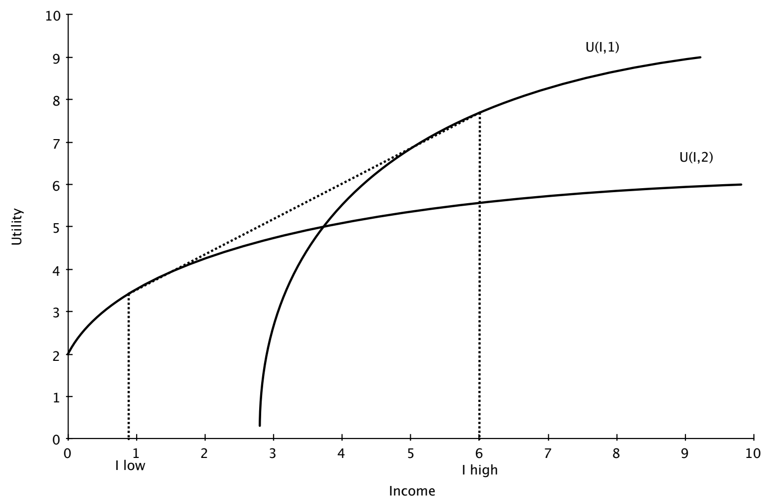


Figure 3: Decisions can create a convex region in the utility function

1.10 Convex region in utility function

In the convex region, people prefer optimal fair lottery: $pI_l' + (1-p)\, I_h' = I$

1.11 Competitive Industry model of exhaustible resources

Profit for the firm if sell today: $\pi_0 = p_0 S^0$
Profit for the firm if sell tomorrow: $\pi_1 = \frac{p_1 S^1}{1+r}$
In equilibrium, if $S^0 = S^1$(no discovery of new oil), $p_0 S^0 = \frac{p_1 S^0}{1+r} \implies p_1 = p_0\,(1+r)$
Equivalently, $p_1 - p_0 = rp_0$ (cost of holding is equal to gain of holding) or $\dot{p} = rp$
To pin down p_0, we know: $\sum_{i=1}^{\infty} Q_i\,(p_i) = \sum_{i=1}^{\infty} S_i = S$
$\implies \sum_{i=1}^{\infty} Q_i \left(p_0\,(1+r)^i\right) = S$

1.12 Exhaustible resources with cost of extraction:

$max \quad \sum_{t=0}^{\infty}(p_t q_t - c(q_t))\,(1+r)^{-t}$ s. t. $\quad \sum_{t=0}^{\infty} q_t \leq S$

FOC: $p_t - c'(q_t) = \lambda(1+r)^{-t}$

$\implies \frac{p_{t+1}-p_t}{p_t} = r + \frac{c'_{t+1} - c'_t(1+r)}{p_t}$

1.13 Trade Theory

- Trade is good
- Firm's problem is independent of the preference problem

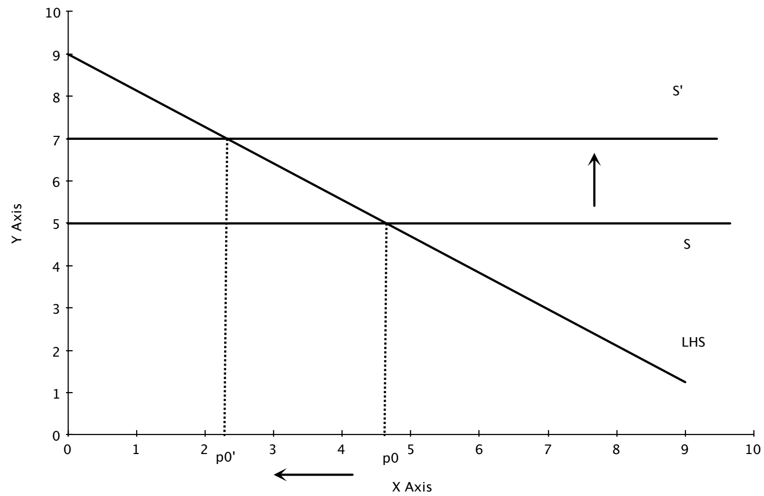


Figure 4: Oil recovery should reduce prices (S $\rightarrow$ S')

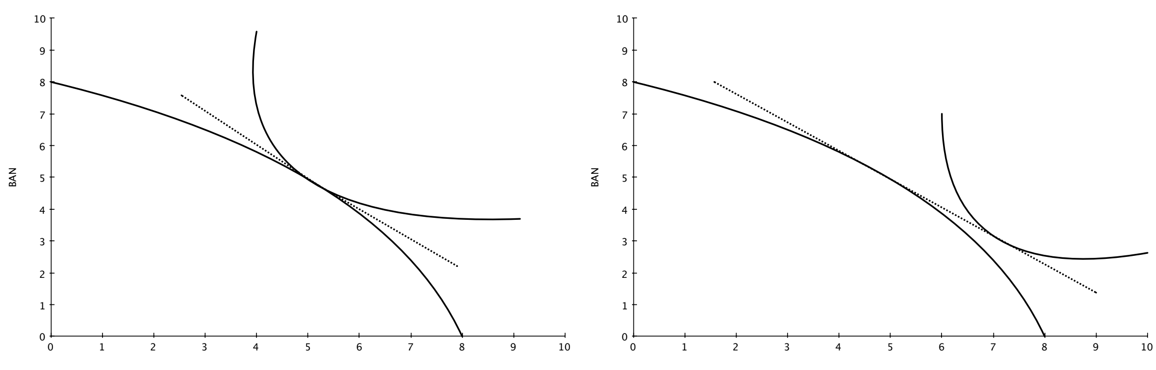


Figure 5: Left to Right: From a Robinson Crusoe Economy to NAFTA

- $P_{NAFTA} \neq P_{LOCAL}$
- Trade doesn't benefit everybody

1.14 Model for durable assets

This section is applicable for goods that are produced once and provides a flow of services over time. The dynamics are summarized in 4 equations:

$$X_t = (1-\delta)X_{t-1} + I_t \tag{9}$$
$$I_t = I(P) \tag{10}$$
$$R_t = P_t - \frac{P_{t+1}(1-\delta)}{1+r} \tag{11}$$
$$X_t = D(P_t) \tag{12}$$

1.15 Pollution and Spill-overs

Spillover: A's action affect B's utility, but everybody is compensated and A internalize that into his/her decision.

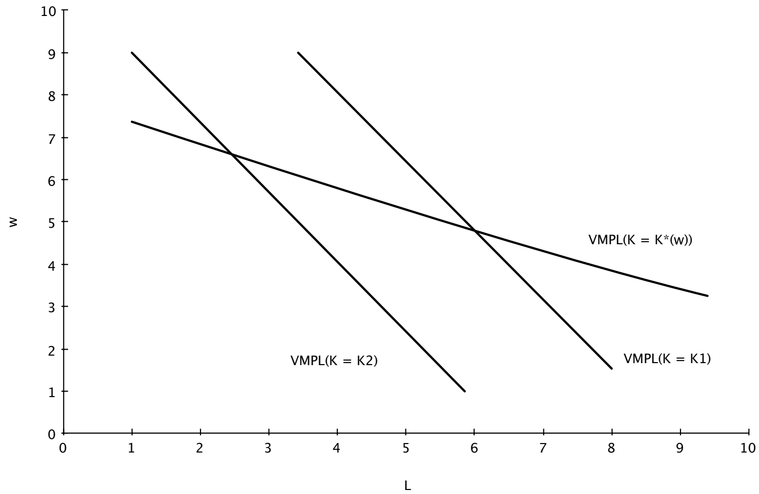
Externality: spillover without any compensation

2 Production Theory

2.1 Individual firm's profit maximization

$$\max_{L,K} PF(K,L) - wL - rK \tag{13}$$
$$[K] : P\frac{\partial F}{\partial K} = R = VMP_K \tag{14}$$
$$[L] : P\frac{\partial F}{\partial L} = w = VMP_L \tag{15}$$

Labor demand is always more elastic in the long-run (when capital is not fixed) than in the short-run.



2.2 Slutsky's Equation:

If $\frac{\partial L}{\partial y} > 0$ (labor is normal input), then $C_{yw} = C_{wy} > 0 \implies \frac{\partial C_y}{\partial w} > 0$ (increase w increase marginal cost)

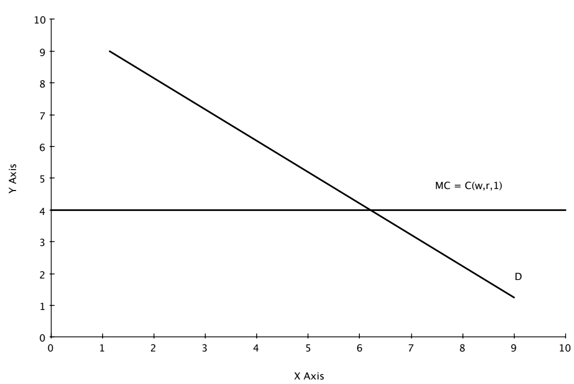
$$L = C_w(w, r, y) \implies \frac{\partial L}{\partial w} = C_{ww} + C_{wy}\frac{\partial y}{\partial w} \tag{16}$$

$$P = C_y(w, r, y) \implies \frac{\partial P}{\partial w} = 0 = C_{yw} + Cyy\frac{\partial y}{\partial w} \tag{17}$$

$$\implies \frac{\partial L}{\partial w} = \underbrace{C_{ww}}_{\text{substitution effect}} \underbrace{-\frac{C_{yw}C_{wy}}{C_{yy}}}_{\text{scale effect}} \tag{18}$$

2.3 Industry Model - Constant Return to Scale

CRS implies $C(w, r, y) = C(w, r, 1).y)$



4 magics equations:

$$L = C_w(w, r, y) \tag{19}$$
$$P = C_y(w, r, y) \tag{20}$$
$$K = C_r(w, r, y) \tag{21}$$
$$Y = D(P) \tag{22}$$

After the cost minimization, we get $C(w_1, \ldots, w_N, y)$ and $X_i^\star(w_1, \ldots, w_N, y) = C_i(w_1, \ldots, w_N, y)$
Cross-price elasticities: $\frac{\partial x_i^H}{\partial w_j} = C_{ij} \implies \epsilon_{ij}^H = \frac{w_j}{x_i}\frac{\partial x_i}{\partial w_j} = \frac{C_{ij}C}{C_iC_j}\frac{w_jx_j}{C} = \sigma_{ij}s_j$ where σ_{ij} is the elasticity of substitution, $\sigma_{ij} = \frac{C_{ij}C}{C_iC_j} = \frac{clog(L/K)}{dlog(r/w)}$

For the industry model, the output is not holding constant, but under constant return to scale, we get:

$$\frac{\partial x_i}{\partial w_j} = C_{ij} + C_{iy}\frac{\partial y}{\partial w_j} \tag{23}$$
$$\frac{\partial P}{\partial w_j} = C_{yj} + C_{yy}\frac{\partial y}{\partial w_j} = C_{yj} \tag{24}$$
$$\frac{\partial y}{\partial w_j} = D'(P)\frac{\partial P}{\partial w_j} \tag{25}$$
$$\tag{26}$$

Slutsky's Equation:

$$\frac{\partial x_i}{\partial w_j} = C_{ij} + C_{iy}D'(P)C_{yj} \tag{27}$$
$$\frac{\partial x_i}{\partial w_i} = C_{ii} + C_{iy}D'(P)C_{yi} \tag{28}$$
$$\epsilon_{ij} = s_j\sigma_{ij} + s_j\epsilon^D \tag{29}$$
$$\epsilon_{ii} = s_i\sigma_{ii} + s_i\epsilon^D \tag{30}$$

2.4 More useful equations

Profit-maximizing firm: $\triangle Y = s_L\triangle L + s_K\triangle K$
Constant Return to Scale: $s_L\triangle w + s_K\triangle r = \triangle P$
Constant Return to Scale: $\triangle L - \triangle K = \sigma(\triangle r - \triangle w)$
Definition: $\triangle Y = \epsilon^D\triangle P$
Average Labor Productivity: $\frac{Y}{L} \implies \triangle labor\, productivity = \triangle Y - \triangle L$
Marginal Labor Productivity: $\frac{w}{P} \implies \triangle MLP = \triangle w - \triangle P$
$\triangle TFP = \triangle Y - (s_L\triangle L + s_K\triangle K)$
$\triangle TFP = (\triangle ws_L + \triangle rs_K) - \triangle P$