Problem Set 1 - Odds

Six Nice People

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Question 1

Prove that if $\succeq$ is a preference relation on $\mathbb{R}^l_+$, then

(a) *$\sim$ and $\succ$ are transitive*

Recall that $x \sim y$ means $x \succeq y$ and $y \succeq x$. Let $x, y, z \in \mathbb{R}^l_+$, such that $x \sim y$ and $y \sim z$. Then by definition of $\sim$, $x \succeq y$ and $y \succeq z$, so by transitivity of $\succeq$, $x \succeq z$. Again, by definition of $\sim$, $y \succeq x$ and $z \succeq y$, so by transitivity of $\succeq$, $z \succeq x$. Then $x \succeq z$ and $z \succeq x$, so $x \sim z$. So $x \sim y$ and $y \sim z$ imply $x \sim z$, so $\sim$ is transitive.

Recall that $x \succ y$ means $x \succeq y$ and $y \not\succeq x$. Let $x, y, z \in \mathbb{R}^l_+$, such that $x \succ y$ and $y \succ z$. Then by definition of $\succ$, $x \succeq y$ and $y \succeq z$, so by transitivity of $\succeq$, $x \succeq z$. Now suppose $z \succeq x$. Then, by transitivity of $\succeq$, we would have $z \succeq y$. This is a contradiction, because $y \succ z$ implies $z \not\succeq y$. Therefore $x \succeq z$ and $z \not\succeq x$, so $x \succ z$. So $x \succ y$ and $y \succ z$ imply $x \succ z$, so $\succ$ is transitive.

(b) *$x \succ y$ and $y \sim z$ imply $x \succ z$*

We have $x \succ y$ implies $x \succeq y$, and $y \sim z$ implies $y \succeq z$, so transitivity of $\succeq$ implies $x \succeq z$. Suppose $z \succeq x$. Then , since $y \sim z$ implies $y \succeq z$, transitivity of $\succeq$ implies $y \succeq x$, which contradicts $x \succ y$. So $z \not\succeq x$. Thus, $x \succ z$.

(c) *For all $x, y \in \mathbb{R}^l_+$, exactly one of $x \succ y$, $x \sim y$, $x \prec y$ holds.*

There are two ways to order x, y ((x, y) and (y, x)) and $\succeq$ is a binary relationship, so we would expect (a priori) 4 possible outcomes (none of which can happen simultaneously due to the binary nature of $\succeq$)

(i) $x \succeq y$ and $y \succeq x$: Then $x \sim y$.
(ii) $x \succeq y$ and $y \not\succeq x$: Then $x \succ y$.
(iii) $x \not\succeq y$ and $y \succeq x$: Then $x \prec y$.
(iv) $x \not\succeq y$ and $y \not\succeq x$: This case is ruled out by the completeness of $\succeq$.

Question 3

Part a

Since $U(\cdot)$ represents $\succsim$, $\forall x^1, x^2 \in \mathbb{R}^l_+$,

$$U(x^1) \geq U(x^2) \iff x^1 \succsim x^2$$

Since $\phi : \mathbb{R} \to \mathbb{R}$ is strictly monotonically increasing it preserves ordering,

$$\phi(U(x^1)) \geq \phi(U(x^2)) \iff U(x^1) \geq U(x^2)$$

Therefore, it follows that $\phi(U(\cdot))$ represents $\succsim$ i.e. $\forall x^1, x^2 \in \mathbb{R}^l_+$,

$$\phi(U(x^1)) \geq \phi(U(x^2)) \iff x^1 \succsim x^2$$

Part b

Towards a contradiction, assume that that the proposition is true i.e. if $U(\cdot)$ represents $\succsim$ and $\phi : \mathbb{R} \to \mathbb{R}$ is weakly monotonically increasing then $\phi(U(\cdot))$ represents $\succsim$.

Define $\phi(x) = 1$, a weakly monotonically increasing function, and choose $x^1, x^2 \in \mathbb{R}^l_+$ such that $x^1 \succ x^2 \implies U(x^1) > U(x^2)$. However,

$$\phi(U(x^1)) = \phi(U(x^2)) \implies x^1 \sim x^2$$

a contradiction.

Question 5

Let $u(x; y) = \sqrt{xy}$: Show that u_x is strictly decreasing in x; i.e. that u exhibits strictly diminishing marginal utility for x. Exhibit a utility function representing the same preferences, but not satisfying strictly diminishing marginal utility for x.

To show that u_x is strictly decreasing in x it is sufficient to show that u_{xx}, the second derivative of of u wrt x, is < 0:

$$\begin{aligned} u(x, y) &= \sqrt{xy} \\ \Rightarrow u_x &= \frac{1}{2}\sqrt{\frac{y}{x}} \\ \Rightarrow u_{xx} &= -\frac{1}{4}\sqrt{y}x^{-\frac{3}{2}} \end{aligned}$$

Where $u_{xx} < 0$ because both $\sqrt{y} \geq 0$ and $x^{-\frac{3}{2}} = \sqrt{\frac{1}{x^2}} \geq 0$ (by definition of the domain of the square root).

Theorem 1.2 (Invariance of the Utility function to positive monotonic transforms) states that if one function represents the preference relation, then any transformation of that function that preserves the ordering of bundles (i.e. any strictly increasing function) will also represent the preference relation.
Let's transform $u(x, y)$ using the strictly increasing (for non-negative v) function $f(v) = v^2$. We now have

$$\begin{aligned} f(u(x)) &= xy \\ \Rightarrow \frac{\partial f(u(x))}{\partial x} &= f_x = y \\ \Rightarrow \frac{\partial f_x}{\partial x} &= f_{xx} = 0 \not< 0 \end{aligned}$$

This utility function does not satisfy strictly diminishing marginal utility for x.

Question 7

Suppose that $\{x^n\}$ and $\{y^n\}$ are two sequences of consumption bundles in $\mathbb{R}^l_+$ converging to x and y respectively. Prove that if $x^n \sim y^n$ for all n and $\succeq$ is complete, transitive, and continuous on $\mathbb{R}^l_+$, then $x \sim y$.

There is either a subsequence such that for all k, $x^{n_k} \succeq x$, or $x^{n_k} \not\succeq x$ (or both may exist within the original sequence). Within either of these subsequences there is either a subsequence such that for all j, $y^{n_{k_j}} \succeq y$, or $y^{n_{k_j}} \not\succeq y$ (or both may exist within the original sequence). In any case, we will show that $x \sim y$.

(i) $x^{n_k} \succeq x$, $y^{n_{k_j}} \succeq y$: Then $x^{n_{k_j}} \sim y^{n_{k_j}}$ implies $y^{n_{k_j}} \succeq x$ for all k, by **transitivity** of $\succeq$. Then $y^{n_{k_j}}$ is a sequence in $\succeq (x)$ converging to y, and by **continuity** of $\succeq$, $\succeq (x)$ is closed, so we must have $y \succeq x$. Symmetrically, $x^{n_{k_j}} \sim y^{n_{k_j}}$ implies $x^{n_{k_j}} \succeq y$ for all k, by **transitivity** of $\succeq$. Then $x^{n_{k_j}}$ is a sequence in $\succeq (y)$ converging to x, and by **continuity** of $\succeq$, $\succeq (y)$ is closed, so we must have $x \succeq y$. Thus, $x \sim y$.

(ii) $x^{n_k} \succeq x$, $y^{n_{k_j}} \not\succeq y$: By **completeness**, we know $y \succeq y^{n_{k_j}}$.

Suppose $x \not\succeq y$.

Also suppose there does not exist a z such that $z \succ x$ and $y \succ z$. Then for any z we either have $z \succeq y$ or $x \succeq z$, but not both, since $x \not\succeq y$. Then $\succeq (y) \cup \preceq (x)$ must cover $\mathbb{R}^l_+$, and if the two sets were disjoint, then they would be complements, and since one is closed, the other would have to be open, which would violate **continuity**. So they must not be disjoint, in which case there exists z such that $z \succeq y$ and $x \succeq z$, implying $x \succeq y$, a contradiction with our $x \not\succeq y$ assumption.

So suppose, instead, there does exist z such that $z \succ x$ and $y \succ z$. Now there exists a further subsequence of n_{k_j} (let's just index with p) that is either in $\succeq (z)$ or not in $\succeq (z)$ (or both may exist). If it is in $\succeq (z)$, then $x^p \succeq z$, and x^p converges to x, but $x \notin \succeq (z)$, so we have a contradiction, by **continuity** ($\succeq (z)$ is not closed). If the new subsequence is not in $\succeq (z)$, then $z \succeq y^p$, and y^p converges to y, but $y \notin \preceq (z)$, so we have a contradiction, by **continuity** ($\preceq (z)$ is not closed).

Then it must be the case that our original assumption $x \not\succeq y$ was wrong, so $x \succeq y$. Obviously, for any n_{k_j} we have $y \succeq y^{n_{k_j}} \sim x^{n_{k_j}} \succeq x$, by **transitivity**. So $x \sim y$.

(iii) $x^{n_k} \not\succeq x$, $y^{n_{k_j}} \succeq y$: The exact argument in (ii) holds if all xs and ys are swapped.

(iv) $x^{n_k} \not\succeq x$, $y^{n_{k_j}} \not\succeq y$: By **completeness**, we know $x \succeq x^{n_k}$, $y \succeq y^{n_{k_j}}$. Then, as in (i), $y^{n_{k_j}}$ is a sequence in $\preceq (x)$ converging to y, so **continuity** implies $y \preceq x$. Also, $x^{n_{k_j}}$ is a sequence in $\preceq (y)$ converging to x, so **continuity** implies $x \preceq y$. Thus, $x \sim y$.

Question 9

Yes, these preferences are continuous. Before proceeding with the proof, it is helpful to establish that $U(x,y) \geq U(x',y') \iff xy \geq x'y'$. To see this consider the strictly monotonically increasing function $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = \begin{cases} x+2 & \text{if } x > 1 \\ x+1 & \text{if } x = 1 \\ x & \text{if } x < 1 \end{cases}$$

Since monotonically increasing functions preserve orderings,

$$xy \geq x'y \iff U(x,y) = f(xy) \geq f(x'y') = U(x',y')$$

To establish the preferences represented by $U(.)$ are continuous begin by considering the complement of the at least as good as set, $\prec (\cdot)$. This set is defined as $\prec (x,y) = \{(x',y') \in \mathbb{R}^2_+ : U(x',y') < U(x,y)\} \iff \{(x',y') \in \mathbb{R}^2_+ : x'y' < xy\}$. Since this set is open, the at least as good set is closed. Similarly the complement of the no better than set is defined as $\succ (x,y) = \{(x',y') \in \mathbb{R}^2_+ : U(x',y') < U(x,y)\} \iff \{(x',y') \in \mathbb{R}^2_+ : x'y' > xy\}$. Since, this set is open, the no better than set is also closed. Therefore the preference relation described by the utility function is continuous.

The utility function $H(x,y) = xy$ is a continuous function which represents these preferences. We can see this function represents these preferences because U is a monotonically increasing transformation of H

$$U(x,y) = f(H(x,y))$$

Question 11

Suppose that $\preceq$ is a preference relation on $\mathbb{R}^l_+$: Show that the continuity axiom implies that for all $x; y; z \in \mathbb{R}^l_+$ such that $x \prec y \prec z$; the line segment joining x and z must contain a point that is indifferent to y.

Let's start by a couple reminders.

- **Axiom 3 (continuity)**: For any $x \in X$, the sets $\preceq (x)$ and $\succeq (x)$ are closed.
- **Line segment**: a closed line segment between a and $a + b$ is defined as

$$L = \{a + tb, t \in [0, 1]\}$$

Setting $a = x$, $b = z - x$ this is equivalent to

$$L = \{x(1 - t) + tz, t \in [0, 1]\}$$

Preliminary remark / intuition: We focus here on continuity because continuity implies that there is no gap between $\preceq (x)$ and $\succeq (x)$, which contain their boundaries, i.e. that there is "no gap" between indifference curves representing $\sim (x)$, $\sim (x')$ for all $x, x' \in \mathbb{R}^l_+$. Without gaps, the set between sets $\prec (x)$ and $\succ (x)$ must represent $\sim (x)$.

Formally (proof): We want to use the fact that preferences are continuous to show that

$$\{L \cap \preceq (y)\} \cap \{L \cap \succeq (y)\} \neq \emptyset$$

To do so, let's define two new sets and show that $S_1 \cap S_2 \neq \emptyset$

$$S_1 = \{L \cap \succeq (y)\} = \{x + t(z - x) | t \in [0, 1], x + t(z - x) \succeq (y)\}$$
$$S_2 = \{L \cap \preceq (y)\} = \{x + t(z - x) | t \in [0, 1], x + t(z - x) \preceq (y)\}$$

We have

1. $S_1 \neq \emptyset$ because $x \in S_1$ $(t = 0)$ as $x \prec y$ and $S_2 \neq \emptyset$ because $z \in S_2$ because of completeness and the definition of a closed line segment.
2. $S_1 \cup S_2 = L$ because of completeness.
3. Both S_1 and S_2 are closed sets because they are the intersection of L a closed line segment and continuous preference relations $\succeq (\cdot)$ and $\preceq (\cdot)$ ($\Rightarrow$ closed by continuity!). The intersection of two closed sets is a closed set itself.

L is a connected set, therefore it cannot be the union of two non-empty separated sets (from Baby Rudin). By our above points S_1 and S_2 are non-empty and their union comprises L, so they cannot be separated, and they are closed, so $\bar{S}_1 \cap S_2 = S_1 \cap \bar{S}_2 = S_1 \cap S_2$ so $S_1 \cap S_2 \neq \emptyset$.

Question 13

For each of the utility functions below, calculate the ordinary (i.e. Marshallian), Slutsky compensated (for a definition, see exercise 1.45 in JR), and Hicksian compensated demand functions.

First I'll clarify the definitions, then I'll complete the calculations.

Marshallian demand $M(\mathbf{p}, \omega)$ is a correspondence which maps prices $\mathbf{p}$ and endowment ω to choices of goods $\mathbf{x}$ which maximize utility.

Slutsky compensated demand $S(\mathbf{p}, \mathbf{y})$ is a correspondence which maps prices $\mathbf{p}$ and bundles $\mathbf{y}$ to choices of goods $\mathbf{x}$ which maximize utility, given that the endowment is adjusted so that $\mathbf{y}$ is always just affordable. Note $S(\mathbf{p}, \mathbf{y}) = M(\mathbf{p}, \mathbf{p}\mathbf{y})$.

Hicksian compensated demand $H(\mathbf{p}, u)$ is a correspondence which maps prices $\mathbf{p}$ and utility levels u to choices of goods $\mathbf{x}$ for which there exists an endowment such that the solution to the maximization problem delivers u utils. Note $H(\mathbf{p}, u) = M(\mathbf{p}, e(\mathbf{p}, u))$, where $e(\cdot)$ denotes the expenditure function, which maps prices and utility levels to the lowest cost possible to achieve u at prices $\mathbf{p}$.

(a) $u(x, y) = x^\alpha y^{1-\alpha}$, $0 < \alpha < 1$

- Marshallian: The Lagrangian for the consumer is

$$\mathcal{L}(x, y, \lambda) = x^\alpha y^{1-\alpha} - \lambda[p_x x + p_y y - \omega]$$

Note that since goods are "good" (increase utility) I here used that all income will be exhausted. Additionally, I assume we are only considering positive prices, since otherwise the solution will be to consume infinite amount of the free good. I do this below as well.
So the first-order necessary conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x} &= \alpha x^{\alpha-1} y^{1-\alpha} - \lambda p_x = 0 \\
\frac{\partial \mathcal{L}}{\partial y} &= (1-\alpha) x^\alpha y^{-\alpha} - \lambda p_y = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= p_x x + p_y y - \omega = 0
\end{aligned}$$

Therefore

$$\begin{aligned}
\lambda &= \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{p_x} = \frac{(1-\alpha) x^\alpha y^{-\alpha}}{p_y} \\
\Rightarrow \frac{p_y}{p_x} &= \frac{(1-\alpha)x}{\alpha y} \\
\Rightarrow x + \frac{(1-\alpha)}{\alpha} x &= \frac{\omega}{p_x} \\
\Rightarrow x &= \frac{\alpha \omega}{p_x} \\
y &= \frac{(1-\alpha)\omega}{p_y}
\end{aligned}$$

$$\boxed{M(p_x, p_y, \omega) = (\frac{\alpha\omega}{p_x}, \frac{(1-\alpha)\omega}{p_y})}$$

- Slutsky: Let (z_x, z_y) be the bundle we wish to keep affordable. We have

$$\begin{aligned} S(p_x, p_y, z_x, z_y) &= M(p_x, p_y, p_x z_x + p_y z_y) \\ &\boxed{= (\frac{\alpha(p_x z_x + p_y z_y)}{p_x}, \frac{(1-\alpha)(p_x z_x + p_y z_y)}{p_y})} \end{aligned}$$

- Hicksian: We need to find the expenditure function. We minimize cost for a given u

$$\mathcal{L}(x, y, \lambda) = p_x x + p_y y - \lambda(x^\alpha y^{1-\alpha} - u)$$

FONCs:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial x} &= p_x - \lambda \alpha x^{\alpha-1} y^{1-\alpha} = 0 \\ \frac{\partial \mathcal{L}}{\partial y} &= p_y - \lambda(1-\alpha) x^\alpha y^{-\alpha} = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= x^\alpha y^{1-\alpha} - u = 0 \end{aligned}$$

Therefore

$$\begin{aligned} \lambda &= \frac{p_x}{\alpha x^{\alpha-1} y^{1-\alpha}} = \frac{p_y}{(1-\alpha) x^\alpha y^{-\alpha}} \\ u &= x^\alpha y^{1-\alpha} \\ \Rightarrow x &= \frac{u}{(\frac{(1-\alpha)p_x}{\alpha p_y})^{1-\alpha}} = (\frac{\alpha}{1-\alpha})^{1-\alpha} (\frac{p_y}{p_x})^{1-\alpha} u \\ y &= \frac{u}{(\frac{(1-\alpha)p_x}{\alpha p_y})^{-\alpha}} = (\frac{1-\alpha}{\alpha})^{\alpha} (\frac{p_x}{p_y})^{\alpha} u \\ \Rightarrow e(p_x, p_y, u) &= p_x x + p_y y \\ &= \Big(\frac{1}{\alpha^\alpha} \frac{1}{(1-\alpha)^{1-\alpha}}\Big) p_x^\alpha p_y^{1-\alpha} u \\ H(p_x, p_y, u) &= M(p_x, p_y, e(p_x, p_y, u)) \\ &\boxed{= (\frac{\alpha}{p_x}\Big(\frac{1}{\alpha^\alpha} \frac{1}{(1-\alpha)^{1-\alpha}}\Big) p_x^\alpha p_y^{1-\alpha} u, \frac{(1-\alpha)}{p_y}\Big(\frac{1}{\alpha^\alpha} \frac{1}{(1-\alpha)^{1-\alpha}}\Big) p_x^\alpha p_y^{1-\alpha} u)} \end{aligned}$$

(b) $u(x,y) = x^\alpha + y^\alpha$, $0 < \alpha < 1$

- Marshallian:

$$\begin{aligned}
\mathcal{L}(x,y)\lambda) &= x^\alpha + y^\alpha - \lambda[p_x x + p_y y - \omega] \\
\frac{\partial \mathcal{L}}{\partial x} &= \alpha x^{\alpha-1} - \lambda p_x = 0 \\
\frac{\partial \mathcal{L}}{\partial y} &= \alpha y^{\alpha-1} - \lambda p_y = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= p_x x + p_y y - \omega = 0 \\
\Rightarrow \lambda &= \frac{\alpha x^{\alpha-1}}{p_x} = \frac{\alpha y^{\alpha-1}}{p_y} \\
y &= (\frac{p_x}{p_y})^{\frac{1}{1-\alpha}} x \\
&\boxed{M(p_x,p_y,\omega) = \left(\frac{\omega}{p_x + p_y(\frac{p_x}{p_y})^{\frac{1}{1-\alpha}}}, \frac{\omega}{p_y + p_x(\frac{p_y}{p_x})^{\frac{1}{1-\alpha}}}\right)}
\end{aligned}$$

- Slutsky:

$$\begin{aligned}
S(p_x,p_y,z_x,z_y) &= M(p_x,p_y,p_x z_x + p_y z_y) \\
&\boxed{= \left(\frac{p_x z_x + p_y z_y}{p_x + p_y(\frac{p_x}{p_y})^{\frac{1}{1-\alpha}}}, \frac{p_x z_x + p_y z_y}{p_y + p_x(\frac{p_y}{p_x})^{\frac{1}{1-\alpha}}}\right)}
\end{aligned}$$

- Hicksian:

$$\begin{aligned}
\mathcal{L}(x,y,\lambda) &= p_x x + p_y y - \lambda[x^\alpha + y^\alpha - u] \\
\frac{\partial \mathcal{L}}{\partial x} &= p_x - \lambda \alpha x^{\alpha-1} = 0 \\
\frac{\partial \mathcal{L}}{\partial y} &= p_y - \lambda \alpha y^{\alpha-1} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= x^\alpha + y^\alpha - u = 0 \\
\Rightarrow \lambda &= \frac{p_x}{\alpha x^{\alpha-1}} = \frac{p_y}{\alpha y^{\alpha-1}} \\
y &= (\frac{p_y}{p_x})^{\frac{1}{1-\alpha}} x \\
x &= \left(\frac{u}{1+(\frac{p_y}{p_x})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}
\end{aligned}$$

$$\boxed{H(p_x,p_y,u) = \left(\left(\frac{u}{1+(\frac{p_x}{p_y})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}, \left(\frac{u}{1+(\frac{p_y}{p_x})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}\right)}$$

(c) $u(x,y) = \min\{x,y\}$

- Marshallian: The lack of differentiability means we need a different approach. We can note that any solution where $x \neq y$ will not be optimal, because in that case, say $y = x + a$, we have $u(x,y) = x$, but since $(x, x+a)$ is affordable, so is $(x + (\frac{p_y}{p_x+p_y})a, x + (\frac{p_y}{p_x+p_y})a)$:

$$\begin{aligned}
p_x x + p_y(x+a) &= p_x x + p_y x + p_y(\frac{p_x+p_y}{p_x+p_y})a \\
&= p_x(x + (\frac{p_y}{p_x+p_y})a) + p_y(x + (\frac{p_y}{p_x+p_y})a)
\end{aligned}$$

which delivers $u(x,y) = x + (\frac{p_y}{p_x+p_y})a > x$. I assumed here that $y > x$, but of course the symmetry of $u(x,y)$ means the argument would work in the other direction as well. So the optimal bundle will be along the line $x = y$. It will also exhaust the budget:

$$\begin{aligned}
\omega &= p_x x + p_y y \\
&= p_x x + p_y x \\
\Rightarrow & \boxed{M(p_x,p_y,\omega) = (\frac{\omega}{p_x+p_y}, \frac{\omega}{p_x+p_y})}
\end{aligned}$$

- Slutsky:

$$S(p_x, p_y, z_x, z_y) = M(p_x, p_y, p_x z_x + p_y z_y)$$
$$\boxed{= (\frac{p_x z_x + p_y z_y}{p_x + p_y}, \frac{p_x z_x + p_y z_y}{p_x + p_y})}$$

- Hicksian: We already know that the optimal choice will be along $x = y$, so we just need to determine how utility levels map to this line.

$$u = \min\{x, y\} = x = y$$

So we need to "give" the consumer just enough endowment to buy u of each good, which is what they will optimally do.

$$\boxed{H(p_x, p_y, u) = (u, u)}$$

(d) $u(x, y) = x + y$

- Marshallian: This is the case of perfect substitutes. If $p_x \neq p_y$, then it will be optimal to spend the entirety of the endowment on the cheaper good. We can show this by considering any other feasible bundle. Since the price of the more expensive good is higher, the total amount of goods $x + y$ must be lower if any of it is bought, so this choice is suboptimal. If $p_x = p_y$, then all bundles that exhaust the budget are equally optimal, since in this case $p_x x + p_y y = p_x(x + y) = pu$. Therefore

$$\boxed{M(p_x, p_y, \omega) = \begin{cases} (\frac{\omega}{p_x}, 0) & p_x < p_y \\ \{(t, \frac{\omega}{p_x} - t) : t \in [0, \frac{\omega}{p_x}]\} & p_x = p_y \\ (0, \frac{\omega}{p_y}) & p_x > p_y \end{cases}}$$

- Slutsky:

$$\boxed{S(p_x, p_y, z_x, z_y) = \begin{cases} (\frac{p_x z_x + p_y z_y}{p_x}, 0) & p_x < p_y \\ \{(t, \frac{p_x z_x + p_y z_y}{p_x} - t) : t \in [0, \frac{p_x z_x + p_y z_y}{p_x}]\} & p_x = p_y \\ (0, \frac{p_x z_x + p_y z_y}{p_y}) & p_x > p_y \end{cases}}$$

- Hicksian: The same decision argument we made for Marshallian demand applies here: the more expensive good will never be purchased. Therefore the consumer will always be compensated so that they can finance their given utility by exclusively purchasing the cheaper good, which is what they will do (the one exception being the continuum of solutions when the prices are the same).

$$\boxed{H(p_x, p_y, u) = \begin{cases} (u, 0) & p_x < p_y \\ \{(t, u-t) : t \in [0, u]\} & p_x = p_y \\ (0, u) & p_x > p_y \end{cases}}$$