**Price Theory, III** | **Professor Lars Stole**
**Economics 33003** | **Spring 2020**

**Assignment 3**
**(Due Friday, May 1, prior to the start of the Review session )**

**Problem 1** Consider a principal-agent model with moral hazard in which the principal is risk neutral and the agent is risk averse with $u(w) = \sqrt{100 + w}$. There are two efforts, $e_h > e_l$, with personal cost to the agent of $\psi(e_h) = 1$ and $\psi(e_l) = 0$. The agent's expected utility from wage contract $w(\cdot)$ and effort $e$ is $E[\sqrt{100 + w(x)} \mid e] - \psi(e)$.

There are two outcomes. The high-output outcome is $x_2 = 200$ and the low-outcome output is $x_1 = 100$. When the agent exerts low effort, each output is equally likely. When the agent exerts high effort, the probability of high output is $\frac{3}{4}$ (and the probability of low output is $\frac{1}{4}$). The agent's reservation utility is $\underline{U} = u(0) = 10$.

(a). Does this distribution satisfy MLRP?

(b). Solve for the optimal output-contingent wage contract, $\{w_1^*, w_2^*\}$.

**Problem 2** Consider a principal-agent model with moral hazard in which the principal is risk neutral and the agent is also risk neutral. There are two efforts, $e_h > e_l$, with personal cost to the agent of $\psi(e_h) = 10$ and $\psi(e_l) = 0$. The agent's expected utility is $E[w|e] - \psi(e)$.

There are two outcomes. The high-output payoff is $x_2 = 200$, but the low-outcome payoff represents a loss, $x_1 = -100$. When the agent exerts low effort, each output is equally likely. When the agent exerts high effort, the probability of high output is $\frac{3}{4}$ (and the probability of low output is $\frac{1}{4}$). The agent's reservation utility is $\underline{U} = 0$.

(a). Solve for an optimal output-contingent wage contract, $\{w_1^*, w_2^*\}$. Does the worker earn any surplus above his reservation value?

(b). Now suppose that the agent is still risk neutral, but legally the principal is not allowed to pay a wage that is negative. [Aside: This is sometimes called the *limited-liability* assumption. It is also equivalent to a situation in which an agent is infinitely risk averse at $w < 0$: i.e., $u(w) = w$ for $w \geq 0$ and $u(w) = -\infty$ for $w < 0$.] Solve for an optimal output-contingent wage contract, $\{w_1^*, w_2^*\}$.

What is the expected cost to the principal of the limited-liability constraint?

**Problem 3** Consider a simple moral hazard where the principal and the agent are risk neutral (i.e., $v'(\cdot) = u'(\cdot) = 1$), there is no uncertainty over output, but now output depends upon the actions of the agent and the principal. Specifically, the agent chooses $e_1 \in [0, 2]$, the principal chooses $e_2 \in [0, 2]$ and output is deterministic:

$$x = e_1 + e_2 \in \mathcal{X} = [0, 4].$$

Assume that the cost of effort for each player is $\frac{1}{2}e_i^2$, so that first-best production requires $e_i^{fb} = 1$ and $x^{fb} = 2$.

(a). Assume that the principal can offer a contract to the agent which promises $s(x)$ in payment for the outcome $x$, and the residual profit, $x - s(x)$, is kept by the principal. After the agent accepts the contract, both the principal and agent simultaneously choose their individual efforts, $e_i$. Once $x$ is revealed, payments are shared as promised with $s(x)$ going to the agent and $x-s(x)$ going to the principal.

If $s(x)$ is required to be continuously differentiable on $\mathcal{X}$, show that the first best cannot be implemented by the principal. [Hint: remember that the principal must also be given incentives. Another way to think of this problem is that there is a team of two players that need to devise a sharing rule $\{s(x), x - s(x)\}$ to give each incentives.]

(b). Suppose that instead of designing a sharing rule, $\{s(x), x-s(x)\}$, the principal can commit to giving some of the output to a third party for some values of $x$. I.e., the principal can offer $s(x)$ to the agent, and can commit to giving $z(x)$ to a third party, keeping $x - s(x) - z(x)$ for herself. Show that the first best can now be implemented as a Nash equilibrium between the principal and agent and that nothing is given to the third party *along the equilibrium path*. For this part, you may construct the implementing functions using discontinuous $s$ and $z$ functions. [Hint: think about simple contracts which split the output between principal and agent for some $x$, and give the entire output to a third party for other $x$.]

(c). Assume we are back to setting (a) without a third party, and thus the share contracts are limited to $\{s(x), x - s(x)\}$. Now, however, assume that the sets of feasible actions are discrete $e_1 \in \{0.5, 1, 1.5\}$ and $e_2 \in \{.67, 1, 1.33\}$. Can the first best $e_1^* = e_2^* = 1$ be implemented?

**Problem 4** Consider a risk-averse individual with utility function of money $u(\cdot)$ with initial wealth $y$ who faces the risk of having an accident and losing an amount $x$ of her wealth. She has access to a perfectly competitive market of risk-neutral insurers who can offer coverage schedules $b(x)$ (i.e., in the event of loss $x$, the insurance company pays out $b(x)$) in exchange for an insurance premium, $p$. Assume that the distribution of $x$, which depends on accident-prevention effort $e$, has an atom at $x = 0$:

$$f(x|e) = \begin{cases} 1 - \phi(e) & \text{if } x = 0 \\ \phi(e)g(x) & \text{if } x > 0, \text{ where } g(\cdot) \text{ is a probability density} \end{cases}$$

Assume $\phi'(e) < 0 < \phi''(e)$. Also assume that the individual's (increasing and convex) cost of effort, separable from her utility of money, is $\psi(e)$.

(a). What is the full information insurance contract when $e$ is contractible? Specifically, what is $e$ and $b(x)$?

(b). If $e$ is not observable, prove that the optimal contract consists of a premium and a deductible; i.e., $b(x) = x - \delta$. You may assume that a solution exists and the first-order approach is valid.

**Problem 5** Consider a principal-agent setting with moral hazard and a finite number of outputs and efforts. There are $n$ possible outputs, $x_i \in \mathcal{X} = \{x_1, \dots, x_n\}$, $x_n > x_{n-1} > \cdots > x_1$. There are $m$ possible effort levels, $e_j \in \mathcal{E} = \{e_1, \dots, e_m\}$, $e_m > e_{m-1} > \cdots > e_1$. The probability of output $x_i$ given effort $e_j$ is $\phi_i(e) = \phi(x_i|e_j) > k > 0$ (i.e., bounded away from zero). Assume that $\phi_i(\cdot)$ satisfies MLRP: i.e., for all $e > \tilde{e}$,

$$\frac{\phi_i(e) - \phi_i(\tilde{e})}{\phi_i(e)}, \quad \text{is increasing in } i.$$

Assume that the principal is risk neutral and the agent is risk averse with preferences $U = u(w) - \psi(e)$, where $u(\cdot)$ is increasing and concave, and $\psi(\cdot)$ is increasing and convex. Assume the agent's outside option is $\underline{U} = 0$ and $\psi(e_1) = 0$. Finally, the principal's wage schedule must satisfy $w_i = w(x_i) \in [\underline{w}, \overline{w}]$, but assume that these constraints are slack at the optimum and ignore them.

(a). State the principal's program for the optimal $e$ and $(w_1, \dots, w_n)$, and write the Lagrangian for the program using $\{\mu_j\}_j$ as multipliers for the IC constraints and $\lambda$ as the multiplier for the IR constraint.

(b). Prove that the agent's IR constraint is binding (i.e., $\lambda > 0$) in the solution to the program.

(c). Suppose that the optimal choice of effort is $e^* = e_m$, the maximum effort level. Prove that the optimal wage schedule which induces $e^m$ is monotonic, $w_1^* \leq w_2^* \cdots \leq w_n^*$ with strict inequality for some $i$ (i.e., $w_i^* < w_{i+1}^*$).

(d). Prove that the MLRP implies first-order stochastic dominance for the case of a discrete distribution, $(\phi_1(e), \dots, \phi_n(e))$.

(e). Suppose that the optimal choice of effort $e^* > e_1$. Prove that there must exist some $i$ such that $w_i^* < w_{i+1}^*$ (i.e., prove that $w_1^* \geq w_2^* \geq \cdots \geq w_n^*$ cannot be optimal). [Hint: use the fact in (d) above.]

**Problem 6** Consider principal-agent problem with three exogenous states of nature, $\theta_1$, $\theta_2$, and $\theta_3$; two effort levels, $e_L$ and $e_H$; and two output levels, distributed as follows as a function of the state of nature, $\theta$, and the effort level, $e$:

| State of nature | $\theta_1$ | $\theta_2$ | $\theta_3$ |
|---|---|---|---|
| Probability of state | $\phi_1 = 0.25$ | $\phi_2 = 0.50$ | $\phi_3 = 0.25$ |
| Output under $e_H$ | 1800 | 1800 | 100 |
| Output under $e_L$ | 1800 | 100 | 100 |

The principal is risk neutral, while the agent has utility function $u(w) = \sqrt{w}$ when receiving monetary compensation $w$, minus the cost of effort, which is normalized to 0 for $e_L$ and to $\Delta = 10$ for $e_H$. The agent's reservation expected utility is $\underline{U} = 10$.

(a). Derive the first-best, full-information contract.

(b). Derive the second-best contract when only output levels are observable.

(c). A salesman comes by (in the second-best world in which only the cash outcome is observed) and offers to sell the principal an information system. The information will be observed by both the principal and the agent after the act is implemented. (We interpret the information system as a monitor.) The system in question will tell whether or not the state is $\theta_3$; in other words it will provide the following partition of information $\{\{\theta_1, \theta_2\}, \{\theta_3\}\}$. How much will the principal be willing to pay for the monitoring system?

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## Answers to Assignment 3

**1** (a). Yes. To check formally,

$$\frac{f(x_2|e_h) - f(x_2|e_l)}{f(x_2|e_h)} = \frac{\frac{3}{4} - \frac{1}{2}}{\frac{3}{4}} = \frac{1}{3} > \frac{f(x_1|e_h) - f(x_1|e_l)}{f(x_1|e_h)} = \frac{\frac{1}{4} - \frac{1}{2}}{\frac{1}{4}} = -1.$$

(b). We will solve the program separately for $e^* = e_l$ and $e^* = e_h$. If the principal wishes to induce low effort, the optimal wage is constant because the agent is risk averse and no incentives need to be given for $e_l$. Thus, $\sqrt{100 + w} = 10$ and therefore $w_1^* = w_2^* = 0$. The principal's profit from inducing $e_l$ is

$$\frac{1}{2}200 + \frac{1}{2}100 - w = 150.$$

Now suppose the principal wishes to induce $e_h$. The Principal's program becomes

$$\max_{w_1, w_2} \frac{3}{4}(200 - w_2) + \frac{1}{4}(100 - w_1),$$

subject to

$$\frac{3}{4}u(w_2) + \frac{1}{4}u(w_1) - 1 \geq 10,$$

$$\frac{3}{4}u(w_2) + \frac{1}{4}u(w_1) - 1 \geq \frac{1}{2}u(w_2) + \frac{1}{2}u(w_1).$$

The second constraint can be further simplified to

$$\frac{1}{4}(u(w_2) - u(w_1)) - 1 \geq 0.$$

Because both constraints will be binding and there are only two outputs, the optimal wages $w_1^*$ and $w_2^*$ can be determined by simply solving them as equalities:

$$\frac{3}{4}u(w_2) + \frac{1}{4}u(w_1) = 11,$$

$$\frac{1}{4}(u(w_2) - u(w_1)) = 1.$$

Substituting in for $u(w) = \sqrt{100 + w}$, we have

$$3\sqrt{100 + w_2} + \sqrt{100 + w_1} = 44,$$

$$\sqrt{100 + w_2} - \sqrt{100 + w_1} = 4.$$

Now we need to do some algebra. Rather than typing all of the steps out, I'll simply note the solution: $w_1^* = -36$ and $w_2^* = 44$. Plugging these into the principal's objective, we obtain

$$\frac{3}{4}(200 - 44) + \frac{1}{4}(100 + 36) = 151.$$

Thus it is optimal to induce the high-effort outcome.

**2** (a). Because the agent and principal are each risk neutral, one optimal wage schedule effectively sells the firm to the agent for its expected value. That is, in the full-information setting, the firm is worth

$$\frac{3}{4}(200) - \frac{1}{4}(100) - 10 = 115 > 50.$$

The firm can offer the worker the following wage

$$w(x) = x - 115.$$

If the worker accepts, he will choose the high effort level, generating an expected revenue of $E[x|e_h] = 125$, less his cost of effort, $\Delta = 10$, less his fixed payment in the wage of 115, yielding zero. The firm captures the full surplus.

(b). Now the firm cannot charge $w_1 < 0$. If we write out and solve the Lagrangian, we would see that indeed, $w_1^* = 0$. Assuming it is still optimal to implement the high effort, the principal will have to provide incentives by raising $w_2$ sufficiently high. In particular, it requires (assuming $w_1^* = 0$)

$$\frac{3}{4}w_2 - \Delta \geq \frac{1}{2}w_2,$$

or $w_2 \geq 40$. Setting $w_2^* = 40$, the principal can obtain $e^* = e_h$, and total profit (net of wages) is

$$E[x|e_h] - \frac{3}{4}(40) = 125 - 30 = 95.$$

This is still greater than the $e_l$ outcome of 50, so the principal prefers to induce $e_h$. On net, the firm is poorer by $115 - 95 = 20$ because of the limited-liability constraint.

**3** (a). This is based on Holmström (1982). In any Nash equilibrium, $\{e_1^*, e_2^*\}$, the agent chooses $e_1$ to maximize $s(e_1 + e_2) - \frac{1}{2}e_1^2$, and so

$$s'(e_1^* + e_2^*) = e_1^*;$$

the principal chooses $e_2$ to maximize $e_1 + e_2 - s(e_1 + e_2) - \frac{1}{2}e_2^2$, and so

$$1 - s'(e_1^* + e_2^*) = e_2^*.$$

Adding these together implies

$$1 = e_1^* + e_2^* < 2.$$

Hence outputs are below first best. Note that the assumption that $s$ is continuously differentiable is not important for this result. A more general result is found in Holmström (1982) (appendix) that applies to discontinuous sharing rules also.

(b). The following schedule will implement the first best.

$$s(x) = \begin{cases} 1 & \text{if } x = 2 \\ 0 & \text{otherwise,} \end{cases}$$

$$z(x) = \begin{cases} 0 & \text{if } x = 2 \\ 2 & \text{otherwise.} \end{cases}$$

With these sharing rules, both the principal and the agent get 1 if $x = 2$ and 0 otherwise. Assuming that $e_j^* = 1$ in equilibrium, then player $i$ will choose $e_i = 1$ because the payoff is $\frac{1}{2}$. Anything else can generate a payoff of zero, at best. The idea here is a general one – if there exists someone who is known not to have shirked (i.e., a third party), the other players can commit to give the output to that party off the equilibrium path, in order to provide incentives for first-best efforts.

(c). This is based on Legros and Matthews (1993) who show, among other things, that if the output function is discrete and generic, the outputs will identify some party that did not shirk, allowing that person to obtain the entire output. This effectively allows the budget breaking that is used in (b). To see how we do it here, notice that if player 1 chooses $e_1^* = 1$, then only three outputs can arise, $\{1.67, 2, 2.33\}$. Furthermore, these outputs cannot arise if player 1 chooses either $e_1 = 0.5$ or $e_1 = 1.5$. Thus, we can assign $s(x) = 1$ if $x = 2$, $s(x) = 1.67$ if $x = 1.67$ and $s(x) = 2.33$ if $x = 2.33$. This effectively punishes player 2 with a payoff of zero for not choosing $e_2 = 2$. Similarly, if player 2 chooses $e_2 = 1$, then outputs can only be $\{1.5, 2, 2.5\}$. For $x = 2$, we have already assigned $s(x) = 1$, so player 2 also gets 1. For $x = 1.5$ or 2.5, player 1 must have shirked, so we give the output to player 2: $s(1.5) = 0$ and $s(2.5) = 0$. What remains is to assign a sharing rule for outputs where both parties have shirked. In these cases, it is enough to split the output 50-50. We now have a Nash equilibrium in which $e_1^* = e_2^* = 1$. If either party chooses an effort other than 1 (while the first party plays the equilibrium effort), they are assured a payoff of zero (at best). Hence, it is optimal to choose $e_i^* = 1$ and obtain a payoff of $\frac{1}{2}$.

**4** (a). Because effort is contractible, there is no moral hazard problem and the optimal contract will offer full insurance: $b(x) = x$. Because there is perfect competition, insurance profits are zero, and hence

$$p = \phi(e)E[b(x)] = \phi(e)E[x],$$

where $E[x] = \int xg(x)dx$. Given these conditions, the efficient level of effort for the agent maximizes

$$u(y - \phi(e)E[x]) - \psi(e),$$

The first-order condition is therefore

$$-u'(y - \phi(e)E[x])\phi'(e)E[x] - \psi'(e) = 0.$$

(b). There are a few ways to solve this problem. Given that there is perfect competition, we can think of the consumer maximizing their expected utility subject to incentive compatibility and a requirement that the insurer makes nonnegative profits in expectation. (You could also solve a problem maximizing firm profit subject to an expected utility constraint for the consumer.) All expectations are taken with respect to $x$ using density $g$:

$$\max_{e,p,b(\cdot)} (1 - \phi(e))u(y - p) + \phi(e)E[u(y - p - x + b(x))] - \psi(e)$$

subject to

$$\phi'(e)\left(E[u(y - p - x + b(x))] - u(y - p)\right) - \psi'(e) = 0$$

$$p \geq \phi(e)E[b(x)].$$

Note that the second-order condition for incentive compatibility is always satisfied in this program if

$$u(y-p) \geq E[u(y-p-x+b(x))],$$

that is, if the consumer always prefers the accident-free state to an accident with random loss given by $g$. The first-order condition requires this property, and so the first-order approach is valid for this problem.

The Lagrangian can be written as

$$\begin{aligned}\mathcal{L} = {} & (1-\phi(e))u(y-p) + \phi(e)E[u(y-p-x+b(x))] - \psi(e) \\ & + \mu\left(\phi'(e)\left(E[u(y-p-x+b(x))] - u(y-p)\right) - \psi'(e)\right) + \lambda\left(p - \phi(e)E[b(x)]\right).\end{aligned}$$

Maximize the benefit function, $b(x)$, pointwise over $x$, by taking the derivative with respect to $b(x)$ for a given $x$:

$$\phi(e)u'(y-p-x+b(x)) + \mu\phi'(e)u'(y-p-x+b(x)) - \lambda\phi(e) = 0.$$

Thus,

$$u'(y-p-x+b(x)) = \frac{\lambda\phi(e)}{\phi(e)+\mu\phi'(e)},$$

which is constant, and so we conclude that $y-p-x+b(x)$ must also be a constant for all $x$. Thus,

$$b(x) = x - \delta,$$

which can be thought of as full insurance less a deductible.

The question didn't ask you to determine $\delta$, but this is straightforward:

**Aside:** Notice that $x > 0$ is not informative about $e$, conditional on an accident. Thus, marginal utilities should be equated across all accident states (i.e., full insurance conditional on an accident). The deductible is used as the single lever for incentives.

**5** (a). Principal solves

$$\max_{e,(w_1,\ldots,w_n)} \sum_i (x_i - w_i)\phi_i(e),$$

subject to

$$\sum_i u(w_i)\phi_i(e) - \psi(e) \geq 0, \quad \text{(IR)}$$

$$\sum_i u(w_i)(\phi_i(e) - \phi_i(e_j)) - \psi(e) + \psi(e_j) \geq 0, \quad \text{(IC)}.$$

Forming the Lagrangian,

$$\begin{aligned}\mathcal{L} = {} & \sum_i (x_i - w_i)\phi_i(e) + \sum_{j=1}^{m} \mu_j \left(-\psi(e) + \psi(e_j) + \sum_i u(w_i)(\phi_i(e) - \phi_i(e_j))\right) \\ & + \lambda\left(-\psi(e) + \sum_i u(w_i)\phi_i(e)\right).\end{aligned}$$

(b). Suppose that the constraint was not binding:

$$\sum_i u(w_i^*)\phi_i(e) - \psi(e) > 0.$$

The principal could choose a new wage schedule, $(w_1^\varepsilon, w_2^\varepsilon, \ldots, w_n^\varepsilon)$ such that for each $i$

$$u(w_i^\varepsilon) = u(w_i^*) - \varepsilon.$$

The new wage schedule has lower expected cost to the principal. For $\varepsilon$ sufficiently small, the new wage schedule is satisfies IR; the IC constraints are unaffected by the variation. Hence, the variation is feasible and increases profit – a contradiction.

(c). Differentiating the Lagrangian in (a) with respect to $w_i$, we have

$$\frac{1}{u'(w_i)} = \lambda + \sum_{j=1}^{m-1} \mu_j \left( \frac{\phi_i(e_m) - \phi_i(e_j)}{\phi_i(e_m)} \right).$$

We know that $\mu_j \geq 0$ for all $j$. Because $e_m > e_j$ for all $j < m$, MLRP implies each likelihood ratio in the parentheses is increasing in $i$. Because $1/u'(w_i)$ is increasing in $w_i$, we conclude that $w_1^* \leq \cdots \leq w_n^*$. Note that if $w_1^* = w_n^*$ (i.e., the wage schedule was constant) then the agent's utility payoff $\sum_i u(w_i^*)\phi_i(e)$ is independent of $e$, and thus to save on the cost of effort the agent would choose $e_1$ – a contradiction to $e^* = e_m$. Thus, there must exist at least one $i$ such that $w_i^* < w_{i+1}^*$.

(d). Define

$$LR_i(e, \tilde{e}) = \frac{\phi_i(e) - \phi_i(\tilde{e})}{\phi_i(e)}.$$

Note that because $\phi$ sums to 1 regardless of $e$,

$$\sum_{i=1}^{n} \phi_i(e) - \phi_i(\tilde{e}) = 0.$$

Rewriting this in terms of the likelihood ratio, we have

$$\sum_{i=1}^{n} LR_i(e, \tilde{e})\phi_i(e) = 0.$$

MLRP implies that $LR_i(e, \tilde{x})$ is increasing in $i$ if $e > \tilde{e}$. Because it's expectation is zero, it must be that for any $k < n$,

$$\sum_{i=1}^{k} LR_i(e, \tilde{e})\phi_i(e) < 0.$$

Rewriting, this is equivalent to

$$\sum_{i=1}^{k} \frac{\phi_i(e) - \phi_i(\tilde{e})}{\phi_i(e)}\phi_i(e) < 0,$$

or

$$\sum_{i=1}^{k}(\phi_i(e) - \phi_i(\tilde{e})) < 0.$$

Defining $\Phi_i(e) = \sum_{i=1}^{k} \phi_i(e)$ as the CDF for $(\phi_1(e), \ldots \phi_n(e))$, we have

$$\Phi_k(e) < \Phi_k(\tilde{e}) \text{ for all } k < n.$$

But this is the discrete form of first-order stochastic dominance.

(e). Suppose to the contrary that $w_1^* \geq w_2^* \geq \cdot \geq w_n^*$. In this case, $u(w(x_i))$ is a nonincreasing function in $x_i$. As such, FOSD implies for any $e > \tilde{e}$

$$\sum_{i=1}^{n} u(w_i)\phi_i(e) \leq \sum_{i=1}^{n} u(w_i)\phi_i(\tilde{e}).$$

Thus, the agent will choose $e = e_1$ which contradicts the assumption that $e^* > e_1$.

**6** [This question is based on Bolton and Dewatripont, Exercise 12.]

(a). An action of $a_H$ costs the principal (via the IR constraint) an amount $z$ such that

$$\sqrt{z} - 10 \geq 10.$$

$z = 400$ satisifies this expression as an equality. Alternatively, for a cost of 100, the principal can implement the low action. Since the expected difference between the two actions is $1375 - 525 = 850$, the best choice for the principal is to implement $a_H$ at a cost of 400. Profit at the first-best contract is 1375-400=975.

(b). Implementing the low effort in the second best world generates expected profit of $525 - 100 = 425$.

We compare this to the expected profit to implementing the high action. Note that

$$\frac{1}{u'(w_i)} = 2\sqrt{w_i} = 2u_i,$$

where $u_i = u(w_i)$ for short. We know, from class, that in a two-action setting such as this, the optimality conditions are therefore

$$2u_2 = \lambda + \mu\left(\frac{0.75 - 0.25}{0.75}\right),$$

$$2u_1 = \lambda + \mu\left(\frac{0.25 - 0.75}{0.25}\right).$$

In addition, we have the two constraints to satisfy with equality: $0.75u_2 + 0.25u_1 = 20$ and $0.5(u_2 - u_1) = 10$. We thus have 4 linear equations with 4 unknowns. It is a simple matter to check that $u_2 = 25$, $u_1 = 5$, $\lambda = 40$ and $\mu = 15$ uniquely solve these four equations. Thus, $w_2 = u_2^2 = 625$ and $w_1 = u_1^2 = 25$. The second-best contract here which implements $a_h$ yields expected profit of $0.75(1800 - 625) + 0.25(100 - 25) = 900 > 425$.

(c). With the finer information partition, the principal can always detect shirking with positive probability as whenever $\theta_3$ is not true, $e_H$ will never produce $x_1 = 100$, while $a_L$ will. Thus, with positive probability, the agent choosing $e_L$ will be detected shirking. If the principal could pay an arbitrarily low wage in this event, she could implement $e_H$ without any risk. The idea is basically that, conditional on $\{\theta_1, \theta_2\}$, there is a shifting support for output. Because our principal is practically limited to $w \geq 0$ (this is in implicit in the assumption that $u(w) = \sqrt{w}$), we need to check if $w = 0$ is sufficiently low to induce $e_H$ at the full-information wage $w = 400$ from part (a). Specifically, consider the following wage schedule: in the event $\{\theta_1, \theta_2\}$ occurs, the agent is paid $w(100) = 0$ and $w(1800) = 400$; in the event $\{\theta_3\}$, the agent is paid $w(100) = 400$. Now we check that it is incentive compatible for the agent to choose $e_H$. If the agent chooses to work hard, then the agent is assured a wage of 400. If the agent shirks, the agent will receive 400 if $\theta_1$ or $\theta_3$ occur, and $w = 0$ if $\theta_2$. Hence, incentive compatibility requires

$$\sqrt{400} - 10 \geq \frac{1}{2}\sqrt{400} + \frac{1}{2}\sqrt{0},$$

which is satisfied. Hence, the full-information outcome can be implemented for the cost of the information system. Because the difference between the full-information outcome and the second-best outcome without monitoring is $975 - 900 = 75$, the principal would pay up to 75 for the monitoring system.