Ph.D. Core Examination

MICROECONOMICS

Friday, July 12, 2019

WRITE in **black ink** and write only on **one side** of each page.

DO NOT WRITE in the margins.

Be sure the "random number" stamped on your paper is the correct number (the number you were given by the department office). Be sure to put your **random number** in the upper left hand corner of **every page** you write on.

On the **first page** of your examination write:

- the name of the examination
- the date of the examination

Put the **page number** in the upper right hand corner of **every page** of your examination.

When you have completed your examination, write the **total number** *of pages on the* ***back*** of the last page.

Put your examination in the envelope to be turned in.

Results of the examination will be posted by your random number outside the Office of Graduate Student Affairs and sent to you by letter.

TIME: THREE HOURS

1. **You must answer ALL questions and justify ALL answers.**
2. **The exam is out of 180 points. There are three parts, I, II, and III, each worth 60 points.**

Note: The envelope you will be given will contain a colored writing pad to be used for notes and a white writing pad with your random number to be used for writing your final answers.

Part I. (Six questions; 60 points total)

Answer each question as TRUE, FALSE, or UNCERTAIN and justify your answer. If the statement makes multiple predictions evaluate each of those predictions as TRUE, FALSE, or UNCERTAIN. Each question is worth 10 points.

1) If a researcher conducts a randomized experiment that gives a unit of physical capital to half of the households in a population (assume that the donated capital cannot be resold and must be employed by the household that received it), then the increase in measured output for those households relative to the control group (households that do not get the extra unit of capital) would measure the marginal product of capital in that population.

2) If the marginal product of fertilizer is higher in areas with more rainfall (for any given level of other inputs) and all other inputs are sold in a world market, then the price of fertilizer should be higher in areas with more rainfall.

3) Farmers will sell their land when land values rise to the point where the present value of an alternative use (for example housing) exceeds the present value of using the land for farming.

4) If there are two groups of consumers (type A and type B) and the demand for a status good (say the latest fashion item) by both types is decreasing in the aggregate consumption of group B (the followers) then there can be more than one equilibrium price for a given level of supply but not more than one equilibrium quantity at the same price.

5) The supply of labor will tend to be more elastic in the aggregate than it is for individual households in that same population.

6) In an economy with a single consumption good and a constant returns to scale production function that has two inputs (capital and labor), a reduction in the tax rate on capital will benefit workers by more than the reduction in taxes collected by the government if the supply of capital is highly elastic in the real return on capital.

Part II. (Four questions; 60 points total)

II.1. (10 points) State and prove the first welfare theorem for a production economy with private ownership.

II.2. (20 points) Suppose that all consumers $i \in \mathcal{I} = \{1, ..., I\}$ in a pure exchange economy are endowed with a positive amount of each of the n goods and that they all have the same utility function $U(x_1, ..., x_n) = \min(x_1, ..., x_n)$. Suppose also that the aggregate endowment of good k is e_k, and that $0 < e_1 < e_2 < ... < e_n$.

(a) (*10 points*) Prove that a *feasible* allocation $(\mathbf{x}^1, ..., \mathbf{x}^I) \in \mathbb{R}_+^{nI}$ is Pareto efficient if and only if $x_k^i \geq x_1^i$ for every consumer $i \in I$ and for every good $k \in \{2, 3, ..., n\}$.

(b) (*5 points*) What is the amount of good 1 that must be consumed by each consumer in any Walrasian equilibrium allocation? (Hint: Use the first welfare theorem.)

(c) (*5 points*) What must be the price vector in any Walrasian equilibrium? (Assume that prices are nonnegative and that they sum to one.)

II.3. (15 points) Consider the following stage game G.

$1\backslash 2$	A	B
A	1,1	5,0
B	0,5	2,2

The questions below refer to the infinitely repeated game G^∞. Both players have common discount factor $\delta \in (0,1)$. So if u_t is the expected utility of a player in stage $t \in \{0, 1, 2...\}$ of the repeated game, then her average discounted expected utility in G^∞ is $(1-\delta)\sum_{t=0}^{\infty} \delta^t u_t$.

(a) (*5 points*) What is the minimum average discounted expected utility that each player must get in any Nash equilibrium of this repeated game.

(b) (*5 points*) Find the smallest common discount factor $\delta \in (0,1)$ for the two players such that the payoff vector $(2,2)$ can be achieved as the average discounted expected utility vector in some symmetric pure strategy subgame perfect equilibrium in Nash reversion strategies (*symmetric* pure strategies means that both players use the same repeated game pure strategy).

(c) (*5 points*) Given the discount factor that you found in part (b), find a pure strategy subgame perfect equilibrium in Nash reversion strategies that Pareto dominates the average discounted expected utility vector $(2,2)$.

II.4. (15 points) Find a Bayesian-Nash equilibrium of the following game and show that it is unique (up to types who are indifferent). Player 1's type t_1 is uniform on $[0,3]$, player 2's type t_2 is uniform on $[1,4]$ and the types are independent.

$1\backslash 2$	L	R
T	$t_1, 1$	$1, 0$
B	$2, 2$	$1, t_2$

Part III. (One question; 60 points total)

The government purchases electricity from n distinct, risk-neutral electricity suppliers, $i = 1, \ldots, n$. Each firm i produces q_i units of electricity at a cost of

$$C(q_i, \theta_i) = (\alpha - \theta_i)q_i + \frac{\delta}{2}q_i^2,$$

where $\alpha > 1$, $\delta \geq 0$, and θ_i is a measure of cost efficiency that is privately known to firm i. The θ_i's are each independently drawn from $[0, 1]$ according to cumulative distribution function $F(\cdot)$, where $F'(\theta) = f(\theta)$ is continuous and strictly positive on $[0, 1]$. A supplier participates if and only if its expected payoff is nonnegative.

The government's benefit from obtaining $\sum_i q_i$ units of electricity is given by an increasing, strictly concave surplus function, $S(\sum_i q_i)$. Assume that $S'(0) = \infty$.

(a) (6 points) Suppose that the profile of types $\theta = (\theta_1, \ldots, \theta_n)$ is observable to the government. Characterize the full-information, efficient production vector $\{q_1(\theta), \ldots, q_n(\theta)\}$. Indicate the marginal rule that applies to firms that produce positive output and indicate an inequality that applies to firms that are not producing.

For the remainder of the question, assume that θ_i is private information to firm i and that firm i knows θ_i at the time of contracting with the government.

(b) (8 points) The government wishes to design a procurement mechanism that maximizes the expected difference between $S(\sum_i q_i)$ and the total payments made to the firms. Consider direct mechanisms of the form $\{(q_i(\hat{\theta}), t_i(\hat{\theta})\}_{i,\hat{\theta}}$ where $q_i(\hat{\theta})$ is the output firm i is required to produce when the vector of reported types is $\hat{\theta} = (\hat{\theta}_1, \ldots, \hat{\theta}_n)$, and $t_i(\hat{\theta})$ *is the corresponding payment the government makes to firm* i. *State a monotonicity condition and an integral condition that together are necessary and sufficient for (Bayesian) incentive compatibility in this problem.* (No proof is needed here; just provide the two conditions.)

(c) (20 points) Show how to solve for the government's optimal allocation, $\{q_1(\cdot), \ldots, q_n(\cdot)\}$. Make explicit any assumptions that you use. In your answer, provide a marginal condition that characterizes the outputs of the producing firms, and an inequality that characterizes the inactive firms.

(d) (8 points) What inefficiencies are introduced in the government's optimal allocation compared to (a), both in terms of aggregate production and in terms of misallocations across firms? Explain.

(e) (8 points) As a special case, consider $\delta = 0$. Which firm(s) are chosen to produce? Explain.

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(f) (10 points) Suppose that the government also cares about the amount of pollution $x_i \in [0, \bar{x}]$ that is generated by firm i, and suppose that x_i is contractible. The government's objective is now $S(\sum_i q_i) - P(\sum_i x_i) - \sum_i t_i$, where P is strictly increasing and convex in aggregate pollution, $\sum_i x_i$. Each firm's cost of producing q_i depends on its type and its pollution level,

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$$C(q_i, x_i, \theta_i) = (\alpha - \theta_i)(\bar{x} - x_i)q_i + \frac{\delta}{2}q_i^2.$$

If the government can design a procurement mechanism over output, pollution, and transfers, $\{(q_i(\hat{\theta}), x_i(\hat{\theta}), t_i(\hat{\theta})\}_{i,\hat{\theta}}$, what additional marginal condition for pollution, $x_i(\theta)$, should be satisfied for a producing firm when $x_i(\theta) \in (0, \bar{x})$? Is pollution higher or lower than the full-information outcome? Explain. (No detailed derivation is needed here.)