Problem Set 1 - Evens

Six Nice People

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1 Question 2 - Sarah

There are two commodities

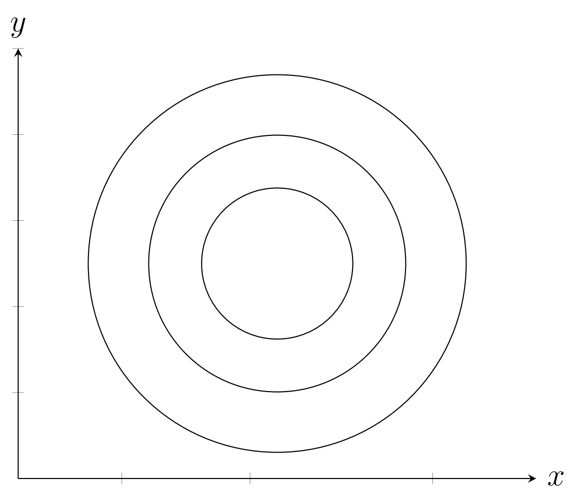
(a) *Sketch some preferences which have a bliss point and which can be represented by a continuous utility function. (A bliss point is a point which is preferred or indifferent to any other point.)*

Consider the utility function

$$u(x, y) = -((x - h)^2 + (y - k)^2).$$

Here utility is measured as the negative of a diameter of a circle. Therefore, as the diameter decreases, utility increases. This means the center of the circle (h, k) is a bliss point. Here, utility is at the highest it can be, 0.

In this plot, $h = k$, and utility increases as one moves toward the center of the concentric indifference curves



(b) *No matter what his income. Jones will demand fifteen units of commodity one provided he can afford it and provided the price of commodity 2 exceeds that of commodity one. Is this consistent with Axioms 1-5 and also Axioms 4' and 5'?*

I will prove this is not consistent with Axioms 1-5 due to strict monotonity (A4) which states for all $x_0, x_1 \in \mathbb{R}^n_+$ if $x_0 \geq x_1$ then $x_0 \succsim x_1$, while if $x_0 > x_1$, then $x_0 \succ x_1$.

Suppose preferences satisfy A1-A5. Let x_2 be some arbitrary consumption for good 2. Let's construct a subsequence $\overline{x_2^{n_k}}$ which converges to x_2 and $\overline{x_2^{n_k}} \geq x^2$. Another subsequence $\underline{x_2^{n_k}}$ which converges to x_2 and $\underline{x_2^{n_k}} \leq x_2$. By A4 $(16, \overline{x_2^{n_k}}) \succsim (15, x_2)$. By Jones' preferences, $(15, x_2) \succsim (16, \underline{x_2^{n_k}})$. By using the fact that these subsequences both converge to x_2, $(16, x_2) \succsim (15, x_2)$ and $(15, x_2) \succsim (16, x_2)$ Since the relation is both complete and transitive, this implies $(15, x_2) \sim (16, x_2)$

By strict convexity (A5), $\forall \lambda \in (0,1), (16\lambda + (1-\lambda)15, x_2) = (15 + \lambda, x_2) \succ (16, x_2)$ but this contradicts strict monotonicity because $\lambda < 1$ and $(15 + \lambda, x_2) \leq (16, x_2)$. Axioms 1-5 cannot hold simultaneously.

Axiom 4' states local non-satiation: For all $x^0 \in \mathbb{R}^n_+$, and for all $\epsilon > 0$, there exists some $x \in B_\epsilon(x^0) \cap \mathbb{R}^n_+$ such that $x \succ x^0$.

Consider the function

$$u(x, y) = -(x - 15)^2 + (y)^{\frac{1}{2}}$$

This upholds local non-satiation and is still convex. However, this function is not strictly monotonic. Normalize utility to 1 then consider any point where $x > 15$. The bundle has strictly more of both goods yet is indifferent to any point at $x \leq 15$.
These preferences are consistent with Jones' behavior where he prefers 15 goods no matter what and is indifferent to more or less of it.

Axiom 5' states convexity (not strict): If $x^1 \succsim x^0$, then $tx^1 + (1-t)x^0 \succsim x^0$ for all $t \in [0, 1]$.

Consider the functions

$$u(x, y) = \log\left(\frac{x}{15}\right) + y, x \leq 15$$
$$u(x, y) = y, x > 15$$

Here we have a utility function that goes flat after $x = 15$. This means Axiom 4 of strict monotonicity is satisfied. However, a flat indifference curve is only convex and not strictly convex which is consistent with Axiom 5'.
These preferences are consistent with Jones' behavior that makes him indifferent to any bundle with more than 15 goods.

So we found that some preferences exist for Jones that uphold Axioms 1-3, 4' and 5 or 1-4 and 5'. This implies some preferences exist for 1-3,4' and 5' because both 4 and 5 are stronger than 4' and 5'.

Question 4

Take $l = 2$ and consider preferences $\succeq$ on $\mathbb{R}^2_+$ defined by

$$(x_1, x_2) \succeq (y_1, y_2) \text{ iff } x_1 1_{\{x_2 > 1\}} \geq y_1 1_{\{y_2 > 1\}}$$

Where $1_{\{\}}$ is the indicator function. Note that the utility function $u(x_1, x_2) := x_1 1_{\{x_2>1\}}$ represents $\succeq$ immediately from the definition.

Suppose for a contradiction that $\succeq$ has a continuous utility representation $\tilde{u}$. Consider the sequence $(x_1^{(n)}, x_2^{(n)}) := \big(1, (1 - \frac{1}{n})\big) \to (1, 1)$. Note that for any $m, n \in \mathbb{N}$ we have that $(x_1^{(n)}, x_2^{(n)}) \sim (x_1^{(m)}, x_2^{(m)})$ since $u(1, 1 - \frac{1}{n}) = u(1, 1 - \frac{1}{m}) = 0$. So in our representation $\tilde{u}$ we must also have that $\tilde{u}(x_1^{(n)}, x_2^{(n)}) = c$ for all $n \in \mathbb{N}$ for some constant $c \in \mathbb{R}$.

But we also have that $(1, 1) \succ (x_1^{(n)}, x_2^{(n)})$ for all $n \in \mathbb{N}$ since $u(1, 1) = 1 > 0 = u(1, 1 - \frac{1}{n})$. So again in our representation we must have that $\tilde{u}(1, 1) > c$. Now note that since it is a constant we have that $\tilde{u}(x_1^{(n)}, x_2^{(n)}) \to c \neq \tilde{u}(1, 1)$. This violates the continuity of $\tilde{u}$ and we have a contradiction as desired.

Question 6

Part 1

Show that for if X is any finite set, and $\succsim$ satisfies Axioms 1 and 2 on X, then $\succsim$ can be represented by a utility function.

We will start by restating the two axioms as:

Axiom 1. *Completeness: For all $x^1, x^2 \in \mathbb{R}^n_+$, either $x^1 \succsim x^2$ or $x^2 \succsim x^1$*

Axiom 2. *Transitivity: For any 3 bundles x^1, x^2, x^3 if $x^1 \succsim x^2$ and $x^2 \succsim x^3$ then $x^1 \succsim x^3$.*

Which, when you have these two Axioms (1 and 2) you call this a preference relation. In question 1 and through proof in class we showed that if you have a preference relation $\succsim$ then you simultaneously have $\precsim$, $\sim$, and $\succ$ and transitivity of those relationships.

With this take a finite collection of N bundles x_i indexed by $i \in 1, 2, 3...N$. Choose any $i, j \in \mathcal{I}$ and by completeness we know the relationship between x_i, x_j for all $i, j \in \mathcal{I}$. Because all bundles have a preference relation, by completeness, and because these relations are transitive we can create an ordering of bundles from most preferred bundle to least preferred bundle.

We define a set of least preferred bundles as the set of bundles in which it is not strictly preferred to any other bundle. More technically define $A_1 = \{x' : x' \in X, s.t. \nexists i \in \mathcal{I}, s.t. x' \succ x_i\}$. Take that/those bundles and assign $u(x') = 1, \forall x' \in A$ and redefine $X_2 = X_1 \setminus A_1$.

For the remaining bundles in X_2 we can repeat the process but assigning the now least preferred bundles, call them $x' \in A_2$ a value of $u(x') = 2$ before removing the bundles from the set. As such we can define the process in which we assign the least preferred bundles of $A_m \in X_m$ a utility of $u(x') = m$ where m is the iteration of such a process.

Because, as we showed above that there exists at least one least preferred bundle in every finite set of bundles with a preference relation satisfying our two axioms, we know that this process has at most N possible iterations before we have successfully assigned a utility value for all bundles in the set X_1. Furthermore, as this process was constructed generally we can use this mapping from $u : \mathbb{R}^n_+ \to \mathbb{R}$ where $u(x_i) \geq u(x_j) \iff x_i \succsim x_j$ for all $i, j \in \mathcal{I}$.

Part 2

But what if we do not have completeness? Suppose $\exists x^1, x^2 \in X_1$ s.t. $x^1 \not\succsim x^2$ and $x^2 \not\succsim x^1$. We will prove that one cannot represent this non-preference relation through a proof by contradiction:

Assume we can project these preferences onto a utility function on the real line. As such we have two possible scenarios:

1. $u(x^1) \geq u(x^2)$
2. $u(x^1) \leq u(x^2)$

However, we can clearly ee that this would lead to a contradiction. Take the first scenario:

By definition of the utility function representation $u(x^1) \geq u(x^2) \Rightarrow x^1 \succsim x^2$, immediately violating our non-completeness. Similarly $u(x^2) \geq u(x^2) \Rightarrow x^2 \succsim x^1$. violates the same non-completeness.

As such we must have completeness in order to turn a preference relation into a utility function and for a utility function to be a proper representation of preferences.

Question 8

Show that if $\succsim$ satisfies Axiom 2 on $\mathbb{R}^l_+$ then $\forall x, y \in \mathbb{R}^l_+$ the sets $\sim(x)$ and $\sim(y)$ are either disjoint or equal.

By 1.a, $\succsim$ being transitive implies $\sim$ is transitive. Let's look at the intersection of $\sim(x)$ and $\sim(y)$ and call it S. Suppose $S \neq \varnothing$ and $\sim(x) \neq \sim(y)$. Then, there exists some element $z \in S$ which implies $z \in \sim(x)$ and $z \in \sim(y)$ Also, there exists some element $w \in \sim(x)$ where $w \notin \sim(y)$. Then we have $w \sim z$ and $z \sim y$ which by transitivity implies $w \sim y$. This is a contradiction that proves a nonempty intersection implies equality. In the other case, if S is the empty set, then the sets $\sim(x)$ and $\sim(y)$ are disjoint. Therefore, either $S = \varnothing$ or $\sim(x) = \sim(y)$.

Question 10

Let $\succeq$ be consumer 2's preferences.

a.

Suppose $\succeq$ is transitive and in what follows let $x = (x_1, x_2)$ and $y = (y_1, y_2)$ be arbitrary $x, y \in \mathbb{R}^2_+$. We show completeness and strict monotonicity in turn.

Completeness (Show: $x \succeq y$ or $y \succeq x$)

There are six mutually exlusive and exhaustive cases. Using the definition from the question we find:

1. $x_1x_2 > y_1y_2$
   Since x lies on a strictly higher Cobb-Douglas indifference curve than y we have $x \succ y$ whether or not either bundle is on the 45° line. So $x \succeq y$.

2. $y_1y_2 > x_1x_2$
   Here analogously $y \succ x$ so $y \succeq x$.

3. $x_1x_2 = y_1y_2, x_1 = x_2, y_1 \neq y_2$
   We have $x \succ y$ so also $x \succeq y$.

4. $x_1x_2 = y_1y_2, x_1 \neq x_2, y_1 = y_2$
   Here we have $y \succ x$ so also $y \succeq x$.

5. $x_1x_2 = y_1y_2, x_1 \neq x_2, y_1 \neq y_2$
   In this case we have $x \sim y$ so both $x \succeq y$ and $y \succeq x$.

6. $x_1x_2 = y_1y_2, x_1 = x_2, y_1 = y_2$
   Here $x_1 = x_2 = y_1 = y_2$ so $x \sim y$ and again both $x \succeq y$ and $y \succeq x$.

So in all six cases at least one of $x \succeq y$ or $y \succeq x$ holds, as desired.

Strict monotonicity (Show: $x \geq y$ entails $x \succeq y$ and $x \gg y$ entails $x \succ y$)

As written, the preferences are not strictly monotonic. To see this, note that $(1,0) \geq (0,0)$ but $(0,0) \succ (1,0)$ since $(0,0)$ lies on the 45° line and $u(1,0) = u(0,0) = 0$ so they lie on the same Cobb-Douglas indifference curve. To proceed, we restrict attention to the strictly positive orthant and consider $x, y \in \mathbb{R}^2_{++}$.

Suppose $x \geq y$. Then $x_1 \geq y_1$ and $x_2 \geq y_2$. So $x_1 x_2 \geq y_1 y_2$. Now either $x = y$ or $x \neq y$. If $x = y$ then $x \sim y$ and $x \succeq y$. If $x \neq y$ then the inequalities hold strictly for one of the coordinates: either $x_1 > y_1$ or $x_2 > y_2$. Given that we are in the strictly positive orthant this implies that $x_1 x_2 > y_1 y_2$ (note that this is not the case when one of the coordinates can be 0 as argued above!). So $x \succ y$ and so $x \succeq y$.

Now suppose that $x \gg y$ so that $x_1 > y_1$ and $x_2 > y_2$. We then have that $x_1 x_2 > y_1 y_2$ and so $x \succ y$ as desired. This then establishes strict monotonicity.

b

No. Suppose for a contradiction that $\tilde{u}$ represents $\succeq$ and is continuous. Now consider the sequence $(\frac{n+1}{n}, \frac{n}{n+1}) \to (1,1)$. Note that $(1,1) \succ (\frac{1}{2}, 2)$ as they are on the same Cobb-Douglas indifference curve but $(1,1)$ is on the 45° line. Given that $\tilde{u}$ represents $\succeq$ we must have

$$\tilde{u}(1,1) > \tilde{u}(\frac{1}{2}, 2) = \tilde{u}\left(\frac{n+1}{n}, \frac{n}{n+1}\right) \forall n \in \mathbb{N}$$

So $\tilde{u}\left(\frac{n+1}{n}, \frac{n}{n+1}\right)$ is constant for all n and does not converge to $\tilde{u}(1,1)$ even though $(\frac{n+1}{n}, \frac{n}{n+1}) \to (1,1)$. This contradicts the continuity of $\tilde{u}$ as desired.

Question 12

Solving for x,y

We will set up the Lagrangian as:

$$\mathcal{L} = x^2 y + xy + \lambda(M - p_x x - p_y y)$$

with $x, y > 0$. Here, p_x and p_y are the price of goods x and y respectively. M is the agent's income. Taking the first order conditions yield the following:

$$[\frac{\partial \mathcal{L}}{\partial x}] : 2xy + y - \lambda p_x = 0 \tag{1}$$

$$[\frac{\partial \mathcal{L}}{\partial y}] : x^2 + x - \lambda p_y = 0 \tag{2}$$

$$[\frac{\partial \mathcal{L}}{\partial \lambda}] : M - p_x x - p_y y = 0 \tag{3}$$

We know these hold with equality by the fact that u is increasing in both x and y. Furthermore, we know that $\lambda > 0$ from equations (1) and (2) from $x, y > 0$ and $p_x, p_y > 0$. We can rearrange these same equations to get the result that:

$$\frac{p_x}{p_y} = \frac{2xy + y}{x^2 + x} \Rightarrow yp_y = \frac{x^2 + x}{2x + 1}p_x$$

Where we can plug this y equation into equation 3 to get

$$M = p_x x + p_y y \Rightarrow M = p_x x + \frac{x^2 + x}{2x + 1}p_x$$

Which after some rearranging yields the expression suitable for using the quadratic formula.

$$3x^2(p_x) + 2x(p_x - M) - M = 0$$

Because $x > 0$ we use the positive roots and eventually we end up with the expression:

$$x = \frac{2(M - p_x) + \sqrt{(2(p_x - M))^2 - 4(3p_x)(-M)}}{2(3p_x)} = \frac{(M - p_x) + \sqrt{(p_x - M)^2 + 3p_x M}}{3p_x}$$

And finally we can plug in $x = \frac{(M-p_x)+\sqrt{(p_x-M)^2+3p_xM}}{3p_x}$ to get y after rearranging $M = p_x x + pyy \Rightarrow y = \frac{M - p_x x}{p_y}$:

$$y = \frac{M}{p_y} - \frac{p_x}{p_y}\frac{(M - p_x) + \sqrt{(p_x - M)^2 + 3p_x M}}{3p_x}$$

Quasi-Concavity

We can prove quasi-concavity by showing that the indifference curves under this utility structure are convex. By doing so we will show that u is quasi-concave by showing that $\succeq (x, y)$ is a convex set. Take:

$$u(x, y) = x^2y + xy$$

We can take the derivative of this expression with respect to x and y to get the following partial derivatives.

$$u_x = 2xy + y, u_{xx} = 2y, u_y = x^2 + x, u_{yx} = 2x + 1$$

We start showing that given a value of utility we know that the u function exhibits:

$$\delta u = u_x \delta x + u_y \delta y = 0 \Rightarrow \frac{\delta y}{\delta x} = -\frac{u_x}{u_y}$$

And we can look at the absolute value of this derivative because the slope of the indifference curves must be decreasing so we want to make sure the magnitude in which they do is getting smaller:

$$\frac{\delta \frac{\delta y}{\delta x}}{\delta x} = \frac{u_{xx}u_y - u_x u_{yx}}{u_y^2} = \frac{2y(x^2 + x) - (2xy + y)(2x + 1)}{(x^2 + x)^2}$$

$$\frac{2y(x^2+x)-(2xy+y)(2x+1)}{(x^2+x)^2} = \frac{2x^2y+2xy-(4x^2y+4xy+y)}{(x^2+x)^2} = -\frac{(2x^2y+2xy+y)}{(x^2+x)^2}$$

$$-\frac{(2x^2y+2xy+y)}{(x^2+x)^2} < 0, x, y \in \mathbb{R}$$

Thus we have found that the indifference curve is necessarily convex and thus the utility function exhibits quasi-concavity. We need the indifference curves to be convex for us to be able to use the Lagrangian/F.O.C. method of deriving the Marshallian demand curve. Quasi-concavity in u allows us to find a solution to a maximization problem.

Question 14

For each of the utility functions above, derive directly the indirect utility function and the expenditure function. Next, verify the identities $v(p, e(p, u)) = u$ and $e(p, v(p, \omega)) = \omega$. Finally, show how the first identity can be used to obtain the expenditure function once the indirect utility function has been derived.

The indirect utility function is defined as $v(p, y) = u(x^m(p, y))$ and the expenditure function is defined as $e(p, u) = px^h(p, u)$.

(a) $u(x, y) = x^\alpha y^{1-\alpha}, 0 < \alpha < 1$

We know from 13a that the Marshallian demand functions are $x(p_x, \omega) = \frac{\omega}{p_x}\alpha$ for good 1 and $y(p_y, \omega) = \frac{\omega}{p_y}(1-\alpha)$ for good 2. Substituting this into the utility function we have,

$$v(p, \omega) = (\frac{\omega}{p_x}\alpha)^\alpha(\frac{\omega}{p_y}(1-\alpha))^{1-\alpha}.$$

We also know from 13a the Hicksian demand functions are $x^h = (\frac{\alpha}{p_x}\left(\frac{1}{\alpha^\alpha}\frac{1}{(1-\alpha)^{1-\alpha}}\right)p_x^\alpha p_y^{1-\alpha}u$ for good 1 and $y^h = \frac{(1-\alpha)}{p_y}\left(\frac{1}{\alpha^\alpha}\frac{1}{(1-\alpha)^{1-\alpha}}\right)p_x^\alpha p_y^{1-\alpha}u)$ for good 2. Substituting this into the expenditure function we have,

$$e(p, u) = p_x(\frac{\alpha}{p_x}\left(\frac{1}{\alpha^\alpha}\frac{1}{(1-\alpha)^{1-\alpha}}\right)p_x^\alpha p_y^{1-\alpha}u + p_y\frac{(1-\alpha)}{p_y}\left(\frac{1}{\alpha^\alpha}\frac{1}{(1-\alpha)^{1-\alpha}}\right)p_x^\alpha p_y^{1-\alpha}u)$$

To show $v(p, e(p, u)) = u$, we simplify the indirect utility function and substitute in the expenditure function. We have

$$(\frac{\alpha}{p_x})^\alpha(\frac{(1-\alpha)}{p_y})^{1-\alpha}\left(p_x u(\frac{p_y}{p_x}\frac{\alpha}{1-\alpha})^{1-\alpha} + p_y u(\frac{p_x}{p_y}\frac{1-\alpha}{\alpha})^\alpha\right)$$

which simplifies to u.

To show $e(p,v(p,y)) = y$, we simplify the expenditure function and substitute in the utility function we have

$$[p_1(\frac{p_1}{p_2}\frac{\alpha}{1-\alpha})^{1-\alpha} + p_2(\frac{p_1}{p_2}\frac{\alpha}{1-\alpha})^{-\alpha}][(\frac{\alpha}{p_1})^{\alpha}(\frac{(1-\alpha)}{p_2})^{1-\alpha}]y$$

which simplifies to y.

(b) $u(x,y) = x^\alpha + y^\alpha$, $0 < \alpha < 1$
We know from 13b that the Marshallian demand functions are $x(p_x,\omega) = \frac{\omega}{p_x + p_y(\frac{p_y}{p_x})^{\frac{1}{1-\alpha}}}$ for good 1 and $y(p_y,\omega) = \frac{\omega}{p_y + p_x(\frac{p_x}{p_y})^{\frac{1}{1-\alpha}}}$ for good 2. Substituting this into the utility function we have,

$$v(p,\omega) = \left(\frac{\omega}{p_x + p_y(\frac{p_y}{p_x})^{\frac{1}{1-\alpha}}}\right)^{\alpha} + \left(\frac{\omega}{p_y + p_x(\frac{p_x}{p_y})^{\frac{1}{1-\alpha}}}\right)^{\alpha}.$$

We also know from 13b the Hicksian demand functions are $x^h = \left(\frac{u}{1+(\frac{p_y}{p_x})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}$ for good 1 and $y^h = \left(\frac{u}{1+(\frac{p_x}{p_y})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}$ for good 2. Substituting this into the expenditure function we have,

$$e(p,u) = p_x\left(\frac{u}{1+(\frac{p_y}{p_x})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}} + p_y\left(\frac{u}{1+(\frac{p_x}{p_y})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}$$

To show $v(p,e(p,u)) = u$, we simplify the indirect utility function and substitute in the expenditure function to get

$$\begin{aligned} v(p,e(p,u)) = &\left(\left(\frac{1}{p_x + p_y(\frac{p_y}{p_x})^{\frac{1}{1-\alpha}}}\right)^{\alpha} + \left(\frac{1}{p_y + p_x(\frac{p_x}{p_y})^{\frac{1}{1-\alpha}}}\right)^{\alpha}\right)\times \\ &\left(p_x\left(\frac{u}{1+(\frac{p_y}{p_x})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}} + p_y\left(\frac{u}{1+(\frac{p_x}{p_y})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}\right)^{\alpha} \end{aligned}$$

which simplifies to u.

To show $e(p,v(p,y)) = y$,we simplify the expenditure function and substitute in the utility function we have

$$\begin{aligned} e(p,u) = &\left(p_x\left(\frac{1}{1+(\frac{p_y}{p_x})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}} + p_y\left(\frac{1}{1+(\frac{p_x}{p_y})^{\frac{\alpha}{1-\alpha}}}\right)^{\frac{1}{\alpha}}\right)\times \\ &\left(\left(\frac{1}{p_x + p_y(\frac{p_y}{p_x})^{\frac{1}{1-\alpha}}}\right)^{\alpha} + \left(\frac{1}{p_y + p_x(\frac{p_x}{p_y})^{\frac{1}{1-\alpha}}}\right)^{\alpha}\right)^{\frac{1}{\alpha}} \times \omega \end{aligned}$$

which simplifies to ω.

(c) $u(x,y) = \min\{x,y\}$
From 13.c we know the $v(p,y) = \frac{\omega}{p_x+p_y}$ and $e(p,u) = (p_x+p_y)u$ so, using substitution, it is easy to see that $v(p,e(p,u)) = u$ and $e(p,v(p,\omega)) = \omega$

(d) $u(x,y) = x+y$

From 13.d we have $v(p,y) = \begin{cases} (\frac{\omega}{p_x}) & p_x < p_y \\ \frac{\omega}{p} & p_x = p_y \\ \frac{\omega}{p_y} & p_x > p_y \end{cases}$ and $e(p,u) = \begin{cases} up_x & p_x < p_y \\ up & p_x = p_y \\ up_y & p_x > p_y \end{cases}$

In each case, it is easy to see by substitution that the identities hold.

(e) *Show how the first identity can be used to obtain the expenditure function once the indirect utility function has been derived*

First, we can separate y from $u(x(p,y))$ to get $u(x(p))y = u$. Then rearranging we have $y = \frac{u}{u(x(p))}$ where $u(x(p))$ is the expression left when separating y from the indirect utility function.