Ph.D. Core Examination

MICROECONOMICS

Wednesday, July 6, 2016

WRITE in **black ink** and write only on **one side** of each page.

DO NOT WRITE in the margins.

Be sure the "random number" stamped on your paper is the correct number (the number you were given by the department office). Be sure to put your **random number** in the upper left hand corner of **every page** you write on.

On the **first page** of your examination write:

- the name of the examination
- the date of the examination

Put the **page number** in the upper right hand corner of **every page** of your examination.

When you have completed your examination, write the **total number** of pages on the **back** of the last page.

Put your examination in the envelope to be turned in.

Results of the examination will be posted by your random number outside the Office of Graduate Student Affairs and sent to you by letter.

TIME: THREE HOURS

1. **You must answer ALL questions and justify ALL answers.**

2. **The exam is out of 180 points. Part I is worth 60 points. Parts II-V are each worth 30 points.**

Note: The envelope you will be given will contain a colored writing pad to be used for notes and a white writing pad with your random number to be used for writing your final answers.

I. BASIC PRICE THEORY QUESTIONS (60 points; 10 points each)

Answer each of the following 6 questions as TRUE, FALSE, or UNCERTAIN, and justify your answer in a half of a page or less. Your grade will depend entirely on your justification.

1) Automobile manufacturers are updating their technologies for locking vehicles to prevent theft. A truly theft-proof locking system would be unfortunate for them, because then they would lose the future automobile sales associated with additional updates to the locking systems.

2) For a competitive industry, a lump sum tax (that requires each firm in an industry that produces a positive level of output to pay a fixed annual amount to the government) will be more efficient than a per unit tax (that taxes each unit of output produced a fixed amount) if the two tax systems raise the same amount of revenue for the government.

3) Factor-augmenting technical progress can change relative factor rental rates, but it cannot actually reduce any of the rental rates (relative to the price of output).

4) A great many manufacturers use machines and labor in fixed proportions. That is inconsistent with an industry or economy wide aggregate Cobb-Douglas production function.

5) An improvement in the quality of cell phones produced by one cell phone manufacturer would be expected to reduce the sales of other cell phone manufacturers' cell phones.

6) Making it easier to quit smoking (say by the introduction of a new pill that reduces the negative effects of quitting) can increase the number of smokers and the total quantity of cigarettes smoked.

II. GENERAL EQUILIBRIUM AND CHOICE THEORY (30 points)

(a) (i) (5 points) Consider a private ownership economy with production. Define the aggregate production set and the aggregate profit function and prove that an aggregate production plan maximizes aggregate profits at the price vector $\mathbf{p} \in \mathbb{R}^n_+$ if and only if there are individual-firm production plans that generate that aggregate production plan and that each maximize the respective individual firm's profits at the price vector $\mathbf{p}$. Point out where, in your proof, you use the assumption of no externalities in production.

(ii) (5 points) Consider a pure exchange economy with two consumers, each with strictly monotonic preferences over bundles in $\mathbb{R}^n_+$. Suppose that there is a per-unit tax $\tau > 0$ placed on consumer 1's (perfectly observable) consumption of good 1 so that tax revenue $T > 0$ can be collected and redistributed as a lump sum income transfer to consumer 2. Suppose further that, given the before tax price vector $\mathbf{p}^* \in \mathbb{R}^n_{++}$, all markets clear in this economy when taxes are taken into account, and that the total tax raised from consumer 1's consumption of good 1 is exactly T. Consider any proof of the first welfare theorem and show the step in that proof that fails in this scenario with taxes. You need not give the entire proof, but you must provide the step that fails and explain why it fails.

(b) (i) (5 points) Define a social choice function and state the Gibbard-Satterthwaite Theorem (you need **not** define the other terms used in the statement of the theorem).

(ii) (15 points) There are three individuals $i = 1, 2, 3$, and the set of social alternatives is $A = \{1, 2, ..., K\}$. The preferences of the three individuals come from the following **restricted domain**. Each individual i has preferences over the K alternatives that can be represented by a utility function $u_i : \{1, ..., K\} \to \mathbb{R}$ that assigns distinct utility numbers to distinct alternatives (i.e., no indifference) and that satisfies the following strict quasi-concavity condition:

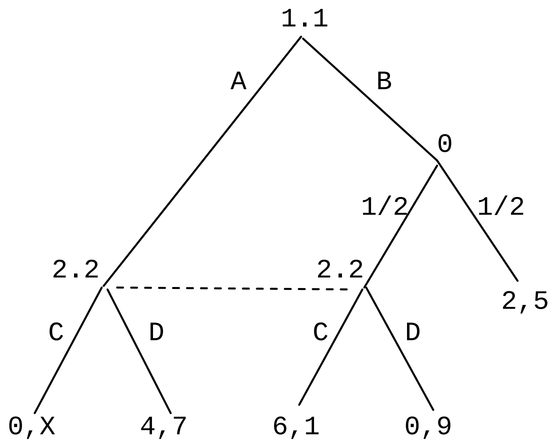
$$u_i(k) > \min(u_i(j), u_i(l)) \text{ for all alternatives } j < k < l.$$

Let $\mathcal{D}$ denote the the set of utility functions on A that satisfy the above conditions (i.e., no indifference and strictly quasi-concave).

Define the social choice function, $f : \mathcal{D} \times \mathcal{D} \times \mathcal{D} \to A$ as follows. For any three utility functions $u_1, u_2, u_3 \in \mathcal{D}$, let $a_i^* \in A$ be the most preferred social alternative according to u_i. If $a_i^* = a_{i'}^*$ for at least two individuals $i \neq i'$, then the social choice is $f(u_1, u_2, u_3) = a_i^*$. But if all three most preferred alternatives are distinct, i.e., $a_i^* < a_{i'}^* < a_{i''}^*$, then the social choice is $f(u_1, u_2, u_3) = a_{i'}^*$, i.e., the social choice is the alternative the lies "between" the other two. Prove that f is strategy-proof on the restricted domain $\mathcal{D} \times \mathcal{D} \times \mathcal{D}$.

III. GAME THEORY QUESTION (*30 points: 10 points for each part*)

Consider the following extensive-form game, which begins with a move by player 1.



(a) Suppose that X=9 (and this fact is common knowledge among the players). Construct the normal representation of this game in strategic form (the normal form).

(b) Find a sequential equilibrium of this game with X=9. Show all move probabilities, and show what player 2 would believe is the probability of 1 having chosen A (the left "2.2" node) if 2 got to move in this equilibrium.

(c) Now suppose that either X=8 or X=10, each with probability 1/2, and the actual value of X is privately known only by player 2 when she moves in the game. (That is, X is player 2's type.) Player 1 only knows that X is equally likely to be 8 or 10. Find a Bayesian equilibrium of this game. Be sure to completely describe both players' strategies.

IV. INFORMATION ECONOMICS AND MECHANISM DESIGN (30 points)

Consider the following specification of the Mussa-Rosen Model. There is a seller who can produce a perfectly divisible good. His cost of producing $x \geq 0$ units is $x^2/2$. There is a single buyer with quasi-linear utility, whose utility from consuming x units of the good at price $p(x)$ is $x\theta - p(x)$, where θ is the buyer's privately known type. The buyer's type distribution, F, is common knowledge. The support of F is the interval $[1, \infty)$ and F has a strictly positive and continuous density f $(= F')$. In addition, the buyer's virtual valuation is strictly increasing in her type. Trade between the buyer and the seller proceeds as follows. The seller posts a price schedule $\{p(x)\}_{x \geq 0}$. Then the buyer chooses x and the seller delivers x units of the good at price $p(x)$.

Suppose that the profit-maximizing price schedule for the seller is $p^*(x) = x^2/2 + x$, for all $x \geq 0$, and that the seller's profits under the price-schedule $p^*(\cdot)$ are finite. What is the buyer's type distribution, F?

V. INFORMATION ECONOMICS AND MECHANISM DESIGN (30 points)

(a) (5 points) State the Myerson Satterthwaite Theorem.

(b) (5 points) Provide an example in which the conclusion of the Myerson-Satterthwaite theorem fails and in which all of the assumptions required for the Myerson-Satterthwaite theorem hold except for the assumption that no value for the buyer or for the seller has positive probability.

(c) (5 points) State the revenue equivalence theorem.

(d) (5 points) Suppose that N bidders have independent private values in $[0, 1]$ each distributed according to the cumulative distribution function F with positive and continuous density f. Suppose that the bidders' common virtual value function $v - \frac{1-F(v)}{f(v)}$ is nondecreasing but not necessarily strictly increasing. In particular, suppose that the virtual value function is positive and constant on the interval of values $[1/4, 3/4]$. Nevertheless, prove that a first-price auction with an optimally chosen reserve price maximizes the seller's expected revenue.

(e) (10 points) Consider a marriage market with N men and N women and in which each individual has strict preferences over the individuals on the other side of the market. Suppose also that all individuals strictly prefer being matched with any individual on the other side of the market to being single. Prove that the matching that results from the male-proposing Gale-Shapley algorithm is the worst stable matching for all of the women.