Price Theory, III | **Professor Lars Stole**
Economics 33003 | **Spring 2020**

Assignment 1: Due Friday, April 17, prior to the start of the review session

Problem 1 (MWG, Exercise 13.B.3 - variation) Consider a *positive-selection* variation of the model discussed in MWG (13.8) in which $r(\cdot)$ is a continuous, *strictly decreasing* function of θ. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \overline{\theta}]$

(a). Show that the *more capable* workers are the ones choosing to work at any wage for which some workers are employed and others are not.

(b). Show that if $r(\theta) > \theta$ for all θ, then the resulting competitive equilibrium is Pareto efficient.

(c). Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves too much employment relative to the Pareto-optimal allocation of workers.

Problem 2 (JR, Exercise 8.7 - variation) Consider the following market for used cars. There are many sellers of used cars. Each seller has exactly one used car to sell and is characterized by the quality of the used car he wishes to sell. Let $\theta \in [0, 1]$ index the quality of a used car and assume that θ is uniformly distributed on $[0, 1]$. If a seller of type θ sells his car (of quality θ) for a price of p, his utility is p. If he does not sell his car, then his utility is $r(\theta)$, which is increasing in θ and $r(0) = 0$. Buyers of used cars receive utility $\theta - p$ if they buy a car of quality θ at price p and receive utility 0 if they do not purchase a car. There is asymmetric information regarding the quality of used cars. Sellers know the quality of the car they are selling, but buyers do not know its quality. Assume that there are not enough cars to supply all potential buyers.

(a). Argue that in a competitive equilibrium under asymmetric information, we must have $E[\theta | p \geq r(\theta)] = p$.

(b). Show that if $r(\theta) = \frac{\theta}{2}$, then every $p \in (0, 1/2]$ is an equilibrium price.

(c). Find the equilibrium price when $r(\theta) = \sqrt{\theta}$. Describe the equilibrium in words. In particular, which cars are traded in equilibrium?

(d). Find an equilibrium price when $r(\theta) = \theta^3$. How many equilibria are there in this case?

(e). Are any of the preceding outcomes Pareto efficient? Describe the inefficiencies (if any) in each setting, relative the the full-information efficient allocations.

Problem 3 A buyer and a seller have valuations v_b and v_s. It is common knowledge that there are gains from trade (i.e., that $v_b > v_s$), but the size of the gains is private information, as follows: the seller's valuation is uniformly distributed on $[0, 1]$; the buyer's valuation $v_b = k \cdot v_s$, where $k > 1$ is common knowledge; the seller knows v_s (and hence v_b) but the buyer does not know v_b or v_s.

(a). Suppose the buyer makes a single offer, p, which the seller either accepts or rejects. What is the perfect Bayesian equilibrium when $k < 2$? When $k > 2$?

(b). Suppose instead the seller makes the offer, p. What is the PBE when $k < 2$? When $k > 2$?

Problem 4 Consider the MWG labor-market signaling game from class. Assume that $c(e, \theta) = e(K - \theta)$ where K is a constant such that $K > \theta_h > \theta_l > 0$. Assume that the probability of the high type is ϕ and the probability of the low type is $(1 - \phi)$.

(a). Characterize a hybrid (mixed-strategy) equilibrium in which the high-type worker randomizes between two levels of education, $e_0 = 0$ and e_1, with probabilities λ and $(1 - \lambda)$ respectively, and the low-type plays a pure-strategy always choosing $e_0 = 0$. In particular, compute w_0^*, w_1^* and e_1^*, and give the equation that ties together the high-type's mixing probability, λ^*, with the education level chosen by the high type, e_1^*.

(b). Discuss the extreme cases of this hybrid equilibrium when $\lambda \to 0$ and $\lambda \to 1$.

Problem 5 Consider a labor-market signaling model from MWG. Recall: there are two types of workers, $\theta_H > \theta_L > 0$, and the probability of the high type is $\phi \in (0, 1)$. Type-θ worker's payoff from wage w is given by $u(w, e, \theta) = w - c(e, \theta)$, where $c(e, \theta) = e/\theta$ is the cost of obtaining (unproductive) education level $e \geq 0$. The worker's reservation utility is zero and there are at least risk-neutral two firms.

(a). What is the range of education levels that can arise in a pooling equilibrium?

(b). In the set of all separating equilibria, what is the range of education levels that are chosen by the low type, θ_L, and what is the range of education levels chosen by the high type, θ_H?

(c). Suppose now that $c = e$ (independent of θ), but the worker's marginal utility of money depends upon type. Specifically, assume the worker's payoff is

$$u(w, e, \theta) = \theta w - e.$$

How does your answer in (b) change with these new preferences? Explain.

(d). Suppose again that $c = e$ (independent of θ) and the worker's marginal utility of money depends upon type, but now the worker's payoff is

$$u(w, e, \theta) = \frac{w}{\theta} - e.$$

How does your answer in (b) change? Explain.

Problem 6 Consider the labor-market signaling model of MWG, but with productive education. Suppose there are two types $\theta_H > \theta_L > 0$, where the proportion of type θ_H are $\phi \in (0, 1)$. If agent θ chooses e, then their productivity is $\theta + e$. The cost of education for type θ is $c(e, \theta) = \frac{e^2}{2\theta}$.

(a). Suppose type θ_i (along with e_i) is observable. How many years of education would type θ_i get?

(b). Suppose $\theta_L > \frac{\theta_H}{3}$. Characterize the least–cost separating equilibrium.

(c). Suppose $\theta_L \leq \frac{\theta_H}{3}$. Characterize the least–cost separating equilibrium.