THE UNIVERSITY OF CHICAGO
Department of Economics
Econ 30200 Problem Set 2

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Due Wednesday January 22

1. In an Edgeworth box two-consumer, two-commodity pure exchange economy with consumption sets equal to $\mathbb{R}^2_+$ sketch a situation in which there is no competitive equilibrium because preferences are not convex.

2. Prove that in a pure exchange economy in which consumer i's preferences are represented by the utility function u^i, an allocation $(\hat{\mathbf{x}}^i)_{i=1}^m$ is Pareto efficient if and only if it solves the following maximization problem:

$$\max_{\mathbf{x}^1,\ldots,\mathbf{x}^m \in \mathbb{R}^n_+} u^1(\mathbf{x}^1) + \ldots + u^m(\mathbf{x}^m) \quad \text{s.t.} \quad u^i(\mathbf{x}^i) \geq u^i(\hat{\mathbf{x}}^i), \text{ all } i \text{ and}$$

$$\mathbf{x}^1 + \ldots + \mathbf{x}^m \leq \mathbf{e}^1 + \ldots + \mathbf{e}^m$$

3. Consider a two-consumer two-good exchange economy in which $u^1(x_1, x_2) = x_1 x_2$, $u^2(x_1, x_2) = x_1(x_2)^2$, $\mathbf{e}^1 = (18, 4)$, and $\mathbf{e}^2 = (3, 6)$. (a) Characterize the set of Pareto efficient allocations as completely as possible. (b) Characterize the core. (c) Find a competitive equilibrium allocation and corresponding price vector. (d) Verify that the competitive equilibrium allocation is in the core.

4. (Scarf) Consider a pure exchange economy with three consumers and three goods. Consumers' utility functions and endowments are:

$$
\begin{aligned}
u^1(x_1, x_2, x_3) &= \min(x_1, x_2) & \mathbf{e}^1 &= (1, 0, 0) \\
u^2(x_1, x_2, x_3) &= \min(x_2, x_3) & \mathbf{e}^2 &= (0, 1, 0) \\
u^3(x_1, x_2, x_3) &= \min(x_1, x_3) & \mathbf{e}^3 &= (0, 0, 1)
\end{aligned}
$$

Find a competitive equilibrium allocation and corresponding price vector for this economy.

5. Prove Theorems 1.2 and 1.3 on p. 17 of JR.

IN ADDITION, complete the following exercises from JR, chapter 5: 5.3, 5.7, 5.14 (a) and (b), 5.21, 5.22.