# Ph.D. Core Examination

## MICROECONOMICS

Wednesday, July 11, 2018

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WRITE in **black ink** and write only on **one side** of each page.

DO NOT WRITE in the margins.

**Be sure the "random number" stamped on your paper is the correct number** (the number you were given by the department office). Be sure to put your **random number** in the upper left hand corner of **every page** you write on.

On the **first page** of your examination write:

- the name of the examination
- the date of the examination

Put the **page number** in the upper right hand corner of **every page** of your examination.

When you have completed your examination, write the **total number** of pages on the **back** of the last page.

Put your examination in the envelope to be turned in.

Results of the examination will be posted by your random number outside the Office of Graduate Student Affairs and sent to you by letter.

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**TIME: THREE HOURS**

1. **You must answer ALL questions and justify ALL answers.**
2. **The exam is out of 180 points. There are three parts, I, II, and III, each worth 60 points.**

Note: The envelope you will be given will contain a colored writing pad to be used for notes and a white writing pad with your random number to be used for writing your final answers.

**Part I.** (Six questions; 60 points total)

Answer each of the following 6 questions as TRUE, FALSE, or UNCERTAIN, and justify your answer in a half of a page or less. Your grade will depend entirely on your justification. Each question is worth 10 points.

1) Knowledge that productivity growth will be higher than previously expected will lead to higher stock market valuations.

2) Consider a good with sales of one million units per year. If technical progress results in that good being replaced by a higher quality version that sells for the same price, the higher quality version will sell more than one million units per year.

3) A price ceiling in a competitive market cannot benefit consumers unless the quantity traded is also regulated.

4) A firm that pays for the MBA degrees of its employees would not be maximizing firm value since MBA studies provide general training.

5) Giving college-aged students access to loans for financing tuition and living expenses would make their college attendance rates less sensitive to the dollar amount of tuition.

6) Studies of the impact of corporate taxes on wages based on exogenous variations in state-level corporate tax rates will tend to underestimate the impact on worker wages of a reduction in the federal corporate income tax rate.

**Part II.** (Three questions; 60 points total)

**II.1. (15 points)** Consider the following private ownership economy with two goods. There is one firm with production set $Y = \{(y_1, y_2) \in \mathbb{R}^2 : y_1 \leq 0,\ y_2 \geq 0,\ y_1 + y_2 \leq 0\}$, where negative quantities are inputs and positive quantities are outputs. There are two consumers. Consumer 1 has utility function and endowment vector, $u_1(x_1, x_2) = 2x_1 + x_2$ and $e^1 = (1, 0)$, and consumer 2 has utility function and endowment vector $u_2(x_1, x_2) = x_1 + 2x_2$ and $e^2 = (1, 0)$. Each consumer has a one-half ownership share of the firm.

**(a)** (*5 points*) Let $p_i$ denote the price of good $i$. What are the firm's maximum profits if $p_1 > p_2$? If $p_1 < p_2$?

**(b)** (*10 points*) Find a Walrasian equilibrium price vector and the associated Walrasian equilibrium allocation for this economy. What is the profit income of each consumer?

**II.2. (20 points)** This question pertains to the independent private values model in which a seller owns a single unit of an indivisible good, there are $N$ buyers $i = 1, 2, ..., N$, and where buyer $i$'s value, $v_i$, is drawn from $[0, 1]$ (independently of the other buyers' values) according to the positive density function $f_i : [0, 1] \to (0, \infty)$.

**(a)** (*5 points*) State the revenue equivalence theorem.

**(b)** (*5 points*) Suppose that the seller has an outside option (not involving any of the $N$ buyers) to sell the good for a price of $v_0 > 0$. Provide a mechanism for selling the good that maximizes the seller's ex-ante expected revenue from the $N$ buyers and the outside option.

**(c)** (*10 points*) Suppose that the seller has no outside option for selling the good and has no use value for the good. Suppose that there are two buyers, i.e., $N = 2$, and suppose that each buyer's value is independently drawn from a uniform distribution on $[0, 1]$.

Suppose that the two buyers arrive at the seller's store one at a time, with buyer 1 arriving first and then, after buyer 1 leaves, buyer 2 arrives second. The two buyers never meet. There is no discounting.

When buyer 1 arrives, the seller can use any mechanism to sell the good to him. If buyer 1 receives the good, the seller closes his store and enjoys the revenue from the sale if any, and there is no buying opportunity for buyer 2. If the good is not transferred to buyer 1, then, after buyer 1 leaves and buyer 2 arrives, the seller can use any mechanism to sell to buyer 2.

Describe a selling mechanism that the seller offers to buyer 1 and a selling mechanism that the seller offers to buyer 2 (if the store is still open) that, taken together, maximize the seller's ex-ante (i.e., before buyer 1 arrives) expected revenue. (You should assume that the seller can commit to the mechanism offered to each buyer. Justify why your mechanism is revenue maximal, including any revelation principle ideas that are used. Be clear, but brief.)

**II.3. (25 points)** $N$ men and $N$ women each have their own continuous, concave, and strictly increasing VNM utility function for money income. There are two dates, 1 and 2, and there is no discounting. On date 2, an income is drawn from $[0, \infty)$ independently for each individual according to a distribution with mean $\overline{w} > 0$ and variance $\sigma^2 > 0$. On date 1, couples can marry and commit to how they will share their uncertain future date 2 income. All decisions are made on date 1.

If on date 1 man $i$ and woman $j$ choose to marry, they can choose any (income-sharing) *contract* $\phi_{ij} : [0, \infty) \times [0, \infty) \to [0, \infty)$ satisfying $0 \leq \phi_{ij}(x_i, y_j) \leq x_i + y_j$ for every $x_i, y_j \in [0, \infty)$. Given $\phi_{ij}$, if man $i$'s date-2 income is $x_i$ and woman $j$'s date-2 income is $y_j$, then $\phi_{ij}(x_i, y_j)$ dollars goes to man $i$ and $x_i + y_j - \phi_{ij}(x_i, y_j)$ dollars goes to woman $j$. For example, the contract $\phi_{ij}(x_i, y_j) = (x_i + y_j)/2$ splits the couple's total date-2 income equally between them.

Married individuals get utility only from the date-2 income they receive according to their contract. There is no utility from the marriage itself! So we can assume that no one remains single because any couple can choose the contract $\phi_{ij}(x_i, y_j) = x_i$, which ensures that both individuals receive exactly their own income.

A *matching* is a collection $\{(i, j, \phi_{ij})\}$ that lists $N$ couples and their contracts, where each $(i, j, \phi_{ij})$ indicates that, on date 1, man $i$ is matched with woman $j$ and that they commit to share their date-2 income according to the contract $\phi_{ij}$. Each man is matched to exactly one woman and vice versa.

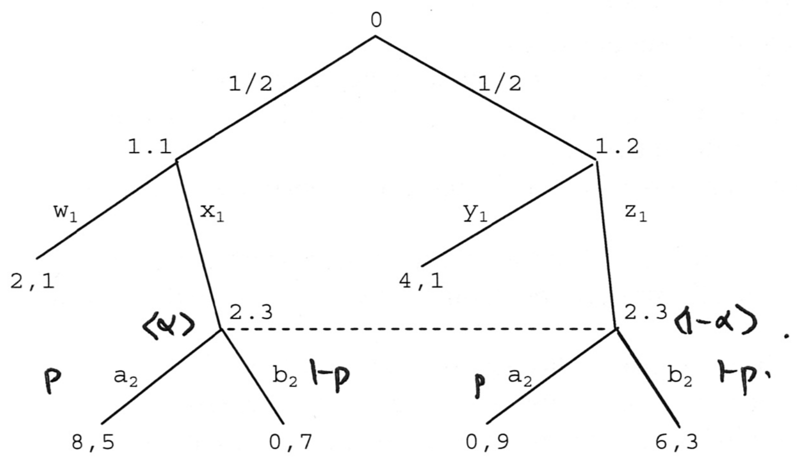
A matching $\{(i, j, \phi_{ij})\}$ is *stable* iff on date 1, (A) no individual strictly prefers being single (and receiving his/her own income on date 2) to what is specified for him/her by the matching $\{(i, j, \phi_{ij})\}$, and (B) there is no man $i'$, woman $j'$, and contract $\phi'_{i'j'}$ such that, on date 1, man $i'$ and woman $j'$ each strictly prefer marrying one another and committing to the contract $\phi'_{i'j'}$ to what is specified for them by the matching $\{(i, j, \phi_{ij})\}$.

**(a)** (*10 points*) Suppose that there are just 2 men and 2 women. Man 1 and woman 1 are risk neutral, and man 2 and woman 2 are strictly risk averse. Prove that in any stable matching it must be the case that man 1 is matched with woman 2 and that woman 1 is matched with man 2. (Hint: Argue by contradiction.) What is the intuition for this result?

**(b)** (*15 points*) For the general model with $N$ men and $N$ women, prove that a stable matching exists. (Hint: Define an appropriate deferred acceptance algorithm. You may assume here that incomes are not infinitely divisible (e.g., the smallest money unit is a penny), and are drawn from a finite set of possible values. Then there are only finitely many possible incomes and finitely many possible income-sharing contracts. So on date 1 each individual can rank the finitely many contracts from highest to lowest in expected utility.)

**Part III.** (Two questions; 60 points total)

**III.1.** (***30 points***) Consider the following extensive-form game, which begins with a chance move that will be observed by player 1 but not by player 2.




**(a)** (*10 points*) Construct the normal representation of this game in strategic form (the normal form).

**(b)** (*5 points*) Show that, in the normal representation, player 1 has a pure strategy which is strictly dominated by other strategies (which might involve some randomization).

**(c)** (*15 points*) Find a sequential equilibrium of this extensive-form game. Show all move probabilities, and show what player 2 would believe is the probability of 1 having been type 1.1 (the left "2.3" node) if player 2 got to move in this equilibrium.
(If you cannot find a sequential equilibrium then, for partial credit, you may try to find a Nash equilibrium of the normal representation in strategic form.)

**III.2** (***30 points***)
An entrepreneur wants to borrow \$100 from a bank to finance a risky short-term project.
If the project succeeds then it will return \$200 next month, but otherwise it will return \$0.
The bank can specify the interest rate r that it will charge for the loan. We may restrict our attention here to interest rates r such that $0<r<1$ (so that the amount due $100(1+r)$ will not be more than the returns from the project in the event of success).
If the project succeeds then the bank will collect \$100(1+r) from the project's returns.
But if the project fails then the entrepreneur will default on the loan, and in this case the bank will collect only collateral worth \$50 from the entrepreneur.
If the entrepreneur invests the \$100 loan appropriately then the project's probability of success will be 0.6.
But the entrepreneur could instead secretly divert 20% of the borrowed funds (that is, \$20) to his personal consumption, and if he did so then the project's probability of success would decrease to 0.4.
If the entrepreneur chose to divert \$20 from the loan to his personal consumption, the bank would not be able to observe any evidence of this diversion (other than in the decreased probability of success), and the bank would be unable to recover any of these diverted funds if the project failed.
(The entrepreneur's liability in default is limited to his \$50 collateral.)
Suppose that the bank and the entrepreneur are both risk neutral, and they do not significantly discount payments over the short term of this project. (Short-term discount factors equal 1.)
(The cost of alternative use of the entrepreneur's time during the project can also be ignored.)

**(a)** (*8 points*) What is the highest interest rate $\bar{r}$ that the entrepreneur would be willing to pay for the loan to finance this project, if the entrepreneur were planning to divert \$20 from the loan to his personal consumption?

**(b)** (*8 points*) What is the highest interest rate $r^*$ at which the entrepreneur would prefer to invest the \$100 loan appropriately (rather than diverting 20% to his personal consumption)?

**(c)** (*14 points*) Show that the bank would prefer not to charge this entrepreneur the highest interest rate that he would be willing to pay for this loan. Compute the maximal expected return that the bank could get from lending at a rate that would make the entrepreneur prefer to invest the loan appropriately, and compare this amount to the expected return that the bank would get from the highest acceptable interest rate from part (a).