Price Theory, III **Professor Lars Stole**
Economics 33003 **Spring 2020**

Assignment 2: Due Friday, April 24, prior to the start of the review session

Problem 1 Consider a certifiable disclosure game with n players, in which each player i's payoff is simply the market expectation of their type, conditional on the information $m \subseteq \Theta$ revealed:

$$u_i(\theta_i) = E[\theta|\theta \in m].$$

There are two types, $\theta_h > \theta_l > 0$, where $\phi \in (0,1)$ is the probability of the high type. As a complication, however, suppose that with probability λ, a player cannot use the free certification mechanism and therefore no message is sent (effectively, $m = \Theta$). What is the unique equilibrium? If it involves less than full revelation by those players who can reveal their types, explain why unraveling does not arise. What key condition is violated?

Problem 2 (MWG, Exercise 13.C.1) Consider a game in which, first, nature draws a worker's type θ from some continuous distribution on $[\underline{\theta}, \overline{\theta}]$. Once the worker observes her type, she can choose whether to submit to a costless test that reveals her ability perfectly. Finally, after observing whether the worker has taken the test and its outcome if she has, two firms bid for the worker's services. Prove that in any subgame perfect Nash equilibrium of this model with probability 1 workers submit to the test, and firms offer a wage no greater than $\underline{\theta}$ to any worker not doing so.

Problem 3 (JR, Exercise 8.9 - major variation) Analyze the insurance signaling game in JR (ch8) when the coverage B is restricted to being equal to L. Assume that the low risk consumer strictly prefers full insurance at the high-risk competitive price to no insurance. For simplicity, assume that there is a single firm.

(a). Characterize the unique pure-strategy sequential equilibrium price(s) when attention is restricted to those equilibria in which the insurance company earns zero profits.

(b). Show that there are pooling sequential equilibria in which the insurance company earns positive profits.

(c). Characterize a mixed-strategy sequential equilibrium strategy profile in which (1) the insurance company earns zero profits and (2) consumers fully separate. [Hint: assume that there are two offers in equilibrium, and the firm randomizes between "Accept" and "Reject" for one of them. If the firm "rejects", the game is over and the consumer is uninsured.]

Problem 4 Consider the quadratic Cheap-talk model of Crawford and Sobel (1982), but with θ uniformly distributed on $\Theta = [0,2]$, with message space $\mathcal{M} = [0,2]$, and with $b = \frac{1}{8}$. Find the most informative equilibrium (the one with the most steps), fully characterizing the equilibrium partition.

Problem 5 Consider a setting of career concerns where a manager has one of two types, $1 > \theta_h > \theta_l > 0$, with prior probabilities ϕ and $(1-\phi)$, respectively. Let $E[\theta] = \phi\theta_h + (1-\phi)\theta_l$ be the players' prior expectation on θ. Neither the market nor the manager knows the manager's type.

In period 1, the manager privately learns of an investment project with some probability. An investment project, i, if available, gives a risky return paying y_i with probability θ and 0 with probability $1-\theta$. A safe asset is always available and pays $r > 0$. Assume that some projects are *ex ante* profitable (i.e., $E[\theta]y_i > r$ for some i and $E[\theta]y_i < r$ for others).

The firm cannot write contracts on investments or outcomes. Rather, it delegates the investment choice to the manager if a project (known only to the manager) becomes available. In period 1, the manager may either choose an investment project, or choose the safe risk-free project. The manager is paid w_1 in the current period and will be paid $w_2 = \text{Prob}[\theta = \theta_h|\cdot]$ after the market observes whether or not she invests and, if she invests, the outcome of the project (success, S, or failure, F). There is no discounting (i.e., $\delta = 1$).

(a). Compute the market's inference of the manager's type if she invests in the safe asset?

(b). Compute the market's inference of the manager's type if she invests in a risky project and it is a success? Compute the market's inference of the manager's type if she invests in a risky project and it is a failure?

(c). Show that a risk-neutral manager is indifferent between all projects (and the safe asset). Formally, show that the market's expected inference of the manager's type, conditional on investing, is the same as if she invests in the safe asset.

(d). Argue that if the manager is only slightly risk averse, she will not choose positive NPV projects.

Problem 6 (MWG, Exercise 13.D.1- variation) Extend the screening model of MWG, Chapter 13.D, to the case in which tasks are productive. Assume that a type θ worker produces $\theta(1+t)$ units of output when her task level is t. Assume that $\theta_h = 3$ and $\theta_l = 1$. Also assume that the cost of task t for type θ is

$$c(t, \theta) = \frac{t^2}{2}(4 - \theta).$$

What is the subgame perfect Nash equilibrium allocation, $\{(w_l^*, t_l^*), (w_h^*, t_h^*)\}$, offered by each firm?

Problem 7 Two life insurance companies offer policies to consumers with unknown health types, $\theta_H > \theta_L > 0$, where the healthy (high) type occurs in proportion $\phi \in (0, 1)$ in the population. Buyers of insurance know which type they are.

The expected cost of a life insurance policy to a type θ customer is $C - \theta$; thus, a life insurance company earns a profit of $p - (C - \theta)$ for each policy it sells to a θ-type customer. The buyers gain value $v > C$ from purchasing a life insurance policy and have net surplus of $v - p$ if they purchase a policy at price p.

Both insurance companies have decided to start using fitbits in their pricing strategies. Specifically, customers will be given free fitbit monitors that will track and certify how many steps they walk each day. The insurance companies can price their policies conditional on the customer reaching a certain number of daily steps.

Assume (unrealistically), that walking has no benefit on prolonging someone's life, but walking is costly for people who have low types, and not as costly for people with high-health types. Specifically, suppose

that the cost of reaching s daily steps to a type-θ consumer is

$$c(s, \theta) = \frac{\sqrt{s}}{\theta}.$$

The insurance companies simultaneously announce a menu of prices and minimum-daily steps for their life insurance policies. The utility of a consumer of type θ who purchases a policy (p, s), at a price of p and a requirement of s steps per day, is

$$u = v - p - \frac{\sqrt{s}}{\theta}.$$

Lastly, assume that $\theta_H = 20$ and $\theta_L = 10$.

(a). Show that the expected profit of each company is zero in any pure-strategy equilibrium.

(b). Show that there does not exist a pure-strategy pooling equilibrium.

(c). Prove that in any pure-strategy separating equilibrium, the price paid by the type-θ consumer is $p = C - \theta$.

(d). Prove that in any pure-strategy separating equilibrium, the θ_L consumer never walks anywhere (i.e., $s_L = 0$).

(e). Characterize the pure-strategy equilibrium of the game assuming that one exists. How many steps per day does a type-θ_H walk to get a discounted policy?