THE UNIVERSITY OF CHICAGO
Department of Economics
Econ 30200 Problem Set 4

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Complete exercises 5.35, 5.36, 6.1, 6.3, 6.9, 6.18-6.23 from JR chapter 6, and also the following exercises.

1. (This formalizes a discussion that we had in class.) Consider a production economy $\mathcal{E}$ with private ownership in which all ownership shares are either 0 or 1, that is, $\theta^{ij} \in \{0, 1\}$ for all $i \in \mathcal{I}$ and all $j \in \mathcal{J}$. Define the core of $\mathcal{E}$ to be the set of all feasible allocations $(\mathbf{x}, \mathbf{y})$ such that there does not exist a nonempty subset S of $\mathcal{I}$ and an allocation $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}})$ with $\tilde{y}^j \in Y^j$ for every $j \in \mathcal{J}$ such that,

   (i) $\sum_{i \in S} \tilde{x}^i = \sum_{i \in S} e^i + \sum_{i \in S} \sum_{j \in J: \theta^{ij}=1} \tilde{y}^j$, and

   (ii) $u^i(\tilde{x}^i) \geq u^i(x^i)$ for every $i \in S$ with at least one strict inequality.

   (Think about how you would interpret this defintion. How does the assumption that θ^{ij} must be either 0 or 1 impact your interpretation? What difficulties surface when trying to define the core when owenrship shares can be fractional?)

   If all utility functions are stricty increasing, prove that every WEA of this production economy is in the core.

   The next two problems concern our marriage matching model.

2. There are three men and three women. Their strict preferences are as follows, where it is assumed that every individual strictly prefers to be matched to any individual on the other side of the market rather than being single.

   $>_{m_1}$: w_2, w_1, w_3

   $>_{m_2}$: w_1, w_3, w_2

   $>_{m_3}$: w_1, w_2, w_3

   $>_{w_1}$: m_1, m_3, m_2

$>_{w_2}$: m_3, m_1, m_2

$>_{w3}$: m_1, m_3, m_2

Consider the matching

$\mu_1 = (m_1, w_1)$, (m_2, w_2), (m_3, w_3).

This matching is unstable because, for example, m_1 and w_2 form a blocking pair. If we let them divorce their partners and marry one another, and then marry their abondoned partners, we get the matching

$\mu_2 = (m_1, w_2)$, (m_2, w_1), (m_3, w_3).

But this matching is blocked by m_3 and w_2. If we rematch in the same way as before, we get the matching

$\mu_3 = (m_1, w_3)$, (m_2, w_1), (m_3, w_2), which is blocked by (m_3, w_1). Rematching yet again yields,

$\mu_4 = (m_1, w_3)$, (m_2, w_2), (m_3, w_1), which is blocked by (m_1, w_1). But rematching once more brings us back to μ_1!

This shows that naively trying to reach a stable match by letting blocking couples marry does not always work! However, show in this example that, by carefully choosing which blocking pairs marry (the blocking pairs are not unique), it is possible to reach stable match using this process. (It turns out that this is true in general.)

3. Use both the man-proposing and then the woman-proposing DAA to find stable matchings given the following strict preferences. Each individual strictly prefers to be married to anyone on the other side of the market rather than being single, except for m_5 who prefers being single to marrying w_3.

   $>_{m_1}$: w_1, w_2, w_3, w_4

   $>_{m_2}$: w_4, w_3, w_2, w_1

   $>_{m_3}$: w_4, w_3, w_1, w_2

   $>_{m_4}$: w_1, w_4, w_3, w_2

   $>_{m_5}$: w_1, w_2, w_4

   $>_{w_1}$: m_2, m_3, m_1, m_4, m_5

   $>_{w_2}$: m_3, m_1, m_2, m_4, m_5

$>_{w_3}$: m_5, m_4, m_1, m_2, m_3

$>_{w4}$: m_1, m_4, m_5, m_2, m_3

4. Prove that the set of unmatched individuals is the same in every stable matching of a marriage market with strict preferences.