A FEW OF MY FAVORITE THINGS–PRICE THEORY

JONATHAN D. HALL

1. Properties of Marshallian Demand

1.1. Adding Up (comes from budget constraint).

$$\sum_{i=1}^{n} p_i x_i = M$$

$$\Rightarrow \sum p_i \frac{\partial x_i}{\partial M} = 1 \Rightarrow \sum \frac{p_i x_i}{M} \frac{M}{x_i} \frac{\partial x_i}{\partial M} = 1 \Rightarrow \sum s_i \eta_i = 1$$

$$\Rightarrow \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} + x_j = 0 \Rightarrow \sum_{i=1}^{n} \frac{p_i x_i}{M} \frac{p_j}{x_i} \frac{\partial x_i}{\partial p_j} + \frac{p_j x_j}{M} = 0 \Rightarrow \sum_{i=1}^{n} s_i \epsilon_{ij} + s_j = 0$$

1.2. Homogeneity (of degree zero).

$$x_j(tp_1, \ldots, tp_n, tM) = x_j(p_1, \ldots, p_n, M)$$

$$\Rightarrow \sum_{i=1}^{n} \frac{\partial x_j}{\partial p_i} p_i + \frac{\partial x_j}{\partial M} M = 0 \Rightarrow \sum_{i=1}^{n} \frac{\partial x_j}{\partial p_i} \frac{p_i}{x_j} + \frac{\partial x_j}{\partial M} \frac{M}{x_j} = 0 \Rightarrow \sum_{i=1}^{n} \epsilon_{ji} + \eta_j = 0$$

(i.e. no money illusion).

2. Properties of Hicksian Demand

From concavity of the cost function:

$$\frac{\partial x_i^h}{\partial p_i} < 0 \ \forall \ i$$

2.1. Adding Up.

$$U(x_1, \ldots, x_n) = \bar{U}$$

$$\Rightarrow \sum_{i=1}^{n} \frac{\partial U}{\partial x_i^h} \frac{\partial x_i^h}{\partial p_j} = 0 \Rightarrow \sum_{i=1}^{n} \frac{x_i^h(\lambda p_i)}{M} \frac{p_j}{x_i^h} \frac{\partial x_i^h}{\partial p_j} = 0 \Rightarrow \sum_{i=1}^{n} \frac{x_i^h p_i}{M} \frac{p_j}{x_i^h} \frac{\partial x_i^h}{\partial p_j} = 0 \Rightarrow \sum_{i=1}^{n} s_j \epsilon_{ij}^h = 0$$

(i.e. you must have at least one substitute).

2.2. Homogeneity (of degree 0).

$$x_i^h(tp_1, \ldots, tp_n, \bar{U}) = x_i^h(p_1, \ldots, p_n, \bar{U})$$

$$\Rightarrow \sum_{i=1}^{n} \frac{\partial x_j}{\partial p_i} p_i = 0 \Rightarrow \sum_{i=1}^{n} \frac{p_i}{x_j} \frac{\partial x_j}{\partial p_i} = 0 \Rightarrow \sum_{i=1}^{n} \epsilon_{ji}^h = 0$$

2.3. Symmetry.

$$\frac{\partial C(p_1, \ldots, p_n, U)}{\partial p_i \partial p_j} = \frac{\partial C(p_1, \ldots, p_n, U)}{\partial p_j \partial p_i} \Rightarrow \frac{\partial x_i^h}{\partial p_j} = \frac{\partial x_j^h}{\partial p_i}$$

$$\Rightarrow \frac{x_i^h p_i}{M} \frac{p_j}{x_i^h} \frac{\partial x_i^h}{\partial p_j} = \frac{x_j^h p_j}{M} \frac{p_i}{x_j^h} \frac{\partial x_j^h}{\partial p_i} \Rightarrow s_i \epsilon_{ij}^h = s_j \epsilon_{ji}^h$$

(i.e. big products are important to little products, but the little products don't matter to big products (e.g. houses and doorknobs).)

3. Slutsky Equation

- $\frac{\partial x_i^m}{\partial p_j} = \frac{\partial x_i^h}{\partial p_j} - \frac{\partial x_i^m}{\partial M} x_j$.
- $\epsilon_{ij}^m = \epsilon_{ij}^h - s_j \eta_i$.

Just let $i = j$ for standard Slutsky.

4. Properties of Constant Returns to Scale Production

(1) $\Delta Y = S_L \Delta L + S_K \Delta K$.[1]
(2) $\Delta P = S_L \Delta W + S_K \Delta R$.
(3) $(\Delta L - \Delta K) = \sigma(K/L)(\Delta R - \Delta W)$.

5. Other Production Theory Identities

- $R = (r + \delta)P - \dot{P}$.
- $\Delta\text{TFP} = \Delta Y - (S_L \Delta L + S_K \Delta K) \Rightarrow \Delta \frac{Y}{L} = S_K \Delta \frac{K}{L} + \Delta\text{TFP}$. This last equality says that change in per capita income is due to capital deepening and productivity growth.
- $\Delta\text{TFP} = (S_L \Delta W + S_K \Delta K) - \Delta P = S_L(\Delta W - \Delta P) + S_K(\Delta R - \Delta P) = S_L \Delta MP_L + S_K \Delta MP_K$.
- $\epsilon_{ij} = s_i \sigma_{ij} + s_j \epsilon^d \Rightarrow \epsilon_{ii} = -\sum_{j \neq i} s_j \sigma_{ij} + s_i \epsilon^d$.

[1] To generalize let S_L and S_K be the shares of revenue.