Price Theory II Problem Set 2

Chase Abram, Sarah Brown,
Manav Chaudhary, Thomas Hierons,
Jeanne Sorin, George Vojta

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Question 2

Recall that that an allocation $(\hat{\mathbf{x}}^i)_{i=1}^m$ is Pareto efficient if $\Sigma_{i=1}^m \hat{\mathbf{x}}^i \leq \Sigma_{i=1}^m \mathbf{e}^i$ (i.e. it is feasible) and there is no other allocation $(\tilde{\mathbf{x}}^i)_{i=1}^m$ with $\Sigma_{i=1}^m \tilde{\mathbf{x}}^i \leq \Sigma_{i=1}^m \mathbf{e}^i$, such that $u^i(\tilde{\mathbf{x}}^i) \geq u^i(\hat{\mathbf{x}}^i)$ for all i and $u^i(\tilde{\mathbf{x}}^i) > u^i(\hat{\mathbf{x}}^i)$ for at least one i.

(Only If) Suppose $(\hat{\mathbf{x}}^i)_{i=1}^m$ is Pareto efficient. Now for a contradiction suppose it does not solve the stated optimisation problem. Note that $(\hat{\mathbf{x}}^i)_{i=1}^m$ statisfies the constraints of the optimisation problem. So there must exist an $(\tilde{\mathbf{x}}^i)_{i=1}^m$ with $u^i(\tilde{\mathbf{x}}^i) \geq u^i(\hat{\mathbf{x}}^i)$ for all i, $\Sigma_i \tilde{\mathbf{x}}^i \leq \Sigma_i \mathbf{e}^i$ and $\Sigma_i u^i(\tilde{\mathbf{x}}^i) > \Sigma_i u^i(\hat{\mathbf{x}}^i)$. But given this, at least one of the inequalities must hold strictly, namely there is an i such that $u^i(\tilde{\mathbf{x}}^i) > u^i(\hat{\mathbf{x}}^i)$. This contradicts $(\hat{\mathbf{x}}^i)_{i=1}^m$ being Pareto optimal since $(\tilde{\mathbf{x}}^i)_{i=1}^m$ is a feasible Pareto improvement.

(If) Now suppose that $(\hat{\mathbf{x}}^i)_{i=1}^m$ solves the optimisation problem. For a contradiction suppose that $(\hat{\mathbf{x}}^i)_{i=1}^m$ is not Pareto efficient. So there exists a feasible Pareto improvement, namely an allocation $(\tilde{\mathbf{x}}^i)_{i=1}^m$ such that $\Sigma_{i=1}^m \tilde{\mathbf{x}}^i \leq \Sigma_{i=1}^m \mathbf{e}^i$, and $u^i(\tilde{\mathbf{x}}^i) \geq u^i(\hat{\mathbf{x}}^i)$ for all i and $u^i(\tilde{\mathbf{x}}^i) > u^i(\hat{\mathbf{x}}^i)$ for at least one i. But then $\Sigma_i u^i(\tilde{\mathbf{x}}^i) > \Sigma_i u(\hat{\mathbf{x}}^i)$ and so $(\tilde{\mathbf{x}}^i)_{i=1}^m$ satisfies the constraints of the optimisation problem and yields a higher value of the objective function. This contradicts the optimality of $(\hat{\mathbf{x}}^i)_{i=1}^m$. So $(\hat{\mathbf{x}}^i)_{i=1}^m$ is indeed Pareto efficient.

Question 4

Problem

Consider a pure exchange economy with three consumers and three goods. Consumers' utility functions and endowments are:

$$u^1(x_1, x_2, x_3) = \min(x_1, x_2), e^1 = (1, 0, 0)$$
$$u^2(x_1, x_2, x_3) = \min(x_2, x_3), e^2 = (0, 1, 0)$$
$$u^3(x_1, x_2, x_3) = \min(x_1, x_3), e^3 = (0, 0, 1)$$

Find a competitive equilibrium allocation and corresponding price vector for this economy.

It will also be useful to redefine a competitive (Walsrasian) equilibrium based on the definition from class:

Definition: a price vector $p^* \in \mathbb{R}^n_+$ is a competitive equilibrium if and only if there exists $\hat{x_1}, ...\hat{x_I} \in \mathbb{R}^n_+$ s.t.:

1. $\hat{x}_i$ maximizes $u^i(x_i)$ s.t. $p^* \cdot x_i \leq p^* \cdot e^i, \forall i \in \mathcal{I}$
2. $\sum_{i \in \mathcal{I}} x_i = \sum_{i \in \mathcal{I}} e_i$

Then: $\hat{x}(\hat{x_1}, ..., \hat{x_n}) \in \mathbb{R}^{\ltimes \cdot \mathcal{I}}_+$ is called a competitive equilibrium.

1. Here

Solution

Note in this economy, each agent is going to try to consume an equal amount of the two goods that matter to them. There is no point in consuming $x_i > x_j$, because the utility function is based on the minimum of goods x_i, x_j depending on the person.

Because the economy is symmetric, i.e. all the agents face similar problems in the sense where they want equal amount of two goods and are given full 1 unit endowments of one of the goods the crave.

Let's conjecture a price vector of $p^* = \{1, 1, 1\}$ for the three goods. And an allocation of:

$$\hat{x^1} = \{\frac{1}{2}, \frac{1}{2}, 0\}$$
$$\hat{x^2} = \{0, \frac{1}{2}, \frac{1}{2}\}$$
$$\hat{x^3} = \{\frac{1}{2}, 0, \frac{1}{2}\}$$

Where $\hat{x^i}$ represents the allocation to agent i. Does this allocation satisfy our two conditions?

1. Clearly $p^* \cdot x_i \leq p^* \cdot e^i$ for all $i \in \mathcal{I}$ as $p^* \cdot x_i = 1 = p^* \cdot e^i$ for each agent in the Economy. But are they maximizing utility? Yes, and to see this condition the perspective of agent $i = 1$. Having used up their entire budget, would they rather sell a little bit of good 1 or 2 to buy more of anything? No, because that would reduce their minimum between the two and thus they'd be strictly worse off. Every agent in the problem shares this similar problem and thus we are at an equilibrium with respect to the first condition.

2. Here the sum of all unit endowments sum up to one and the consumed units are also all equal to one. So this is trivially satisfied. ($1 = 1$ for each good and $3 = 3$ for the aggregate)

More Rigorous Solution

Based on the structure of the utility function, where we know each agent won't demand the goods in which they get no utility from we know the agents face the following budget constraints:

$$\begin{aligned} i = 1 &: p^1 x_1 + p^2 x_2 = p^1 \\ i = 2 &: p^2 x_2 + p^3 x_3 = p^2 \\ i = 3 &: p^1 x_1 + p^3 x_3 = p^3 \end{aligned}$$

Note here each agent has a maximum budget of the price of the good in which they are endowed. Now here we need to pick a price in which to fix as our normalization anchor. Lets choose $p^1 = \lambda$:

$$\begin{aligned} i = 1 &: \lambda x_1 + p^2 x_2 = \lambda \\ i = 2 &: p^2 x_2 + p^3 x_3 = p^2 \\ i = 3 &: \lambda x_1 + p^3 x_3 = p^3 \end{aligned}$$

Because in this setting consumers consume equal amounts of both goods:

$$\begin{aligned} i = 1 &: x_1 = x_2 \\ i = 2 &: x_2 = x_3 \\ i = 3 &: x_1 = x_3 \end{aligned}$$

For ease, Where the superscript now identifies the consumer for ease:

$$i = 1 : \lambda x_1^1 + p^2 x_2^1 = \lambda \tag{1}$$
$$i = 2 : p^2 x_2^2 + p^3 x_3^2 = p^2 \tag{2}$$
$$i = 3 : \lambda x_1^3 + p^3 x_3^3 = p^3 \tag{3}$$

Finally market clearing gives us that all of the demanded x's must equal their endowments (which are conveniently 1):

$$x_1^1 + x_1^3 = 1 \tag{4}$$
$$x_2^1 + x_2^2 = 1 \tag{5}$$
$$x_3^2 + x_3^3 = 1 \tag{6}$$

Equations (1) through (6) represent a system of equations which yield our above prices if you set $\lambda = 1$. But actually this works for all $\lambda \in \mathbb{R}_{++}$ giving a price vector of $\{\lambda, \lambda, \lambda\}$.

Question 5.3

Consider an exchange economy. Let p be a vector of prices in which the price of at least one good is non-positive. Show that if consumers' utility functions are strongly increasing then aggregate excess demand cannot be zero in every market.

(Def A1.17) - Let $f : D \to \mathbb{R}$, where D is a subset of $\mathbb{R}^n$. Then f is strongly increasing if $f(x^0) > f(x^1)$ whenever x^1 and x^0 are distinct and $x^0 \geq x^1$.

Suppose $p_{k^*} = 0$ for some good k^*. If u is strongly increasing then $x^0 \neq x^1$ and $x^0 \geq x^1$ implies $u(x^0) > u(x^1)$. Let x^0 be a bundle with 1 more unit of good k^* than x^1. Then $u(x^0) > u(x^1)$ and the consumer will prefer x^0 to x^1. If excess demand was 0 then demand for that good should equal supply across consumers.

$$0 = z_{k^*}(p) = \sum_{i \in \mathcal{I}} x^i_{k^*}(\mathbf{p}, \mathbf{p}e^i) - \sum_{i \in \mathcal{I}} e^i_{k^*}$$
$$\sum_{i \in \mathcal{I}} x^i_{k^*}(\mathbf{p}, \mathbf{p}e^i) = \sum_{i \in \mathcal{I}} e^i_{k^*}$$

However, a consumer will always prefer more of a good than less by strongly increasing utility. Then for good k^*, $\sum_{i \in \mathcal{I}} x^i_{k^*}$ will be unbounded above since $p_{k^*} = 0$. In other words, if it costs nothing to the consumer to have one more unit of good k^* and utility will increase by (Def A1.17), then demand for that good will be unbounded. This implies the economy can't hold its aggregate goods constraint with equality since endowment in finite. Therefore, aggregate excess demand cannot be zero in every market if $p_{k^*} = 0$.

Suppose $p_{k^*} < 0$ for some good k^*. Due to strongly increasing utility, the consumer should hold their budget constraint with equality. With $p_{k^*} < 0$,

$$p_{k^*} e^i_{k^*} + \mathbf{p}\mathbf{x}^\mathbf{i} = \mathbf{p}\mathbf{e}^\mathbf{i} + p_{k^*} x^i_{k^*}$$

This is peculiar since the endowment is a finite number and demand is unbounded for k^*. So the right hand side is unbounded. Intuitively, this is saying that the consumer actually expands the budget constraint by demanding more of good k^*. This implies the budget constraint cannot hold with equality which contradicts (Def A1.17). Similarly the aggregate excess demand equation, $\sum_{i \in \mathcal{I}} x^i_{k^*}$ will be unbounded above. Again, since initial endowment is finite, this implies $z_{k^*}(p) \neq 0$ Therefore, aggregate excess demand cannot be zero in every market if $p_{k^*} < 0$.

Question 5.14 (a) and (b)

a.

Recall that a function $f : \mathbb{R}^m \to \mathbb{R}$ is strongly increasing if $\mathbf{x} \geq \mathbf{y}$ and $\exists k$ s.t. $x_k > y_k$ entails that $f(\mathbf{x}) > f(\mathbf{y})$ (JR p.203).

Take $\mathbf{x} = (1, 0, \ldots, 0)$ and $\mathbf{y} = (0, \ldots, 0)$. Then $\mathbf{x} \geq \mathbf{y}$ and $x_1 > y_1$ but $u(\mathbf{x}) = u(\mathbf{y}) = 0$ so u is not strongly increasing.

b.

First, for completeness we briefly rederive the Cobb-Douglas Marshallian demands for consumer i of good k. The Lagrangean for consumer i in an exchange economy with prices $\mathbf{p}$ and endowments $\mathbf{e}^i$ is

$$\mathcal{L}^i = x_1^{\alpha_1^i} x_2^{\alpha_2^i} \ldots x_n^{\alpha_n^i} + \lambda^i [\Sigma_k p_k (e_k^i - x_k)]$$

The F.O.C.'s are sufficient ($\alpha_k^i > 0$ and $\Sigma_k \alpha_k = 1 \forall i$ ensures concavity of the objective and the constraint qualification holds). These are:

$$\forall k : \frac{\alpha_k^i}{x_k} u(\mathbf{x}) = \lambda^i p_k$$

and

$$\Sigma_k p_k e_k^i = \Sigma_k p_k x_k$$

Taking the ratio of the first equation for k, k', substituting into the budget constraint and solving yields:

$$x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) = \frac{\alpha_k^i \Sigma_{k'} p_{k'} e_{k'}^i}{p_k}$$

We now check each of the conditions in turn.

Condition 1

By definition: $z_k(\mathbf{p}) = \Sigma_i \left(x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - e_k^i \right)$ and from our derivation above we see that each of the individual Marshallian demands $x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$ is continuos in $\mathbf{p}$ since it the ratio of two continuous functions of $\mathbf{p}$ with the denominator strictly positive and both numerator and denominator linear functions of $\mathbf{p}$. So $z_k(\mathbf{p})$ is the sum of continuous functions and therefore continuous. And since each component of $\mathbf{z}(\mathbf{p})$ is continuous and it is itself continuous.

Condition 2

Let $\mathbf{p} \gg 0$. Then using our Marshallian demands from above we find:

$$\begin{aligned}
\mathbf{p} \cdot \mathbf{z}(\mathbf{p}) &= \Sigma_i \Sigma_k p_k x_k^i(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i) - \Sigma_i \Sigma_k p_k \\
&= \Sigma_i \Sigma_k \alpha_k^i (\Sigma_{k'} p_{k'} e_{k'}^i) - \Sigma_i \Sigma_k p_k e_k^i \\
&= (\Sigma_i \Sigma_{k'} p_{k'} e_{k'}^i) \underbrace{\Sigma_k \alpha_k^i}_{=1} - \Sigma_i \Sigma_k p_k e_k^i \\
&= \Sigma_i \Sigma_k p_k e_k^i - \Sigma_i \Sigma_k p_k e_k^i = 0
\end{aligned}$$

Condition 3
Let $(\mathbf{p}^m) \to \bar{\mathbf{p}} \neq 0$ with $\mathbf{p}^m \in \mathbb{R}^n_{++}$ for all m. Suppose that $\bar{p}_k = 0$. It suffices to show that $z_k(\mathbf{p}^m) \to \infty$ as $(\mathbf{p}^m) \to \bar{\mathbf{p}}$.

Using our expressions for Marshallian demand we have:

$$\begin{aligned} z_k(\mathbf{p}^m) &= \Sigma_i x_k^i(\mathbf{p}^m, \mathbf{p}^m \cdot \mathbf{e}^i) - \Sigma_i e_k^i \\ &= \frac{\Sigma_i \alpha_k^i (\Sigma_{k'} p_{k'}^m e_{k'}^i)}{p_k^m} - \Sigma_i e_k^i \end{aligned}$$

Noting that $p_k^m \to 0$, to show that $z_k(\mathbf{p}^m) \to \infty$ it suffices to show that the numerator $\Sigma_i \alpha_k^i (\Sigma_{k'} p_{k'}^m e_{k'}^i)$ is bounded away from zero by a fixed positive constant as $(\mathbf{p}^m) \to \bar{\mathbf{p}}$. Now since $\bar{\mathbf{p}} \neq 0$ one of its components is non-zero so there is a $\bar{p}_l \neq 0$.

Now:

$$\begin{aligned} \Sigma_i \alpha_k^i (\Sigma_{k'} p_{k'}^m e_{k'}^i) &\geq \alpha_l^1 p_l^m e_l^1 && \text{all terms non-negative} \\ &> \alpha_l^1 (\frac{1}{2}\bar{p}_l) e_l^1 && \text{for large enough } m \text{ since } p_l^m \to \bar{p}_l \text{ and } \bar{p}_l > 0 \\ &> 0 && \text{which is a constant that does not depend on } m \text{ as desired.} \end{aligned}$$

Text Book - Question 5.22

5.22 This exercise will guide you through a proof of a version of Theorem 5.4 when the consumer's utility function is quasiconcave instead of strictly quasiconcave and strictly increasing instead of strongly increasing.

(a) If the utility function $u\colon \mathbb{R}^n_+ \to \mathbb{R}$ is continuous, quasiconcave and strictly increasing, show that for every $\varepsilon \in (0,1)$ the approximating utility function $v_\varepsilon : \mathbb{R}^n_+ \to \mathbb{R}$ defined by

$$v_\varepsilon(\mathbf{x}) = u\left(x_1^\varepsilon + (1-\varepsilon)\sum_{i=1}^n x_i^\varepsilon, \ldots, x_n^\varepsilon + (1-\varepsilon)\sum_{i=1}^n x_i^\varepsilon\right),$$

is continuous, strictly quasiconcave and strongly increasing. Note that the approximation to $u(\cdot)$ becomes better and better as $\varepsilon \to 1$ because $v_\varepsilon(\mathbf{x}) \to u(\mathbf{x})$ as $\varepsilon \to 1$.

(b) Show that if in an exchange economy with a positive endowment of each good, each consumer's utility function is continuous, quasiconcave and strictly increasing on $\mathbb{R}^n_+$, there are approximating utility functions as in part (a) that define an exchange economy with the same endowments and possessing a Walrasian equilibrium. If, in addition, each consumer's endowment gives him a positive amount of each good, show that any limit of such Walrasian equilibria, as the approximations become better and better (e.g., as $\varepsilon \to 1$ in the approximations in part (a)) is a Walrasian equilibrium of the original exchange economy.

(c) Show that such a limit of Walrasian equilibria as described in part (b) exists. You will then have proven the following result.
If each consumer in an exchange economy is endowed with a positive amount of each good and has a continuous, quasiconcave and strictly increasing utility function, a Walrasian equilibrium exists.

(d) Which hypotheses of the Walrasian equilibrium existence result proved in part (b) fail to hold in the exchange economy in Exercise 5.21?

Figure 1: Problem From Textbook

Part A

We start with u which we know to be a continuous, quasiconcave and strictly increasing function. Also given is the fact that u is a linear sum of continuous functions. We know that the sum of continuous function is, itself, continuous, and thus it follows that v_ϵ is continuous. We restate v_ϵ below:

$$v^i_\epsilon(x) = u^i(x_1^\epsilon + (1-\epsilon)\sum_{i=1}^n x_i^\epsilon, ..., x_n^\epsilon + (1-\epsilon)\sum_{i=1}^n x_i^\epsilon)$$

Now consider a minute shift in one of the components in the form of a vector $\alpha\epsilon_k = (0, 0, ...\alpha, 0, ..., 0)$ is an α in the kth component. By the fact that u^i is strictly increasing (see that the added α comes through in every component of the utility function):

$$v^i_\epsilon(x) = u^i(x_1^\epsilon + (1-\epsilon)\sum_{i=1}^n x_i^\epsilon, ..., x_n^\epsilon + (1-\epsilon)\sum_{i=1}^n x_i^\epsilon)$$

$$<$$

$$u^i(x_1^\epsilon + (1-\epsilon)[\sum_{i\neq k}^n x_i^\epsilon + (x_k+\alpha)^\epsilon], ..., (x_k+\alpha)^\epsilon + [\sum_{i\neq k}^n x_i^\epsilon + (x_k+\alpha)^\epsilon], x_n^\epsilon + [\sum_{i\neq k}^n x_i^\epsilon + (x_k+\alpha)^\epsilon])$$

$$= v^i_\epsilon(x + \alpha\epsilon_k)$$

Which shows that in this structure v^i, an approximation for u^i is strongly increasing. This comes from the fact we showed $v^i_\epsilon(y) > v^i_\epsilon(x) \iff y = x + \alpha\epsilon > x$. Now lets define $t \in (0,1)$ and define $z_m := tx_m + (1-t)y_m$. Because $\epsilon < 1$ we get that the $(tx_m + (1-t)y_m)^{>}tx^\epsilon_m + (1-t)y^\epsilon_m$ by mapping $z| \to z^\epsilon$ for any $t \in (0,1)$ As such it is clear to see that:

$$v^i_\epsilon(z_i = tx_i + (1-t)y_i) =$$

$$= u^i(z^\epsilon_1 + (1-\epsilon)\sum_{i=1}^n z^\epsilon_i, ..., z^\epsilon_n + (1-\epsilon)\sum_{i=1}^n z^\epsilon_i) >$$

$$u^i(tx^\epsilon_1 + (1-t)y^\epsilon_1 + (1-\epsilon)\sum_{i=1}^n tx^\epsilon_i$$

$$+(1-t)y^\epsilon_i, ..., tx^\epsilon_n + (1-t)y^\epsilon_n + (1-\epsilon)\sum_{i=1}^n tx^\epsilon_i + (1-t)y^\epsilon_i) = ...$$

And we can separate the x's from the y's in this scenario to get:

$$... = u^i(t[x^\epsilon_1 + (1-\epsilon)\sum_{i=1}^n x^\epsilon_i] + (1-t)[y^\epsilon_1 + (1-\epsilon)\sum_{i=1}^n y^\epsilon_i], ...,$$

$$..., t[x^\epsilon_n + (1-\epsilon)\sum_{i=1}^n x^\epsilon_i] + (1-t)[y^\epsilon_n + (1-\epsilon)\sum_{i=1}^n y^\epsilon_i])$$

$$\geq \min\{v^i_\epsilon(x), v^i_\epsilon(y)\}$$

Which we get by the quasiconcavity in u^i. As such we have prove that v^i is continuous, strictly increasing, and itself quasiconcave.

Part B

We have shown that v^i satisfies Assumptions 5.1 and clearly in this economy $e^i >> 0 \Rightarrow \sum_{i\in\mathcal{I}} e^i >> 0$, implied by the fact we have endowments in this economy. So we have all the conditions necessary in theorem 5.5 to prove that a Walrasian Equilibrium exists s.t. $p^* >> 0$, with associated demand functions $x^i_\epsilon(p) = x^i_\epsilon(p, p \cdot e^i)$.

By construction as $\epsilon \to 1$ our v^i_ϵ converges to u^i. Now consider an approximating economy $\mathcal{E}(\epsilon) = (v^i_\epsilon, e^i)_{i\in\mathcal{I}}$ which converges to the true economy $\mathcal{E}$ point-wise. By Walras' Law lets normalize $p^*_\epsilon, \forall\epsilon$. Suppose $\{p^*_\epsilon\} \to p^* \in \mathbb{R}^n_+$. Now take the demand functions along this sequence of price vectors $x^i_\epsilon(p^*_\epsilon, p^*_\epsilon \cdot e^i)$, which is bounded because $p^*_\epsilon >> 0$. Without loss of generality we can take another sub-sequence that converges $\{x^i_\epsilon(p^*_\epsilon, p^*_\epsilon \cdot e^i)\} \to x^i \in \mathbb{R}^n_+$.

For every economy in our sequence $\{\mathcal{E}(\epsilon)\}_{\epsilon\in(0,1)}$ markets must clear by Walras law. By the fact that $\{x^i_\epsilon(p^*_\epsilon, p^*_\epsilon \cdot e^i)\}$ and that $e^i >> 0$ then markets also clear at the limiting point: i.e. $\sum_{i\in\mathcal{I}} x^i = \sum_{i\in\mathcal{I}} e^i$. At the same time equilibrium prices at the converged on economy and associated demand functions converge on p^*, or $\lim_{\epsilon\to1} p^*_\epsilon = p^*$.

Suppose, for the sake of conjuring up a contradiction later that one of the people in this economy has a bundle that is not optimal for them given the price vector and their endowment, e^i, in the economy. Thus there must be a better bundle, call it $x^{i\prime}$ s.t. $u^i(x^{i\prime}) > u^i(x^i)$ where $p^* \cdot x^{i\prime} \leq p^* \cdot x^i$). Remembering that we proved that u^i is continuous lets choose one of our $t \in (0,1)$ s.t. $u^i(tx^{i\prime}) > u^i(x^i)$ and because endowments are strictly positive $p^* \cdot tx^{i\prime} \leq p^* \cdot x^i$) where due to the converging property of our pricing vectors we know that $x^{i\prime}$ is in our approximating and true economies once $\epsilon \in B_\delta(1), \exists \delta > 0$. We would also like to show that within this δ ball around 1 $v^i_\epsilon(tx^{i\prime}) > v^i_\epsilon(x^i)$ and to do this we want to show that these v functions converge to $u_i(tx^{i\prime}_\epsilon)$ and $u_i(x^i_\epsilon)$ respectively. By this argument $\forall \delta > 0, \exists \epsilon' \in (0,1)$ s.t. $\forall \epsilon' \geq \epsilon$.

$$|v^i_\epsilon(tx^{i\prime}) - u_i(tx^{i\prime}_\epsilon)| < \delta$$
$$|v^i_\epsilon(x^i_\epsilon) - u_i(xi)| < \delta$$

But this is a contradiction of the fact that x^i_ϵ was a utility maximizer for the aforementioned agent in the Economy! Therefore, the limiting allocations must be the Walrasian equilibrium allocations of the true economy $\mathcal{E}$.

Part C

When making the arguments above, we subtley made the assumption that price vectors converged at the same time as the allocations without ever explicitly proving the price vectors p^*_ϵ truly existed. We were able to do this W.L.O.G. because as we can normalize prices generally in n dimensions of $\Delta^n := \{p \in [0,1]^n | \sum_{i=1}^n p_i = 1\}$. We can then reach all the way back in our real analysis tool box to pull out the Bolzano-Weierstrass theorem to state that there must exist a convergence sub-sequence where $p_\epsilon, k^* \to p^*_\epsilon$ as ϵ_k converges to 1. Thus we have all the necessary components of a Walrasian Equilibrium.

Part D

We also previously showed above for question 5.21 that $\epsilon^2 = (1,0)$ and thus is NOT $>> 0$.