Ph.D. Core Examination

MICROECONOMICS

Friday, July 11, 2014

WRITE in **black ink** and write only on **one side** of each page.

DO NOT WRITE in the margins.

Be sure the "random number" stamped on your paper is the correct number (the number you were given by the department office). Be sure to put your **random number** in the upper left hand corner of **every page** you write on.

On the **first page** of your examination write:

- the name of the examination
- the date of the examination

Put the **page number** in the upper right hand corner of **every page** of your examination.

When you have completed your examination, write the **total number** of pages on the **back** of the last page.

Put your examination in the envelope to be turned in.

Results of the examination will be posted by your random number outside the Office of Graduate Student Affairs and sent to you by letter.

TIME: THREE HOURS

1. **You must answer ALL questions and justify ALL answers.**

2. **The exam is out of 180 points. Part I is worth a total of 90 points. Each of Parts II, III, and IV are worth 30 points.**

Note: The envelope you will be given will contain a colored writing pad to be used for notes and a white writing pad with your random number to be used for writing your final answers.

I. PRICE THEORY QUESTIONS (90 minutes – 10 points each)

Answer each question as TRUE, FALSE or UNCERTAIN. Justify your answer. Your score will depend entirely on your justification. If the question makes several claims, address each of those claims in your answer. Penalties will be assessed for irrelevant material.

1. A 10% increase in the wage rate due to technological progress in the non-household sector combined with a 10% increase in household productivity across all household commodities would increase real household income by 10%.
2. Suppose a marriage market where there is transferable utility between men and women in a marriage. Consider a marriage market competitive equilibrium where there are benefits to any couple of being married (i.e. the utility of two individuals when married exceeds that of being single), there is positive assortative mating and there are more women than men. The introduction of a tax benefit to married couples of $M per couple financed by a uniform per-person tax would raise the utilities of all men and lower the utilities of all women.
3. Consider a good X subject to social influences. Assume that there are two groups of consumers, leaders (L) and followers (F). Assume that the demand for X by F is increasing in the consumption of X by L but the demand for X by L is decreasing in the consumption of X by F and that each group's demand for X is decreasing in the price of X holding the other group's consumption of X fixed. Aggregate demand for this good could be backward bending (i.e. a fall in price could lead to less aggregate consumption) but there would not be multiple equilibria at the same price.
4. Suppose that some consumers underestimate the value received from a good and as a result purchase less of that good than they would otherwise. If a small number of consumers under-estimate the value of the good then those consumers will be worse off but if many consumers do so they may all be better off.
5. Many low income areas suffer from a lack of policing. A law which mandates that the government provide greater levels of policing would make residents of those communities better off if the increased police activity is financed by taxes on third parties (i.e. individuals that do not live in the community).
6. A new invention which makes capital goods more durable that applies to both new capital goods produced as well as the existing capital stock would reduce rental prices for the capital good in the long run but would have an uncertain effect on long-run capital good prices.
7. A law that limits the annual catch of fish by stopping fishing once a set limit has been reached for the year would lower the price of fish relative to a system that yields the same annual catch by taxing fish.
8. If advertising by a monopolist increases a good's value to all consumers by the same percentage but does not change the monopolist's profit-maximizing price then it would be socially efficient to increase advertising above the level chosen by a profit-maximizing monopolist.
9. With two factors and CRS, capital augmenting technological progress will benefit workers in both the short-run (capital fixed) and the long-run (a constant real return to capital) and the benefit to workers will be greater in the long-run than in the short-run.

II. GENERAL EQUILIBRIUM AND CHOICE THEORY (30 points)

(a) (15 points) Consider an exchange economy with two consumers and two goods. Each consumer i is endowed with a strictly positive amount of good k, i.e., $e_k^i > 0$ for $i, k = 1, 2$. Let $\hat{x}_k^i(p_1, p_2) \geq 0$ denote consumer i's utility-maximizing demand for good k when the prices of goods 1 and 2 are $p_1 > 0$ and $p_2 > 0$, respectively, where we have supressed the dependence of $\hat{x}^i(\cdot)$ on consumer i's income. Assume the following for $i, k = 1, 2$.

(A) $\lim_{p_2 \to 0^+} \hat{x}_2^i(p_1, p_2) = +\infty$, $\forall p_1 > 0$.

(B) $\lim_{p_2 \to \infty} \hat{x}_2^i(p_1, p_2) = 0$, $\forall p_1 > 0$.

(C) $\hat{x}_2^i(p_1, p_2)$ is continuous in (p_1, p_2) on $\mathbb{R}_{++}^2$.

(D) Each consumer's utility function is strictly increasing on $\mathbb{R}_+^2$.

Suppose that the government has imposed a marginal tax of $\tau > 0$ dollars per unit of good 1 purchased by consumers, and that it intends to use the tax revenue to purchase as much good 1 as possible and then donate all of its purchases of good 1 to another country. Let G_1 denote the amount of good 1 purchased by the government. Our two-consumer exchange economy is closed and, apart from sending good 1 to another country for free, does not interact in any other way with any other country.

(i) (1 point) Write down the budget constraint for consumer i.

(ii) (1 point) Write down the government's budget constraint as a function of τ, p_1, p_2, G_1.

(iii) (3 points) Prove an appropriate version of Walras' law for this economy that includes the government's purchases, G_1, of good 1.

(iv) (10 points) Prove that, for any given $\tau > 0$, there exist prices $\hat{p}_1 > 0$ and $\hat{p}_2 > 0$ and government purchases $\hat{G}_1$ of good 1 such that both markets clear and the government's budget constraint is satisfied with equality. (*Hint*: Use Walras' law.)

(b) (15 points)

(i) (10 points) State Arrow's theorem (you need **not** define the terms used in the statement of the theorem).

(ii) (5 points) Show that Arrow's conditions **can** all be satisfied when there are just two alternatives. (Hint: You may assume an odd number of individuals if this is helpful.)

III. GAME-THEORY QUESTION (30 points)

In this game, player 1 privately knows his type t_1, and player 2 privately knows her type t_2, where t_1 and t_2 are independently drawn from a Uniform distribution on the interval from 0 to 1.

Then each player i must choose an action a_i.
Player 1's action a_1 can be any real number (positive or negative), that is: $a_1 \in \mathbb{R}$.
But player 2's action a_2 must be a number between 0 and 1, that is: $0 \leq a_2 \leq 1$.

The players choose their actions independently of each other, each knowing his or her own type, but each knowing only that the other's type is drawn from a Uniform distribution on [0,1].

The utility payoffs (u_1,u_2) for players 1 and 2 respectively depend on their types and actions according to the following formulas:

$$u_1(a_1,a_2,t_1,t_2) = a_1a_2 + 3a_1t_1 - (a_1)^2,$$

$$u_2(a_1,a_2,t_1,t_2) = 2a_2 - a_1a_2 - a_2t_2.$$

(a) (10 points) Suppose that player 1 used a linear strategy that would choose his action as some linear function of his type of the form $a_1 = \beta+\gamma t_1$ for some constants β and γ.
In response to this linear strategy of player 1, what would be player 2's optimal strategy, to maximize her expected utility given her information?

(b) (10 points) What strategy for player 1 would be his best response to the strategy for player 2 that you identified in part (a)?

(c) (10 points) Find a Bayesian equilibrium of this game in which each player's strategy is optimal against the other's strategy.

IV. INFORMATION ECONOMICS (30 points)

An investor (she) has $B = \$8$ with which to finance an entrepreneur (he) who is developing a product that, if successful will generate $R = \$60$ in total revenue. The entrepreneur's development costs are $c = \$4$ per day and so the investor can support the entrepreneur for at most 2 days. The entrepreneur has no other source of funding.

There are two equally likely states of the world, "good" and "bad." The state is chosen before day 1 and is the same on both days. Neither the investor nor the entrepreneur knows the state. On any day that the entrepreneur works on his product, he observes one of three possible signals, s_1, s_2 or "success." The signals s_1 and s_2 produce no information that can be verified by the investor, hence each of those signals, when it occurs, is the entrepreneur's private information. If the signal "success' occurs, then it becomes common knowledge that the product is a success and revenue of \$60 is produced and the relationship ends.

The table below provides the probabilities of the signals conditional on the state. For example, the first column shows that, conditional on the bad state, the signal "success" has probability zero (and so is impossible), while the signal s_1 occurs with proability 5/6. The second column shows that, conditional on the good state, only signal s_2 and "success" occur with positive probability and, in particular, the probability of "success" is 2/3.

	bad state	good state
signal s_1	$\frac{5}{6}$	0
signal s_2	$\frac{1}{6}$	$\frac{1}{3}$
success	0	$\frac{2}{3}$

On any day that the entrepreneur receives the \$4 in funding offered by the investor, the entrepreneur can keep the \$4 for himself instead of spending it on development work. The investor cannot observe whether or not the entrepreneur is working. No signal is generated on any day that the entrepreneur does not work. Both the investor and the entrepreneur are risk neutral with respect to money/wealth and neither discount the future. The entrepreneur maximizes his lifetime expected utility.

A contract $(n; x_1, x_2)$ specifies the number of days, $n \in \{1, 2\}$, that the investor will fund the project, and the amount of revenue, x_i, that will go to the entrepreneur if "success" (i.e., \$60 revenue) occurs on day i. The investor receives the remainder of the revenue, i.e., $60 - x_i$. Note that "success" occurs at most once, i.e., either on day 1 or on day 2, but not both, or not at all. If "success' does not occur, then no additional payments are made to either party and the investor does not recoup any of her investment.

(a) (5 points) What contract of the form $(1; x_1, 0)$ – i.e., what value of x_1 – maximizes the investor's expected profits? What are her profits?

(b) (5 points) (i) If the entrepreneur works on day 1 and generates the signal s_1, would he work on day 2 if he were given funding? (ii) If the entrepreneur works on day 1 and generates the signal s_2, what would x_2 have to be in order for him to work also on day 2 when funded?

(c) (15 points) Find a contract of the form $(2; x_1, x_2)$ – i.e., find x_1 and x_2 – that maximizes the investor's lifetime expected profits subject to the constraints that the investor funds the project for 2 days and that the entrepreneur works on day 1 and also works on day 2 if he observes the signal s_2 on day 1. Explain why x_1 here is larger than in your answer to part (a). (*Hint*: Displaying correct incentive constraints will earn significant partial credit.)

(d) (5 points) Show that your contract in part (c) is expected profit-maximizing among all contracts $(n; x_1, x_2)$ for the investor.