McCall Model

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Motivation

- We would like to study basic labor market data:
  - unemployment and its duration
  - wage heterogeneity among seemingly identical workers
  - job to job transitions
  - how do policies affect those variables?
- Frictionless models of the labor market cannot talk about these issues.
- We need models in which workers must search for jobs.

Search Models

- Unemployment is a productive activity: search for a new job.
- Types of models:
  1. Decision theoretic (McCall model).
  2. Matching: A matching function creates new jobs.
  3. Search: Random encounters and bargaining.

McCall Model

- A partial equilibrium model of a worker searching for a job.
- The worker lives forever, in discrete time.
- Preferences:

$$\sum_{t=0}^{\infty} \beta^t y_t$$

- y_t is income.
- When employed: $y = w$. When unemployed: $y = c$.

Timing

- Enter the period as unemployed worker.
- Draw a wage offer w from the distribution $F(W) = \Pr(w \leq W)$.
- Support: $[0, B]$.
- Choose whether to accept or reject.
- If accept: work forever at wage w with lifetime income $\frac{w}{1-\beta}$.
- If reject: start over next period.

Bellman equation

- State: current wage offer w.
- Control: accept / reject.
- Bellman:

$$v(w) = \max\left\{\frac{w}{1-\beta}, Q\right\}$$

- Q is the expected continuation value:

$$Q = c + \beta \int_0^B v(w')\, dF(w')$$

Reservation wage property

- If reject: get the same continuation value regardless of w.
- If accept: get the present value of earnings: $\frac{w}{1-\beta}$
- The reservation wage makes the worker indifferent between accepting and rejecting:

$$v(\bar{w}) = \frac{\bar{w}}{1-\beta} = Q \tag{1}$$

- Note: For $w < \bar{w}$ the worker still gets $v(\bar{w})$.

Decision Rule

- Write the reservation wage as (proof below):

$$\begin{aligned}\bar{w} - c &= \beta \int_{\bar{w}}^{B} \frac{w' - \bar{w}}{1-\beta} dF(w') \\ &= \beta E\left\{ \frac{w' - \bar{w}}{1-\beta} | w' \geq \bar{w} \right\} \Pr(w' \geq \bar{w})\end{aligned}$$

- In words:
  - the surplus from working now $(\bar{w} - c)$ equals
  - the surplus from searching: the expected lifetime wage gain from perhaps finding a better job

Proof

Write the indifference condition as

$$\frac{\bar{w}}{1-\beta} = c + \beta \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF\left(w'\right) + \beta \int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF\left(w'\right)$$

Simplify:

$$\begin{aligned} & \int_0^{\bar{w}} \frac{\bar{w}}{1-\beta} dF\left(w'\right) + \int_{\bar{w}}^{B} \frac{w'}{1-\beta} dF\left(w'\right) \\ = \quad & \int_0^{B} \frac{\bar{w}}{1-\beta} dF\left(w'\right) + \int_{\bar{w}}^{B} \frac{w' - \bar{w}}{1-\beta} dF\left(w'\right) \\ = \quad & \frac{\bar{w}}{1-\beta} + \int_{\bar{w}}^{B} \frac{w' - \bar{w}}{1-\beta} dF\left(w'\right) \end{aligned}$$

Therefore:

$$\frac{\bar{w}}{1-\beta} - c = \beta \frac{\bar{w}}{1-\beta} + \beta \int_{\bar{w}}^{B} \frac{w' - \bar{w}}{1-\beta} dF\left(w'\right)$$

Implications: Unemployment Benefits

- What is the effect of more generous unemployment benefits? (higher c).
- Expected surplus shrinks when $\bar{w}$ rises.
- Optimality: $c = \bar{w} - \text{expected surplus}$
- RHS increases in $\bar{w}$.
- Higher $c \rightarrow$ higher reservation wage $\rightarrow$ longer unemployment.

More dispersed wage offers

- Result: A mean preserving spread in the wage offer distribution raises the reservation wage and ex ante utility.
- Intuition:
  - Making bad wage offers worse is costless - they are rejected anyway.
  - Making good wage offers better is valuable.
- Proof: Ljunqvist & Sargent.

Firing

- Each period the worker is fired with probability α.
- A fired worker must wait 1 period before drawing a new wage.
- Value when unemployed:

$$v_U = c + \beta \int v(w') dF(w')$$

- Value when employed at wage w:

$$v_E(w) = w + \beta(1-\alpha)v(w) + \beta\alpha v_U$$

Firing: Reservation wage

- Bellman equation

$$v(w) = \max\{v_E(w), v_U\}$$

- Reservation wage solves

$$\begin{aligned} v_E(\bar{w}) &= v_U \\ \bar{w} + \beta(1-\alpha)v(\bar{w}) + \beta\alpha v_U &= v_U \end{aligned}$$

- With $v(\bar{w}) = v_U$:

$$\bar{w} = \frac{v_U}{1-\beta} = c + \beta \int v(w')dF(w')$$

Firing: Implications

- How does the firing probability affect unmployment?
- The reservation wage equations are the "same" with and without firing:

$$\bar{w} = c + \beta \int v(w')dF(w')$$

- The value function is lower with firing
  - because quitting is never optimal
- Therefore $\bar{w}$ is lower with firing.
- If jobs do not last as long, there is no point holding out for the perfect offer.

Equilibrium: Bath Tub Model

- An easy way of embedding the decision problem into a partial equilibrium.
- Exit: Assume the worker "dies" with probability α.
- Layoffs: Workers are laid off with probability ξ.
- Entry: α identical unemployed workers enter each period.

Laws of motion

- Total population: constant at 1.
- Keep track of the unemployment rate U_t.
- Inflows:
  - α new entrants.
  - $\xi(1-\alpha)(1-U)$ layoffs.
- Outflows:
  - αU deaths.
  - $[1-F(\bar{w})](1-\alpha)U$ job matches.
- Law of motion

$$U_{t+1} - U_t = \alpha + \xi(1-\alpha)(1-U_t) - \alpha U_t - (1-\alpha)[1-F(\bar{w})]U_t$$

What is missing?

- Not satisfactory: The job finding rate / wage offer distribution should be endogenous.
  - Think about analyzing policies...
- Matching and search models address this.
  - by introducing endogenous supply of jobs
  - and wage bargaining.

Reading

- Ljunqvist & Sargent, Recursive Macroeconomic Theory, 2nd ed., ch. 6.3.
- Williamson (2006), "Notes on macroeconomic theory," ch. 7, works out a similar model with exogenous job separations.