Aortic Clamping

1 Introduction

Lumped parameter model is the first level of modeling. They are an analogy of electrical models and are constituted of an assembly of resistances, compliances and inductances. Electrical circuits typically link tension to intensity. Lumped parameter models allow to connect the inlet flow rate to the pressure gradient between the inlet and the outlet.

2 Windkessel model

In this section, we study the Windkessel model which is the most commonly used zero-dimensional (0D) parameter model to study blood flow. The windkessel model is composed of two resistances, one compliance and one inductance. In our case, this inductance will be set to zero and therefore does not appear on Fig. 1.

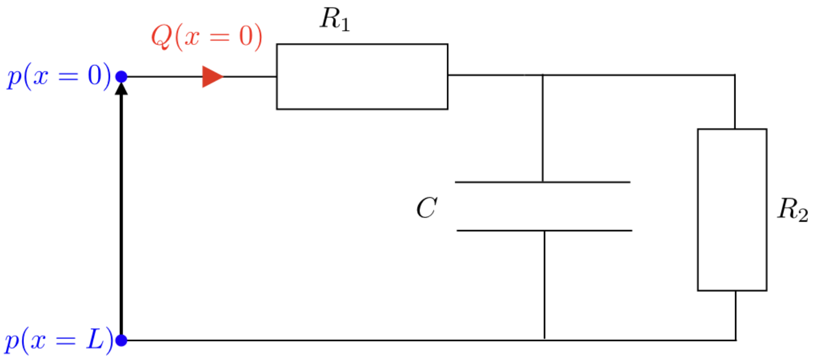

FIGURE 1 – Electrical representation of the Windkessel 0D model.

To model a vessel of lenght L,the general governing equation of a Windkessel model is the following:

$$\left(1 + R_2C\frac{\mathrm{d}.}{\mathrm{d}t} + IC\frac{\mathrm{d}^2.}{\mathrm{d}t^2}\right)p(x=0) - p(x=L) = \left((R_1+R_2) + R1\left(R_2C\frac{\mathrm{d}.}{\mathrm{d}t} + IC\frac{\mathrm{d}^2.}{\mathrm{d}t^2}\right) + I\frac{\mathrm{d}.}{\mathrm{d}t}\right)Q(x=0). \tag{1}$$

Considering that $p(x=L)=0$ and $I=0$, we can discretize this equation with an explicit Euler scheme that gives:

$$p^n + R_2C\frac{p^{n+1}-p^n}{\Delta t} = (R_1+R_2)Q^n + R_1R_2C\frac{Q^{n+1}-Q^n}{\Delta t}. \tag{2}$$

In Eq. (2) the unknown is p^{n+1} that is the pressure at $x=0$ at time t_{n+1}. The flow rate Q^n and Q^{n+1} correpond to the inlet flow in $x=0$ that we impose.

3 Method

3.1 Input flow rate

The inflow boundary condition $Q(x = 0)$ is a flow rate thats mimics a heart pulse. The input flow rate is a half sine signal (see Fig. 2) described by two parameters: the amplitude Q_0 and the ejection time T_{ej}, which is a portion of heart beat T fixed by the experimental data. We defined the following function:

$$f(t, \alpha) = \begin{cases} \sin(\pi t/\alpha) & \text{if } \ t < \alpha \\ 0 & \text{if } \ t > \alpha \end{cases} \tag{3}$$

The input flow rate is then defined as:

$$Q_{input}(t) = Q_0.f(t/T - E(t/T), T_{ej}) \tag{4}$$

where Q_0 is the amplitude, the function $E(t)$ is the integer-valued function, T is the heart period and T_{ej} is the ejection fraction. This function is represented in Fig. 2.

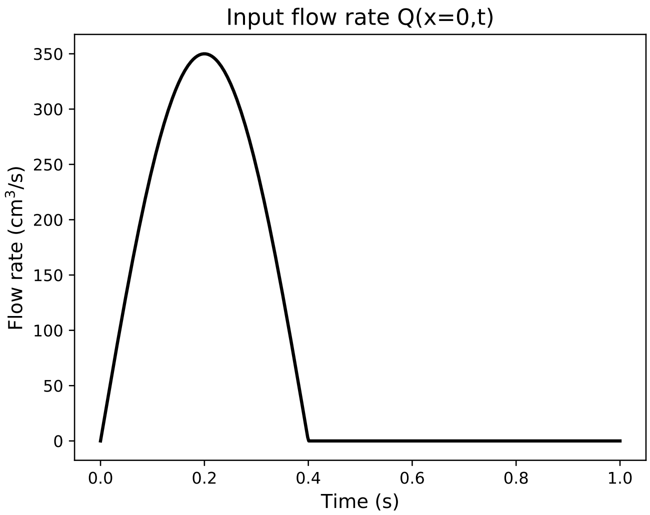

FIGURE 2 – Input flow rate Q as a function of time with amplitude $Q_0 = 350$ cm^3/s and ejection time $T_{ej} = 40\%$ of heart time period $T = 1$ s .

In the following, we decide not to estimate Q_0 the amplitude of input flow rate since it is obviously related to the value of resistance through Ohm's law ($P = RQ$). Therefore the chosen value for Q_0 is such that the stroke volume (i.e ventricular ejection volume over a heart beat) is comprised between 70 and 90 mL.

3.2 Cost function

Now the objective is for both pre clamp and post clamp to estimate the values R_1, R_2, C along with the ejection fraction T_{ej} for each patient. Therefore, we define a cost function J that depends on these parameters as follows:

$$J(\mathrm{p}) = \left(\int_0^T (P_{exp} - P_{num}(\mathrm{p}))^2 \mathrm{d}t \right)^{1/2} \tag{5}$$

where P_{exp} is the experimental pressure wave and P_{num} is the solution of Eq. (1) and p is the set of parameterss (R_1, R_2, C, T_{ej}).

We use a gradient-based algorithm L-BFGS-B that allows bound type of constraints and combine it with a Basin-Hopping method to find the global extrema for each situation.

4 Results

Table 1 below sums up the results of parameter estimation on all patients for pre clamp and post clamp. We added the values of the total resistance $R_{tot} = R_1 + R_2$ and the amplitude of input flow rate Q_0. This last parameter is not estimated with the algorithm for obvious reasons but is adjusted so that the stroke volume V_s is constant during clamping. We chose to keep the stroke volume constant because we expect that the heart has not responded yet to clamping within the period of acquisition of data. The last column corresponds to the calculated relaxation time $\tau = R_2 C$.

Patient 6	R_1	R_2	C	T_{ej}	R_{tot}	Q_0	V_s	τ
Pre Clamp	220	1294	1.31e-3	32.8	1514	350	83.3	1.70
Post Clamp	287	1428	9.49e-4	31.6	1715	317	83.4	1.36
Pre/Post ratio	0.77	0.91	1.38	1.04	0.88	1.10	—	1.25
Patient 7	R_1	R_2	C	T_{ej}	R_{tot}	Q_0	V_s	τ
Pre Clamp	152	1209	1.18e-3	33.7	1361	400	88.4	1.43
Post Clamp	194	1214	1.15e-3	34.9	1408	380	88.5	1.40
Pre/Post ratio	0.78	0.99	1.03	0.97	0.65	1.05	—	1.02
Patient 9	R_1	R_2	C	T_{ej}	R_{tot}	Q_0	V_s	τ
Pre Clamp	122	1059	1.8e-3	39.6	1180	300	73.3	1.91
Post Clamp	165	1240	1.4e-3	40.2	1405	278	73.3	1.74
Pre/Post ratio	0.74	0.85	1.29	0.99	0.84	1.08	—	1.09
Patient 14	R_1	R_2	C	T_{ej}	R_{tot}	Q_0	V_s	τ
Pre Clamp	110	698	2.50e-3	52.1	808	400	77	1.75
Post Clamp	219	778	1.91e-3	53.4	997	359	76.9	1.49
Pre/Post ratio	0.50	0.90	1.31	0.98	0.81	1.11	—	1.17
Mean ratio	0.70	0.91	1.25	0.99	0.80	1.09	—	1.13

TABLE 1 – Sum up of parameter estimation results of Windkessel model. Resistances R_1 and R_2 are in g.cm^{-4}.s^{-1}, compliance is in g^{-1}.cm^4.s^2, amplitude of flow rate is in cm^3/s, ejection fraction is in percentage of heart period, stroke volume is in cm^3 and τ is in s. R_{tot} corresponds to the total resistance $R_1 + R_2$.

We observe that for patient 6, the ejection time decreases with clamping whereas it increases for other patients under the condition that thee stroke volume remains constant. It is probably due to the fact that patient 6 has a significant difference of heart beat before and after clamping (1.14 s before clamping vs 1.31s after clamping). However the mean change in ejection time is very small so this result is not truly relevant.

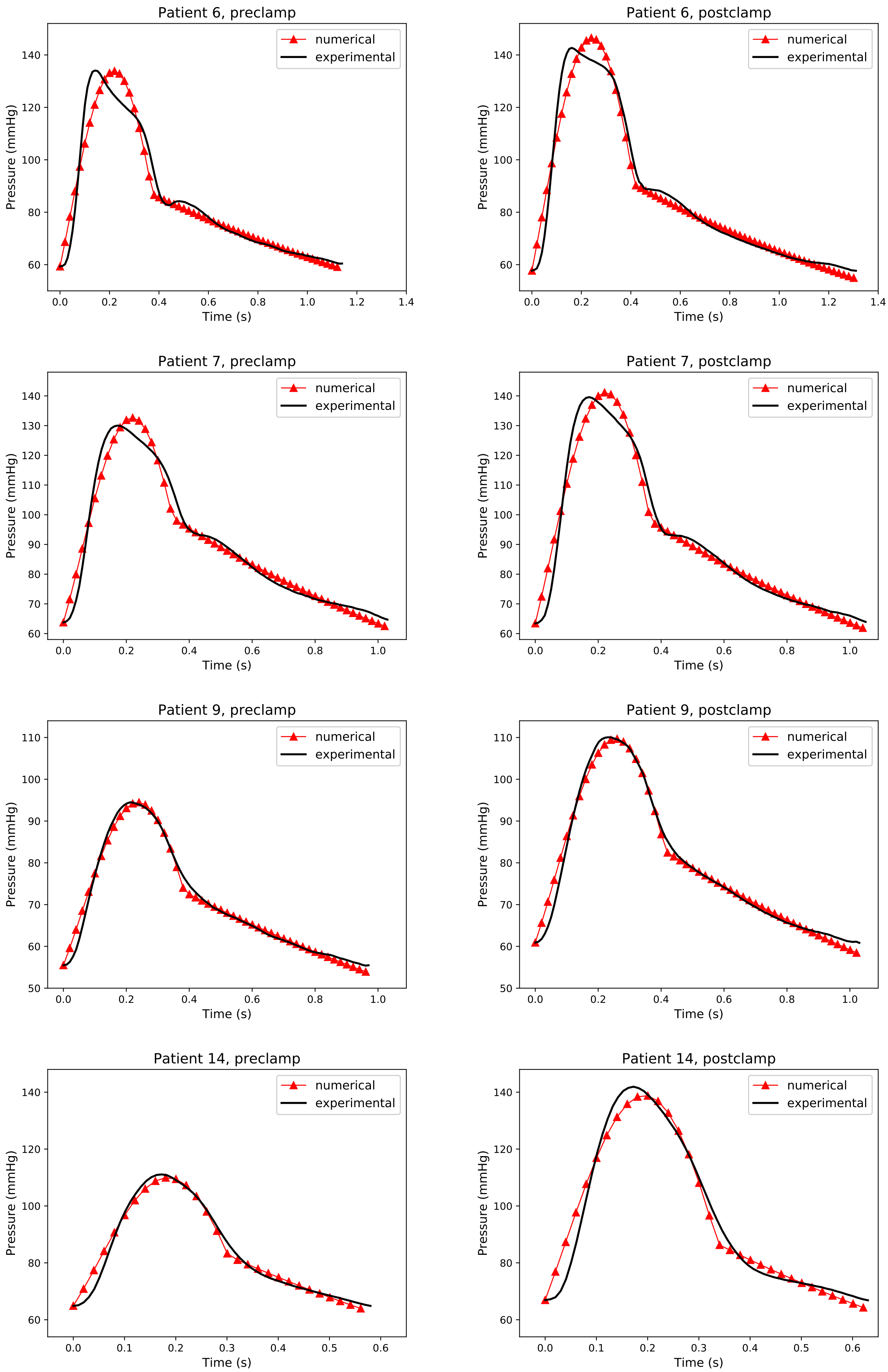

FIGURE 3 – Pressure wave signal for 4 patients undergoing aortic clamping during vascular surgery. Black lines are the experimental measurements and red corresponds to the numerical solution of Eq. 2 .