Exercises

1. Let $G = (V, E)$ be an undirected graph with n nodes. Recall that a subset of the nodes is called an *independent set* if no two of them are joined by an edge. Finding large independent sets is difficult in general; but here we'll see that it can be done efficiently if the graph is "simple" enough.

   Call a graph $G = (V, E)$ a *path* if its nodes can be written as $v_1, v_2, \ldots, v_n$, with an edge between v_i and v_j if and only if the numbers i and j differ by exactly 1. With each node v_i, we associate a positive integer *weight* w_i.

   Consider, for example, the five-node path drawn in Figure 6.28. The *weights* are the numbers drawn inside the nodes.

   The goal in this question is to solve the following problem:

   Find an independent set in a path G whose total weight is as large as possible.

   (a) Give an example to show that the following algorithm *does not* always find an independent set of maximum total weight.

```
The "heaviest-first" greedy algorithm
  Start with S equal to the empty set
  While some node remains in G
    Pick a node v_i of maximum weight
    Add v_i to S
    Delete v_i and its neighbors from G
  Endwhile
  Return S
```

   (b) Give an example to show that the following algorithm also *does not* always find an independent set of maximum total weight.

```
Let S_1 be the set of all v_i where i is an odd number
Let S_2 be the set of all v_i where i is an even number
(Note that S_1 and S_2 are both independent sets)
Determine which of S_1 or S_2 has greater total weight,
  and return this one
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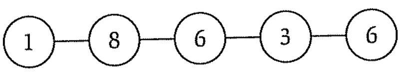

Figure 6.28 A paths with weights on the nodes. The maximum weight of an independent set is 14.