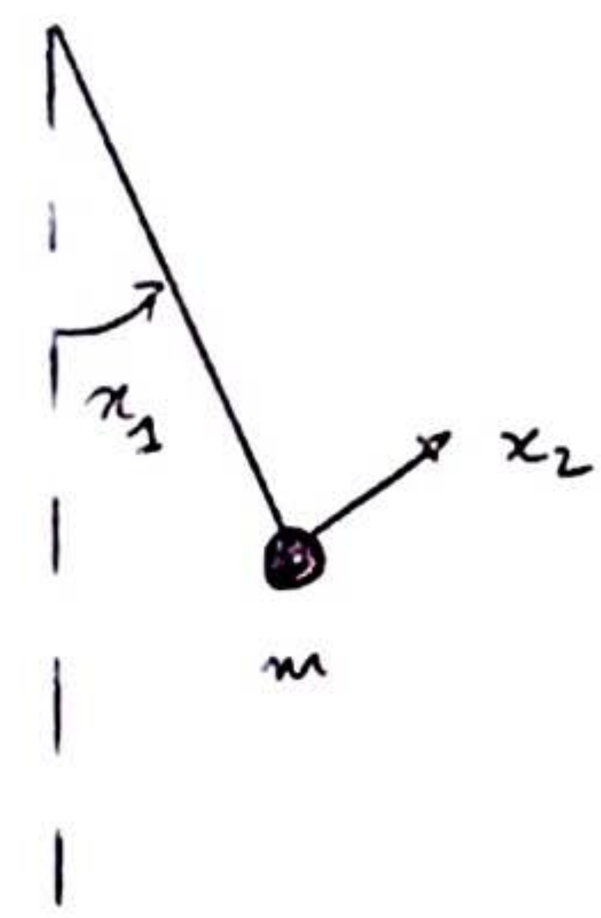
(113) Sliding pendulum.



$$u \rightarrow \boxed{\begin{cases}\dot x_1 = x_2\\ \dot x_2 = -\sin x_1 + u\end{cases}}$$

(i) bouclage linéarisant : $y = x_1$ (l'angle que l'on veut contrôler)

$\dot y = \dot x_1 = x_2$

$\ddot y = \dot x_2 = -\sin x_1 + u$

on prendra $u = \sin x_1 + \overbrace{\underbrace{(w-y)}_{e} + 2\cdot\underbrace{(\dot w - \dot y)}_{\dot e}}^{\text{PID}} + \underbrace{\ddot w}_{\ddot e}$

$\Rightarrow \quad e + 2\dot e + \ddot e = 0$

$P(s) = s^2 + 2s + 1 = (s+1)^2$

$\lambda = -1 \rightarrow e^{-1\cdot t}$

. on a $w(t) = 0, \forall t$ d'où $w = \dot w = \ddot w = 0$

$$\Rightarrow \begin{cases}\dot x_1 = x_2\\ \dot x_2 = -\cancel{\sin}\, x_1 - 2x_2\end{cases}$$

② glissant.

$$\begin{cases} y = x_1 \\ \dot{y} = x_2 \end{cases}$$

$\overset{2}{\downarrow}$ $n-1$

$$s(x,t) = \underbrace{(\dot{w} - \dot{y})}_{\dot{e}} + \underbrace{(w - y)}_{e} = 0$$

$e \to 0$

$\dot{e} + e = 0$

$$u = \sin x_1 + K \cdot \mathrm{sign}(s(x,t))$$

$$w(t) = 0, \forall t \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -K \,\mathrm{sign}(\cancel{\alpha}\, x_1 + x_2) \end{cases}$$

③ $V(x) = \frac{1}{2} s^2(x)$

$$\dot{V}(x) = s(x) \cdot \dot{s}(x) = (\dot{w} - x_2) + (w - x_1))((\ddot{w} - \dot{x}_2) + \dot{w} - \dot{x}_1))$$

or $w = 0$ $\forall t$ d'où $= (-x_2 - x_1)(-\dot{x}_2 - \dot{x}_1)$

$$= (x_2 + x_1)(\dot{x}_2 + \dot{x}_1)$$

$\dot{x}_1 = x_2$

avec $\dot{x}_2 = -\sin x_1 + u = -\sin(x_1) + \cancel{\mathrm{sign}}\ \sin x_1 - K\,\mathrm{sign}(x_1 + x_2)$

$$\dot{V}(x) = (x_2 + x_1)(-K\,\mathrm{sign}(x_1 + x_2))$$

$S = \{ x /\ \dot{V}(x) > 0 \} \longrightarrow$ analyse d'intervalle