(112) Field following.

équation d'état du robot :

$$\begin{cases} \dot{x}_1 = \cos x_3 \\ \dot{x}_2 = \sin x_3 \\ \dot{x}_3 = u \end{cases}$$

avec

x_3 : cap du robot.

$p = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ coordonnés du robot.

$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ vecteur d'état.

(1) $y = x_3 + \arctan(x_2)$

$$\dot{y} = \dot{x}_3 + \frac{\dot{x}_2}{1+x_2^2} = u + \frac{\sin x_2}{1+x_2^2} \qquad (k=1)$$

on prend $\dot{y} = -y$

x

on a $u = \dot{y} - \frac{\sin x_2}{1+x_2^2} = -y - \frac{\sin x_2}{1+x_2^2} = \boxed{-x_2 - \operatorname{atan} x_2 - \frac{\sin x_2}{1+x_2^2}}$ u

u

on a $\begin{cases} \dot{x}_1 = \cos x_3 = \cos(-\operatorname{atan} x_2) \\ \dot{x}_2 = \sin x_3 = \sin(-\operatorname{atan} x_2) \end{cases}$ (car $x_3 + \operatorname{atan} x_2 = 0$)

(3) Van der Pol equation :

$$\begin{cases} \dot{x}_1 = x_2 = a \\ \dot{x}_2 = -(0{,}01 \times x_1^2 - 1)\, x_2 - x_1 = b \end{cases}$$

on a • $y = x_3 - \operatorname{arctan2}(b, a) = 0$

• $\dot{y} = -y$

• $\dot{y} = \underset{\downarrow\\ u}{\dot{x}_3} - \left(\underset{\downarrow\\ \frac{\partial \operatorname{atan2}}{\partial a}}{\frac{-b}{a^2+b^2}} \dot{a} + \underset{\downarrow\\ \frac{\partial \operatorname{atan2}}{\partial a}}{\frac{a}{a^2+b^2}} \dot{b} \right)$

• $\dot{y} = u + \dfrac{b\dot{a} - a\dot{b}}{a^2+b^2}$ $(k=1)$ pas besoin de dériver une nouvelle fois.

• $a = x_2$

$\dot{a} = \dot{x}_2 = \sin x_2$

• $b = -(0{,}01\, x_1^2 - 1)\, x_2 - x_1 = -0{,}02 \cdot x_1 \underset{\downarrow\\ \cos x_3}{\dot{x}_1}\, x_2 - (0{,}01\, x_1^2 - 1) \underset{\downarrow\\ \sin x_3}{\dot{x}_2} - \underset{\downarrow\\ \cos x_3}{\dot{x}_1}$

$$\boxed{u = -\left(x_3 - \operatorname{atan2}(b, a)\right) - \left(\frac{b\dot{a} - a\dot{b}}{a^2+b^2}\right)}$$

sawtooth pour filtrer le % 2π !