109 Group of robots

$$\begin{cases} \dot{x}_1 = x_4 \cdot \cos x_3 \\ \dot{x}_2 = x_4 \cdot \sin x_3 \\ \dot{x}_3 = u_1 \\ \dot{x}_4 = u_2 \end{cases}$$

$$y = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

- $\dot{y}_1 = x_4 \cdot \cos x_3$
- $\ddot{y}_1 = -x_4 \cdot \sin x_3 \cdot \dot{x}_3 + \dot{x}_4 \cos x_3 .$

$$\ddot{y}_1 = -u_1 \cdot x_4 \sin x_3 + u_2 \cdot \cos x_3$$

- $\ddot{y}_2 = \dot{x}_4 \sin x_3 + x_4 \dot{x}_3 \cos x_3$

$$= u_2 \cdot \sin x_3 + u_1 \cos x_3 \cdot x_4$$

- $\cancel{\ddot{y}_2} =$

sous forme matricielle on obtient :

$$\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} = \underbrace{\begin{pmatrix} -x_4 \cdot \sin x_3 & \cos x_3 \\ x_4 \cos x_3 & \sin x_3 \end{pmatrix}}_{A(x)} \underbrace{\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}}_{u}$$

$\Rightarrow u = A^{-1}(x) \cdot v$ avec v : nouvelle entrée v.

on prend $v = c - \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + 2 \cdot \left(\dot{c} - \begin{pmatrix} x_4 \cos x_3 \\ x_4 \sin x_3 \end{pmatrix} \right) + \ddot{c}(t)$

avec • $c(t) = \begin{pmatrix} \cos(at + \frac{2i\pi}{m}) \\ \sin(at + \frac{2i\pi}{m}) \end{pmatrix}$

$$\dot{c}(t) = a \cdot \begin{pmatrix} -\sin\left(at + \frac{2i\pi}{n}\right) \\ \cos\left(at + \frac{2i\pi}{n}\right) \end{pmatrix}$$

$$\ddot{c}(t) = -a^2 \cdot c(t)$$

② $$w(t) = \underbrace{\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}}_{\text{matrice de rotation} \; = R} \underbrace{\begin{pmatrix} 20 + 15 \cdot \sin(at) & 0 \\ 0 & 20 \end{pmatrix}}_{\text{mat. diagonale} \; = D} \cdot c(t)$$

$$\dot{w}(t) = R \cdot D \cdot \dot{c} + R\dot{D}c + \dot{R}D \cdot c$$

avec
$$\dot{D} = \begin{pmatrix} 15 \cdot a \cdot \cos(at) & 0 \\ 0 & 0 \end{pmatrix}$$

$$\dot{R} = \begin{pmatrix} -\dot{\theta}\sin\theta & -\dot{\theta}\cos\theta \\ \dot{\theta}\cos\theta & -\dot{\theta}\sin\theta \end{pmatrix} = \dot{\theta} \cdot \begin{pmatrix} -\sin\theta & -\cos\theta \\ \cos\theta & -\sin\theta \end{pmatrix}$$

or $\theta = at$ d'où par identification

$$\dot{\theta} = a$$

$$\ddot{w}(t) = R \cdot D \cdot \ddot{c} + R\ddot{D}c + \ddot{R}Dc + 2\dot{R}D\dot{c} + 2R\dot{D}\dot{c} + 2 \cdot \dot{R}\dot{D}c$$

avec
$$\ddot{D} = \begin{pmatrix} -15a^2 \sin(at) & 0 \\ 0 & 0 \end{pmatrix}$$

$$\Rightarrow \ddot{R} = -a^2 \cdot R$$