

# Purpose of this exercise: First-arrival travel time tomography

## FRECHET FORMULATION

This exercise will illustrate the least-square inversion of first-arrival times (onset frequency-independent picking) using Fréchet derivatives (or, in other words, the sensitivity matrix).

We shall compare results with those obtained by using the adjoint approach where the Fréchet derivatives are not built up explicitly. The delayed travel time  $\delta t$  is given by the expression

$$\delta t(s, r) = \int_{ray_0}^{\square} \sum_{cube} \delta s_{i,j,k} h_{i,j,k} dl = \sum_{cube} \delta s_{i,j,k} \int_{ray_0}^{\square} h_{i,j,k} dl$$

along the ray between the source and the receiver. We assume that the ray is independent of the slowness perturbation  $\delta s$  which is described with interpolation function  $h$ . We consider a constant background such that the ray is a straight line. The ray tracing is quite simple. The linear forward problem

$$\delta t(s, r) = \sum_{i,j,k} \frac{\partial t}{\partial s_{i,j,k}} \delta s_{i,j,k}$$

can be solved by any solver and the often used one is the conjugate gradient method. We need to compute Fréchet derivatives  $\frac{\partial t}{\partial s}$  which are the weighted ray segment, making the matrix  $A$ . The rectangular matrix has a complexity  $N \times M$  where the number of data is denoted by  $N$  and the number of degrees of freedom for the model by  $M$ . Instead of using the LSQR algorithm, we shall consider a more recent one named LSMR (we shall use the damping option as for LSQR). Moreover, this algorithm is provided with a reverse communication engine which makes the optimization tool external to the forward problem, leading to a non-intrusive writing of computer codes.

The LSMR will solve the damped system

$$\begin{bmatrix} A \\ damp.I \end{bmatrix} x = \begin{bmatrix} b \\ 0 \end{bmatrix}$$

in a least-squares sense, i.e. normal equation is going to be solved iteratively. The structure of the code is not invasive and specific actions have to be performed by the user when asked by the optimization tool. It performs an initialization phase (action=0) with a further output requested action to be performed by the user. If the action=1, the user has to perform the following computation  $v = v + A^t u$  and, if the action=2, the user has to perform the following computation  $u = u + Av$ . They are set to zero at the beginning. The vector  $v$  is in the model space and the vector  $u$  in the data space. The vector  $b$  is the delayed travel times to be fitted by the optimization LSMR.

- For a cross-hole experiment, specify the description of the acquisition, the description of the data and the description of the model (files read\_\*.)
- Let us consider a mesh in a 2D space for which you can only ask where you are: design an algorithm to computing ray segment lengths.
- Algorithm ray\_over\_data: global task - building the matrix  $A = model \% frechet$  and the vector  $b = data \% time_{syn}$  using d)
- Algorithm ray\_sngl\_data\_src\_rec: building one line of the matrix  $A$  for a couple (src,rec) and the travel time using e)
- Algorithm ray\_segment: recursive code writing with dichotomy for computing each element (ray segment) of the current data line and the travel time using trapezoidal rule for estimating

the integral: we use a simpler constant interpolation  $h=1$  as usually done with a bilinear one making the velocity model continuous.

- f) Call the LSMR for solving the damped least-squares problem

**To be done:** please, write down the workflow of this algorithm.

**Workflow:** (see presentations if needed)

**Remark:** LSMR allows orthogonalization with respect to previous updated models (new compared to LSQR) and a similar strategy for considering previous models as done when performing adjoint tomography.

# TOMO2D\_RAYS\_FRECHET

(inside the directory SRC\_UTIL of the TOMO2D\_RAYS\_FRECHET)

We assume that you have typed the command “cd SRC\_UTIL” for being in the utility source directory. Simple programs are there for building models, sources and receivers distributions.

For compiling these codes, type “make” at the prompt. It will put an executable binary file in the directory BIN. You need the gfortran compiler.

(inside the directory SRC\_FRECHET of the TOMO2D\_RAYS\_FRECHET)

We assume that you have typed the command “cd SRC\_FRECHET” for being in the source directory.

This is a simple program where the full sensitivity matrix is completely built in spite of its strong sparsity. One can use a more compact strategy for storing this sparse matrix than the one used in this toy program.

For compiling the code, type “make” at the prompt. It will put an executable binary file in the directory BIN. You need the gfortran compiler.

Please have a look at the organization of the code and the successive calls to subroutines performing algorithms mentioned above.

## Computing synthetic data to be used as observed data

Copy the directory RUN\_SYNT\_HETERO\_TEMPLATE into RUN\_SYNT\_HETERO by the command “cp -r RUN\_SYNT\_HETERO\_TEMPLATE RUN\_SYNT\_HETERO”

Go inside this new directory by the command “cd RUN\_SYNT\_HETERO”

Launch the command “sh run\_synth2D\_hetero.sh 101 201” which will create acquisition files, a model file with a decaying exponential anomaly to be reconstructed and an output file data\_fwd\_hetero.dat which are the “observed” data we shall use.

A binary file “model\_hetero\_ref.bin” is created and can be seen using gnuplot by the following command “gnuplot dessin.gnu”. A jpeg file “MAP.jpg” will appear and can be plotted by any ad-hoc software. For example the application “display” from Image Magick.

Once done, type “cd ..” to go back to the project directory.

## Performing a simple inversion test

Copy the directory RUN\_HETERO\_TEMPLATE into RUN\_HETERO by the command “cp -r RUN\_HETERO\_TEMPLATE RUN\_HETERO”

Go inside this new directory by the command “cd RUN\_HETERO”

Launch the command “sh run\_tomo2D\_hetero.sh 101 201” and analyze what you will see in this directory, using three gnuplot files target.gnu, init.gnu and final.gnu.

The data to be fitted comes from the directory RUN\_SYNT\_HETERO..

## Performing your own inversion test

Design your own velocity anomaly to be reconstructed: the perturbation should be small compared to the background velocity.

Remark: Of course, for more complex approaches, rays will not be straight and one should iterate over ray tracing, following a linearized optimization: update the velocity, trace rays, evaluate Fréchet derivatives and residues in the current model, solve this linear system for a new velocity which could be used for another iteration.

Congratulations if you arrive here 🏖️