Quiz 3

1. 請框出答案. 2. 禁止作弊!

1. Use the Newton's Forward Difference Formula to approximate $\sqrt{5}$ with the function $f(x) = 5^x$ and the values list on the table. Also, compute the absolute error and relative error in this approximation.

Answer.

i	x_i	$f(x_i)$	$\Delta f(x_i)$	$\Delta^2 f(x_i)$	$\Delta^3 f(x_i)$
0	-1	$0.2 = f(x_0)$	$0.8 = \Delta f(x_0)$	$3.2 = \Delta^2 f(x_0)$	$12.8 = \Delta^3 f(x_0)$
1	0	1	4	16	
2	1	5	20		
3	2	25			

$$x = 1/2 = 0.5 \Rightarrow s = \frac{x - x_0}{h} = \frac{0.5 - (-1)}{1} = 1.5,$$

$$\begin{aligned} f(0.5) \approx P_3(0.5) &= f(x_0) + s\Delta f(x_0) + \frac{s(s-1)}{2!}\Delta^2 f(x_0) + \frac{s(s-1)(s-2)}{3!}\Delta^3 f(x_0) \\ &= 0.2 + (1.5)0.8 + \frac{(1.5)(0.5)}{2}3.2 + \frac{(1.5)(0.5)(-0.5)}{6}12.8 \\ &= 1.8 \end{aligned}$$

$$f(0.5) = 5^{0.5} = \sqrt{5} = 2.2361 \Rightarrow \text{Absolute Error :} |2.2361 - 1.8| = 0.4361.$$

2. Neville's method is used to approximate $f(0.5)$ as follows. Complete the table.

i	x_i	$Q_{i,0}$	$Q_{i,1}$	$Q_{i,2}$
0	0	$Q_{0,0} = 0$		
1	0.4	$Q_{1,0} = 2.8$	$Q_{1,1} = 3.5$	
2	0.7	$Q_{2,0} =?$	$Q_{2,1} =?$	$Q_{2,2} = \frac{27}{7}$

Answer.

$$
\begin{aligned}
Q_{2,2}(0.5) = \frac{27}{7} &= \frac{(0.5 - x_0)Q_{2,1} - (0.5 - x_2)Q_{1,1}}{x_2 - x_0} \\
\frac{27}{7} &= \frac{(0.5 - 0)b - (0.5 - 0.7)3.5}{0.7 - 0} = \frac{0.5b + 0.7}{0.7} \Rightarrow \\
\frac{27}{7} &= \frac{5b + 7}{7} \Rightarrow b = 4,
\end{aligned}
$$

$$
\begin{aligned}
Q_{2,1}(0.5) = 4 &= \frac{(0.5 - x_1)Q_{2,0} - (0.5 - x_2)Q_{1,0}}{x_2 - x_1} \\
4 &= \frac{(0.5 - 0.4)a - (0.5 - 0.7)2.8}{0.7 - 0.4} = \frac{0.1a + 0.56}{0.3} \Rightarrow \\
4 &= \frac{10a + 56}{30} \Rightarrow a = 6.4.
\end{aligned}
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