姓名: SOLUTION 　　　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________

Quiz 3

考試日期: 2020/10/15

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. (50%) Find a basis for the solution set of the given homogeneous linear system.

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 = 0 \\ x_1 - 6x_2 + x_3 = 0 \\ 3x_1 + 5x_2 + 2x_3 + x_4 = 0 \\ 5x_1 - 4x_2 + 3x_3 + 2x_4 = 0 \end{cases}$$

Answer: the basis set is $\left\{ \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

Let $A = \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & -6 & 1 & 0 \\ 3 & 5 & 2 & 1 \\ 5 & -4 & 3 & 2 \end{bmatrix}$, and $H = rref(A) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Assume $x_4 = r$, plug into $[H|0]$. We have $\begin{cases} r + x_1 = 0 \\ x_2 = 0 \\ -r + x_3 = 0 \end{cases}$.

Hence, $x_1 = -r, x_3 = r$. We have solution set $\left\{ \begin{bmatrix} -r \\ 0 \\ r \\ r \end{bmatrix} \middle| r \in \mathbb{R} \right\} = sp\left(\begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right)$

2. (50%) Solve the given linear system and express the solution set.

$$\begin{cases} 2x_1 + x_2 + x_3 + x_4 = 1 \\ x_1 - 6x_2 + x_3 = 12 \\ 3x_1 + 5x_2 + 2x_3 + x_4 = -7 \\ 5x_1 - 4x_2 + 3x_3 + 2x_4 = 14 \end{cases}$$

Answer: the solution set is $\left\{ \begin{bmatrix} 3 \\ -2 \\ -3 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \middle| r \in \mathbb{R} \right\}$

Let $[A|\vec{b}] = \left[\begin{array}{cccc|c} 2 & 1 & 1 & 1 & 1 \\ 1 & -6 & 1 & 0 & 12 \\ 3 & 5 & 2 & 1 & -7 \\ 5 & -4 & 3 & 2 & 14 \end{array}\right]$, and $[H|\vec{c}] = rref([A|\vec{b}]) = \begin{bmatrix} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & -1 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Assume $x_4 = 0$, plug into $[H|\vec{c}]$. We have $\begin{cases} x_1 = 3 \\ x_2 = -2 \\ x_3 = -3 \end{cases}$.

Hence, We have a particular solution $\begin{bmatrix} 3 \\ -2 \\ -3 \\ 0 \end{bmatrix}$.

The solution set is $\left\{ \begin{bmatrix} 3 \\ -2 \\ -3 \\ 0 \end{bmatrix} + r \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} \middle| r \in \mathbb{R} \right\}$