Section 1.4 Solving Systems of Linear Equations

Thm 1.7 Let $A\vec{x} = \vec{b}$ be a linear system, and let $[A|\vec{b}] \sim [H|\vec{c}]$, where H is in row-echelon form.

(a) The system $A\vec{x} = \vec{b}$ is inconsistent if and only the augmented matrix $[H|\vec{c}]$ has a row with all entries 0 to the left of the partition and a non-zero entry to the right of the partition.

(b) If $A\vec{x} = \vec{b}$ is consistent and every column of H contains a pivot, the system has a unique solution.

(c) If $A\vec{x} = \vec{B}$ is consistent and some column of H has no pivot, the system has infinitely many solutions, with as many free variables as there are pivot-free columns in H.

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We need a vector $\vec{b}$ so that $[A|\vec{b}]$ has solution, where

$$[A|\vec{b}] = \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 3 & 6 & b_2 \end{array}\right], \text{ and } rref[A|\vec{b}] = \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 0 & 0 & b_2 - 3b_1 \end{array}\right]$$

Therefore, the linear system is consistent only if $b_2 - 3b_1 = 0$ by Theorem 1.7 (3).

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We need a vector $\vec{b}$ so that $[A|\vec{b}]$ has solution, where

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 0 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Since $rref(A) = I_3$, the linear system is consistent with all possible b_i by Theorem 1.7 (2).