

Section 9-4

課本第 33 題

Let A be an $n \times n$ matrix with eigenvalue λ . Prove that the algebraic multiplicity of λ is at least as large as its geometric multiplicity.

解答：

Assume the Jordan Canonical form of A contains k λ -Jordan blocks, which has size m_1, m_2, \dots, m_k and $m_1 + m_2 + \dots + m_k = M$. Since the characteristic polynomial the Jordan Canonical form has factor $(x - \lambda)^M$ and the characteristic polynomial of A and its the Jordan Canonical form are the same, the algebraic multiplicity of λ is M .

The geometric multiplicity of λ is the dimension of its eigenspace. Therefore, the geometric multiplicity of λ is the number of λ -Jordan blocks, which is k .

Obviously, $k \leq M$.

$$A \sim J = \begin{bmatrix} J_{m_1}(\lambda) & & & & \\ & J_{m_2}(\lambda) & & & \\ & & \ddots & & \\ & & & J_{m_k}(\lambda) & \\ & & & & \text{other jordan blocks} \end{bmatrix}$$