姓名: SOLUTION 葉均承 應數三科學計算

學號: _____ **Quiz 7** 考試日期: 2021/04/13

1. 請框出答案. 2. 禁止作弊!

1. Find the first three iterations obtained by the Power method applied to the following matrices.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}. \text{ Use } \vec{x}^{(0)} = (1, -1, 2)^T$$

Answer:
1^{st} iteration: $\mu^{(1)} = 2$, 2^{nd} iteration: $\mu^{(2)} = 3$, 3^{rd} iteration: $\mu^{(3)} = 3.6667$.

1. 照證明流程走

$$p_0 = 3,\ x_3^{(0)} = 2,\ \vec{x}^{(0)} \to \frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

$$\vec{y}^{(1)} = A\vec{x}^{(0)} = \frac{1}{2}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix},\ \mu^{(1)} = \frac{y_3^{(1)}}{1} = \frac{4}{2} = 2,$$

$$p_1 = 3,\ y_3^{(1)} = 2,\ \vec{x}^{(1)} = \frac{1}{2}\vec{y}^{(1)} = \frac{1}{4}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix},$$

$$\vec{y}^{(2)} = A\vec{x}^{(1)} = \frac{1}{4}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix},\ \mu^{(2)} = \frac{y_3^{(2)}}{1} = \frac{12}{4} = 3,$$

$$p_2 = 3,\ y_3^{(2)} = 3,\ \vec{x}^{(2)} = \frac{1}{3}\vec{y}^{(2)} = \frac{1}{12}\begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix},$$

$$\vec{y}^{(3)} = A\vec{x}^{(2)} = \frac{1}{12}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix} = \frac{1}{12}\begin{bmatrix} 43 \\ 41 \\ 44 \end{bmatrix},\ \mu^{(3)} = \frac{y_3^{(3)}}{1} = \frac{44}{12} = 3.6667,$$

背面還有

2. 筆試推薦作法

$$\vec{x}^{(0)} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix},$$

$$\vec{x}^{(1)} = A\vec{x}^{(0)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}, \vec{x}^{(2)} = A\vec{x}^{(1)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix},$$

$$\vec{x}^{(3)} = A\vec{x}^{(2)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 11 \\ 9 \\ 12 \end{bmatrix} = \begin{bmatrix} 43 \\ 41 \\ 44 \end{bmatrix}, \vec{x}^{(3)} = A\vec{x}^{(2)} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 43 \\ 41 \\ 44 \end{bmatrix} = \begin{bmatrix} 171 \\ 179 \\ 172 \end{bmatrix},$$

$$\mu^{(1)} = \frac{4}{2} = 2,\ \mu^{(2)} = \frac{12}{4} = 3,\ \mu^{(3)} = \frac{44}{12} = 3.6667,\ \mu^{(4)} = \frac{172}{44} = 3.9091,$$