數學二離散數學 2023 秋, 第一次期中考

學號: _____, 姓名: _____

本次考試共有 9 頁 (包含封面)，有 12 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號。

- **不可**翻閱課本或筆記。

- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。　**敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。　　　**遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Find integers a and b such that

$$m^2 = a\binom{m}{2} + b\binom{m}{1}$$

   for all m. Then use it to find the sum the series $1^2 + 2^2 + ... + n^2$.

   Answer: $a = $ _____, $b = $ _____, $1^2 + 2^2 + ... + n^2 = $ _____.

2. (10 points) Show that if n + 1 integers are chosen from the set 1, 2, . . . , 4n then there are always two which differ by at most 3.

3. (10 points) A group of mn people are to be arranged into m teams each with n players.

   (a) Determine the number of ways if each team has a different name.

   (b) Determine the number of ways if the teams don't have names.

   Answer: (a) ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯, (b) ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯.

4. (10 points) A bakery sells five different kinds of donuts, includes original, glazed, chocolated, mango and strawberry. If the bakery has at least two dozens of each kind, how many different options for twenty of donuts are there? What if a box is to contain at least five of chocolated donuts and two strawberry donuts?

   Answer: (a) ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯, (b) ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯.

5. (10 points) A student has 35 days to prepare for an examination. From past experience she knows that she will require no more than 53 hours of study. She also wishes to study at least 1 hour per day. Show that no matter how she schedules her study time (a whole number of hours per day, however), there is a succession of days during which she will have studied exactly 16 hours.

6. (10 points) Determine the mobile integers in

$$\overrightarrow{4}\ \overleftarrow{8}\ \overrightarrow{3}\ \overleftarrow{1}\ \overrightarrow{6}\ \overleftarrow{7}\ \overleftarrow{2}\ \overrightarrow{5}$$

and then use the algorithm for generating the next six permutations.

Answer: the mobile integers are _____.

(1) _____. (2) _____.

(3) _____. (4) _____.

(5) _____. (6) _____.

7. (10 points) Construct a permutation whose inversion sequences are 6, 2, 4, 4, 1, 1, 2, 0, 0.

   Answer: _____.

8. (10 points) Show that an m-by-n chessboard has a perfect cover by dominoes if and only if at least one of m and n is even.

9. (10 points) Determine the number of integral solutions of the equation

$$x + y + z + w = 20$$

that satisfy

$$3 \leq x \leq 8, \ -6 \leq y \leq 1, \ 5 \leq z \leq 9, \ 5 \leq w \leq 9.$$

Answer: _____.

10. (10 points) Determine the number of ways to place six non-attacking rooks on the following 6-by-6 board, with forbidden positions as shown.

x	x				
x	x	x			
		x			
			x		
			x	x	

Answer: _____.

11. (10 points) A collection of subsets of $1, 2, ..., n$ has the property that each pair of subsets has at least one element in common. Prove that there are at most 2^{n-1} subsets in the collection.

12. (10 points) Use combinatorial reasoning (組合解釋) to prove the identity (in the form given)

$$\binom{n}{k} - \binom{n-3}{k} = \binom{n-1}{k-1} + \binom{n-2}{k-1} + \binom{n-3}{k-1}.$$

(Hint: Let S be a set with three distinguished elements a, b and c and count certain k-subsets of S.)

學號: ——————, 姓名: ——————, **以下由閱卷人員填寫**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	10	10	10	10	10	10	10	10	10	10	10	120
Score:													