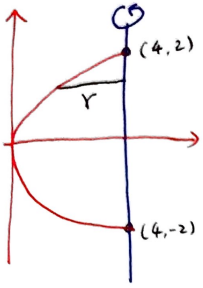
姓名: Sol.　　葉均承　化學一微積分

學號: _____　　**Quiz 1**　　考試日期: 2020/03/16

不可使用手機、計算器，禁止作弊!

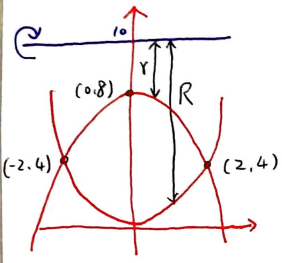
1. (30%) Set up, but do not evaluate, an integral to find the volume of the solid found by rotating the region bounded by $x = y^2$ and $x = 4$ about the line $x = 4$.



$r = 4 - y^2$　　by symmetry.

$\int_{-2}^{2} \pi (4-y^2)^2 dy$　　or　$2\int_{0}^{2} \pi (4-y^2)^2 dy$

2. (40%) Set up, but do not evaluate, an integral to find the volume of the solid found by rotating the region bounded by $y = x^2$ and $y = 8 - x^2$ about the line $y = 10$.



$R = 10 - x^2$, $r = 10 - (8 - x^2) = 2 + x^2$

$x^2 = 8 - x^2 \Rightarrow x = \pm 2$

Ans: $\int_{-2}^{2} \pi \left[(10-x^2)^2 - (2+x^2)^2 \right] dx$

or　$2\int_{0}^{2} \pi \left[(10-x^2)^2 - (2+x^2)^2 \right] dx$

3. (30%) Set up, but do not evaluate, an integral to find the volume of the solid whose base is the region bounded by the parabola $y = 16 - x^2$ and the x-axis, and whose cross-sections perpendicular to the x-axis are semicircles.

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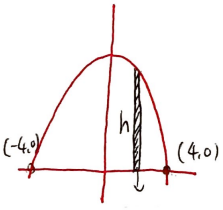

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(\frac{1}{2}(16-x^2) \right)^2 = \frac{1}{8}\pi (16-x^2)^2$

$r = \frac{1}{2}h = \frac{1}{2}(16-x^2)$

Ans: $\int_{-4}^{4} \frac{1}{8}\pi (16-x^2)^2 dx$

or　$2\int_{0}^{4} \frac{1}{8}\pi (16-x^2)^2 dx$

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學號: sol **Quiz 2** 考試日期: 2020/03/23

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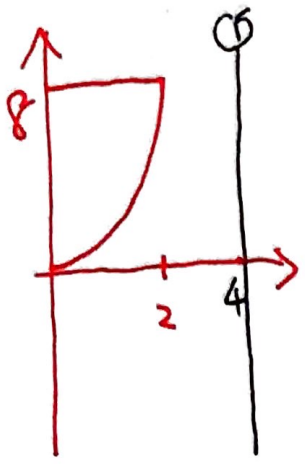
1. (30%) Set up an integral to find the volume of the solid found by rotating the region bounded by $y = x^3$, $y = 8$ and the y-axis about the line $x = 4$. Do not evaluate.



Shell: $\int_0^2 2\pi(8-x^3)(4-x)\,dx$

Washer: $\int_0^8 \pi\left[4^2-(4-\sqrt[3]{y})^2\right]dy$

2. (35%) The *work* required to stretch a spring from its natural length to 3 m beyond its natural length is 18 J. How much *force* is required to hold the spring stretched 5 m beyond its natural length?

$18 = \int_0^3 kx\,dx = \frac{1}{2}kx^2\Big|_0^3 = \frac{9}{2}k \quad \Rightarrow k=4.$

$f(x)=4x \quad \therefore f(5)=4\times5=20\ (N)$

3. (35%) A 20 foot rope that weights 100 pounds is hanging off a cliff with a 10 pound weight attached. Find the work required to pull the rope and weight up 10 feet.

$100/20=5\ (lb/ft) \quad \therefore f(y)=110-5y$

$\int_0^{10} 110-5y\,dy = 110y-\frac{5}{2}y^2\Big|_0^{10} = 1100-250 = 850\ (lb)$