

1. 請框出答案. 2. 不可使用手機、計算器, 禁止作弊!

1. Using the Gram-Schmidt process to find an orthonormal basis for the subspace of \mathbb{R}^4 spanned by $[1, 0, 1, 0]$, $[1, 1, 1, 0]$, $[1, -1, 0, 1]$.

Answer: $\frac{1}{\sqrt{2}}[1, 0, 1, 0], [0, 1, 0, 0], \frac{1}{\sqrt{6}}[1, 0, -1, 2]$

$$\vec{a}_1 = [1, 0, 1, 0], \vec{a}_2 = [1, 1, 1, 0], \vec{a}_3 = [1, -1, 0, 1],$$

$$\vec{v}_1 = [1, 0, 1, 0], \vec{q}_1 = \frac{\vec{v}_1}{\|\vec{v}_1\|} = \frac{1}{\sqrt{2}}[1, 0, 1, 0],$$

$$\vec{v}_2 = \vec{a}_2 - \frac{\vec{a}_2 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1 = [0, 1, 0, 0], \vec{q}_2 = \frac{\vec{v}_2}{\|\vec{v}_2\|} = [0, 1, 0, 0],$$

$$\vec{v}_3 = \vec{a}_3 - \frac{\vec{a}_3 \cdot \vec{v}_1}{\vec{v}_1 \cdot \vec{v}_1} \vec{v}_1 - \frac{\vec{a}_3 \cdot \vec{v}_2}{\vec{v}_2 \cdot \vec{v}_2} \vec{v}_2 = [0.5, 0, -0.5, 1], \vec{q}_3 = \frac{\vec{v}_3}{\|\vec{v}_3\|} = \frac{1}{\sqrt{6}}[1, 0, -1, 2],$$