Section 6-2

課本 proble 3, 5, 9, 13, 17, 19, 24, 25, 27, 28

Ans: 奇數題見課程網頁

24 Let B be the ordered orthonormal basis $\left(\vec{b}_1 = [\frac{1}{3}, \frac{2}{3}, \frac{-2}{3}], \vec{b}_2 = [\frac{2}{3}, \frac{1}{3}, \frac{2}{3}], \vec{b}_3 = [\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}]\right)$ for $\mathbb{R}^3$

(a) Let

$$A = \begin{bmatrix} \vec{b}_1^T & \vec{b}_2^T & \vec{b}_3^T \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix}$$

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix} = \begin{bmatrix} 13/3 \\ -4/3 \\ -2/3 \end{bmatrix}$$

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} 1/3 & 2/3 & -2/3 \\ 2/3 & 1/3 & 2/3 \\ -2/3 & 2/3 & 1/3 \end{bmatrix} \begin{bmatrix} 5 \\ -3 \\ 3 \end{bmatrix} = \begin{bmatrix} -5/3 \\ 11/3 \\ -14/3 \end{bmatrix}$$

(b)

$$[1, 2, -4] \cdot [5, -3, 2] = -9$$

$$[13/3, -4/3, -2/3] \cdot [-5/3, 11/3, -14/3] = -9$$

Noticed that the results of inner product are the SAME, which should known by Theorem 6.6 property 1.

28 Problem 11, find the orthonormal basis of $sp([1, 0, 1, 0], [1, 1, 1, 0], [1, -1, 0, 1])$ is
By Gram-Schmidt process. Let

$$\vec{a}_1 = [1, 0, 1, 0], \vec{a}_2 = [1, 1, 1, 0], \vec{a}_3 = [1, -1, 0, 1] \tag{1}$$

$$\vec{v}_1 = \vec{a}_1 = [1, 0, 1, 0], \vec{q}_1 = \frac{\vec{v}_1}{|\vec{v}_1|} = [\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0] \tag{2}$$

$$\vec{v}_2 = \vec{a}_2 - (\vec{a}_2 \cdot \vec{q}_1)\vec{q}_1 = \vec{a}_2 - \sqrt{2}\vec{q}_1 = [0, 1, 0, 0] \tag{3}$$

$$\vec{q}_2 = \frac{\vec{v}_2}{|\vec{v}_2|} = [0, 1, 0, 0] \tag{4}$$

$$\vec{v}_3 = \vec{a}_3 - (\vec{a}_3 \cdot \vec{q}_1)\vec{q}_1 - (\vec{a}_3 \cdot \vec{q}_2)\vec{q}_2 = \vec{a}_3 - \frac{\sqrt{2}}{2}\vec{q}_1 - \vec{q}_2 = [\frac{1}{2}, 0, \frac{-1}{2}, 1] \tag{5}$$

$$\vec{q}_3 = \frac{\vec{v}_3}{|\vec{v}_3|} = [\frac{1}{\sqrt{6}}, 0, \frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}] \tag{6}$$

By (1)

$$\vec{a}_1 = \sqrt{2}\,\vec{q}_1 \Rightarrow \begin{bmatrix}\vec{a}_1^T\end{bmatrix} = \begin{bmatrix}\vec{q}_1^T\end{bmatrix}\begin{bmatrix}\sqrt{2}\end{bmatrix} \tag{7}$$

By (3) and (4)

$$\vec{a}_2 = \sqrt{2}\,\vec{q}_1 + \vec{v}_2 = \sqrt{2}\,\vec{q}_1 + \vec{q}_2 \Rightarrow \begin{bmatrix}\vec{a}_2^T\end{bmatrix} = \begin{bmatrix}\vec{q}_1^T & \vec{q}_2^T\end{bmatrix}\begin{bmatrix}\sqrt{2} \\ 1 \\ 0\end{bmatrix} \tag{8}$$

By (5) and (6)

$$\vec{a}_3 = \frac{\sqrt{2}}{2}\vec{q}_1 + \vec{q}_2 + \vec{v}_3 = \frac{\sqrt{2}}{2}\vec{q}_1 + \vec{q}_2 + \frac{\sqrt{3}}{\sqrt{2}}\,\vec{q}_3 \Rightarrow \begin{bmatrix}\vec{a}_3^T\end{bmatrix} = \begin{bmatrix}\vec{q}_1^T & \vec{q}_2^T & \vec{q}_3^T\end{bmatrix}\begin{bmatrix}\frac{\sqrt{2}}{2} \\ 1 \\ \frac{\sqrt{3}}{\sqrt{2}}\end{bmatrix} \tag{9}$$

Therefore,

$$\begin{bmatrix}\vec{a}_1^T & \vec{a}_2^T & \vec{a}_3^T\end{bmatrix} = \begin{bmatrix}\vec{q}_1^T & \vec{q}_2^T & \vec{q}_3^T\end{bmatrix}\begin{bmatrix}\sqrt{2} & \sqrt{2} & \frac{\sqrt{2}}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{\sqrt{3}}{\sqrt{2}}\end{bmatrix}$$

That is

$$A = \begin{bmatrix}1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix} = QR = \begin{bmatrix}1/\sqrt{2} & 0 & 1/\sqrt{6} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{6} \\ 0 & 0 & 2/\sqrt{6}\end{bmatrix}\begin{bmatrix}\sqrt{2} & \sqrt{2} & \frac{\sqrt{2}}{2} \\ 0 & 1 & 1 \\ 0 & 0 & \frac{\sqrt{3}}{\sqrt{2}}\end{bmatrix}$$