姓名: SOLUTION　　葉均承　　化學一微積分

學號: _____　　**Quiz 5**　　考試日期: 2020/04/20

不可使用手機、計算器，禁止作弊!

1. (50%) Write down the general partial fraction decomposition of

$$\frac{x+1}{(x-1)(x+2)^3(x^2+4)(x^2-4)^2}$$

We have $x^2-4=(x-2)(x+2)$.
Therefore, $(x-1)(x+2)^3(x^2+4)(x^2-4)^2=(x-1)(x-2)^2(x+2)^5(x^2+4)$.

$$\begin{aligned}\frac{x+1}{(x-1)(x+2)^3(x^2+4)(x^2-4)^2} &= \frac{x+1}{(x-1)(x-2)^2(x+2)^5(x^2+4)}\\ &= \frac{A}{(x-1)}+\frac{B_1}{(x-2)}+\frac{B_2}{(x-2)^2}\\ &+\frac{C_1}{(x+2)}+\frac{C_2}{(x+2)^2}+\frac{C_3}{(x+2)^3}+\frac{C_4}{(x+2)^4}+\frac{C_5}{(x+2)^5}\\ &+\frac{Dx+E}{x^2+4}\end{aligned}$$

2. (50%) Evaluate the integral.

$$\int \frac{16x-8}{x^3+2x^2-8x}\,dx$$

We have $x^3+2x^2-8x=x(x-2)(x+4)$.

$$\begin{aligned}\frac{16x-8}{x^3+2x^2-8x} &= \frac{16x-8}{x(x-2)(x+4)} = \frac{A}{x}+\frac{B}{(x-2)}+\frac{C}{x+4}\\ &= \frac{A(x-2)(x+4)+Bx(x+4)+Cx(x-2)}{x(x-2)(x+4)}\end{aligned}$$

Hence, $16x-8=A(x-2)(x+4)+Bx(x+4)+Cx(x-2)$

When $\boxed{x=0}$: $-8=A(0-2)(0+4)=-8A\to A=1$.
When $\boxed{x=2}$: $16*2-8=B*2(2+4)\to 24=12B\to B=2$.
When $\boxed{x=-4}$: $16*(-4)-8=C(-4)(-4-2)\to -72=24C\to C=-3$.

$$\begin{aligned}\int \frac{16x-8}{x^3+2x^2-8x}\,dx &= \int \frac{1}{x}+\frac{2}{(x-2)}+\frac{-3}{x+4}\,dx\\ &= \ln|x|+2\ln|x-2|-3\ln|x+4|+C\end{aligned}$$