

1. 請框出答案. 2. 禁止作弊!

1. Show that for any positive integers  $i$  and  $j$  with  $i > j$ , we have

$$T_i(x)T_j(x) = \frac{1}{2}[T_{i+j}(x) + T_{i-j}(x)].$$

Note: 1.  $T_n(x) = \cos(n \cdot \arccos(x))$

2.  $T_0(x) = 1, T_1(x) = x, T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ .

3. If  $\theta = \arccos(x)$ , then  $d\theta = -\frac{1}{\sqrt{1-x^2}}dx$

**Answer:**

let  $\theta = \arccos(x)$

$$T_i(x)T_j(x) = \cos(i\theta) \cos(j\theta) = \frac{\cos((i+j)\theta) + \cos((i-j)\theta)}{2} = \frac{1}{2}[T_{i+j}(x) + T_{i-j}(x)].$$