葉均承
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Week 5

沒有星號題的答案見課本後面

Section 1-6

課本 proble 19, 23, 31, 37, 38*, 45*, 47*

38 F T T T F F T T F T

45

a) Clearly $\mathbf{v}_1$, $2\mathbf{v}_1 + \mathbf{v}_2 \in sp(\mathbf{v}_1, \mathbf{v}_2)$ and therefore

$$sp(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2) \subseteq sp(\mathbf{v}_1, \mathbf{v}_2) .$$

Also, $\mathbf{v}_1 = 1\mathbf{v}_1 + 0(2\mathbf{v}_1 + \mathbf{v}_2)$ and $\mathbf{v}_2 = (-2)\mathbf{v}_1 + 1(2\mathbf{v}_1 + \mathbf{v}_2)$ showing that $\mathbf{v}_1, \mathbf{v}_2 \in sp(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2)$ and therefore

$$sp(\mathbf{v}_1, \mathbf{v}_2) \subseteq sp(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2).$$

Thus $sp(\mathbf{v}_1, \mathbf{v}_2) = sp(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2)$.

b) Clearly $\mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{v}_1 - \mathbf{v}_2 \in sp(\mathbf{v}_1, \mathbf{v}_2)$ and therefore

$$sp(\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2) \subseteq sp(\mathbf{v}_1, \mathbf{v}_2).$$

Also, $\mathbf{v}_1 = \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) + \frac{1}{2}(\mathbf{v}_1 - \mathbf{v}_2)$ and $\mathbf{v}_2 = \frac{1}{2}(\mathbf{v}_1 + \mathbf{v}_2) - \frac{1}{2}(\mathbf{v}_1 - \mathbf{v}_2)$ so $\mathbf{v}_1, \mathbf{v}_2 \in sp(\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2)$ and therefore

$$sp(\mathbf{v}_1, \mathbf{v}_2) \subseteq sp(\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2).$$

Thus $sp(\mathbf{v}_1, \mathbf{v}_2) = sp(\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 - \mathbf{v}_2)$.

47 Clearly $W_1 \cap W_2$ is nonempty; it contains 0. Let $\vec{v}, \vec{w} \in (W_1 \cap W_2)$. Then $\vec{v}, \vec{w} \in W_1$ and $\vec{v}, \vec{w} \in W_2$, so $\vec{v} + \vec{w} \in W_1$ and $\vec{v} + \vec{w} \in W_2$ since W_1 and W_2 are subspaces. Thus $\vec{v} + \vec{w} \in (W_1 \cap W_2)$. Similarly, $r\vec{v} \in W_1$ and $r\vec{v} \in W_2$. Since W_1 and W_2 are subspaces. Thus $r\vec{v} \in (W_1 \cap W_2)$. Thus W_1 and W_2 are subspaces. Thus $W_1 \cap W_2$ is a subspace of $\mathbb{R}^n$