姓名: SOLUTION

應數三科學計算

學號:

Quiz 5

考試日期: 2021/03/30

1. 請框出答案. 2. 禁止作弊!

1. Show that for any positive integers i and j with i > j, we have

$$T_i(x)T_j(x) = \frac{1}{2}[T_{i+j}(x) + T_{i-j}(x)].$$

Note: 1. $T_n(x) = \cos(n \cdot \arccos(x))$

2.
$$T_0(x) = 1, T_1(x) = x, T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$$
.
3. If $\theta = \arccos(x)$, then $d\theta = -\frac{1}{\sqrt{1-x^2}}dx$

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Answer:

let $\theta = \arccos(x)$

$$T_i(x)T_j(x) = \cos(i\theta)\cos(j\theta) = \frac{\cos((i+j)\theta) + \cos((i-j)\theta)}{2} = \frac{1}{2}[T_{i+j}(x) + T_{i-j}(x)].$$