3. $\mathbf{w}_2 = \begin{bmatrix} 1 \\ \frac{5}{7} \end{bmatrix}$, $\mathbf{w}_3 = \begin{bmatrix} 1 \\ \frac{19}{29} \end{bmatrix}$, $\mathbf{w}_4 = \begin{bmatrix} 1 \\ \frac{65}{103} \end{bmatrix}$

   Rayleigh quotients: 6, $\frac{298}{74} \approx 4$, $\frac{4222}{1202} \approx 3.5$

   Maximum eigenvalue 3, eigenvector $\begin{bmatrix} 1 \\ \frac{3}{5} \end{bmatrix}$

5. $5\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

7. $2\begin{bmatrix} \frac{2}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{6} & \frac{1}{6} \end{bmatrix} - \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} + 0\mathbf{b}_3\mathbf{b}_3{}^T$

9. $\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

11. $\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} & -\frac{1}{3} \end{bmatrix}$

3. $\lambda = 12$, $\mathbf{v} = [-.7059, 1, -.4118]$

5. $\lambda = 6$, $\mathbf{v} = [-.9032, 1, -.4194]$

7. $\lambda = .1883$, $\mathbf{v} = [1, .2893, .3204]$

9. $\lambda_1 = 4.732050807568877$,
   $\mathbf{v}_1 = r[1, -.7320508, 1]$

   $\lambda_2 = 1.267949192431123$,
   $\mathbf{v}_2 = s[.3660254, 1, .3660254]$

   $\lambda_3 = -4$, $\mathbf{v}_3 = t[-1, 0, 1]$,
   for nonzero r, s, t

1. $\lambda_1 = 16.87586339619508$,
   $\mathbf{v}_1 = r[.9245289, 1, .3678643, .7858846]$

   $\lambda_2 = -15.93189429348535$,
   $\mathbf{v}_2 = s[-.3162426, .6635827, -1, -.004253739]$

   $\lambda_3 = 6.347821447472841$,
   $\mathbf{v}_3 = t[-.5527083, .9894762, .8356429, -1]$

   $\lambda_4 = -.291790550182573$,
   $\mathbf{v}_4 = u[-1, .06924058, .3582734, .9206089]$

   for nonzero r, s, t, and u

3. a. The characteristic polynomial $|A - \lambda|$
   $= \begin{vmatrix} a-\lambda & b \\ b & c-\lambda \end{vmatrix} = \lambda^2 + (-a-c)\lambda + (ac - b^2)$ has roots

   $$\lambda = \frac{1}{2}\left(a + c \pm \sqrt{(a+c)^2 - 4(ac - b^2)}\right)$$
   $$= \frac{1}{2}\left(a + c \pm \sqrt{(a-c)^2 + 4b^2}\right).$$

   b. If we use part a, the first row vector of $A - \lambda I$ is
   $[a - \lambda, b] =$
   $\left[\frac{1}{2}\left(a - c \mp \sqrt{(a-c)^2 + 4b^2}\right), b\right]$
   $= [g \mp \sqrt{g^2 + b^2}, b]$.

   c. From part a, eigenvectors for the matrix are $[-b, g \pm \sqrt{g^2 + b^2}] = [-b, g \pm h]$. Normalizing, we obtain $\frac{(-b, g \pm h)}{\sqrt{b^2 + (g \pm h)^2}}$. Using the upper choice of sign and setting $r = \sqrt{b^2 + (g+h)^2}$, we obtain $[-b/r, (g+h)/r]$ as the first column of C. Using the lower choice of sign and setting $s = \sqrt{b^2 + (g-h)^2}$, we obtain $[-b/s, (g-h)/s]$ as the second column of C.

   d. $\det(C) = \frac{-b(g-h)}{rs} + \frac{b(g+h)}{rs} = \frac{2bh}{rs}$; because $h, r, s \geq 0$, we see that the algebraic sign of $\det(C)$ is the same as that of b.

25. $\lambda_1 = -12.00517907692924$,
    $\lambda_2 = 7.906602974286551$,
    $\lambda_3 = 17.09857610264269$

27. $\lambda_1 = -5.210618568922174$,
    $\lambda_2 = 2.856693936892428$,
    $\lambda_3 = 3.528363748899602$,
    $\lambda_4 = 7.825560883130143$

29. 5.823349919059785,
    $-11.91167495952989 \pm 1.357850063519836i$

31. 57.22941613544168,
    -92.88108454947197,
    -54.25594801085533,
    $47.45380821244281 \pm 44.48897425527453i$

CHAPTER 9

Section 9.1

1. a. $z + w = 4 + i$, $zw = 5 + 5i$
   b. $z + w = 3 + 2i$, $zw = -1 + 3i$

3. a. $|z| = \sqrt{13}$, $\bar{z} = (3 - 2i)$, $z\bar{z} = (3 + 2i)(3 - 2i) = 13 = |z|^2$

b. $|z| = \sqrt{17}$, $\bar{z} = 4 + i$, $z\bar{z} = (4 - i)(4 + i) = 17 = |z|^2$

7. a. $\frac{3}{2} + \frac{1}{2}i$ **b.** $\frac{13}{25} + \left(-\frac{9}{25}\right)i$

9. a. Modulus $2\sqrt{2}$, principal argument $3\pi/4$

11. 16 **17.** F T F F T F F T F T

19. $\sqrt{2} + \sqrt{2}i, -\sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i, \sqrt{2} - \sqrt{2}i$

21. $1, i, -1, -i$

23. $2, \sqrt{2} + \sqrt{2}i, 2i, -\sqrt{2} + \sqrt{2}i, -2, -\sqrt{2} - \sqrt{2}i, -2i, \sqrt{2} - \sqrt{2}i$

Section 9.2

3. $AB = \begin{bmatrix} -3 + 2i & 2i & 2i \\ 2 & 2i & 1 \\ 2 + 3i & -1 + i & 2 + i \end{bmatrix}$,

$BA = \begin{bmatrix} -2 + 2i & i & 2 - i \\ 2 + 3i & 1 + 3i & 0 \\ 2i & -1 + i & 0 \end{bmatrix}$

5. $\frac{1}{3}\begin{bmatrix} 2 + i & -i \\ -1 - i & 1 \end{bmatrix}$

7. $\frac{1}{10}\begin{bmatrix} 9 - 3i & 1 + 3i & -4 + 8i \\ -3 + i & 3 - i & -2 - 6i \\ -2 + 4i & 2 - 4i & 2 - 4i \end{bmatrix}$

9. $\mathbf{z} = \frac{1}{10}\begin{bmatrix} -7 + 9i \\ 9 - 3i \\ 6 - 2i \end{bmatrix}$ **11.** $\text{sp}\left(\begin{bmatrix} 1 + i \\ 1 + 3i \\ 2 \end{bmatrix}\right)$

13. 3 **15. a.** $\langle \mathbf{u}, \mathbf{v}\rangle = 0$, $\langle \mathbf{v}, \mathbf{u}\rangle = 0$

b. $\langle \mathbf{u}, \mathbf{v}\rangle = 5 - 3i$, $\langle \mathbf{v}, \mathbf{u}\rangle = 5 + 3i$

21. a. Perpendicular **d.** Parallel

b. Parallel **e.** Perpendicular

c. Neither

23. $\frac{2}{\sqrt{7}}[i, 1 - i, 1 + i, 1 - i]$

25. $[-3i, 1, 2 + 2i]$

27. $\{[2 + i, 1 + i], [1 - i, -2 + i]\}$

29. $\{[1, i, i], [1 + 3i, 3 - 2i, i], [1 + i, i, 1 - 2i]\}$

31. a. Both

b. Hermitian but not unitary

c. Not Hermitian but unitary

d. Neither

33. T T F F T T T T F F

41. Diagonal matrices with entries of modul 1 on the diagonal.

M1. See answer to Exercise 3.

M3. $\begin{bmatrix} -i & 1 + i & 0 \\ 1 + i & -1 + i & 1 \\ -1 + i & -1 - 2i & i \end{bmatrix}$ **M5.** $\begin{bmatrix} 2 + \\ -4 + \\ -6i \end{bmatrix}$

M7. 2

M9. Entering **[Q, R] = qr(A)**, where A is the matrix having the given vectors $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3$ column vectors, returns a matrix Q havin as column vectors an *orthonormal* basis $\{\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3\}$, where

$\mathbf{q}_1 \approx [-0.5774, -0.5774i, -0.5774i]$,

$\mathbf{q}_2 \approx [-0.4695 - 0.1719i, -0.4695 - 0.1719i, 0.2977 + 0.6414i]$,

$\mathbf{q}_3 \approx [0.4695 + 0.429i, 0.0726 - 0.6414i, 0.3703 + 0.1719$

To check, using MATLAB, the Student's Solutions Manual's answers

$\mathbf{v}_1 = \mathbf{a}_1$, $\mathbf{v}_2 = \mathbf{a}_2$, $\mathbf{v}_3 = [1 - 3i, -3 + i, -2$

for an *orthogonal* basis, enter **((1−i)/Q(1,1))*Q(:,1)** to check $\mathbf{v}_1$, enter **(1/Q(1,2))*Q(:,2)** to check $\mathbf{v}_2$, and enter **((1−3*i)/Q(1,3))*Q(:,3)** to check $\mathbf{v}_3$.

M11. a. $\sqrt{274}$, $\sqrt{476}$, $\sqrt{458}$, and $\sqrt{353}$ for rows 1, 2, 3, and 4, respectively.

b. $\sqrt{277}$, $\sqrt{192}$, $\sqrt{529}$, $\sqrt{124}$, and $\sqrt{439}$ for columns 1, 2, 3, 4, and 5, respectively.

c. $-45 - 146i$

d. $31 + 14i$

Section 9.3

1. $U = \frac{1}{\sqrt{2}}\begin{bmatrix} -i & i \\ 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix}$

3. $U = \begin{bmatrix} (1+i)/\sqrt{3} & (1+i)/\sqrt{6} \\ -1/\sqrt{3} & 2/\sqrt{6} \end{bmatrix}$, $D = \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix}$

5. $U = \dfrac{1}{\sqrt{2}}\begin{bmatrix} -i & 0 & i \\ 1 & 0 & 1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

7. $U = \begin{bmatrix} (1-i)/\sqrt{6} & 0 & (1-i)/\sqrt{3} \\ -2\sqrt{6} & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$,

$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

9. $U = \begin{bmatrix} (1+i)/\sqrt{6} & 0 & (1+i)/\sqrt{3} \\ 0 & 1 & 0 \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \end{bmatrix}$,

$D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

11. $U = \begin{bmatrix} (-1-i)/\sqrt{8} & 0 & (3+3i)/\sqrt{24} \\ (1-i)/\sqrt{8} & (1+i)/\sqrt{3} & (1-i)/\sqrt{24} \\ 2/\sqrt{8} & -i/\sqrt{3} & 2/\sqrt{24} \end{bmatrix}$,

$D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

13. $\{a \in \mathbb{C} \mid |a| = 4\}$ **15.** $a = -1$

19. F T T F T F T T F F

tion 9.4

1. Yes 3. No 5. No

7. **a.** $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = -2$.

b. $J + 2I$ has rank 3 and nullity 1,
$(J + 2I)^2$ has rank 2 and nullity 2,
$(J + 2I)^3$ has rank 1 and nullity 3,
$(J + 2I)^k$ has rank 0 and nullity 4 for $k \geq 4$.

c. $J + 2I$: $\mathbf{e}_4 \to \mathbf{e}_3 \to \mathbf{e}_2 \to \mathbf{e}_1 \to \mathbf{0}$.

d. $J\mathbf{e}_1 = -2\mathbf{e}_1$, $J\mathbf{e}_2 = \mathbf{e}_1 - 2\mathbf{e}_2$, $J\mathbf{e}_3 = \mathbf{e}_2 - 2\mathbf{e}_3$, $J\mathbf{e}_4 = \mathbf{e}_3 - 2\mathbf{e}_4$.

9. **a.** $\lambda_1 = -1$, $\lambda_2 = \lambda_3 = \lambda_4 = \lambda_5 = 2$.

b. $(J + I)^k$ has rank 4 and nullity 1 for $k \geq 1$.
$(J - 2I)$ has rank 3 and nullity 2,
$(J - 2I)^k$ has rank 1 and nullity 4 for $k \geq 2$.

c. $J + I$: $\mathbf{e}_1 \to \mathbf{0}$,
$J - 2I$: $\mathbf{e}_3 \to \mathbf{e}_2 \to \mathbf{0}$, $\mathbf{e}_5 \to \mathbf{e}_4 \to \mathbf{0}$.

d. $J\mathbf{e}_1 = -\mathbf{e}_1$, $J\mathbf{e}_2 = 2\mathbf{e}_2$, $J\mathbf{e}_3 = \mathbf{e}_2 + 2\mathbf{e}_3$, $J\mathbf{e}_4 = 2\mathbf{e}_4$, $J\mathbf{e}_5 = \mathbf{e}_4 + 2\mathbf{e}_5$.

11. $\begin{bmatrix} 3 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix}$

15. $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \left\{\begin{bmatrix} 2 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}\right\}$ (Other bases are possible.)

17. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 1 \\ 0 & 0 & 4 \end{bmatrix}, \left\{\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}\right\}$ (Other answers are possible.)

19. $\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \left\{\begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}\right\}$

(Other answers are possible.)

21. $\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$, $\{\mathbf{e}_1 + \mathbf{e}_3, \mathbf{e}_5, \mathbf{e}_2, \mathbf{e}_4, \mathbf{e}_1 - \mathbf{e}_3\}$

(Other answers are possible.)

23. T F T T F T T F F F 25. O 27. O

29. $A^4 + (3 - i)A^3 + (3 - 3i)A^2 + (1 - 3i)A - iI$