

## Section 5-2 Diagonalization

17. Prove that, for every square matrix  $A$  all of whose eigenvalues are real, the product of its eigenvalues is  $\det(A)$

**Answer:**

If the characteristic polynomial of  $A$  is  $p(\lambda) = |A - \lambda I|$ , then  $p(0) = |A| = \det(A)$ .

Also,

$$p(\lambda) = (-1)^n(\lambda - \lambda_1)(\lambda - \lambda_2) \cdots (\lambda - \lambda_n)$$

, so

$$p(0) = (-1)^n \lambda_1 \lambda_2 \cdots \lambda_n = \lambda_1 \lambda_2 \cdots \lambda_n = \det(A).$$