

1. 請框出答案. 2. 不可使用手機、計算器, 禁止作弊!

1. (50%) Find the eigenvalues λ_i and the corresponding eigenvectors v_i of the linear transformation T. T defined on \mathbb{R}^3 by $T([x_1, x_2, x_3]) = [x_1, 4x_2 + 7x_3, 2x_2 - x_3]$

Answer: eigenvalues: -3, 6, 1, eigenvectors: $\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 7 \\ 0 & 2 & -1 \end{bmatrix}, |A - \lambda I| = (\lambda + 3)(\lambda - 6)(\lambda - 1)$$

$$\lambda = -3$$

$$[A - (-3) * I] \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v}_1 = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

$$\lambda = 6$$

$$[A - 6 * I] \sim \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v}_1 = \begin{bmatrix} 0 \\ 7 \\ 2 \end{bmatrix}$$

$$\lambda = 1$$

$$[A - 1 * I] \sim \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \vec{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$