姓名: SOLUTION 葉均承 應數一線性代數

學號: _____ **Quiz 2** 考試日期: 2020/03/11

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find a formula for the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ that reflects vectors in the line $y = mx$.

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2x+2my \\ 2mx+(m^2-1)y \end{bmatrix}$$

$$T(\begin{bmatrix} 1 \\ m \end{bmatrix}) = \begin{bmatrix} 1 \\ m \end{bmatrix}, T(\begin{bmatrix} -m \\ 1 \end{bmatrix}) = \begin{bmatrix} m \\ -1 \end{bmatrix}$$

That is the s.m.r. of T is $A = CDC^{-1}$, where

$$C = \begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A = CDC^{-1} = \begin{bmatrix} 1 & -m \\ m & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\frac{1}{1+m^2}\begin{bmatrix} 1 & m \\ -m & 1 \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}$$

$$T(\begin{bmatrix} x \\ y \end{bmatrix}) = A\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{1+m^2}\begin{bmatrix} 1-m^2x+2my \\ 2mx+(m^2-1)y \end{bmatrix}$$