

2-2

題號: 9, 14(a)(b)

2-2 #9

Use Theorem 2.3 to show that $g(x) = \pi + 0.5 \sin(\frac{x}{2})$ has a unique fixed point on $[0, 2\pi]$. Use fixed-point iteration to find an approximation to the fixed point that is accurate to within 10^{-2} . Use Corollary 2.5 to estimate the number of iterations required to achieve 10^{-2} accuracy, and compare this theoretical estimate to the number actually needed.

Answer:

Since $g'(x) = \frac{1}{4} \cos x$, g is continuous and g' exists on $[0, 2\pi]$. Further, $g'(x) = 0$ only when $x = \pi$, so that $g(0) = g(2\pi) = \pi \leq g(x) \leq g(\pi) = \pi + \frac{1}{2}$ and $|g'(x)| \leq \frac{1}{4}$, for $0 \leq x \leq 2\pi$. Theorem 2.3 implies that a unique fixed point p exists in $[0, 2\pi]$. With $k = \frac{1}{4}$ and $p_0 = \pi$, we have $p_1 = \pi + \frac{1}{2}$. Corollary 2.5 implies that

$$|p_n - p| \leq \frac{k^n}{1 - k} |p_1 - p_0| = \frac{2}{3} \left(\frac{1}{4}\right)^n$$

For the bound to be less than 0.1, we need $n \geq 4$. However, $p_3 = 3.626996$ is accurate to within 0.01.

2-2 #14

For each of the following equations, use the given interval or determine an interval $[a, b]$ on which fixed-point iteration will converge. Estimate the number of iterations necessary to obtain approximations accurate to within 10^{-5} , and perform the calculations.

(a) $2 + \sin x - x = 0$ use $[2, 3]$

(b) $x^3 - 2x - 5 = 0$ use $[2, 3]$

Answer:

The inequalities in Corollary 2.4 give $|p_n - p| < k^n \max(p_0 - a, b - p_0)$.

We want $k^n \max(p_0 - a, b - p_0) < 10^{-5}$, so we need $n > \frac{\ln(10^{-5}) - \ln(\max(p_0 - a, b - p_0))}{\ln k}$.

(a) Using $g(x) = 2 + \sin x$ we have $k = 0.9899924966$, so that with $p_0 = 2$ we have $n > \frac{\ln(0.00001)}{\ln k} = 1144.663221$. However, our tolerance is met with $p_{63} = 2.5541998$.

(b) Using $g(x) = \sqrt[3]{2x + 5}$ we have $k = 0.1540802832$, so that with $p_0 = 2$ we have $n > \frac{\ln(0.00001)}{\ln k} = 6.155718005$. However, our tolerance is met with $p_6 = 2.0945503$.

2-3

題號: 13, 17, 18, 19

2-3 #13

The fourth-degree polynomial $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$ has two real zeros, one in $[-1, 0]$ and the other in $[0, 1]$. Attempt to approximate these zeros to within 10^{-6} using the

(b) Secant method

(c) Newton's method

Use the endpoints of each interval as the initial approximations in part (a) and (b) and the midpoints as the initial approximation in part (c).

Answer:

(b) For $p_0 = -1$ and $p_1 = 0$, we have $p_5 = -0.04065929$, and for $p_0 = 0$ and $p_1 = 1$, we have $p_12 = 0.04065929$

(c) For $p_0 = -0.5$, we have $p_5 = -0.04065929$, and for $p_0 = 0.5$, we have $p_21 = 0.9623989$.

2-3 #17

The function described by $f(x) = \ln(x^2 + 1) - e^{0.4x} \cos \pi x$ has an infinite number of zeros.

(a) Determine, within 10^{-6} , the only negative zero.

(b) Determine, within 10^{-6} , the four smallest positive zeros.

(c) Determine a reasonable initial approximation to find the n th smallest positive zero of f . [Hint: Sketch an approximate graph of f .]

(d) Use part (c) to determine, within 10^{-6} , the 25^{th} smallest positive zero of f .

Answer:

For $f(x) = \ln(x^2 + 1) - e^{0.4x} \cos \pi x$, we have the following roots.

(a) For $p_0 = -0.5$, we have $p_3 = 0.4341431$.

(b) For $p_0 = 0.5$, we have $p_3 = 0.4506567$.

For $p_0 = 1.5$, we have $p_3 = 1.7447381$.

For $p_0 = 2.5$, we have $p_5 = 2.2383198$.

For $p_0 = 3.5$, we have $p_4 = 3.7090412$.

(c) The initial approximation $n - 0.5$ is quite reasonable.

(d) For $p_0 = 24.5$, we have $p_2 = 24.4998870$.

2-3 #18

Use Newton's method to solve the equation $0 = \frac{1}{2} + \frac{1}{4}x^2 - x \sin x - \frac{1}{2} \cos 2x$, with $p_0 = \frac{\pi}{2}$. Iterate using Newton's method until an accuracy of 10^{-5} is obtained. Explain why the result

seems unusual for Newton' s method. Also, solve the equation with $p_0 = 5\pi$ and $p_0 = 10\pi$.

Answer:

Newton' s method gives $p_{15} = 1.895488$, for $p_0 = \frac{\pi}{2}$; and $p_{19} = 1.895489$, for $p_0 = 5\pi$. The sequence does not converge in 200 iterations for $p_0 = 10\pi$. The results do not indicate the fast convergence usually associated with Newton' s method.

2-3 #19

Use Newton' s method to approximate, to within 10^{-4} , the value of x that produces the point on the graph of $y = x^2$ that is closest to $(1, 0)$. [Hint: Minimize $[d(x)]^2$, where $d(x)$ represents the distance from (x, x^2) to $(1, 0)$.]

Answer:

For $p_0 = 1$, we have $p_5 = 0.589755$. The point has the coordinates $(0.589755, 0.347811)$.