

姓名: sol.

葉均承 化學一微積分

學號: \_\_\_\_\_

## Quiz 9

考試日期: 2020/06/01

不可使用手機、計算器，禁止作弊!

1. (30%) Determine whether the series  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$  is absolutely convergent, conditionally convergent, or divergent.

1. check abs. conv. i.e.  $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

$$\because \ln(n) < n \text{ for } n \geq 2 \quad \therefore \frac{1}{\ln n} > \frac{1}{n}$$

$$\therefore \sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum_{n=2}^{\infty} \frac{1}{n} = \text{div. by P-test} \quad \therefore \sum_{n=2}^{\infty} \frac{1}{\ln n} = \text{div. by C.T.}$$

2. check conv. i.e.  $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$$\textcircled{1} \lim_{n \rightarrow \infty} \frac{1}{\ln n} = 0 \quad \textcircled{2} \ln(n+1) > \ln(n) \quad \therefore \frac{1}{\ln(n+1)} < \frac{1}{\ln(n)} \quad \text{i.e. } \left\{ \frac{1}{\ln n} \right\} \downarrow$$

$$\therefore \text{by AST, } \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n} = \text{conv.}$$

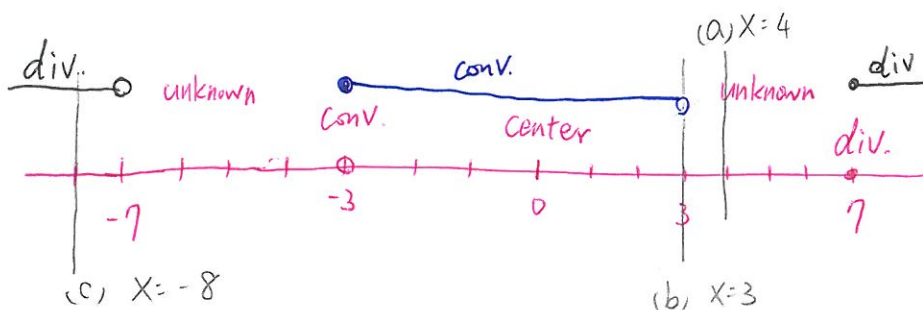
$$\therefore \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n} = \text{conditionally convergent.}$$

2. (30%) Given that the power series  $\sum_{n=0}^{\infty} c_n x^n$  converges when  $x = -3$  and diverges when  $x = 7$ , what can be said about the following series (converge/diverge/inconclusive)?

(a)  $\sum_{n=0}^{\infty} c_n 4^n$  inconclusive

(b)  $\sum_{n=0}^{\infty} c_n 3^n$  inconclusive

(c)  $\sum_{n=0}^{\infty} c_n (-8)^n$  diverge.



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3. (40%) Determine the radius and interval of convergence for the power series  $\sum_{n=1}^{\infty} \frac{(x+3)^n}{n(-2)^n}$ .  
(Make sure you check the endpoints.)

$$a_n = \frac{(x+3)^n}{n(-2)^n}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x+3)^{n+1} n (-2)^n}{(n+1)(-2)^{n+1} (x+3)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(x+3) n}{(-2)(n+1)} \right| = \left| \frac{x+3}{-2} \right| = \frac{1}{2} |x+3|$$

$$\text{need } \frac{1}{2} |x+3| < 1$$

$$\therefore |x+3| < 2 \quad \therefore \boxed{R=2}$$

$$-5 < x < -1$$

check:

$$\boxed{x=-5}$$

$$\sum_{n=1}^{\infty} \frac{(-2)^n}{n(-2)^n} = \sum_{n=1}^{\infty} \frac{1}{n} \quad \text{div by P-test}$$

$$\boxed{x=-1}$$

$$\sum_{n=1}^{\infty} \frac{2^n}{n(-2)^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$$

$$\because b_n \downarrow \text{ and } \lim_{n \rightarrow \infty} b_n = 0$$

$$\therefore \text{conv. by AST}$$

$$\therefore \boxed{I = (-5, -1]}$$