

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. Determinant whether the given 4 points lie in a plane in \mathbb{R}^3

$$A(0, 0, 0), B(1, 4, 3), C(2, 5, 8), D(-1, 2, -5)$$

$$\overrightarrow{AB} = [1, 4, 3], \overrightarrow{AC} = [2, 5, 8], \overrightarrow{AD} = [-1, 2, -5]$$

$$\begin{vmatrix} 1 & 4 & 3 \\ 2 & 5 & 8 \\ -1 & 2 & -5 \end{vmatrix} = -6 \neq 0$$

So the points are not coplanar.

2. Let $T : P_3 \rightarrow P_3$ be defined by $T(p(x)) = \frac{d}{dx}p(x)$, the ordered basis for P_3 is $(x^3, x^2, x, 1)$.
Find the standard matrix representation of T .

$$T(x^3) = 0x^3 + 3x^2 + 0x + 0$$

$$T(x^2) = 0x^3 + 0x^2 + 2x + 0$$

$$T(x^1) = 0x^3 + 0x^2 + 0x + 1$$

$$T(1) = 0x^3 + 0x^2 + 0x + 0$$

Hence the standard matrix representation A is

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$