2-1 $f(x)=0$ bisec.

2-2 $\rightsquigarrow g(x)=x$

$\uparrow$

choice

fixed-point Thm

conv. $\left\{\begin{array}{l} 2.4 \text{ conv linear} \\ \text{qu} \end{array}\right.$

2-3 newton method

$g(x)=x$

$\quad\ \ \|$

$x-\phi(x)f(x)$

where $\phi(x)=1/f'(x)$ if $f'(x)\neq 0$

$\Rightarrow$ $\boxed{g(x)=x-\psi^{-1}(x)f(x) \quad \psi(x)=f'(x)}$

10-1 $\vec{F}(\vec{x})=\vec{0} \rightsquigarrow \vec{G}(\vec{x})=\vec{x}$

fixed-point Thm

10-2 newton method

$\vec{G}(\vec{x})=\vec{x}-A^{-1}(x)\vec{F}(\vec{x})$

Thm 2.9

let $p=g(p)$ 求$\psi(x)$

if ① $g'(p)=0$ and

② g'': conti with $|g''(x)|<M$ on $\overset{p}{\underset{I}{\cap}}$

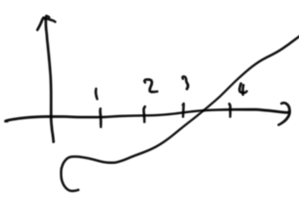
$\Rightarrow \delta>0,\ \forall p_0\in[p-\delta,p+\delta]$

$p_n=g(p_{n-1})$ conv. to p

at least quadratically

Moreover, n: large

$$|p_{n+1}-p|<\frac{M}{2}|p_n-p|^2$$



猜 $p=3.5$

Thm 10.7

let $\vec{p}=\vec{G}(\vec{p})$. $\delta>0$

if ① (i) $\dfrac{\partial g_i(\vec{x})}{\partial x_k}$: contin on $N_\delta=\{\vec{x}\mid \|\vec{x}-\vec{p}\|<\delta\}$

(ii) $\dfrac{\partial g_i}{\partial x_k}(\vec{p})=0$ 求$A(x)$

② $\dfrac{\partial^2 g_i}{\partial x_j\partial x_k}(\vec{x})$: contin, with $\left|\dfrac{\partial^2 g_i}{\partial x_j\partial x_k}(\vec{x})\right|\leq M$ $\forall \vec{x}\in N_\delta$

$\Rightarrow \exists\hat{\delta}<\delta,\ \forall \vec{p}_0$ s.t. $\|\vec{p}_0-\vec{p}\|<\hat{\delta}$

$\vec{p}_n=\vec{G}(\vec{p}_{n-1})$ conv. to $\vec{p}$

at least quadratically

Moreover, n: large.

$$\|\vec{p}^{\,n}-\vec{p}\|_\infty\leq\frac{n^2M}{2}\|\vec{p}_{n-1}-\vec{p}\|_\infty^2$$

$$\vec{G}(\vec{x}) = \vec{x} - A^{-1}(x)\,\vec{F}(\vec{x}) \qquad A^{-1}(\vec{x}) = [b_{ij}(\vec{x})]$$

$$\vec{G}(\vec{x}) = \begin{bmatrix} g_1(\vec{x}) \\ g_2(\vec{x}) \\ \vdots \\ g_n(\vec{x}) \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} - \begin{bmatrix} b_{11}(\vec{x}) & b_{12}(\vec{x}) & \cdots & b_{1n}(\vec{x}) \\ b_{21}(\vec{x}) & & & \\ \vdots & & & \\ b_{n1}(\vec{x}) & \cdots & & b_{nn}(\vec{x}) \end{bmatrix} \begin{bmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \\ \vdots \\ f_n(\vec{x}) \end{bmatrix}$$

$$\frac{\partial g_i}{\partial x_k}(\vec{x}) = \begin{cases} 1 - \sum_{j=1}^{n} \left(b_{ij}(\vec{x}) \frac{\partial f_j(\vec{x})}{\partial x_k} + \frac{\partial b_{ij}(\vec{x})}{\partial x_k} f_j(\vec{x}) \right) & \text{if } i = k \\ 0 - \sum_{j=1}^{n} \left(b_{ij}(\vec{x}) \frac{\partial f_j(\vec{x})}{\partial x_k} + \frac{\partial b_{ij}(\vec{x})}{\partial x_k} f_j(\vec{x}) \right) & \text{if } i \neq k \end{cases}$$

☆ $\frac{\partial g_i}{\partial x_k}(\vec{p}) = 0$

☆ $\vec{G}(\vec{p}) = \vec{p}$, $\vec{F}(\vec{p}) = \vec{0}$

① $\boxed{i = k}$

$$\frac{\partial g_i}{\partial x_i}(\vec{p}) = 0 = 1 - \sum_{j=1}^{n} \left(b_{ij}(\vec{p}) \frac{\partial f_j(\vec{p})}{\partial x_i} + \underbrace{\boxed{\frac{\partial b_{ij}(\vec{p})}{\partial x_i} \underbrace{f_j(\vec{p})}_{=0}}}_{=0} \right)$$

$$0 = 1 - \boxed{\sum_{j=1}^{n} b_{ij}(\vec{p}) \frac{\partial f_j(\vec{p})}{\partial x_i}}$$

$$\boxed{\sum_{j=1}^{n} b_{ij}(\vec{p}) \frac{\partial f_j(\vec{p})}{\partial x_i}} = 1$$

☆ $\vec{G}(\vec{p}) = \vec{p}$, $\vec{F}(\vec{p}) = \vec{0}$

② $\boxed{i \neq k}$

$$\frac{\partial g_i}{\partial x_k}(\vec{p}) = 0 = 0 - \sum_{j=1}^{n} \left(b_{ij}(\vec{p}) \frac{\partial f_j(\vec{p})}{\partial x_k} + \underbrace{\boxed{\frac{\partial b_{ij}(\vec{p})}{\partial x_k} \underbrace{f_j(\vec{p})}_{=0}}}_{=0} \right)$$

$$0 = 0 - \boxed{\sum_{j=1}^{n} b_{ij}(\vec{p}) \frac{\partial f_j(\vec{p})}{\partial x_k}}$$

$$\boxed{\sum_{j=1}^{n} b_{ij}(\vec{p}) \frac{\partial f_j(\vec{p})}{\partial x_k}} = 0$$

①, ②

$\Downarrow$

$$\therefore \sum_{j=1}^{n} \underbrace{\boxed{b_{ij}(\vec p)}}_{i,j}\ \underbrace{\boxed{\frac{\partial f_j(\vec p)}{\partial x_k}}}_{j,k} = \begin{cases} 1 & \text{if } i=k \quad ① \\ 0 & \text{if } i\neq k \quad ② \end{cases}$$

$$\left[b_{ij}(\vec p) \right] \left[\frac{\partial f_j(\vec p)}{\partial x_k} \right] = \begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$$

$$(A(\vec p))^{-1} (J(\vec p)) = I$$

$$\therefore (J(\vec p)) = (A(\vec p))$$

$$\therefore A(\vec p) = J(\vec p) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(\vec p) & \frac{\partial f_1}{\partial x_2}(\vec p) & \cdots & \frac{\partial f_1}{\partial x_n}(\vec p) \\ \frac{\partial f_2}{\partial x_1}(\vec p) & \cdots & \cdots & \frac{\partial f_2}{\partial x_n}(\vec p) \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1}(\vec p) & \cdots & \cdots & \frac{\partial f_n}{\partial x_n}(\vec p) \end{bmatrix}$$

$\therefore$ let $\vec G(\vec x) = \vec x - (J(\vec x))^{-1} \vec F(\vec x)$

Newton Method.

Ch 2 Newton Method

$$g(x) = x - \frac{1}{f'(x)} f(x)$$

pick x_0

$$\left.\begin{aligned} x_1 &= g(x_0) \\ x_2 &= g(x_2) \\ &\vdots \\ x_n &= g(x_{n-1}) \end{aligned}\right\} \rightarrow \text{fixed-point iteration}$$

△ error $= |x_n - x_{n-1}|$

error $<$ tol ex: 10^{-6}

Ch 10 Newton Method.

$$\vec G(\vec x) = \vec x - (J(\vec x))^{-1} \vec F(\vec x)$$

※ 每步的 $\vec F(\vec x_0)$ $J(\vec x_0)$ 不同

pick $\vec x_0$

$$\vec x_1 = \vec G(\vec x_0) \longrightarrow \begin{cases} ① \text{ 求 } \vec y_0 \\ \quad \text{s.t. } (J(\boxed{\vec x_0}))\, \vec y_0 = \vec F(\vec x_0) \\ ② \ \vec x_1 = \vec x_0 - \vec y_0 \end{cases}$$

$$\vec x_2 = \vec G(\vec x_1)$$

$$\vdots$$

$$\vec x_n = \vec G(\vec x_{n-1})$$

$\ell = 1, 2, \infty$ or else

△ error $= \| \vec x_n - \vec x_{n-1} \|_{\ell}$ ∞

error $<$ tol