姓名: SOLUTION　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________　　　　**Quiz 1**　　　　考試日期: 2025/09/17

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

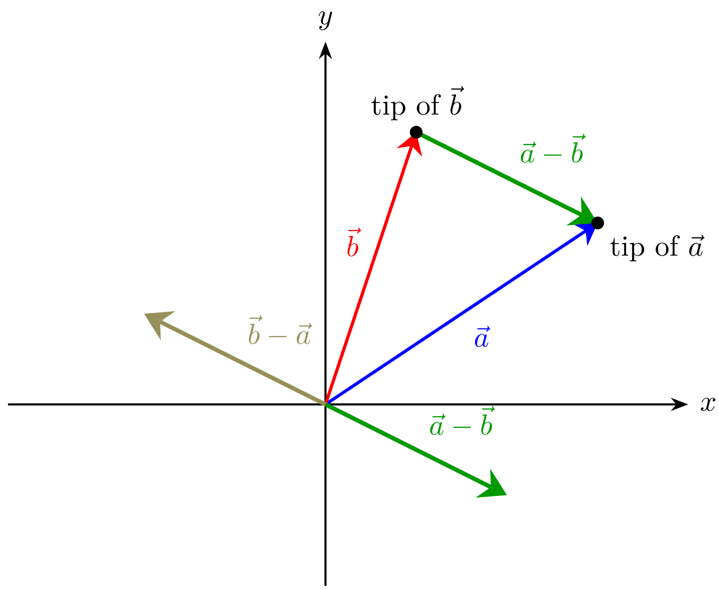
1. For the following, mark True or False. Justify your answer with a graph if true, or with a counterexample if false. (若為真，提供圖例顯示；若為否，提供反例)

   True $\boxed{\textbf{False}}$ If $\vec{a}$ and $\vec{b}$ are two vectors in standard position in $\mathbb{R}^n$, then the arrow from the tip of $\vec{b}$ to the tip of $\vec{a}$ is a translated repersentation of the vector $\vec{b}-\vec{a}$.

   Solution：

   Similar with 1-1 problem 39, (d), (e).
   Should be "$\vec{a}-\vec{b}$".

   Let $\vec{a}=[3,\ 2]$, $\vec{b}=[1,\ 3]$
   $\vec{b}-\vec{a}=[-2,\ 1]$
   $\vec{a}-\vec{b}=[2,\ -1]$



2. Given $\vec{u}=[1,2]$, $\vec{v}=[5,1]$, $\vec{w}=[13,8]$.

   (a) Is $\vec{w}\in sp(\vec{u},\vec{v})$? $\boxed{\textbf{True}}$ False.

   (b) If so, find $r=$ **3**, $s=$ **2** $\in\mathbb{R}$ such that $\vec{w}=r\vec{u}+s\vec{u}$.

   Solution：

   Assume there exist $r,s\in\mathbb{R}$, such that $\vec{w}=r\vec{u}+s\vec{u}$.

$$[13,8]=r[1,2]+s[5,1]=[r+5s,\ 2r+s]$$

$$\begin{cases}13=r+5s\\ 8=2r+s\end{cases}\quad\Rightarrow\quad r=3,\quad s=2$$

3. Let $\vec{v}$ and $\vec{w}$ are any two vectors in $\mathbb{R}^n$, and let r be any scalar in $\mathbb{R}$. Please prove the following property.

$$r(\vec{v}+\vec{w}) = r\vec{w} + r\vec{v}.$$

Solution：

Similar with example 4 from 1-1. Notice that the order of $\vec{v}$ and $\vec{u}$ is not the same on both sides of the equation.

Let $\vec{v} = [v_1,\ v_2, \ldots,\ v_n]$ and $\vec{w} = [w_1,\ w_2, \ldots,\ w_n]$.

$$\begin{aligned}
LHS &= r(\vec{v}+\vec{w}) \\
&= r([v_1,\ v_2,\ldots,\ v_n] + [w_1,\ w_2,\ldots,\ w_n]) \\
&= r[v_1+w_1,\ v_2+w_2,\ldots,\ v_n+w_n] && \text{by Definition of Vector addition} \\
&= [r(v_1+w_1),\ r(v_2+w_2),\ldots,\ r(v_n+w_n)] && \text{by Definition of Scalar multiplication} \\
&= [rv_1+rw_1,\ rv_2+rw_2,\ldots,\ rv_n+rw_n] \\
&= [rv_1,\ rv_2,\ldots,\ rv_n] + [rw_1,\ rw_2,\ldots,\ rw_n] && \text{by Definition of Vector addition} \\
&= r[v_1,\ v_2,\ldots,\ v_n] + r[w_1,\ w_2,\ldots,\ w_n] && \text{by Definition of Scalar multiplication} \\
&= r\vec{v} + r\vec{w} \\
&= r\vec{w} + r\vec{v} = RHS && \text{by A2}
\end{aligned}$$

Definition 1.1: Vector Algebra in $\mathbb{R}^n$
Let $\mathbf{v} = [v_1, v_2, \ldots, v_n]$ and $\mathbf{w} = [w_1, w_2, \ldots, w_n]$ be vectors in $\mathbb{R}^n$. Let r is any scalar. We define the following:

Vector addition/subtraction: $\mathbf{v} \pm \mathbf{w} = [v_1 \pm w_1, v_2 \pm w_2, \ldots, v_n \pm w_n]$

Scalar multiplication: $r\mathbf{v} = [rv_1, rv_2, \ldots, rv_n]$

Theorem 1.1: Properties of Vector Algebra in $\mathbb{R}^n$
Let $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ be any vectors in $\mathbb{R}^n$, and let r and s be any scalars in $\mathbb{R}$.

Properties of Vector Addition

A1: $(\mathbf{u}+\mathbf{v})+\mathbf{w} = \mathbf{u}+(\mathbf{v}+\mathbf{w})$ A3: $\mathbf{0}+\mathbf{v} = \mathbf{v}$

A2: $\mathbf{v}+\mathbf{w} = \mathbf{w}+\mathbf{v}$ A4: $\mathbf{v}+(-\mathbf{v}) = \mathbf{0}$

Properties Involving Scalar Multiplication

S1: $r(\mathbf{v}+\mathbf{w}) = r\mathbf{v}+r\mathbf{w}$ S3: $r(s\mathbf{v}) = (rs)\mathbf{v}$

S2: $(r+s)\mathbf{v} = r\mathbf{v}+s\mathbf{v}$ S4: $1\mathbf{v} = \mathbf{v}$