沒有星號題的答案見課本後面

Section 3-1

課本 problem 6*, 7, 11, 13, 16*, 18*

6. Define $(f \oplus g) = \max\{f(x), g(x)\}$, for all $x \in \mathbb{R}$ and $(rf)(x) = rf(x)$, for all $x \in \mathbb{R}$. Assume $z(x)$ is the $\vec{0}$, that is for all $f(x)$, $z(x) = f(x) \oplus (-f)(x) = \max\{f(x), (-f)(x)\} = \max\{f(x), -f(x)\}$.
   Let $f(x) = 1, z(x) = f(x) \oplus (-f)(x) = \max\{1, -1\} = 1$. However, by **A3**, $z(x) \oplus (-f)(x) = (-f)(x) = -1 \neq \max\{1, -1\}$. Therefore, $\vec{0}$ does not exists.

16. The set P_n of all polynomials in x, with real coefficients and of degree less or equal to n, together with zero polynomial. Noticed that the set P of all polynomials in x with real coefficients is a vector space. (Example 2 in textbook 3-1) Since P_n is a subset of P. P_n is a vector space if $\vec{0} \in P_n$ and P_n is closed under vector addition and scalar multiplication.
    Let $p(x) = p_n x^n + ... + p_1 x + p_0$, $q(x) = q_n x^n + ... + q_1 x + q_0$ are two polynomials of degree $\leq n$ and let r is a real number.
    Then

$$\begin{aligned} (rp)(x) &= rp_n x^n + ... + rp_1 x + rp_0 \\ (p+q)(x) &= (p_n + q_n)x^n + ... + (p_1 + q_1)x + (p_0 + q_0) \end{aligned}$$

are polynomials of degree $\leq n$. Hence, the set P_n is closed under vector addition and scalar multiplication.

18 (a) Matrix multiplication is a vector space operation on the set $M_{m \times n}$ of $m \times n$ matrices.
False. Vector space operations are just scalar multiplication and vector addition.

(b) Matrix multiplication is a vector space operation on the set $M_{n \times n}$ of square $n \times n$ matrices.
False. Vector space operations are just scalar multiplication and vector addition.

(c) Multiplication of any vector by the zero scalar always yields the zero vector.
True.

(d) Multiplication of a non-zero vector by a non-zero scalar always yields a non-zero vector.
True.

(e) No vector is its own additive inverse.
False. The zero vector $\vec{0}$ is its own additive inverse.

(f) The zero vector is the only vector that is its own additive inverse.
True.

(g) Multiplication of two scalars is of no concern to the definition of a vector space.
False. Check **S3**.

(h) Every vector spaces has at least two vectors.
False. $\{\vec{0}\}$ with normal vector addition and scalar multiplication is a vector space.

(i) Every vector space has at least one vector.
True. Every vector space contains a zero vector.

Section 3-2
課本 problem 2*, 3, 4*, 5, 8*, 12* , 13, 20*, 23, 25, 26*

2 The set is NOT a subspace of P since it is not closed under vector addition. For example,$p(x) = x^4 + x^3$ and $q(x) = -x^4$ are both in the set, but $p(x) + q(x) = x^3$ is not in the set.

4 $W = \{f|f(1) = 0\}$ is a subspace of F. You should verify that W contains zero vector, and closed under vector addition, and closed under scalar multiplication, which is proved below.
$z(x) = 0$ is the zero vector in F. $z(x) \in W$ since $g(1) = 0$.
Suppose f, g are functions satisfying $f(1) = g(1) = 0$ and r is a real number.

$$\begin{aligned} (rf)(1) &= rf(1) = 0 \\ (f+g)(1) &= f(1) + g(1) = 0 + 0 = 0 \end{aligned}$$

8 Note that

$$1 = 1(1 + 2x) + (-2)x$$

and

$$x = 0(1 + 2x) + 1(x)$$

,so $sp(1, x)$ is contained in $sp(1 + 2x, x)$. Next,

$$1 + 2x = 1(1) + 2(x)$$

and

$$x = 0(1) + 1(x),$$

so $sp(1+2x, x)$ is contained in $sp(1, x)$. Thus we conclude that $sp(1, x) = sp(1+2x, x)$.

12 The set of vectors is dependent. Supposer

$$1 + r_2(4x + 3) + r_3(3x - 4) + r4(x^2 + 2) + r_5(x - x^2) = 0.$$

Then

$$(r_4 - r_5)x^2 + (4r_2 + 3r_3 + r_5)x + (r_1 + r_2 - 4r_3 + 2r_4) = 0.$$

Thus we solve the system $\begin{cases} r_4 - r_5 = 0 \\ 4r_2 + 3r_3 + r_5 = 0 \\ r_1 + 3r_2 - 4r_3 + 2r_4 = 0 \end{cases}$.

We row reduce the augmented matrix

$$\left[\begin{array}{ccccc|c} 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 4 & 3 & 0 & 1 & 0 \\ 1 & 3 & -4 & 2 & 0 & 0 \end{array}\right] \simeq \left[\begin{array}{ccccc|c} 1 & 0 & -25/4 & 0 & 5/4 & 0 \\ 0 & 1 & 3/4 & 0 & 1/4 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{array}\right]$$

Since the third and fifth columns do not contain a pivot, r_3 and r_5 are free variables, so we can easily find a non-trivial solution for r_1, r_2, r_3, r_4, r_5. Thus the set is dependent.

20 $(x-1)^2 = (x^2+1) + (-2)x$, so the set of vectors is dependent and hence is NOT a basis for P_2.

26 (a) Every independent set of vectors in V is a basis for subspace the vectors span.
True. (Any linearly independent set is a basis for its span.)

(b) If $\{v_1, v_2, ..., v_n\}$ generates V,then each $v \in V$ is a linear combination of vectors in this set.
True.

(c) If $\{v_1, v_2, ..., v_n\}$ generates V, then each $v \in V$ is a unique linear combination of vectors in this set.
False. The vectors $\{v_1, v_2, ..., v_n\}$ need not be linearly independent; so there maybe more than one way of writing the zero vector as a linear combination of the vectors in $\{v_1, v_2, ..., v_n\}$.

(d) If $\{v_1, v_2, ..., v_n\}$ generates V and is independent ,then each $v \in V$is a linear combination of vectors in this set.
True.

(e) If $\{v_1, v_2, ..., v_n\}$ generates V,then this set of vectors is independent.
False.(See Part c).

(f) If each vector in V is a unique linear combination of the vectors in the set $\{v_1, v_2, ..., v_n\}$, then this set is independent.
True.

(g) If each vector in V is a unique linear combination of the vectors in the set $\{v_1, v_2, ..., v_n\}$, then this set is a basis for V.
True.

(h) All vector spaces having a basis are finitely generated.
False. The set P of all polynomials in x with real coefficients is a vector space but not finitely generated. (Check ILLUSTRATION 1 in 3-2.)

(i) Every independent subset of a finitely generated vector space is a part of some basis for V.
True.

(j) Any two bases in a finite-dimensional vector space V have the same number of elements.
True.