

應數一線性代數 2023 秋, 期末考

學號: _____, 姓名: _____

本次考試共有 10 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- 不可翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。
- 書寫空間不夠時，可利用試卷背面，但須標記清楚。

高師大校訓：誠敬宏遠

誠: 一生動念都是誠實端正的。 敬: 就是對知識的認真尊重。
宏: 開拓視界，恢宏心胸。 遠: 任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: _____

1. (10 points) Find the coordinate vector of the given vector relative to the indicated ordered basis.

$$\begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix} \text{ in } M_2 \text{ relative to } \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ 3 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \right).$$

Answer: the coordinate vector is _____

2. (10 points) Let $\vec{a} = \vec{i} - 3\vec{k}$, $\vec{b} = -\vec{i} + 4\vec{j}$, $\vec{c} = \vec{i} + 2\vec{j} + \vec{k}$.

Find $\vec{a} \cdot (\vec{b} \times \vec{c}) =$ _____

3. (10 points) Let $T : P_2 \rightarrow P_3$ be defined by $T(ax^2 + bx + c) = (4a + b - c)x^3 + (2a + 2b)x^2 + (6b + c)x + (2a + b + 3c)$, the ordered basis for P_2 is $B = (x^2, x, 1)$ and the ordered basis for P_3 is $B' = (x^3, x^2, x, 1)$. Find the matrix representation A of T relative to the ordered bases B and B' .

Answer: (a) $A = \underline{\hspace{10cm}}$

(b) find $p(x)$ such that $T(p(x)) = 7x^3 + 4x^2 + 4x - 3$. $p(x) = \underline{\hspace{10cm}}$

4. (10 points) Find the determinant of the given matrix.

$$\begin{bmatrix} 1 & 2 & 0 & -1 & 2 & 4 \\ 2 & 3 & 1 & 4 & 2 & 4 \\ 4 & 6 & 0 & 8 & 2 & 4 \\ -1 & 1 & 0 & -1 & 3 & -5 \\ 0 & 0 & 0 & 0 & 5 & 7 \\ 1 & 2 & 0 & -1 & 2 & 5 \end{bmatrix}$$

Answer: $\det(A) =$ _____

5. (10 points)

$$A = \begin{bmatrix} 5 & -2 & 1 \\ 3 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

The inverse of A = _____, and the adjoint matrix of A = _____

6. (10 points) Determine the set S_1 of all functions f such that $f(0) = 1$ is a subspace in the vector space F of all functions mapping \mathbb{R} into \mathbb{R} .

Answer: Is S_1 a subspace of F ? _____

7. (10 points) Determine whether the given 4 points lie in a plane in \mathbb{R}^4 . If so, find its area. If not, find its volume.

$$A(2, 1, 1, 1), B(3, 1, -1, 2), C(2, 0, 2, 3), D(2, -1, 2, 0)$$

Answer:

- $ABCD$ are coplanar(共平面), and the area of the quadrilateral (四邊形) is _____.
- $ABCD$ are NOT coplanar, and the volume of the tetrahedron(四面體) is _____.

8. (10 points) Consider the set $\{(x, y) | x + y = 0\} \in \mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x + a, y + b]$, and with scalar multiplication defined by $r \otimes [x, y] = [ry, rx]$.

- a. Is this set a vector space? _____

Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

- b. If it is a vector space, then what is the requested vectors in this vector space?

Hint: The zero vector may NOT be the vector $[0, 0]$.

Answer: the zero vector is _____, for any vectors $[x,y]$, the $-[x,y]$ is _____

9. (10 points) Consider the set $\{(x, y) | x+y = 1\} \in \mathbb{R}^2$, with the addition defined by $[x, y] \oplus [a, b] = [x+a+1, y+b]$, and with scalar multiplication defined by $r \otimes [x, y] = [rx + r - 1, ry]$.

- a. Is this set a vector space? _____

Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

- b. If it is a vector space, then what is the requested vectors in this vector space?

Hint: The zero vector may NOT be the vector $[0, 0]$.

Answer: the zero vector is _____, for any vectors $[x,y]$, the $-[x,y]$ is _____

10. (10 points) Determine the dimension of the given set S . Then reduce the given set to be a basis for $sp(S)$.

$S = sp(1, 4x + 5, 5x - 4, x^2 + 2, x - 2x^2)$ is a subspace in a vector space P .

Answer: $\dim(S) = \underline{\hspace{10cm}}$.

A basis for S is $\underline{\hspace{10cm}}$

11. (10 points) Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ and $\hat{T} : \mathbb{R}^m \rightarrow \mathbb{R}^k$ be linear transformations. Prove directly from its definition that $(\hat{T} \circ T) : \mathbb{R}^n \rightarrow \mathbb{R}^k$ is also a linear transformation.

學號: _____, 姓名: _____, 以下由閱卷人員填寫