應數一線性代數 2020 秋, 期末考

學號: ______________________________, 姓名: ______________________________

本次考試共有 8 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。　**敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。　**遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Find the determinant of

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 & 0 \\ 0 & 4 & 1 & -1 & 2 \\ 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -1 & 3 \end{bmatrix}$$

Answer: ____________

2. (10 points) Suppose that A is a 5×5 matrix with determinant 7.

   (a) Find $\det(3A) =$ ________________

   (b) Find $\det(A^{-1}) =$ ________________

   (c) Find $\det(2A^{-1}) =$ ________________

   (d) Find $\det((2A)^{-1}) =$ ________________

3. (5 points) Suppose that A is a 3×3 matrix with row vectors $\vec{a}, \vec{b}$, and $\vec{c}$, and that $det(A) = 3$. Find the determinant of the matrix having $\vec{a}, \vec{b}, 2\vec{a} + 3\vec{b} + 2\vec{c}$ as its row vectors

   Determinant = ________________

4. (10 points)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ -2 & 1 & 2 \end{bmatrix}$$

   The inverse of A = ________________ , and the adjoint matrix of A = ________________

5. (5 points) Let $\vec{a} = \vec{i} - 3\vec{k}$, $\vec{b} = -\vec{i} + 4\vec{j}$, $\vec{c} = \vec{i} + \vec{j} + \vec{k}$.

   Find $\vec{a} \cdot (\vec{b} \times \vec{c}) =$ ________________

6. (10 points) Find out whether points (1, 2, 1), (3, 3, 4), (2, 2, 2) and (4, 3, 5) lie in a plane in $\mathbb{R}^3$

   Answer: ________________

7. (10 points) Using **Cramer's rule** to find the component y of the solution vector for the given linear system.

$$\begin{cases} 2x - 3y = 1 \\ -4x + 6y = -2 \end{cases}$$

$y =$ ____________________

8. (10 points) Circle True or False. Read each statement in original Greek before answering.

(a) True False There's an unique coordinate vector associated with each vector $\vec{v} \in V$ relatve to a basis for V

(b) True False A linear transformation $T : V \to V'$ carries the zero vector of V into the zero vector of V'.

(c) True False The parallelogram (平行四邊形) in $\mathbb{R}^2$ determined by non-zero vectors $\vec{a}, \vec{b}$ is a square (正方形) if and only if $\vec{a} \cdot \vec{b} = 0$

(d) True False The product of a square matrix and its adjoint is the identity matrix.

(e) True False There is no square matrix A such that $\det(A^T A) = -1$.

9. (10 points) Let V and V'' be vector spaces with ordered bases $B = ([1,3,-2],[4,1,2],[-1,1,0])$ and $B' = ([1,0,1,0],[2,1,1,-1],[0,1,1,-1],[2,0,3,1])$, respectively, and let $T : V \longrightarrow V'$ be the linear transformation having the given matrix A as matrix representation relative to B, B'. Find $T([0,3,-6])$.

$$A = \begin{bmatrix} 0 & 4 & -1 \\ 1 & 1 & 2 \\ 2 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix},$$

(a) If $\vec{v} = [0,3,-6]$, then $\vec{v}_B =$ ____________________.

(b) $T([0,3,-6]) =$ ____________________.

10. (10 points) Let $T : P_3 \longrightarrow P_2$ be defined by $T(p(x)) = D(p(x+1))$, and let $B = (x^3, x^2, x, 1)$ and $B' = (x^2, x, 1)$.

(a) Find the matrix A as matrix representation of T relative to B, B'. $A =$ ____________________.

(b) Use A to compute $T(4x^3 - 5x^2 + 3x - 2) =$ ____________________.

11. (10 points) Let $S = \{1, \sin(x), \cos(x), \sin(2x), \cos(2x)\}$ is a set of functions in the vector space F of all functions mapping $\mathbb{R}$ into $\mathbb{R}$.

   (a) Prove that S is an independent set in F.

   (b) Find a basis for the subspace of F generated by the functions $\{f_1, f_2, f_3, f_4\}$, where

$$f_1(x) = 1 - 2\sin(x) + 4\cos(x) - \sin(2x) - 3\cos(2x), \quad f_2(x) = 1 - 2\sin(x),$$

$$f_3(x) = 4\cos(x) - 5\sin(2x) + 3\cos(2x), \quad f_4(x) = 1 + 2\sin(2x)$$

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Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	10	10	5	10	5	10	10	10	10	10	10	100
Score:												