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葉均承 化學一微積分

學號:

Quiz 7

考試日期: 2020/05/04

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (30%) Evaluate $\int_3^\infty \frac{x+1}{x^2-4}dx$ if possible.

$\frac{X+1}{X^2-4} \approx \frac{1}{X}$ $\therefore$ hope ">"

$X(X+1) \overset{?}{>} X^2-4$

$\| $

$X^2+X \overset{?}{>} X^2-4$

$X > -4$ (Correct since $X\in[3,\infty)$)

$\because \frac{X+1}{X^2-4} > \frac{1}{X}$, where $X\in[3,\infty)$

and $\int_3^\infty \frac{1}{X}dx$ diverge by p-test

$\therefore \int_3^\infty \frac{X+1}{X^2-4}dx$ is diverge by comparison to $\int_3^\infty \frac{1}{X}dx$

2. (35%) Find the limit of $a_n = (\sqrt{n+1}-\sqrt{n})\sqrt{n+\frac{1}{2}}$ $\frac{(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})}$

$$a_n = \frac{\sqrt{n+\frac{1}{2}}\,(n+1-n)}{\sqrt{n+1}+\sqrt{n}}$$

$$\therefore \lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{\sqrt{n+\frac{1}{2}}\cdot\frac{1}{\sqrt{n}}}{(\sqrt{n+1}+\sqrt{n})\cdot\frac{1}{\sqrt{n}}} \cancel{= \lim_{n\to\infty}\frac{\sqrt{n}}{\sqrt{n}+\sqrt{n}} = \frac{1}{2}}$$

$$= \lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{2n}}}{\sqrt{1+\frac{1}{n}}+\sqrt{1}} = \frac{1}{1+1} = \frac{1}{2}$$

3. (35%) Given the sequence defined recursively by $a_1 = 1$, $a_{n+1} = \sqrt{3 + a_n}$ is increasing and bounded above by 3, find the limit.

let $\lim_{n\to\infty} a_n = L$, since $\{a_n\}$ increasing and bounded above by 3

$\therefore$ L exists. and $\lim_{n\to\infty} a_{n+1} = L$

$$\underbrace{\lim_{n\to\infty} a_{n+1}}_{= L} = \lim_{n\to\infty} \sqrt{3+a_n} = \sqrt{3+L}$$

$\therefore L = \sqrt{3+L}$

$L^2 = 3 + L$

$L^2 - L - 3 = 0$

$L = \frac{1+\sqrt{13}}{2}$ or $\frac{1-\sqrt{13}}{2}$

$a_1 = 1$ $*$ $\because \{a_n\}$: increasing

$\therefore a_n \to \frac{1+\sqrt{13}}{2}$