化學一微積分 **2020** 春, 第一次期中考

學號: Sol. ____________________, 姓名: ____________________

本次考試共有 8 頁 (包含封面)，有 19 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- 不可翻閱課本或筆記。
- 第一部份是~~本次段考範圍~~。每題 4 分。
- 第二部份是~~額外加分題~~。每題 ~~2.5~~ 10 分。

高師大校訓：誠敬弘遠

誠，一生動念都是誠實端正的。敬，就是對知識的認真尊重。宏，開拓視界，恢宏心胸。遠，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名: ____________________

第一部份，選擇題

B 1. Compute $\int_0^{\frac{\pi}{2}} \cos^2(\theta)\sin^3(\theta)d\theta$. $= \int_0^{\frac{\pi}{2}} \cos^2\theta \sin^2\theta \sin\theta \, d\theta = \int_1^0 -u^2(1-u^2)\,du$

A.$-2/15$.

B.$2/15$.

C.$-\pi^2/30$.

D. 0.

E.$\pi^2/30$

θ	u
$\frac{\pi}{2}$	0
0	1

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$\sin^2\theta = 1 - \cos^2\theta$

$= \int_0^1 u^2(1-u^2)\,du$

$= \frac{2}{15}$

D 2. Which integral gives the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$,and $x = 1$ about the line $x = 2$?

cylindrical shell method

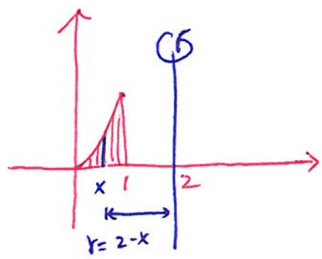
A. $2\pi \int_0^2 x^3(1+x)dx$.

B. $2\pi \int_0^1 x^3(1+x)dx$.

C. $2\pi \int_0^2 x^3(2-x)dx$.

D. $2\pi \int_0^1 x^3(2-x)dx$.

E. $\pi \int_0^1 (2-y^{1/3})^2 dx$.



$r = 2-x$

$h = x^3$

$\int_0^1 2\pi x^3(2-x)\,dx$

E 3. A rope that is 80 feet long and weighs 60 pounds hangs over a building 500 feet tall. How much work is done in pulling only the first 4 feet of rope to the top of the building?

A. 246 foot pounds.

B 6 foot pounds.

C. 688/3 foot pounds.

D. 116/3 foot pounds.

E. 234 foot pounds

rope weight $\frac{60}{80} = \frac{3}{4}$ lb/ft

$\int_0^4 (60 - \frac{3}{4}x)\,dx = (60x - \frac{3}{8}x^2)\Big|_0^4$

$= 240 - \frac{3}{8}\times 16 = 234$

B 4. A spring has a natural length of 1 m. If a 40-N force is required to keep the spring stretched to a length of 3 m,how much work is done in stretching the spring from 1 m to 5 m?

A. 240 J

B. 160 J

C. 320/3 J

D. 180 J

E. None of the above

force: $f(x) = kx$

$f(3-1) = f(2) = 40 \Rightarrow k = 20$

$W = \int_{1-1}^{5-1} f(x)\,dx = \int_0^4 20x\,dx = 10x^2\Big|_0^4 = 160$ (J)

A

5. Which of the following is equal to $\int_0^1 x^2 \sin(x-1)dx$ $= \int_{-1}^{0} (1+u)^2 \sin(u)\,du$

A. $\int_{-1}^{0} (u+1)^2 \sin(u)du$.

B. $\int_{0}^{1} (u+1)^2 \sin(u)du$.

C. $\int_{-1}^{0} u^2 \sin(u)du$.

D. $\int_{0}^{1} x^2 \sin(u)du$.

E. $\int_{0}^{1} u^2 \sin(u)du$.

$u = x-1$

$du = dx$

x	u
1	0
0	-1

C

6. Compute $\int_1^e \ln(x^2)dx$ $= 2\int_1^e \ln|x|\,dx = 2\left(x\ln|x|\Big|_1^e - \int_1^e 1\,du\right) = 2(e-(e-1))$

$= 2$

$\ln(x^2) = 2\ln|x|$

$u = \ln x$, $du = \frac{1}{x}$

$dv = 1$, $v = x$

A. $2\frac{1-e}{e}$.

B. $2e-1$.

C. 2.

D. $4e-2$.

E. 1.

A

7. Which of the following integrals gives the area of the region bounded by the curves $y = \frac{1}{x}$, $y = \frac{x-1}{e(e-1)}$ and $x = 1$. (Hint: the curves $y = \frac{1}{x}$ and $y = \frac{x-1}{e(e-1)}$ intersect at the point $(e, \frac{1}{e})$).

$y = f(x)$ $y = g(x)$

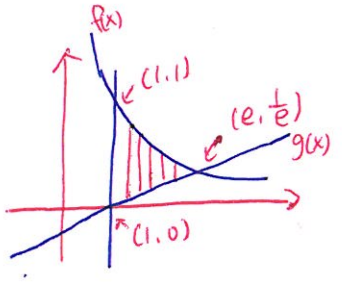
A. $\int_1^e (\frac{1}{x} - \frac{x-1}{e(e-1)})dx$.

B. $\int_{1/e}^{1} \frac{1}{y}dy + \int_0^{\frac{1}{e}} (e(e-1)y+1)dy$.

C. $\int_0^e (\frac{1}{x} - \frac{x-1}{e(e-1)})dx$.

D. $\int_1^e (-\frac{1}{x} + \frac{x-1}{e(e-1)})dx$.

E. $\int_0^1 (\frac{1}{y} - 1)dy$.



$\int_1^e f(x) - g(x)\,dx$

$= \int_1^e \frac{1}{x} - \frac{x-1}{e(e-1)}dx$

C

8. Compute $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$ $= 2\int_1^2 e^u\,du = 2(e^2 - e)$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

A. -6.

B. $-2e^2 + 2e$.

C. $2e^2 - 2e$.

D. 6.

E. $2e - 2$.

x	u
4	2
1	1

A 9. Find the volume of the solid obtained by rotating the region bounded by $y = x^2$ and $y = x^3$ (in the first quadrant) around the x-axis.

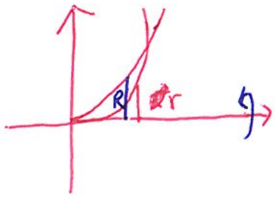
A. $\pi\frac{2}{35}$.

B. $\frac{1}{12}$.

C. $\pi\frac{1}{12}$.

D. $\pi\frac{1}{105}$.

E. $\frac{2}{35}$.



Washer.

$R = x^2$, $r = x^3$

$V = \int_0^1 \pi (R^2 - r^2)\,dx = \int_0^1 \pi (x^4 - x^6)\,dx$

$= \pi\left(\frac{1}{5} - \frac{1}{7}\right) = \frac{2\pi}{35}$

E 10. Which of the following is equal to $\int_0^{\pi/4} \tan^2(x)\sec^4(x)dx$ $= \int_0^1 u^2(1+u^2)\,du$

A. $\int_0^{\pi/4} u^2(1-u^2)du$.

B. $\int_0^{\pi/4} u^2(1+u^2)du$.

C. $-\int_0^1 u^2(1+u^2)du$.

D. $\int_0^1 u^2(1-u^2)du$.

E. $\int_0^1 u^2(1+u^2)du$.

x	u
$\frac{\pi}{4}$	1
0	0

$u = \tan x$

$du = \sec^2 x\,dx$

$\sec^2 x = 1 + \tan^2 x$

B 11. Which integral gives the volume of the solid obtained by rotating the region bounded by $y = \sin x$ and $y = \frac{2}{\pi}x$ **in the first quadrant**(第一象限) around the y-axis?

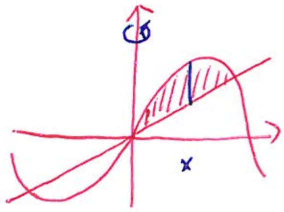
A. $2\pi\int_{-\pi/2}^{\pi/2} (\sin x - \frac{2}{\pi}x)dx$.

B. $2\pi\int_0^{\pi/2} x(\sin x - \frac{2}{\pi}x)dx$.

C. $\pi\int_0^{\pi/2} x(\sin x - \frac{2}{\pi}x)dx$.

D. $2\pi\int_{-\pi/2}^{\pi/2} x(\sin x - \frac{2}{\pi}x)dx$.

E. $\pi\int_0^{\pi/2} (\sin^2 x - \frac{4}{\pi^2}x^2)dx$.



cylindrical shell method.

$h = \sin x - \frac{2}{\pi}x$

$r = x$

$\int_0^{\frac{\pi}{2}} 2\pi x(\sin x - \frac{2}{\pi}x)\,dx$

A

12. Compute $\int_0^1 xe^{2x}$ $= x\cdot\frac{1}{2}e^{2x}\Big|_0^1 - \int_0^1 \frac{1}{2}e^{2x}dx = \frac{1}{2}e^2 - \frac{1}{4}(e^2-1) = \frac{e^2+1}{4}$

$u = x \quad , \quad du = dx$

$dv = e^{2x}dx \quad , \quad v = \frac{1}{2}e^{2x}$

A. $\dfrac{e^2+1}{4}$.

B. $\dfrac{3e^2+1}{4}$.

C. $6e^2$.

D. $-2e^2$.

E. $\dfrac{e+1}{4}$.

E

13. Compute $\displaystyle\int_0^{\sqrt{\pi}} x\sin(\pi - x^2)dx$ $= \frac{1}{2}\int_0^{\pi} \sin(u)\,du = 1$

$u = \pi - x^2$

$du = -2x\,dx$

x	u
$\sqrt{\pi}$	0
0	π

A. -1.

B. $-\dfrac{\sin\sqrt{\pi}}{2}$.

C. 2.

D. -2.

E. 1.

D

14. An ideal spring (i.e. it obeys Hooke' s Law) has a natural length of 10 meters. The work done in stretching the spring from 14 meters to 18 meters is 24 J. Determine the spring constant.

A. $k = \frac{1}{2}\frac{N}{m}$.

B. $k = 3\frac{N}{m}$.

C. $k = \frac{3}{8}\frac{N}{m}$.

D. $k = 1\frac{N}{m}$.

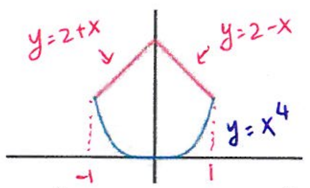
E. $k = \frac{3}{14}\frac{N}{m}$.

$W = 24 = \int_4^8 kx\,dx = k\frac{1}{2}(64-16) = 24k$

$\therefore k = 1$

D

15. Which of the following is the correct set up for the area of the region bounded by $y = x^4$ and $y = 2-|x|$, as shown in the figure below?



A. $\displaystyle\int_{-1}^{0}(2-x-x^4)dx + \int_0^1(2+x-x^4)dx$.

B. $\displaystyle\int_{-2}^{0}(2+x-x^4)dx + \int_0^2(2-x-x^4)dx$.

C. $\displaystyle\int_{-2}^{0}(2-x-x^4)dx + \int_0^2(2+x-x^4)dx$.

D. $\displaystyle\int_{-1}^{0}(2+x-x^4)dx + \int_0^1(2-x-x^4)dx$.

E. None of the above.

第二部份，計算題

16. (10 points) Compute $\int x^5 \sin(x^2)dx$.

$$\int x^5 \sin(x^2)\,dx = \frac{1}{2}\int w^2 \sin(w)\,dw$$

$w = x^2$

$dw = 2x\,dx$

$\frac{1}{2}dw = x\,dx$

$$= \frac{1}{2}\left[-w^2\cos(w) - 2w(-\sin(w)) + 2\cos(w)\right] + C$$

$$= \frac{-1}{2}x^4\cos(x^2) + x^2\sin(x^2) + \cos(x^2) + C$$

u	dv
w^2	$\sin(w)$
$2w$	$-\cos(w)$
2	$-\sin(w)$
0	$\cos(w)$

17. (10 points) Compute $\int e^{2x}\sin(x)dx$. $=A$

$$A = \int e^{2x}\sin(x)\,dx = \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\int e^{2x}\cos x\,dx$$

$u = \sin x$, $du = \cos x\,dx$

$dv = e^{2x}dx$, $v = \frac{1}{2}e^{2x}$

$u = \cos x$, $du = -\sin x\,dx$

$dv = e^{2x}dx$, $v = \frac{1}{2}e^{2x}$

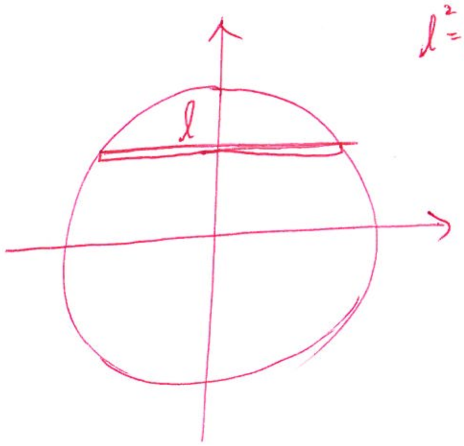
$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\left(\frac{1}{2}e^{2x}\cos x + \frac{1}{2}\int e^{2x}\sin x\,dx\right)$$

$$= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}A$$

$$\therefore \frac{5}{4}A = \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x$$

$$\therefore \int e^{2x}\sin x\,dx = A = \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + C$$

18. (10 points) Find the volume of the solid whose base is the circle $x^2+y^2=1$ and whose cross sections perpendicular to the y axis are equilateral triangles(正三角形).



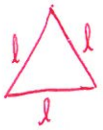
$l^2=(2x)^2=4(1-y^2)$

$$V=\int_{-1}^{1}\frac{\sqrt{3}}{4}l^2\,dy$$

$$=\int_{-1}^{1}\frac{\sqrt{3}}{4}(4(1-y^2))\,dy$$

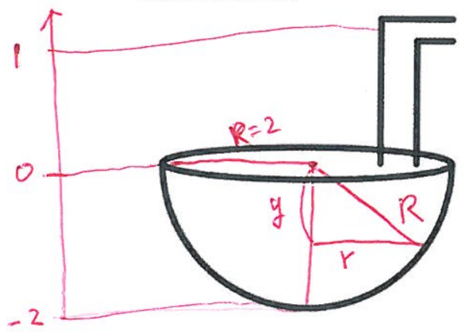
$$=\sqrt{3}\int_{-1}^{1}(1-y^2)\,dy$$

$$=\frac{4}{\sqrt{3}}$$



Area $=\frac{\sqrt{3}}{4}l^2$

19. (10 points) A hemispherical(半球形) tank(水塔) of radius 2 ft is filled with a foot of a liquid with weight density $\rho g \frac{lb}{ft^3}$. There is a one foot spout(噴嘴) mounted(安裝) at the top of the tank through which water is drained(耗盡). Set up the integral that gives the work required to pump the water out of the tank through the spout. Draw the picture and indicate where you are placing the axes(軸) and which direction is positive. DO NOT INTEGRATE.



$\Delta W = \Delta \text{Volume} \times \rho g \times d$ $\quad d = 1-y$

$= \text{Area} \times \rho \times g \times d \times \Delta y$ $\quad \text{Area} = \pi r^2$

$\therefore W = \int_{-2}^{0} \pi(4-y^2)\rho g(1-y)\,dy \quad (lb \cdot ft)$

$R^2 = y^2 + r^2$

$r^2 = R^2 - y^2 = 4 - y^2$

以下由閱卷人員填寫

Question:	1-15	16	17	18	19	Total
Points:	60	10	10	10	10	100
Score:						