

姓名: sol.

葉均承 化學一微積分

學號: _____

Quiz 11

考試日期: 2020/06/15

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (50 points) Find the surface area generated by rotating the curve $\begin{cases} x = e^t - t \\ y = 4e^{t/2} \end{cases}, 0 \leq t \leq 1$ about the y-axis. $r=x$

$$\begin{cases} \frac{dx}{dt} = \frac{d}{dt}(e^t - t) = e^t - 1 \\ \frac{dy}{dt} = \frac{d}{dt}(4e^{t/2}) = 2e^{t/2} \end{cases}$$

$$S = \int_0^1 2\pi r \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt = \int_0^1 2\pi (x(t)) \sqrt{(e^t - 1)^2 + (2e^{t/2})^2} dt$$

$$= \int_0^1 2\pi (e^t - t) \sqrt{e^{2t} - 2e^t + 1 + 4e^t} dt = \int_0^1 2\pi (e^t - t) (e^t + 1) dt$$

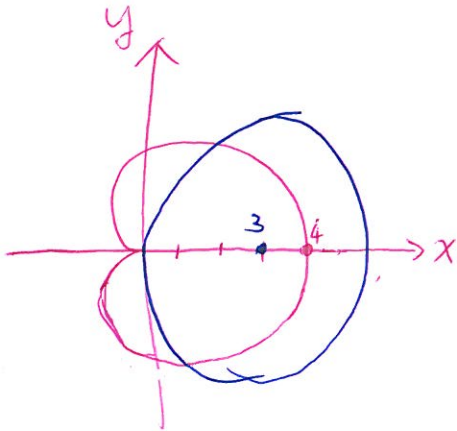
$$= 2\pi \int_0^1 (e^{2t} - te^t + e^t - t) dt = 2\pi \left[\frac{1}{2}e^{2t} - e^t(t-2) - \frac{1}{2}t^2 \right] \Big|_0^1$$

$$= 2\pi \left[\left(\frac{1}{2}e^2 + e - \frac{1}{2} \right) - \left(\frac{1}{2} + 2 \right) \right] = 2\pi \left(\frac{1}{2}e^2 + e - 3 \right)$$

Quiz 11

化學—微積分

2. (50 points) Find the area inside the circle(圓) $r = 6 \cos(\theta)$ and outside the cardioid(愛心) $r = 2 + 2 \cos(\theta)$.



Intersection

$$6 \cos \theta = 2 + 2 \cos \theta$$

$$4 \cos \theta = 2$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \dots$$

$$\therefore A = \int_{-\pi/3}^{\pi/3} \frac{1}{2} \left[(6 \cos \theta)^2 - (2 + 2 \cos \theta)^2 \right] d\theta$$

$$= \int_0^{\pi/3} (6 \cos \theta)^2 - (2 + 2 \cos \theta)^2 d\theta$$

$$= \int_0^{\pi/3} 32 \cos^2 \theta - 8 \cos \theta - 4 d\theta$$

$$= \int_0^{\pi/3} 32 \left(\frac{\cos(2\theta) + 1}{2} \right) - 8 \cos \theta - 4 d\theta$$

$$= \int_0^{\pi/3} 16 \cos(2\theta) - 8 \cos \theta + 12 d\theta$$

$$= \left[8 \sin(2\theta) - 8 \sin \theta + 12\theta \right]_0^{\pi/3}$$

$$= \left[8 \frac{\sqrt{3}}{2} - 8 \frac{\sqrt{3}}{2} + 12 \cdot \frac{\pi}{3} \right] - 0$$

$$= 4\pi$$