姓名: SOLUTION　　　　　　　　　　　　　　　　　　葉均承　　應數三科學計算

學號: ______________　　**Quiz 10**　　考試日期: 2021/05/18

1. 請框出答案. 2. 禁止作弊!

1. The nonlinear system

$$\begin{aligned} x_1^2 - 10x_1 + x_2^2 + 8 &= 0, \\ x_1x_2^2 + x_1 - 10x_2 + 8 &= 0 \end{aligned}$$

can be transformed into the fixed-point problem

$$\begin{aligned} x_1 = g_1(x_1, x_2) &= \frac{x_1^2 + x_2^2 + 8}{10}, \\ x_2 = g_2(x_1, x_2) &= \frac{x_1x_2^2 + x_1 + 8}{10} \end{aligned}$$

Use Theorem 10.6 to show that $\mathbf{G} = (g_1, g_2)^T$ mapping $\mathbf{D} \subset \mathbb{R}^2$ into $\mathbb{R}^2$ has a unique fixed point in

$$D = \{(x_1, x_2)t \mid 0 \le x_1, x_2 \le 1.5\}.$$

Answer:

Continuity properties can be easily shown. Moreover,

$$\frac{8}{10} \le \frac{x_1^2 + x_2^2 + 8}{10} \le 1.25$$

and

$$\frac{8}{10} \le \frac{x_1x_2^2 + x_1 + 8}{10} \le 1.2875$$

so $\mathbf{G}(\mathbf{x}) \in D$, whenever $\mathbf{x} \in D$.

Further,

$$\frac{\partial g_1}{\partial x_1} = \frac{2x_1}{10}, \text{ so } \left|\frac{\partial g_1}{\partial x_1}\right| \le \frac{3}{10}, \ \frac{\partial g_1}{\partial x_2} = \frac{2x_2}{10}, \text{ so } \left|\frac{\partial g_1}{\partial x_2}\right| \le \frac{3}{10},$$

$$\frac{\partial g_2}{\partial x_1} = \frac{x_2^2 + 1}{10}, \text{ so } \left|\frac{\partial g_2}{\partial x_1}\right| \le \frac{3.25}{10}, \ \frac{\partial g_2}{\partial x_2} = \frac{2x_1x_2}{10}, \text{ so } \left|\frac{\partial g_2}{\partial x_2}\right| \le \frac{4.5}{10},$$

Therefore,

$$\left|\frac{\partial g_i}{\partial x_j}\right| \le \frac{4.5}{10} = \frac{0.9}{2}, \ \text{ for } i, j = 1, 2$$

Since all hypothesis of Theorem 10.6 have been satisfied, and G has an unique fixed point in D.