

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Express z/w in the form $a + bi$, where $a, b \in \mathbb{R}$, if

$$z = 1 + 3i, \quad w = 5 - 2i$$

Answer: $z/w = \underline{\frac{-1 + 17i}{29}}$

2. Find the three cube roots of $(\sqrt{3}i - 1)$.

Answer: $\underline{w_k = \sqrt[3]{2} \left(\cos\left(\frac{2\pi}{9} + \frac{2k\pi}{3}\right) + i \sin\left(\frac{2\pi}{9} + \frac{2k\pi}{3}\right) \right), \quad k = 1, 2, 3}.$

3. Given $z = a + bi$ and $w = c + di$ be nonzero complex numbers. Please find all possible a, b, c, d such that $zw \in \mathbb{R}$.

Solution :

計算過程在此省略，只給最終答案。

(a) $a = c = 0, b, d \in \mathbb{R}$.

(b) $b = d = 0, a, c \in \mathbb{R}$.

(c) $a = c \in \mathbb{R}, b = -d \in \mathbb{R}$.