

1. 請框出答案. 2. 禁止作弊!

1. The nonlinear system

$$\begin{aligned}x_1^2 - 10x_1 + x_2^2 + 8 &= 0, \\x_1x_2^2 + x_1 - 10x_2 + 8 &= 0\end{aligned}$$

can be transformed into the fixed-point problem

$$\begin{aligned}x_1 &= g_1(x_1, x_2) = \frac{x_1^2 + x_2^2 + 8}{10}, \\x_2 &= g_2(x_1, x_2) = \frac{x_1x_2^2 + x_1 + 8}{10}\end{aligned}$$

Use Theorem 10.6 to show that $\mathbf{G} = (g_1, g_2)^T$ mapping $\mathbf{D} \subset \mathbb{R}^2$ into \mathbb{R}^2 has a unique fixed point in

$$D = \{(x_1, x_2) \mid 0 \leq x_1, x_2 \leq 1.5\}.$$