應數一線性代數 2023 春, 期末考

學號: ________________, 姓名: ________________

本次考試共有 10 頁 (包含封面)，有 11 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一及最後一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- **不可**翻閱課本或筆記。
- 計算題請寫出計算過程，閱卷人員會視情況給予部份分數。
  沒有計算過程，就算回答正確答案也不會得到滿分。
  答卷請清楚乾淨，儘可能標記或是框出最終答案。

高師大校訓：**誠敬宏遠**

誠，一生動念都是誠實端正的。 **敬**，就是對知識的認真尊重。
宏，開拓視界，恢宏心胸。 **遠**，任重致遠，不畏艱難。

請尊重自己也尊重其他同學，考試時請勿東張西望交頭接耳。

1. (10 points) Express $\frac{z}{w}$ in the form $a + bi$, where $a, b \in \mathbb{R}$, if

$$z = -1 + i, \quad w = 3 + 4i$$

Answer: $\frac{z}{w} =$ ______________________________.

2. (10 points) Find the five fifth roots of $\sin(30°) + i\cos(30°)$. (need not simplify)

3. (10 points) Let A is an 3×3 complex matrix with $\det(A) = 2 + 3i$. Please the value for $\det(iA)$ and $\det(A^*)$.

Answer: $\det(iA) =$ __________, $\det(A^*) =$ __________, $\det(A^2) =$ __________.

4. (10 points) Given the coordinate vector $\vec{v}_B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$. Please find the $\vec{v}$ and $\vec{v}'_B$ when the ordered basis B and B' for P_2 are

$$B = (x^2 - x,\ 2x + 1,\ -x - 5),\ B' = (1,\ (2 + x),\ (2 + x)^2)$$

Answer: $\vec{v} =$ ____________________, $\vec{v}'_B =$ ____________________

5. (10 points) Find the matrix representations $R_{B,B}$, $R_{B',B'}$ and an invertible C such that $R_{B',B'} = C^{-1}R_{B,B}C$ for linear transformation $T : P_2 \to P_2$ defined by $T(p(x)) = \frac{d}{dx}p(x-1)$, $B = (x^2, x, 1)$, $B' = (x^2-1, x-3, 2)$.

   $C_{B,B'}=$ ____________, $C_{B',B}=$ ____________ , $R_{B',B'}=$ ____________ and $R_{B,B}=$ ____________.

   Is C=$C_{B,B'}$ or $C_{B',B}$? ____.

6. (10 points) Find an unitary matrix U and a diagonal matrix D such that $D = U^{-1}AU$. Also find where

$$A = \begin{bmatrix} 2 & 0 & 1-i \\ 0 & -3 & 0 \\ 1+i & 0 & 1 \end{bmatrix}$$

Answer: D = ____________________________, U = ____________________________

7. (10 points) Find a Jordan canonical form and a Jordan basis for the matrix A

$$A = \begin{bmatrix} 2 & 5 & 0 & 0 & -1 \\ 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

Answer: Jordan canonical form = ______________________________,
Jordan basis = ______________________________

8. (10 points) Find a polynomial in A that gives the zero matrix.

$$A = \begin{bmatrix} 4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9i & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9i & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 9i & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9i & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9i & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

Answer: ______________________________.

9. (10 points) Prove that every 2×2 real matrix that is unitarily diagonalizable has one of the following forms: $\begin{bmatrix} a & b \\ b & d \end{bmatrix}$, $\begin{bmatrix} a & b \\ -b & d \end{bmatrix}$, for $a, b, d \in \mathbb{R}$.

10. (30 points) Prove or disprove the following statement:

(a) every unitarily diagonalizable matrix is Hermitian.

(b) If U is unitary, then $(\bar{U})^{-1} = U^T$.

(c) every unitary matrix is normal.

(d) If $A^* = -A$, then A is normal.

(e) $\det(C_{BB'}) = 1$ if and only if $B = B'$.

(f) If $C_{B,B'}$ is an orthogonal matrix and B is an orthonormal basis, then B' is an orthonormal basis.

11. (10 points) Please give a $n \times n$ matrix (不需化簡，但需要理由) such that

(a) is diagonalizable but NOT a normal matrix.

(b) is diagonalizable but NOT unitarily diagonalizable.

(c) is unitarily diagonalizable matrix but NOT Hermitian.

(d) all eigenvalues of algebraic multiplicity 1 but NOT unitarily diagonalizable.

(e) two diagonalizable matrices having the same eigenvectors but NOT similar.

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	10	10	10	10	10	10	10	10	10	30	10	130
Score:												