

不可使用手機、計算器，禁止作弊!

1. Let  $A$  is a  $5 \times 5$  matrix.

$$A = \begin{bmatrix} 73 & 30 & 2 & 3 & 7 \\ -6 & 92 & 4 & -2 & 6 \\ -2 & 4 & 76 & 10 & 2 \\ -1 & 2 & -2 & 85 & 1 \\ 6 & 4 & -4 & 2 & 74 \end{bmatrix}$$

(a) Find a Jordan canonical form and a Jordan basis for the given matrix.

(b) Find the  $\det(A^{80}) =$  \_\_\_\_\_.

Notice that

$$\begin{aligned} & rref(A - 80I) \\ &= \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} & rref((A - 80I)^2) \\ &= \begin{bmatrix} 1 & -2 & \frac{-2}{3} & \frac{1}{3} & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} & rref((A - 80I)^3) \\ &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$