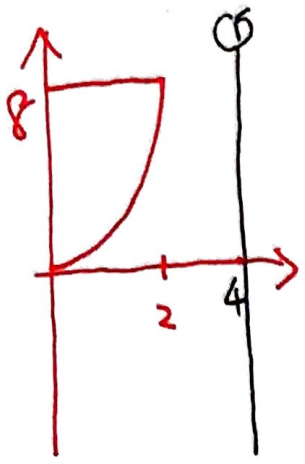
姓名: _____ 葉均承　化學一微積分

學號: sol **Quiz 2** 考試日期: 2020/03/23

不可使用手機、計算器，禁止作弊!

1. (30%) Set up an integral to find the volume of the solid found by rotating the region bounded by $y = x^3$, $y = 8$ and the y-axis about the line $x = 4$. Do not evaluate.



Shell: $\int_0^2 2\pi(8-x^3)(4-x)\,dx$

Washer: $\int_0^8 \pi\left[4^2-(4-\sqrt[3]{y})^2\right]dy$

2. (35%) The *work* required to stretch a spring from its natural length to 3 m beyond its natural length is 18 J. How much *force* is required to hold the spring stretched 5 m beyond its natural length?

$18 = \int_0^3 kx\,dx = \frac{1}{2}kx^2\Big|_0^3 = \frac{9}{2}k \quad \Rightarrow k=4.$

$f(x)=4x \quad \therefore f(5)=4\times5=20\ (N)$

3. (35%) A 20 foot rope that weights 100 pounds is hanging off a cliff with a 10 pound weight attached. Find the work required to pull the rope and weight up 10 feet.

$100/20=5\ (lb/ft) \quad \therefore f(y)=110-5y$

$\int_0^{10} 110-5y\,dy = 110y-\frac{5}{2}y^2\Big|_0^{10} = 1100-250 = 850\ (lb)$