

1. 請框出答案. 2. 禁止作弊!

1. Use the Steepest Descent Method with $\mathbf{x}^{(0)} = [0, 0, 0]$ to find a reasonable $\mathbf{x}^{(1)}$ for the following nonlinear systems.

$$\begin{cases} 15x_1 + x_2^2 - 4x_3 = 13 \\ x_1^2 + 10x_2 - x_3 = 11 \\ x_2^3 - 25x_3 = -22 \end{cases}.$$

Answer:

Rewrite the equation as:

$$\begin{cases} f_1(\mathbf{x}) = 15x + y^2 - 4z - 13 \\ f_2(\mathbf{x}) = x^2 + 10y - z - 11 \\ f_3(\mathbf{x}) = y^3 - 25z + 22 \end{cases}.$$

$$J(\mathbf{x}) = \begin{bmatrix} 15 & 2y & -4 \\ 2x & 10 & -1 \\ 0 & 3y^2 & -25 \end{bmatrix}, \quad F(\mathbf{x}) = \begin{bmatrix} 15x + y^2 - 4z - 13 \\ x^2 + 10y - z - 11 \\ y^3 - 25z + 22 \end{bmatrix}$$

$$\nabla g(\mathbf{x}) = 2J^T(\mathbf{x})F(\mathbf{x}),$$

$$\mathbf{z} = \frac{\nabla g(\mathbf{x}^{(0)})}{|\nabla g(\mathbf{x}^{(0)})|} = [-0.3638 \quad -0.2052 \quad -0.9086]$$

$$g_1 = g(\mathbf{x}^{(0)} - 0 * \mathbf{z}) = 774,$$

$$g_2 = g(\mathbf{x}^{(0)} - \frac{\mathbf{z}}{2}) = 367.2276$$

$$g_3 = g(\mathbf{x}^{(0)} - \mathbf{z}) = 219.04501$$

Use the Newton forward divided diff. interpolating polynomial. First, get the table.

774		
367.2276	-813.5448	
219.0451	-296.3651	517.1797

Second, we have $P(x) = 517.1797x(x - 0.5) - 813.5448x + 774$. Find $\alpha = 1.0365$ so that $P'(\alpha) = 0$. Let $g_0 = P(\alpha) = 218.3531$. Since g_0 is smaller than g_1, g_3 . We set

$$\mathbf{x}^{(1)} = \mathbf{x}^{(0)} - \alpha \mathbf{z} = [0.3771, 0.2127, 0.9418]$$