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Homework 3

應數一線性代數

Section 5-3 課本 proble 3, 9, 13

1. Find det(A) given that A has the characteristic polynomial $p(\lambda)$
   (a) $p(\lambda) = \lambda^3 + 2\lambda - 4\lambda - 5$
   (b) $p(\lambda) = \lambda^5 + 3\lambda^2 - 2\lambda + 12$

2. Compute $A^1 3$.

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

3. Show A and B are similar matrices or not.

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$$

1. Find det(A) given that A has the characteristic polynomial $p(\lambda)$
(a) $p(\lambda) = \lambda^3 + 2\lambda - 4\lambda - 5$

Ans: $\det(A - \lambda I) = p(\lambda) = \lambda^3 + 2\lambda - 4\lambda - 5$, 所以 $\det(A) = \det(A - 0I) = p(0) = -5$

(b) $p(\lambda) = \lambda^5 + 3\lambda^2 - 2\lambda + 12$
Ans: $\det(A) = p(0) = 12$

2. Compute A^13.

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

Ans: A is diagonalized ($A = C^{-1}DC$) by

$$C = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Thus,

$$A^{13} = CD^{13}C^{-1} = \begin{bmatrix} -1 & 0 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2^{13} & 0 & 0 \\ 0 & 2^{13} & 0 \\ 0 & 0 & 1^{13} \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 1 \\ -1 & 0 & -1 \end{bmatrix} = \begin{bmatrix} -8190 & 0 & -16382 \\ 8191 & 8192 & 8191 \\ 8191 & 0 & 16383 \end{bmatrix}$$

3. Show A and B are similar matrices or not.

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 1 & 3 \end{bmatrix}$$

Ans: The characteristic polynomial $p_A(\lambda)$ of A is $\lambda^2 - 6\lambda + 8 = (\lambda - 3 + \sqrt{3})(\lambda - 3 - \sqrt{3})$, and the characteristic polynomial $p_B(\lambda)$ of B is $\lambda^2 - 6\lambda + 5 = (\lambda - 1)(\lambda - 5)$.

Thus A and B are diagonalizable to D_A and D_B.

$$D_A = \begin{bmatrix} 3 + \sqrt{3} & 0 \\ & 3 - \sqrt{3} \end{bmatrix}, D_B = \begin{bmatrix} 1 & 0 \\ 0 & 5 \end{bmatrix}$$

Since D_A and D_B are not similar matrices, so does A and B.