

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. Enlarge the independent set $\{[1, 1, -1], [1, 2, -2]\}$ to be a basis for \mathbb{R}^3 .

Answer: $\{[1, 1, -1], [1, 2, -3], [0, 1, 0]\}$.

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 \\ -1 & -2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

The pivots in column 1, 2, and 4. Thus there's a basis $\{[1, 1, -1], [1, 2, -3], [0, 1, 0]\}$

2. Find a basis for the subspace $W = \text{sp}([-2, 3, 1], [3, -1, 2], [1, 2, 3], [-1, 5, 4])$ in \mathbb{R}^3 .

Answer: $\{[-2, 3, 1], [3, -1, 2]\}$ is a basis for W .

$$\begin{bmatrix} -2 & 3 & 1 & -1 \\ 3 & -1 & 2 & 5 \\ 1 & 2 & 3 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The pivots in column 1 and 2. Thus there's a basis $\{[-2, 3, 1], [3, -1, 2]\}$