姓名:<u>SOLUTION</u> 葉均承 應數一線性代數

學號: \_\_\_\_\_\_ 考試日期: 2023/09/27

## 1. 請框出答案. 2. 不可使用手機、計算器,禁止作弊!

1. Prove that the given relation holds for all real matrices A and B if the expression is defined.

$$(AB)^T = B^T A^T$$

## Solution:

1-3 #32 (上課有證), 或是 109-1 quiz 1 也有證。

2. Determine whether the vector  $\vec{b}$  is in the span of the vectors  $\vec{v_i}$ . If so, write  $\vec{b}$  into the linear combination form.

p.s. Please solve the problem with the corresponding augmented matrix. Also mark the row-echlon form and reduced row-echlon form of the augmented matrix.

Answer: 
$$\vec{b} = \underline{35}$$
  $\vec{v_1} + \underline{-14}$   $\vec{v_2} + \underline{0}$   $\vec{v_3}$ 

$$\vec{b} = \begin{bmatrix} 14\\28\\7 \end{bmatrix}, \vec{v_1} = \begin{bmatrix} 2\\4\\-1 \end{bmatrix}, \vec{v_2} = \begin{bmatrix} 4\\8\\-3 \end{bmatrix}, \vec{v_3} = \begin{bmatrix} -2\\3\\0 \end{bmatrix}$$

## **Solution:**

augmented matrix: 
$$\begin{bmatrix} 2 & 4 & -2 & 14 \\ 4 & 8 & 3 & 28 \\ -1 & -3 & 0 & 7 \end{bmatrix}$$
, reduced row-echlon form: 
$$\begin{bmatrix} 1 & 0 & 0 & 35 \\ 0 & 1 & 0 & -14 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Yes! the vector  $\vec{b}$  is in the span of the vectors  $\vec{v_i}$ .

$$\vec{b} = 35 \cdot \vec{v_1} - 14 \cdot \vec{v_2} + 0 \cdot \vec{v_3}$$

```
octave:1> b=[14;28;7];
octave: 2> v1=[2;4;-1];
octave:3> v2=[4;8;-3];
octave:4> v3=[-2;3;0];
octave:5> A=[v1 v2 v3 b]
A =
                                              14
                                              28
                                               7
octave:6> rref(A)
ans =
           1
                       0
                                   0
                                              35
           0
                       1
                                   0
                                             -14
           0
                                   1
                                               0
```