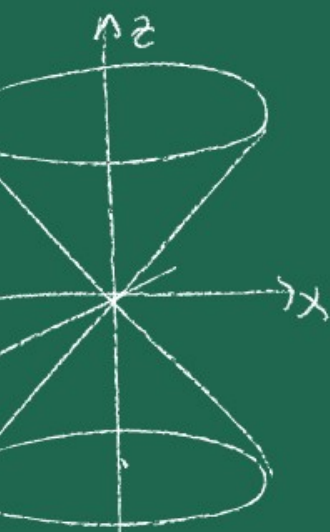


$$\int a^m dx = \frac{a^m}{\ln a} + C$$

# 古爾丁定理

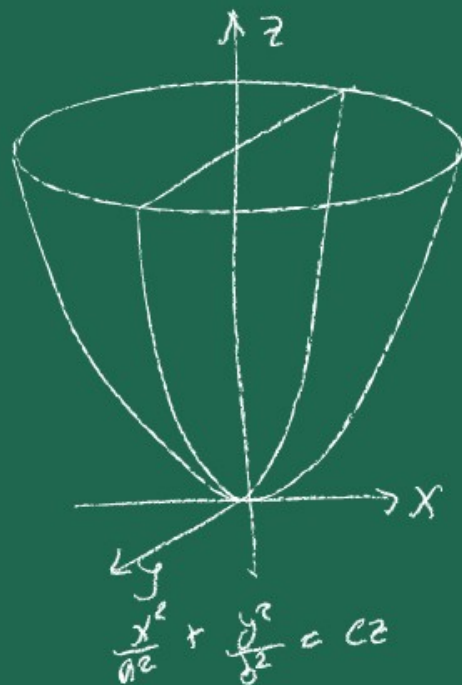
$$y = f(x) \rightarrow f'(x)$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int e^m dx = e^m + C$$

$$y = f(x) \pm g(x) \rightarrow f'(x) \pm g'(x)$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

Math

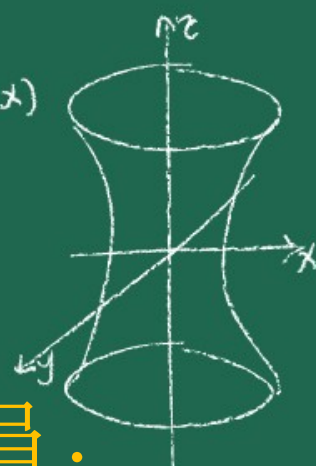
$$\int \frac{dx}{x^2 - a^2}$$

$$y = kf(x) \rightarrow kf'(x)$$

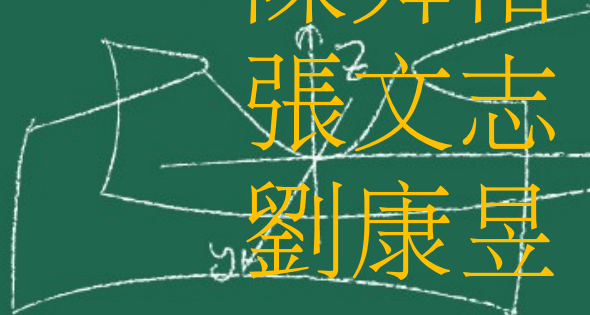
$$\int \frac{dx}{\sqrt{x}} = \ln |x|$$

$$y = f(x)g(x) \rightarrow f'(x)g(x) + f(x)g'(x)$$

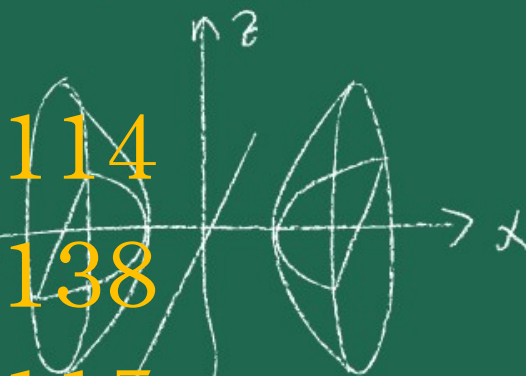
組員:  
黃柏勳 410931114  
陳舜佑 410931138  
張文志 410931115  
劉康昱 410931111



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int a^m dx = \frac{a^m}{\ln a} + C$$

Pappus-Guldinus theorem

(帕普斯—古爾丁定理)

一、定義

Pappus-Guldinus theorem，中文譯作帕普斯—古爾丁定理。以下簡稱為古爾丁定理（為避免和幾何的帕普斯定理混淆）。

古爾丁定理說：

一個平面圖形繞著軸旋轉出的旋轉體體積，恰等於此圖形面積乘以此圖形質心所走路徑長。

Math

$$\int \frac{dx}{x^2 - a^2}$$

$$y = f(x) \pm g(x) = f'(x) \pm g'(x)$$

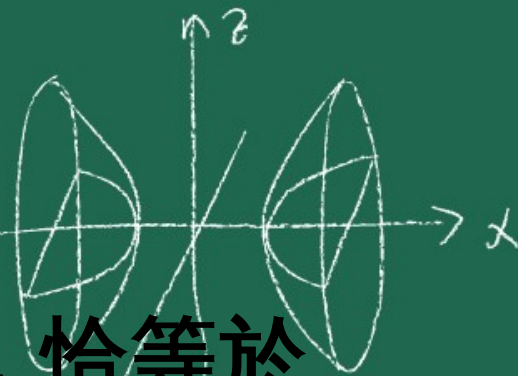
$$y = kf(x) = kf'(x)$$

$$\int \frac{dx}{\sqrt{u}} = \ln |u|$$

$$f'(x)g(x) + f(x)g'(x)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int \sin u \, du = -\cos u + C$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$



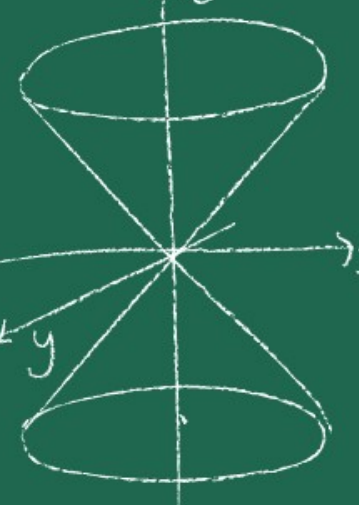
旋轉示意圖：

$$\int e^m \ln a \, du = e^m + c$$

Math

$$\int \frac{du}{u^2 - a^2}$$

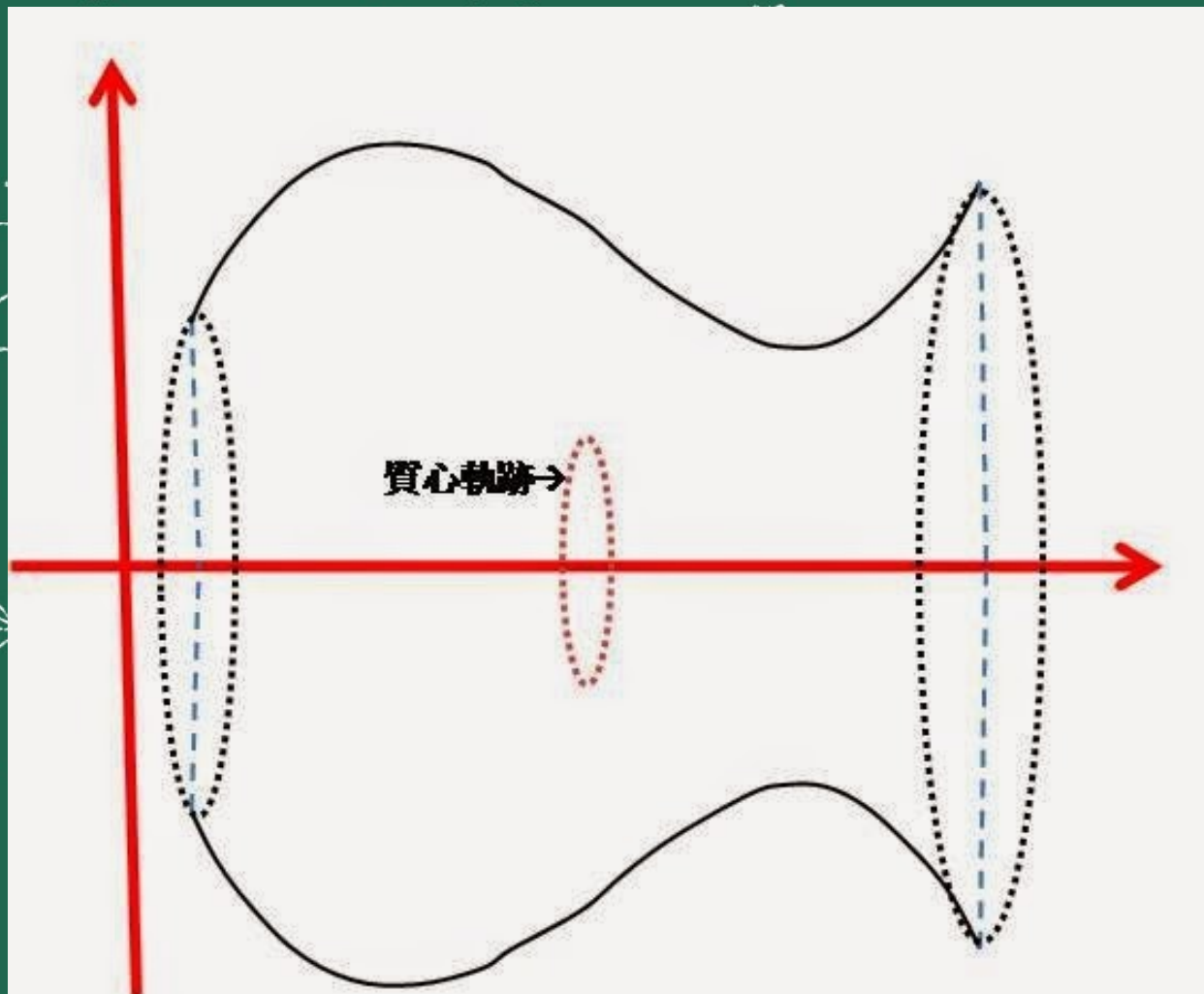
$y = f(x) = f'(x)$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$



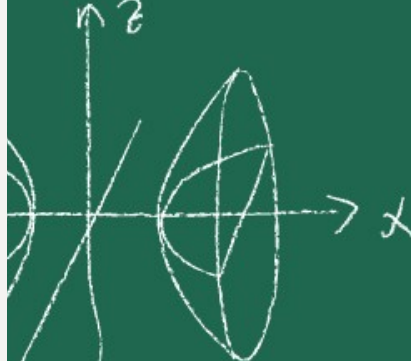
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$



$$f(x) = k f'(x)$$

$$\int \frac{du}{u} = \ln |u|$$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

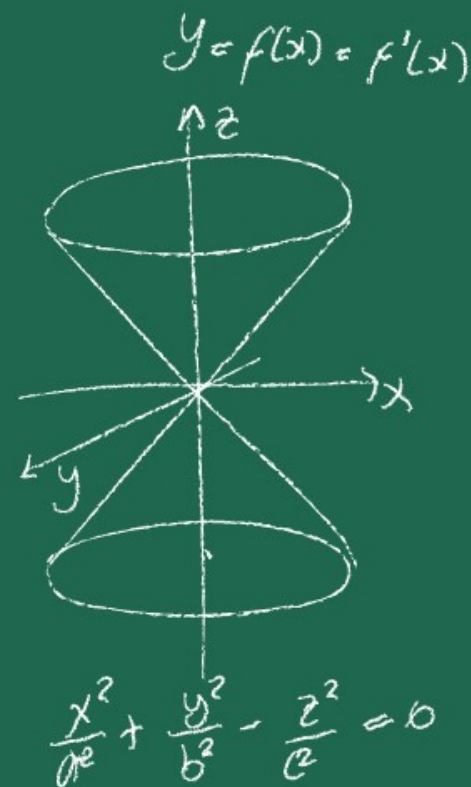
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int u^m du = \frac{u^{m+1}}{m+1} + C \quad \text{二、證明:}$$

$$\int e^m du = e^m + C$$

Math

$$\int \frac{du}{u^2 - a^2}$$



先備定理:

1.  $y = f(x), a \leq x \leq b$ , 這段曲線以及  $x = a, x = b, x$  軸為成一個圖形  
此圖形繞  $x$  軸旋轉的旋轉體體積:

$$V = \int_a^b \pi [f(x)]^2 dx$$

2. 平面圖形重心座標:

$$(x_{CM}, y_{CM}) = \left( \frac{\iint x dA}{\iint dA}, \frac{\iint y dA}{\iint dA} \right)$$

3.  $y = f(x), a \leq x \leq b$ , 與  $x$  軸圍出面積

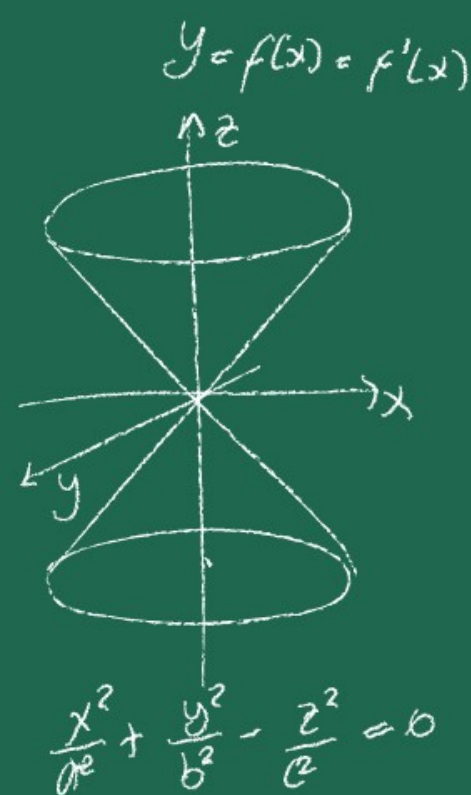
$$A = \int_a^b f(x) dx$$

$$\int u^n du = \frac{u^{n+1}}{n+1}$$

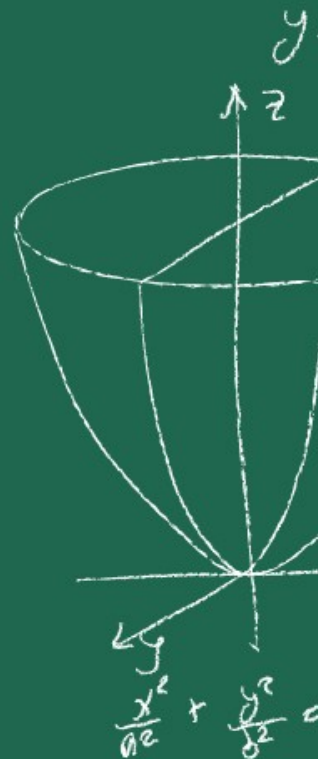
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int \frac{dx}{x^2} = -\frac{1}{x} + C$$

正式開始證明：



$$\int M^n du = \frac{M^{n+1}}{n+1} + c$$



質心路徑長

$$= 2\pi \cdot y_{CM}$$

$$= 2\pi \cdot \frac{\int_a^b \int_0^{f(x)} y dy dx}{\int_a^b f(x) dx}$$

$$= 2\pi \cdot \frac{\int_a^b \frac{1}{2} y^2 \Big|_{y=0}^{f(x)} dx}{\int_a^b f(x) dx}$$

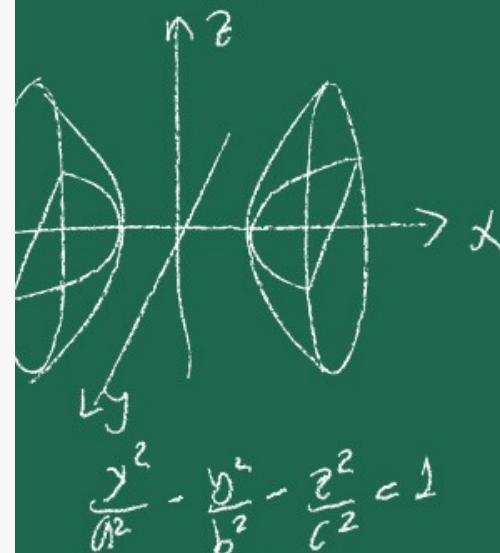
$$= \pi \cdot \frac{\int_a^b [f(x)]^2 dx}{\int_a^b f(x) dx}$$

$$\int \frac{du}{u^2 - 1}$$

$$kf(x) = kf'(x)$$

$$\int \frac{du}{\sqrt{u}} = \ln |u|$$

$$y = f(x)g(x) = f'(x)g(x) + f(x)g'$$



$$\int x^n dx = \frac{x^{n+1}}{n+1} +$$



圖形面積

$$= \int_a^b f(x) dx$$

旋轉體體積

$$= \int_a^b \pi \cdot [f(x)]^2 dx$$

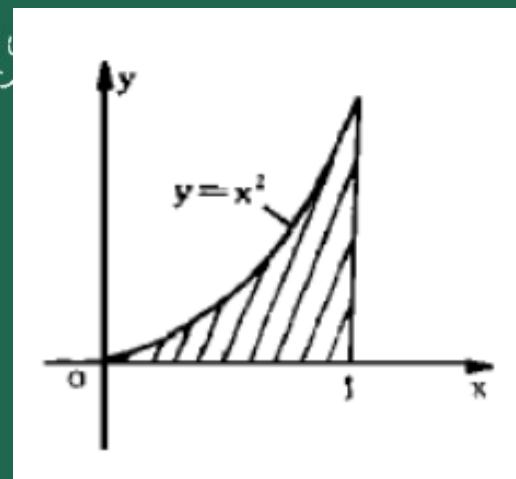
# 三、題目講解

$$\int e^u du = e^u + c$$

Math

$$\int \frac{du}{u^2 - 1}$$

(1)  $y = x^2$  和  $x$  轴、 $x = 1$  所围图形，绕  $y$  轴



ln 1M  
f(x)g

解：

$$V = \int_0^1 2\pi x \cdot x^2 dx = 2\pi \int_0^1 x^3 dx = 2\pi \left. \frac{x^4}{4} \right|_0^1 = \frac{\pi}{2}$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + c$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\int \sec u du = -\cos u + c$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c$$

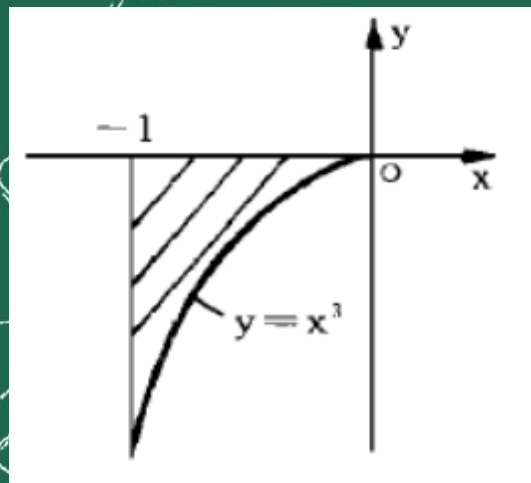
$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int e^u du = e^u + C$$

Math

$$\int \frac{du}{u^2 - a^2}$$

(2)  $y = x^3$  和  $x$  轴,  $x = -1$  所围图形, 绕  $y$  轴



$$f(x) = k f'(x)$$

$$\int \frac{du}{\sqrt{u}} = \ln |u|$$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

解:

$$V = \int_{-1}^0 2\pi |x| |x^3| dx = \int_{-1}^0 2\pi (-x)(-x^3) dx = \int_{-1}^0 2\pi x^4 dx = 2\pi \frac{x^5}{5} \Big|_{-1}^0 = \frac{2}{5}\pi$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

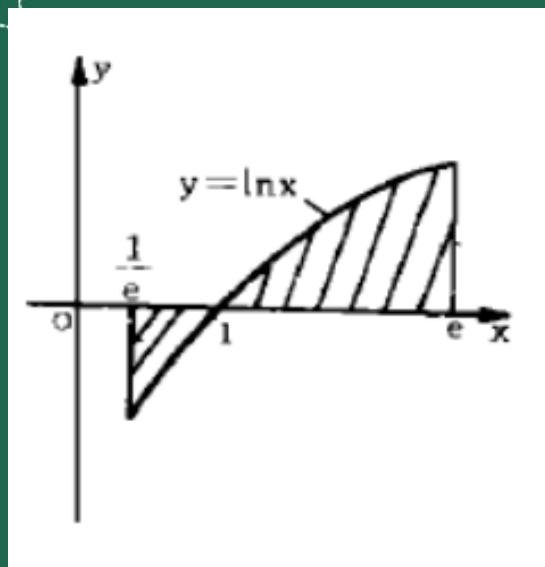
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$



$x_1 = e^{-1}, x_2 = e$  和  $x$  轴所围图形绕  $y$  轴

解:

$$\begin{aligned}
 V &= \int_{e^{-1}}^e 2^c x |\ln x| dx = \int_{e^{-1}}^1 2^c x (-\ln x) dx + \int_1^e 2^c x \ln x dx \\
 &= -2^c \left( \frac{x^2}{2} \ln x - \frac{x^2}{4} \right) \Big|_{e^{-1}}^1 + 2^c \left( \frac{x^2}{2} \ln x - \frac{x^2}{4} \right) \Big|_1^e = c \left( 1 + \frac{e^2}{2} - \frac{3}{2} e^{-1} \right)
 \end{aligned}$$



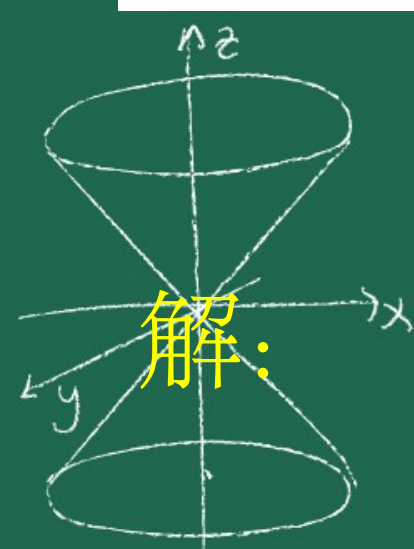
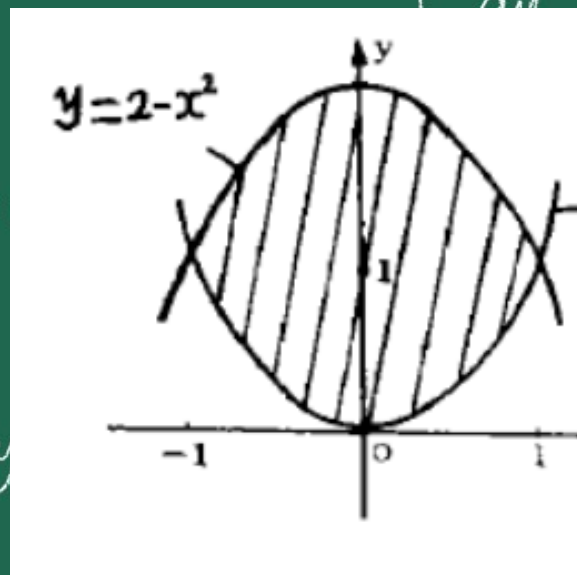
$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int e^u du = e^u + C$$

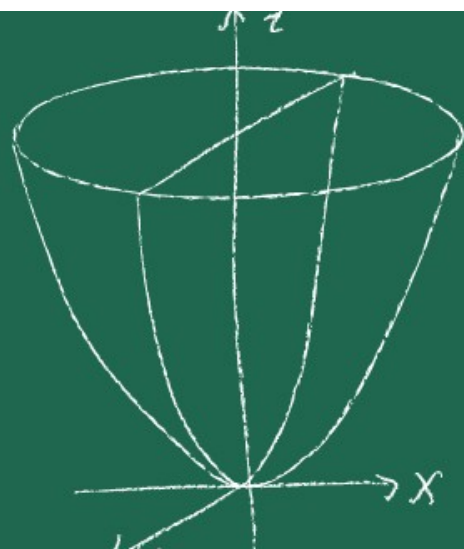
Math

(4)  $y = x^2$  和  $y = 2 - x^2$  所围图形, 绕  $x$  轴.

$y =$



解:



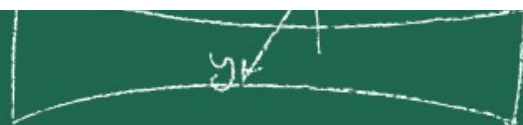
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int \sec u du = -\cos u + C$$

$$\frac{x^2}{a^2}$$

$$V = 2 \int_0^1 2x [(2 - x^2) - x^2] dx = 4 \int_0^1 x (2 - 2x^2) dx = 4 \left( x^2 - \frac{1}{2} x^4 \right) \Big|_0^1$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2}$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

說例 9

漂亮題

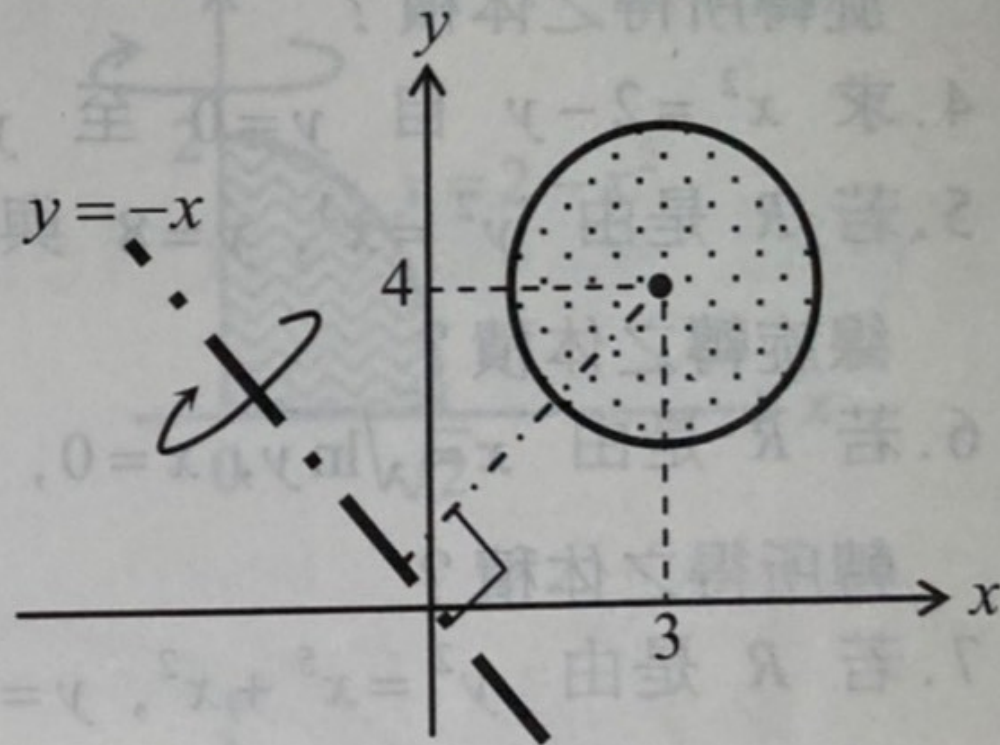
求圓  $(x-3)^2 + (y-4)^2 = 4$  繞直線  $y = -x$  一圈所得之旋轉體  
體積？ (中興轉)

[解] 先求圓心  $(3, 4)$  到  $y = -x$  之距離

$$\text{得 } d = \frac{|3+4|}{\sqrt{1+1}} = \frac{7}{\sqrt{2}}$$

利用 Pappus 定理得

$$V = (\pi \cdot 2^2) \cdot \left( 2\pi \cdot \frac{7}{\sqrt{2}} \right) = 28\sqrt{2}\pi^2。$$





$$\int a^m dx = \frac{a^m}{\ln a} + C$$

$$\int e^m dx = e^m + C$$

Math

$$\int \frac{dx}{x^2 - a^2}$$

參考資料：

$$y = f(x) \pm g(x) = f'(x) \pm g'(x)$$

$$y = kf(x) = kf'(x)$$

$$\int \frac{dx}{x} = \ln |x|$$

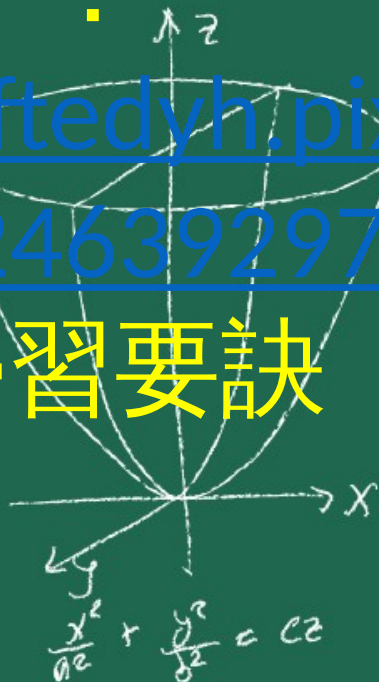
<https://giftedyh.pixnet.net/blog/post/375633428>

<http://m24639297.blogspot.tw/>

微積分學習要訣



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

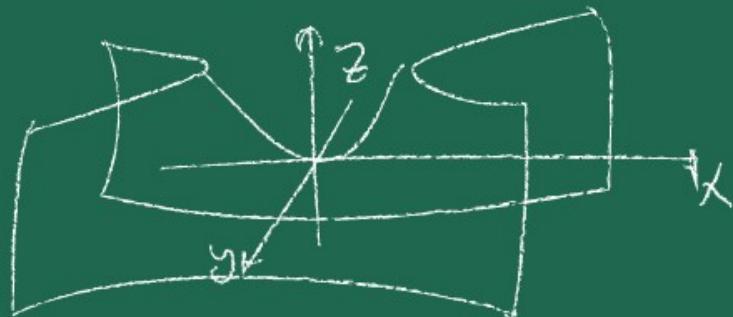


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$$

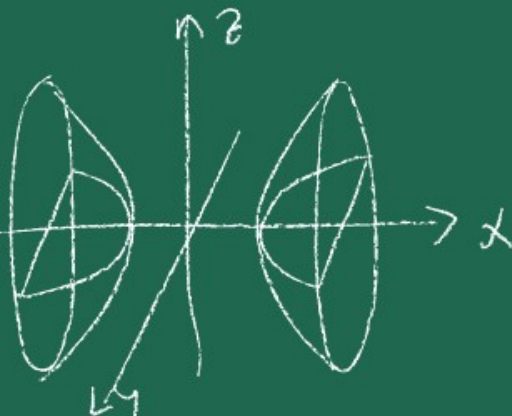


$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1$$

$$\int \sin u \, du = -\cos u + C$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz$$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$