11.2 這觀念我忘了講，這裡補上，所以這次小考不考這個，但是期考還是會考。(我下次上課會講)

EXAMPLE 5 Find the sum of the series $\sum_{n=0}^{\infty} x^n$, where $|x| < 1$.

SOLUTION Notice that this series starts with $n = 0$ and so the first term is $x^0 = 1$. (With series, we adopt the convention that $x^0 = 1$ even when $x = 0$.) Thus

$$\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + x^3 + x^4 + \cdots$$

This is a geometric series with $a = 1$ and $r = x$. Since $|r| = |x| < 1$, it converges and (4) gives

5 $$\sum_{n=0}^{\infty} x^n = \frac{1}{1 - x}$$ □

這題利用到的觀念是下面這個

4 The geometric series

$$\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \cdots$$

is convergent if $|r| < 1$ and its sum is

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1 - r} \qquad |r| < 1$$

If $|r| \geq 1$, the geometric series is divergent.

11.2 題目

11–20 Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

11. $3 + 2 + \frac{4}{3} + \frac{8}{9} + \cdots$

13. $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

17. $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n}$

21–34 Determine whether the series is convergent or divergent. If it is convergent, find its sum.

21. $\sum_{n=1}^{\infty} \frac{1}{2n}$

25. $\sum_{n=1}^{\infty} \frac{1 + 2^n}{3^n}$

29. $\sum_{n=1}^{\infty} \ln\left(\frac{n^2 + 1}{2n^2 + 1}\right)$

33. $\sum_{n=1}^{\infty} \left(\frac{1}{e^n} + \frac{1}{n(n+1)}\right)$

35–40 Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum (as in Example 6). If it is convergent, find its sum.

35. $\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$

39. $\sum_{n=1}^{\infty} \left(e^{1/n} - e^{1/(n+1)}\right)$

47–51 Find the values of x for which the series converges. Find the sum of the series for those values of x.

47. $\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

49. $\sum_{n=0}^{\infty} 4^n x^n$

51. $\sum_{n=0}^{\infty} \frac{\cos^n x}{2^n}$

11.2 答案

EXERCISES 11.2 ■ PAGE 694

11. 9 **13.** D **17.** $\frac{1}{7}$ **21.** D **25.** $\frac{5}{2}$ **29.** D

33. $e/(e-1)$ **35.** $\frac{3}{2}$ **39.** $e - 1$

47. $-3 < x < 3; \frac{x}{3 - x}$ **49.** $-\frac{1}{4} < x < \frac{1}{4}; \frac{1}{1 - 4x}$

51. All x; $\frac{2}{2 - \cos x}$

11.3 题目

3–8 Use the Integral Test to determine whether the series is convergent or divergent.

3. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n}}$ **5.** $\sum_{n=1}^{\infty} \frac{1}{(2n+1)^3}$

9–26 Determine whether the series is convergent or divergent.

9. $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$ **11.** $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \cdots$

13. $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{9} + \cdots$ **19.** $\sum_{n=1}^{\infty} \frac{\ln n}{n^3}$

23. $\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$ **25.** $\sum_{n=1}^{\infty} \frac{1}{n^3 + n}$

27–30 Find the values of p for which the series is convergent.

27. $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$ **29.** $\sum_{n=1}^{\infty} n(1+n^2)^p$

33. (a) Use the sum of the first 10 terms to estimate the sum of the series $\Sigma_{n=1}^{\infty} 1/n^2$. How good is this estimate?

(b) Improve this estimate using (3) with $n = 10$.

(c) Find a value of n that will ensure that the error in the approximation $s \approx s_n$ is less than 0.001.

35. Estimate $\Sigma_{n=1}^{\infty} (2n+1)^{-6}$ correct to five decimal places.

11.3 答案

EXERCISES 11.3 ■ PAGE 703

3. D **5.** C **9.** D **11.** C **13.** D **19.** C

23. C **25.** C **27.** $p > 1$ **29.** $p < -1$

33. (a) 1.54977, error ≤ 0.1 (b) 1.64522, error ≤ 0.005

(c) $n > 1000$

35. 0.00145