練習(7-1, 7-2)

Sec 7.1

3–32 Evaluate the integral.

7. $\int x^2 \sin \pi x \, dx$ **9.** $\int \ln(2x + 1) \, dx$ **11.** $\int \arctan 4t \, dt$ **17.** $\int e^{2\theta} \sin 3\theta \, d\theta$

19. $\int_0^{\pi} t \sin 3t \, dt$ **23.** $\int_1^2 \frac{\ln x}{x^2} \, dx$

33–38 First make a substitution and then use integration by parts to evaluate the integral.

33. $\int \cos \sqrt{x} \, dx$ **35.** $\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3 \cos(\theta^2) \, d\theta$ **37.** $\int x \ln(1 + x) \, dx$

45. (a) Use the reduction formula in Example 6 to show that

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n - 1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx$$

where $n \geqslant 2$ is an integer.

(b) Use part (a) to evaluate $\int_0^{\pi/2} \sin^3 x \, dx$ and $\int_0^{\pi/2} \sin^5 x \, dx$.

(c) Use part (a) to show that, for odd powers of sine,

$$\int_0^{\pi/2} \sin^{2n+1} x \, dx = \frac{2 \cdot 4 \cdot 6 \cdot \cdots \cdot 2n}{3 \cdot 5 \cdot 7 \cdot \cdots \cdot (2n + 1)}$$

57–60 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

59. $y = e^{-x}$, $y = 0$, $x = -1$, $x = 0$; about $x = 1$

7.1 Answers

7. $-\frac{1}{\pi}x^2 \cos \pi x + \frac{2}{\pi^2} x \sin \pi x + \frac{2}{\pi^3} \cos \pi x + C$

9. $\frac{1}{2}(2x + 1) \ln(2x + 1) - x + C$ **11.** $t \arctan 4t - \frac{1}{8} \ln(1 + 16t^2) + C$

17. $\frac{1}{13} e^{2\theta}(2 \sin 3\theta - 3 \cos 3\theta) + C$ **19.** $\pi/3$ **23.** $\frac{1}{2} - \frac{1}{2} \ln 2$

33. $2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C$ **35.** $-\frac{1}{2} - \pi/4$

37. $\frac{1}{2}(x^2 - 1) \ln(1 + x) - \frac{1}{4}x^2 + \frac{1}{2}x + \frac{3}{4} + C$

45. (b) $\frac{2}{3}, \frac{8}{15}$ **59.** $2\pi e$

Sec 7.2

1–49 Evaluate the integral.

3. $\int_{\pi/2}^{3\pi/4} \sin^5 x \cos^3 x \, dx$

7. $\int_0^{\pi/2} \cos^2\theta \, d\theta$

11. $\int (1 + \cos\theta)^2 \, d\theta$

17. $\int \cos^2 x \tan^3 x \, dx$

25. $\int \sec^6 t \, dt$

33. $\int \dfrac{\tan^3\theta}{\cos^4\theta} \, d\theta$

35. $\int x \sec x \tan x \, dx$

45. $\int \sin 5\theta \sin\theta \, d\theta$

61–64 Find the volume obtained by rotating the region bounded by the given curves about the specified axis.

61. $y = \sin x,\ y = 0,\ \pi/2 \leq x \leq \pi$; about the x-axis

63. $y = \sin x,\ y = \cos x,\ 0 \leq x \leq \pi/4$; about $y = 1$

7.2 Answers

3. $-\frac{11}{384}$ **7.** $\pi/4$ **11.** $\frac{3}{2}\theta + 2\sin\theta + \frac{1}{4}\sin 2\theta + C$

17. $\frac{1}{2}\cos^2 x - \ln|\cos x| + C$

25. $\frac{1}{5}\tan^5 t + \frac{2}{3}\tan^3 t + \tan t + C$

33. $\frac{1}{6}\tan^6\theta + \frac{1}{4}\tan^4\theta + C$

35. $x\sec x - \ln|\sec x + \tan x| + C$

45. $\frac{1}{8}\sin 4\theta - \frac{1}{12}\sin 6\theta + C$

61. $\pi^2/4$ **63.** $\pi\left(2\sqrt{2} - \frac{5}{2}\right)$