

## 練習(6.3 – 6.4) :

### Sec: 6-3

**3–7** Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the  $y$ -axis. Sketch the region and a typical shell.

**7.**  $y = 4(x - 2)^2$ ,  $y = x^2 - 4x + 7$

**9–14** Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the  $x$ -axis. Sketch the region and a typical shell.

**9.**  $x = 1 + y^2$ ,  $x = 0$ ,  $y = 1$ ,  $y = 2$

**13.**  $x = 1 + (y - 2)^2$ ,  $x = 2$

**15–20** Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.

**19.**  $y = x^3$ ,  $y = 0$ ,  $x = 1$ ; about  $y = 1$

**21–26** Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis.

**23.**  $y = x^4$ ,  $y = \sin(\pi x/2)$ ; about  $x = -1$

**29–32** Each integral represents the volume of a solid. Describe the solid.

**31.**  $\int_0^1 2\pi(3 - y)(1 - y^2) dy$

**37–42** The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

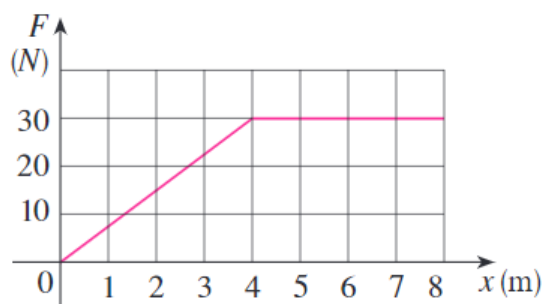
**37.**  $y = -x^2 + 6x - 8$ ,  $y = 0$ ; about the  $y$ -axis

**39.**  $y = 5$ ,  $y = x + (4/x)$ ; about  $x = -1$

**41.**  $x^2 + (y - 1)^2 = 1$ ; about the  $y$ -axis

## Sec: 6.4

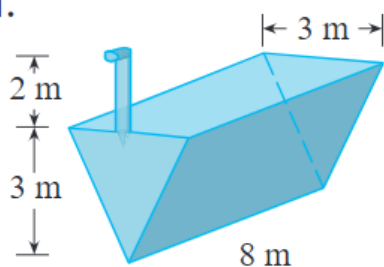
5. Shown is the graph of a force function (in newtons) that increases to its maximum value and then remains constant. How much work is done by the force in moving an object a distance of 8 m?



9. Suppose that 2 J of work is needed to stretch a spring from its natural length of 30 cm to a length of 42 cm.
- How much work is needed to stretch the spring from 35 cm to 40 cm?
  - How far beyond its natural length will a force of 30 N keep the spring stretched?
11. A spring has natural length 20 cm. Compare the work  $W_1$  done in stretching the spring from 20 cm to 30 cm with the work  $W_2$  done in stretching it from 30 cm to 40 cm. How are  $W_2$  and  $W_1$  related?
13. A heavy rope, 50 ft long, weighs 0.5 lb/ft and hangs over the edge of a building 120 ft high.
- How much work is done in pulling the rope to the top of the building?
  - How much work is done in pulling half the rope to the top of the building?

**21–24** A tank is full of water. Find the work required to pump the water out of the spout.

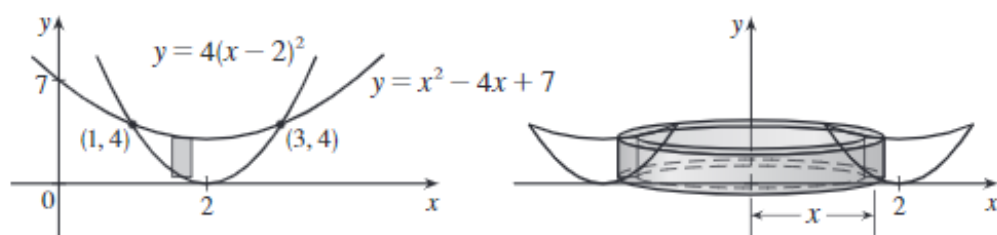
21.



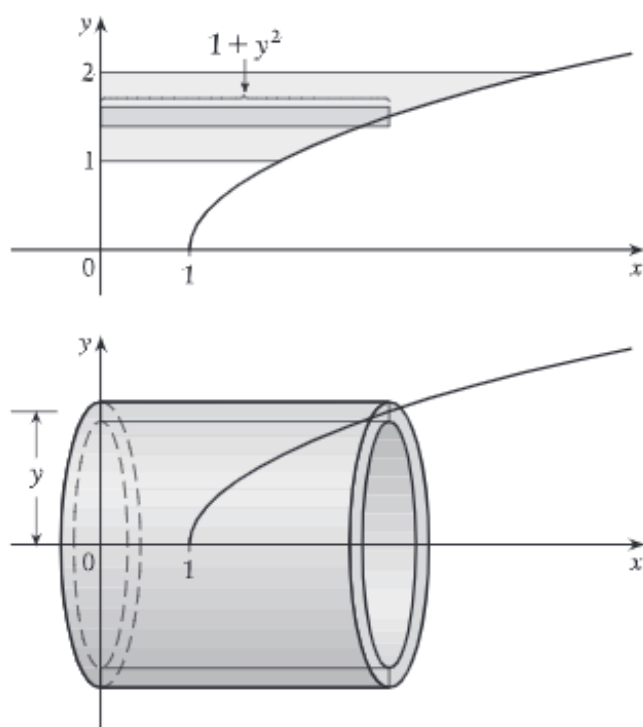
25. Suppose that for the tank in Exercise 21 the pump breaks down after  $4.7 \times 10^5$  J of work has been done. What is the depth of the water remaining in the tank?

## 6.3 ANSWERS:

7.  $16\pi$



9.  $21\pi/2$



13.  $16\pi/3$

19.  $5\pi/14$

23.  $\int_0^1 2\pi(x+1)[\sin(\pi x/2) - x^4] dx$

31. Solid obtained by rotating the region bounded by

(i)  $x = 1 - y^2$ ,  $x = 0$ , and  $y = 0$ , or (ii)  $x = y^2$ ,  $x = 1$ , and  $y = 0$  about the line  $y = 3$

37.  $8\pi$

39.  $2\pi(12 - 4 \ln 4)$

41.  $\frac{4}{3}\pi$

#### 6.4 Answers:

5. 180 J

9. (a)  $\frac{25}{24} \approx 1.04$  J      (b) 10.8 cm

11.  $W_2 = 3W_1$

13. (a) 625 ft-lb      (b)  $\frac{1875}{4}$  ft-lb

21.  $\approx 1.06 \times 10^6$  J

25. 2.0 m