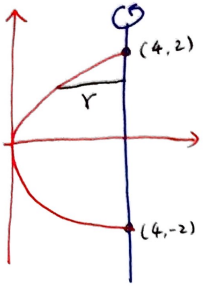
姓名: Sol.　　葉均承　化學一微積分

學號: _____　　**Quiz 1**　　考試日期: 2020/03/16

不可使用手機、計算器，禁止作弊!

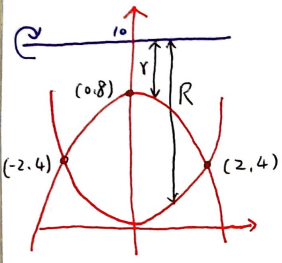
1. (30%) Set up, but do not evaluate, an integral to find the volume of the solid found by rotating the region bounded by $x = y^2$ and $x = 4$ about the line $x = 4$.



$r = 4 - y^2$　　by symmetry.

$\int_{-2}^{2} \pi (4-y^2)^2 dy$　　or　$2\int_{0}^{2} \pi (4-y^2)^2 dy$

2. (40%) Set up, but do not evaluate, an integral to find the volume of the solid found by rotating the region bounded by $y = x^2$ and $y = 8 - x^2$ about the line $y = 10$.



$R = 10 - x^2$, $r = 10 - (8 - x^2) = 2 + x^2$

$x^2 = 8 - x^2 \Rightarrow x = \pm 2$

Ans: $\int_{-2}^{2} \pi \left[(10-x^2)^2 - (2+x^2)^2 \right] dx$

or　$2\int_{0}^{2} \pi \left[(10-x^2)^2 - (2+x^2)^2 \right] dx$

3. (30%) Set up, but do not evaluate, an integral to find the volume of the solid whose base is the region bounded by the parabola $y = 16 - x^2$ and the x-axis, and whose cross-sections perpendicular to the x-axis are semicircles.

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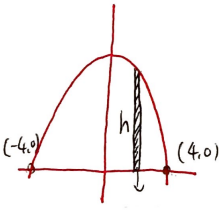

$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi \left(\frac{1}{2}(16-x^2) \right)^2 = \frac{1}{8}\pi (16-x^2)^2$

$r = \frac{1}{2}h = \frac{1}{2}(16-x^2)$

Ans: $\int_{-4}^{4} \frac{1}{8}\pi (16-x^2)^2 dx$

or　$2\int_{0}^{4} \frac{1}{8}\pi (16-x^2)^2 dx$