化學一微積分 **2020** 春, 第二次期中考

學號: Sol, 姓名:

本次考試共有 9 頁 (包含封面)，有 20 題。如有缺頁或漏題，請立刻告知監考人員。

考試須知:

- 請在第一頁填上姓名學號，並在每一頁的最上方屬名，避免釘書針斷裂後考卷遺失。
- 不可翻閱課本或筆記。
- 第一部份是選擇題，請在題號前面寫上正確的選項。每題 4 分。
- 第二部份是計算題，請寫出計算過程，閱卷人員會視情況給予部份分數。沒有計算過程，就算回答正確答案也不會得到滿分。答卷請清楚乾淨，儘可能標記或是框出最終答案。。

高師大校訓：誠敬弘遠

誠，一生動念都是誠實端正的。敬，就是對知識的認真尊重。宏，開拓視界，恢宏心胸。遠，任重致遠，不畏艱難。

請簽名保證以下答題都是由你自己作答的，並沒有得到任何的外部幫助。

簽名:

第一部份，選擇題

D

1. If we use the appropriate trigonometric substitution to evaluate $\int_0^{\sqrt{2}/3} \sqrt{4-9x^2}dx$, which of the following is the correct result?

   A. $\frac{2}{3}\int_0^{\pi/2} \cos^2\theta d\theta$.

   B. $\frac{4}{3}\int_0^{\pi/2} \cos^2\theta d\theta$.

   C. $\frac{2}{3}\int_0^{\pi/4} \cos^2\theta d\theta$.

   D. $\frac{4}{3}\int_0^{\pi/4} \cos^2\theta d\theta$.

   E. $2\int_0^{\pi/4} \cos\theta d\theta$.

$\int_0^{\frac{\sqrt{2}}{3}} \sqrt{4-9x^2}\,dx = \int_0^{\frac{\pi}{4}} \sqrt{4-4\sin^2\theta}\,\frac{2}{3}\cos\theta\, d\theta = \frac{4}{3}\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$

$3x = 2\sin\theta$

$x = \frac{2}{3}\sin\theta$

$dx = \frac{2}{3}\cos\theta\, d\theta$

x	θ
0	0
$\frac{\sqrt{2}}{3}$	$\frac{\pi}{4}$

A

2. Which of the following is the correct partial fraction decomposition of $\frac{x^2+2x-2}{x^3-x^2-2x}$. $= \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+1}$

   A. $\frac{1}{x} - \frac{1}{x+1} + \frac{1}{x-2}$.

   B. $\frac{1}{x} + \frac{1}{3}(\frac{1}{x-1}) - \frac{1}{3}(\frac{1}{x+2})$.

   C. $\frac{1}{x} + \frac{1}{x+1} - \frac{1}{x-2}$.

   D. $\frac{1}{x} - \frac{1}{3}(\frac{1}{x-1}) + \frac{1}{3}(\frac{1}{x+2})$.

   E. $-\frac{1}{x} + \frac{1}{x+1} - \frac{1}{x-2}$.

$x^2+2x-2 = A(x-2)(x+1) + Bx(x+1) + C\,x(x-2)$

$\boxed{x=0} \Rightarrow A=1$

$\boxed{x=-1} \Rightarrow C=-1$

$\boxed{x=2} \Rightarrow B=1$

B

3. The improper integral $\int_{-3}^{0} \frac{1}{3+4x}dx$. $= \int_{-3}^{-\frac{3}{4}} + \int_{-\frac{3}{4}}^{0}$

   A. Converges to $\frac{1}{4}\ln(\frac{1}{3})$.

   B. Diverges.

   C. Converges to $\ln(\frac{1}{3})$.

   D. Converges to $\ln(27)$.

   E. Converges to $\frac{1}{4}(\frac{\ln(3)}{\ln(9)})$.

$\lim_{t\to -\frac{3}{4}^+} \int_t^0 \frac{1}{3+4x}dx = \lim_{t\to -\frac{3}{4}^+} \frac{1}{4}\left(\ln(3) - \ln(3+4t)\right) = \infty \;\therefore$ div.

B

4. Which of the following is true for the three sequences shown below?

   (I) $a_n = \frac{(-1)^n}{e^n}$ (II) $a_n = \frac{5\cos n}{n}$ (III) $a_n = \frac{\ln n}{n}$

   A. (I) diverges and both (II) and (III) converge.

   B. All three converge.

   C. (I) and (III) both converge and (II) diverges.

   D. (I) and (II) both converge and (III) diverges.

   E. Only (III) converges.

(I) $\lim_{n\to\infty} \frac{(-1)^n}{e^n} = 0 \quad \therefore$ conv.

(II) $\lim_{n\to\infty} \frac{5\cos(n)}{n} = 0 \quad \therefore$ conv.

(III) $\lim_{n\to\infty} \frac{\ln n}{n} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{n\to\infty} \frac{\frac{1}{n}}{1} = 0$ conv.

A 5. The sequence $a_n = \ln(2n) - \ln(3n+1)$ $= \ln\left(\frac{2n}{3n+1}\right) \to \ln\left(\frac{2}{3}\right)$

A. Converges to $\ln(\frac{2}{3})$.

B. Converges to 0.

C. Converges to $\frac{\ln(2)}{\ln(3)}$.

D. Converges to 1.

E. Diverges.

C 6. Consider the recursive sequence $a_1 = 3/2, a_{n+1} = \frac{(a_n)^2+2}{2a_n}$. You may assume the sequence is decreasing and bounded. Find the $\lim_{n\to\infty} a_n$. $= L$

A. $\pm\sqrt{2}$.

B. 1.

C. $\sqrt{2}$.

D. $-\sqrt{2}$.

E. None of the above.

$\lim_{n\to\infty} a_{n+1} = L = \lim_{n\to\infty} \frac{(a_n)^2+2}{2a_n} = \frac{L^2+2}{2L}$ $\therefore 2L^2 = L^2+2$

$L = \pm\sqrt{2} \Rightarrow L = \sqrt{2}$

C 7. $\int_1^2 \frac{x^2+1}{x^2+x}dx =$ $\int_1^2 \left(1 + \frac{1-x}{x^2+x}\right)dx = \int_1^2 1 + \frac{1}{x} - \frac{2}{1+x}\,dx = \left[x + \ln(x) - 2\ln(x+1)\right]\Big|_1^2 = 1 + 3\ln 2 - 2\ln 3$

A. $2 + 3\ln(2) - \ln(3)$.

B. $1 - 3\ln(2) + 2\ln(3)$.

C. $1 + 3\ln(2) - 2\ln(3)$.

D. $2 - 3\ln(2) + 2\ln(3)$.

E. None of the above.

$\frac{1-x}{x^2+x} = \frac{A}{x} + \frac{B}{1+x}$ $\quad \boxed{x=0} \Rightarrow A = 1$

$\boxed{x=-1} \Rightarrow B = -2$

$1 - x = A(1+x) + Bx$

D 8. Let $s = \sum_{n=1}^{\infty} \frac{1}{n^4}$. Using The Remainder Estimate for the Integral Test, find the smallest value of n that will ensure that $R_n = s - s_n \leq 1/81$

A. n=2 .

B. n=4.

C. n=5.

D. n=3.

E. n=6.

$R_n \le \int_n^\infty \frac{1}{x^4}dx = \lim_{t\to\infty} \int_n^t x^{-4}dx = \lim_{t\to\infty} \frac{-1}{3x^3}\Big|_n^t = \frac{1}{3n^3}$

$\frac{1}{3n^3} \le \frac{1}{81} \Rightarrow 81 \le 3n^3 \Rightarrow n = 3$

B 9. When applying The Comparison Test for Improper Integrals, which of the following statements is true regarding $\int_1^\infty \frac{1}{x+e^{5x}}dx$

A. $\int_1^\infty \frac{1}{x+e^{5x}}dx$ converges since $\frac{1}{x+e^{5x}} < \frac{1}{x}$ and $\int_1^\infty \frac{1}{x}dx$ converges.

B. $\int_1^\infty \frac{1}{x+e^{5x}}dx$ converges since $\frac{1}{x+e^{5x}} < \frac{1}{e^{5x}}$ and $\int_1^\infty \frac{1}{e^{5x}}dx$ converges.

C. $\int_1^\infty \frac{1}{x+e^{5x}}dx$ diverges since $\frac{1}{x+e^{5x}} > \frac{1}{e^{5x}}$ and $\int_1^\infty \frac{1}{e^{5x}}dx$ diverges.

D. $\int_1^\infty \frac{1}{x+e^{5x}}dx$ diverges since $\frac{1}{x+e^{5x}} > \frac{1}{x}$ and $\int_1^\infty \frac{1}{x}dx$ diverges.

E. $\int_1^\infty \frac{1}{x+e^{5x}}dx$ converges to 0

A

10. Which of the following series diverges by the Test for Divergence?

(I) $\sum_{n=1}^{\infty} \frac{3}{4+e^{-2n}}$ (II) $\sum_{n=1}^{\infty} \frac{1}{\arctan n}$ (III) $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$

A. (I) and (II) only.
B. (II) only.
C. (I) only.
D. (I) and (III) only.
E. (II) and (III) only.

(I) $\lim_{n\to\infty} \frac{3}{4+e^{-2n}} = \frac{3}{4} \neq 0$ div

(II) $\lim_{n\to\infty} \frac{1}{\arctan(n)} = \frac{2}{\pi} \neq 0$ div

(III) $\lim_{n\to\infty} \sin(\frac{1}{n}) = 0$ test fails

B

11. Which of the following three tests will establish that the series $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3+25}}$ diverges?

(I) The limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(II) The comparison test with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

(III) The test for divergence.

A. (I) and (II) only.
B. (I) only.
C. (II) only.
D. (I) and (III) only.
E. (II) and (III) only.

$\boxed{\text{LCT}}$ $\lim_{n\to\infty} \left(\frac{\frac{n}{\sqrt{n^3+25}}}{\frac{1}{\sqrt{n}}} \right) = \lim_{n\to\infty} \frac{n}{\sqrt{n^3+25}} = 1 > 0$

$\therefore$ both div by LCT

$\boxed{\text{CT}}$ $\frac{n}{\sqrt{n^3+25}} < \frac{1}{\sqrt{n}}$, but $\sum \frac{1}{\sqrt{n}}$: div. CT fails

$\boxed{\text{TD}}$ $\lim_{n\to\infty} \frac{n}{\sqrt{n^3+25}} = 0$ TD fails

D

12. Let $s = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$. Use The Alternating Series Estimation Theorem to estimate $|R_{10}|$, the absolute value of the error, in approximating the sum of the series with the 10^{th} partial sum.

A. $|R_{10}| < 1/100$.
B. $|R_{10}| < 1/81$.
C. $|R_{10}| < 1/144$.
D. $|R_{10}| < 1/121$.
E. $|R_{10}| < 1/169$.

$|R_n| < a_{n+1}$ $\therefore$ $|R_{10}| < a_{11} = \frac{1}{11^2} = \frac{1}{121}$

B

13. Which of the following statements is true for the series $\sum_{n=1}^{\infty} \frac{3+\sin n}{n^5+1}$?

A. The series converges by the comparison test with $\sum_{n=1}^{\infty} \frac{3}{n^5}$.

B. The series converges by the comparison test with $\sum_{n=1}^{\infty} \frac{4}{n^5}$.

C. The series converges by the comparison test with $\sum_{n=1}^{\infty} \frac{2}{n^5}$.

D. The series converges by the comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^5}$.

E. None of the above.

$\frac{3+\sin n}{n^5+1} < \frac{4}{n^5}$

$\sum \frac{4}{n^5}$: conv. by p-series

D 14. The integral $\int_{-3}^{0} \frac{2}{x^3}$ $= \lim_{t \to 0^-} \int_{-3}^{t} 2x^{-3} dx = \lim_{t \to 0^-} \frac{2x^{-2}}{-2} \Big|_{-3}^{t} = \lim_{t \to 0^-} \frac{-1}{t^2} - \frac{-1}{9} = -\infty$

A. None of these.

B. diverges to ∞.

C. converges to 1/4.

D. diverges to $-\infty$.

E. converges to -1

A 15. Find the value of x for the series $\sum_{n=0}^{\infty} 5(\frac{x-2}{3})^n$ converges.

A. -1 < x <5.

B. -5< x <1.

C. -1/5 < x < 1.

D. -1 < x < 1/5.

E. None of above.

$\left|\frac{x-2}{3}\right| < 1$

$\therefore |x-2| < 3$

$-3 < x-2 < 3$

$-1 < x < 5$

第二部份，計算題

16. (5 points) The series $\sum_{n=1}^{\infty} a_n$ has partial sums given by $s_n = \frac{2n^2+3}{7n^2+5}$. Determine if the series converges or diverges.If it converges, give the sum.

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{2n^2+3}{7n^2+5} = \frac{2}{7}$ $\therefore$ conv. to $\frac{2}{7}$

17. (5 points) Consider the series $\sum_{n=1}^{\infty} \frac{8}{n(n+2)}$ $= \sum_{n=1}^{\infty} 4\left(\frac{1}{n} - \frac{1}{n+2}\right)$

$= 4\left[\frac{1}{1} - \cancel{\frac{1}{3}}\right] + 4\left[\frac{1}{2} - \cancel{\frac{1}{4}}\right] + 4\left[\cancel{\frac{1}{3}} - \cancel{\frac{1}{5}}\right] + 4\left[\cancel{\frac{1}{4}} - \cancel{\frac{1}{6}}\right] + \cdots$

$= 4 + 4\times\frac{1}{2} = 6$

18. (10 points) Compute $\int \frac{7x^2+3x+11}{(x+1)(x^2+4)}dx$.

$$\frac{7x^2+3x+11}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4}$$

$$7x^2+3x+11 = A(x^2+4)+(Bx+C)(x+1)$$

$$= A(x^2+4)+Bx(x+1)+C(x+1)$$

$$= (A+B)x^2+(B+C)x+(4A+C)$$

$$\therefore \begin{cases} 7 = A+B \\ 3 = B+C \\ 11 = 4A+C \end{cases} \Rightarrow \text{let } \boxed{x=-1} \Rightarrow 7-3+11 = A((-1)^2+4) \quad \therefore A=3$$

$$\Rightarrow B=4, \quad C=-1$$

$$\int \frac{7x^2+3x+11}{(x+1)(x^2+4)}dx = \int \frac{3}{x+1} + \frac{4x}{x^2+4} - \frac{1}{x^2+4}\,dx$$

$$= 3\ln|x+1| + 2\ln|x^2+4| - \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

19. (10 points) Determine whether the series below converges or diverges. Fully support your answer.

$$\sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2}$$

① $\boxed{CT + IT}$

$e^{1/n} < e^1$ for $n > 1$ $\therefore \sum_{n=1}^{\infty} \frac{e^{1/n}}{n^2} \le \sum_{n=1}^{\infty} \frac{e^1}{n^2}$

$f(x) = \frac{e^1}{x^2}$: conti, positive, decreasing $\therefore$ suits for IT.

$\int_1^{\infty} \frac{e^1}{x^2} dx = e^1 \int_1^{\infty} \frac{1}{x^2} dx$ = conv. by p-test $\therefore \sum \frac{e^1}{n^2}$ conv. by IT

$\therefore \sum \frac{e^{1/n}}{n^2}$: conv by CT

② $\boxed{IT}$ $f(x) = \frac{e^{1/x}}{x^2}$: conti for $x > 0$, positive for $x \neq 0$.

$f'(x) = \frac{x^2 \cdot \frac{-1}{x^2} e^{1/x} - e^{1/x} \cdot 2x}{x^4} = \frac{-e^{1/x} - 2x e^{1/x}}{x^4} < 0$ for $x > 0$ $\therefore f(x)$: decreasing

$\int_1^{\infty} \frac{e^{1/x}}{x^2} dx = \lim_{t \to \infty} \int_1^t \frac{e^{1/x}}{x^2} dx = \lim_{t \to \infty} \int_{\frac{1}{1}}^{\frac{1}{t}} -u^u du = \lim_{t \to \infty} -e^{-1/x} \Big|_1^t$

$u = \frac{1}{x}$

$du = -\frac{1}{x} dx$

$= \lim_{t \to \infty} e^{-1/t} - e^{-1} = e^1 - 1$ $\therefore$ conv

by IT, $\sum \frac{e^{1/n}}{n^2}$: conv.

20. (10 points) Find $\int \frac{x^3+6x-6}{x^4+2x^2}dx$. $= \frac{A}{X} + \frac{B}{X^2} + \frac{CX+D}{X^2+2}$

$X^3+6X-6 = AX(X^2+2) + B(X^2+2) + (CX+D)X^2$

$\boxed{X=0} \Rightarrow B=-3$

$X^3+6X-6 = AX^3+2AX-3X^2-6+CX^3+DX^2$

$= (A+C)X^3 + (-3+D)X^2 + 2AX - 6$

$\begin{cases} 1 = A+C \\ 0 = -3+D \\ 6 = 2A \end{cases} \Rightarrow \begin{matrix} C=-2 \\ D=3 \\ A=3 \end{matrix}$

$\int\left(\frac{3}{X} - \frac{3}{X^2} + \frac{-2X+3}{X^2+2}\right)dx$

$= \int\left(\frac{3}{X} - \frac{3}{X^2} - \frac{2X}{X^2+2} + \frac{3}{X^2+2}\right)dx$

$= 3\ln|X| + \frac{3}{X} - \ln|X^2+2| + \frac{3}{\sqrt{2}}\arctan\left(\frac{X}{\sqrt{2}}\right) + C.$

化學一微積分第二次期中考, 學號: __________, 姓名: __________, 以下由閱卷人員填寫

Question:	1-15	16	17	18	19	20	Total
Points:	60	5	5	10	10	10	100
Score:							