姓名: sol.

葉均承　　化學一微積分

學號: ____________

Quiz 8-1

考試日期: 2020/05/11

不可使用手機、計算器，禁止作弊!
背面還有題目

A 1. (20 points) The series $\sum_{n=0}^{\infty}(-1)^n\frac{3^{2n}}{2^{3n}}$ $= \sum_{n=0}^{\infty}(-1)^n\left(\frac{3^2}{2^3}\right)^n = \sum_{n=0}^{\infty}\left(\frac{-9}{8}\right)^n$

(a) is divergent.
(b) is converge to 8/17
(c) is converge to 9/17
(d) is converge to -8/9
(e) is converge to -9/8

$\because \lim_{n\to\infty}\left(\frac{-9}{8}\right)^n \neq 0 \quad \therefore$ div by T.D.

C 2. (20 points) The series $\sum_{n=0}^{\infty}(-1)^n\frac{2^{3n}}{3^{2n}}$ $= \sum_{n=0}^{\infty}\left(\frac{-8}{9}\right)^n$: geo. series with $r=\frac{-8}{9}$

(a) is divergent.
(b) is converge to 8/17
(c) is converge to 9/17
(d) is converge to -8/9
(e) is converge to -9/8

$\therefore$ conv. to $\frac{1}{1-\left(\frac{-8}{9}\right)} = \frac{9}{17}$

C 3. (20 points) Which of the following converge?

I. $\sum_{n=1}^{\infty}\frac{n^3-n^2}{n^3+n^2}$ II. $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}} = \sum\frac{1}{n^{3/2}}$ III. $\sum_{n=1}^{\infty}\frac{1}{2^n-1} \leq \sum\frac{1}{2^{n-1}}$ IV. $\sum_{n=1}^{\infty}\frac{\sin^2(14n)}{n^3} \leq \sum\frac{1}{n^3}$: $P=3>1$ $\therefore$ conv. by P-test

(a) All convergent.
(b) All divergent.
(c) All but I are convergent.
(d) Only II, III are convergent.
(e) Only III, IV are convergent.

II: $P=\frac{3}{2}>1$ $\therefore$ conv. by P-test

III: $\sum\frac{1}{2^{n-1}}$: conv. geo. series $\therefore$ conv. by C.T.

I. $a_n=\frac{n^3-n^2}{n^3+n^2}$, $\lim_{n\to\infty}a_n = 1 \neq 0$ $\therefore$ div. by T.D.

4. (40 points) Determine if the series $\sum_{n=1}^{\infty}\frac{\ln(n)}{n}$ converges or diverges and say why.

$\boxed{\text{I.T.}}$ let $f(x)=\frac{\ln(x)}{x}$, f: continuous, positive, decreasing ($\because f'(x)=\frac{1-\ln(x)}{x^2}<0$, when $x>e^1$)

$\lim_{t\to\infty}\int_1^t \frac{\ln(x)}{x}dx = \int_0^t u\,du = \frac{1}{2}u^2\Big|_0^t = \frac{1}{2}t^2 \to \infty$ $\therefore \sum_{n=1}^{\infty}\frac{\ln(n)}{n}$ div by I.T.

$u=\ln(x)$
$du=\frac{1}{x}dx$

$\boxed{\text{C.T.}}$ $\frac{\ln(n)}{n}>\frac{1}{n}$ for $n>3$, $\therefore \sum_{n=3}^{\infty}\frac{\ln(n)}{n} > \sum_{n=3}^{\infty}\frac{1}{n}$: div. by P-test ($P=1$)

$\therefore \sum_{n=1}^{\infty}\frac{\ln(n)}{n}$ div. by C.T.

$\boxed{\text{T.D.}}$ $\lim_{n\to\infty}\frac{\ln(n)}{n} \overset{L'H}{\underset{\frac{\infty}{\infty}}{=}} \lim_{n\to\infty}\frac{\frac{1}{n}}{1}=0$ $\therefore$ T.D. doesn't work

不夠

not conclusion, use other method

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背面還有題目

1. Find the sum of the following series if they converge or show they diverge.

(a) (30%) $\sum_{n=0}^{\infty} \frac{(-1)^n 3^{2n}}{10^{n+1}}$ $= \frac{1}{10} \sum_{n=0}^{\infty} \left(\frac{(-1)3^2}{10}\right)^n = \frac{1}{10} \sum_{n=0}^{\infty} \left(\frac{-9}{10}\right)^n$

$= \frac{1}{10} \times \frac{1}{1-\left(\frac{-9}{10}\right)} = \frac{1}{19}$

(b) (30%) $\sum_{n=4}^{\infty} \frac{3}{n(n+2)}$ $= \frac{3}{2} \sum_{n=4}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right)$

① $\left(\frac{1}{n} - \frac{1}{n+2}\right) = \frac{2}{n(n+2)}$

② $\sum_{n=4}^{\infty} \left(\frac{1}{n} - \frac{1}{n+2}\right) = \left(\frac{1}{4} - \cancel{\frac{1}{6}}\right) + \left(\frac{1}{5} - \cancel{\frac{1}{7}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{8}}\right) + \left(\cancel{\frac{1}{7}} - \frac{1}{9}\right) + \left(\cancel{\frac{1}{8}} - \frac{1}{10}\right) \ldots\ldots$

③ $S_n = \frac{3}{2}\left[\frac{1}{4} + \frac{1}{5} - \frac{1}{n+1} - \frac{1}{n+2}\right]$

$\therefore S = \lim_{n\to\infty} S_n = \frac{3}{2}\left[\frac{1}{4} + \frac{1}{5}\right] = \frac{3}{2} \times \frac{9}{20} = \frac{27}{40}$

Sol

Quiz 8

2. (40%) Determine if the series below converges or diverges. Clearly state your reasoning and any tests used.

$$\sum_{n=1}^{\infty} n^2 e^{-n^3} = \frac{n^2}{e^{n^3}}$$

① $\lim_{n\to\infty} \frac{n^2}{e^{n^3}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{n\to\infty} \frac{2n}{3n^2 e^{n^3}} \overset{\frac{\infty}{\infty}}{\underset{L'H}{=}} \lim_{n\to\infty} \frac{2}{\infty} = 0$ $\therefore$ T.D. fail

② $f(x) = x^2 e^{-x^3}$ = continuous, positive. decreasing

$f'(x) = -x(3x^3-2)e^{-x^3} < 0$ when $x > 1$ $\therefore f(x) \downarrow$

③ $\int_1^{\infty} x^2 e^{-x^3} dx = -\frac{1}{3} e^{-x^3} \Big|_1^{\infty} = 0 + \frac{1}{3} e^{-1}$ = conv.

$\therefore \sum_{n=1}^{\infty} n^2 e^{-n^3}$ = conv. by I.T.