41. The 0-flat $\mathbf{x} = \begin{bmatrix} -43 \\ -12 \\ 7 \\ 1 \end{bmatrix}$

43. T F F T F T F T F T

CHAPTER 3

Section 3.1

1. Not a vector space 3. A vector space

5. Not a vector space 7. Not a vector space

9. A vector space 11. A vector space

13. A vector space 15. A vector space

17. a. $[-1, 0]$ is the "zero vector."
    b. Part 5 of Theorem 3.1 in this vector space becomes $r[-1, 0] = [-1, 0]$, for all $r \in \mathbb{R}$. That is, $[0, 0]$ is not the zero vector $\mathbf{0}$ in this vector space.

27. Both 2×6 matrices and 3×4 matrices contain 12 entries. If we number entries in some fashion from 1 to 12, say starting at the upper left-hand corner and proceeding down each column in turn, then each matrix can be viewed as defining a function from the set $S = \{1, 2, \ldots, 12\}$ into the set $\mathbb{R}$. The rules for adding matrices and multiplying by a scalar correspond in each case to our definitions of addition and scalar multiplication in the space of functions mapping S into $\mathbb{R}$. Thus we can identify both $M_{2,6}$ and $M_{3,4}$ with this function space, and hence with each other.

29. $\mathbb{R}^{24}$ $\mathbb{R}^{25}$ $\mathbb{R}^{26}$ P_{26} $M_{4,7}$
    $M_{2,12}$ P_{24} P_{25} $M_{3,9}$
    $M_{4,6}$ $M_{5,5}$ $M_{2,13}$
    $M_{3,8}$

Section 3.2

1. Not a subspace 3. Not a subspace

5. A subspace

7. a. Because $1 = \sin^2 x + \cos^2 x$, we have $c = c(\sin^2 x) + c(\cos^2 x)$, which shows that $c \in \mathrm{sp}(\sin^2 x, \cos^2 x)$.
   b. Now $\cos 2x = \cos^2 x - \sin^2 x = (-1)\sin^2 x + (1)\cos^2 x$, which shows that $\cos 2x \in \mathrm{sp}(\sin^2 x, \cos^2 x)$
   c. Now $\cos 4x = \cos^2 2x - \sin^2 2x = (1 - \sin^2 2x) - \sin^2 2x = \frac{1}{7}(7) + (-2)\sin^2 2x$, which shows that $\cos 4x$, and thus $8 \cos 4x$, is in $\mathrm{sp}(7, \sin^2 2x)$.

9. a. We see that $\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2 \in \mathrm{sp}(\mathbf{v}_1, \mathbf{v}_2)$; and therefore,
$$\mathrm{sp}(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2) \subseteq \mathrm{sp}(\mathbf{v}_1, \mathbf{v}_2).$$
   Furthermore, $\mathbf{v}_1 = 1\mathbf{v}_1 + 0(2\mathbf{v}_1 + \mathbf{v}_2)$ and $\mathbf{v}_2 = (-2)\mathbf{v}_1 + 1(2\mathbf{v}_1 + \mathbf{v}_2)$, showing that $\mathbf{v}_1, \mathbf{v}_2 \in \mathrm{sp}(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2)$; and therefore,
$$\mathrm{sp}(\mathbf{v}_1, \mathbf{v}_2) \subseteq \mathrm{sp}(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2).$$
   Thus, $\mathrm{sp}(\mathbf{v}_1, \mathbf{v}_2) = \mathrm{sp}(\mathbf{v}_1, 2\mathbf{v}_1 + \mathbf{v}_2)$.

11. Dependent 13. Dependent

15. Independent 17. Dependent

19. Independent 21. Not a basis

23. $\{1, 4x + 3, x^2 + 2\}$

25. T F T T T F F F T T

35. Let $W = \mathrm{sp}(\mathbf{e}_1, \mathbf{e}_2)$ and $U = \mathrm{sp}(\mathbf{e}_3, \mathbf{e}_4, \mathbf{e}_5)$ in $\mathbb{R}^5$. Then $W \cap U = \{\mathbf{0}\}$ and each $\mathbf{x} \in \mathbb{R}^5$ has the form $\mathbf{x} = \mathbf{w} + \mathbf{u}$, where $\mathbf{w} = x_1\mathbf{e}_1 + x_2\mathbf{e}_2$ and $\mathbf{u} = x_3\mathbf{e}_3 + x_4\mathbf{e}_4 + x_5\mathbf{e}_5$.

39. In deciding whether $\sin x$ and $\cos x$ are independent, we consider linear combinations of them *with scalar coefficients*. The given coefficients $f(x)$ and $g(x)$ are not scalars. For a counterexample, consider $f(x) = -\cos x$ and $g(x) = \sin x$. We have $(-\cos x)(\sin x) + (\sin x)(\cos x) = 0$.

41. The set of solutions consists of all functions of the form $h(x) + p(x)$, where $h(x)$ is the general solution of the corresponding homogeneous equation $f_n(x)y^{(n)} + f_{n-1}(x)y^{(n-1)} + \cdots + f_2(x)y'' + f_1(x)y' + f_0(x)y = 0$.

43. a. $\{a \sin x + b \cos x \mid a, b \in \mathbb{R}\}$
    b. $\{a \sin x + b \cos x + x \mid a, b \in \mathbb{R}\}$

45. a. $\{ae^{3x} + be^{-3x} + c \mid a, b, c \in \mathbb{R}\}$
    b. $\{ae^{3x} + be^{-3x} + c - \frac{x^3}{27} - \frac{1}{9}x^2 - \frac{2}{81}x \mid a, b, c \in \mathbb{R}\}$

7. One basis B for W consists of those f_a for a in S defined by $f_a(a) = 1$ and $f_a(s) = 0$ for $s \neq a$ in S. If $f \in W$ and $f(s) \neq 0$ only for $s \in \{a_1, a_2, \ldots, a_n\}$, then $f = f(a_1)f_{a_1} + f(a_2)f_{a_2} + \cdots + f(a_n)f_{a_n}$. Now the linear combination $g = c_1 f_{b_1} + c_2 f_{b_2} + \cdots + c_m f_{b_m}$ is a function satisfying $g(b_j) = c_j$ for $j = 1, 2, \ldots, m$ and $g(s) = 0$ for all other $s \in S$. Thus $g \in W$, so B spans only W. (The crucial thing is that all linear combinations are sums of only *finite* numbers of vectors.)

Section 3.3

1. $[1, -1]$ 3. $[2, 6, -4]$ 5. $[2, 1, 3]$

7. $[4, 1, -2, 1]$ 9. $\left[\frac{1}{2}, \frac{1}{2}, 1, -\frac{1}{2}, 0\right]$

11. $[1, 2, -1, 5]$

13. $$\begin{aligned} p(x) &= [(x+1)-1]^3 + [(x+1)-1]^2 - [(x+1)-1] - 1 \\ &= (x+1)^3 - 3(x+1)^2 + 3(x+1) - 1 \\ &\quad + (x+1)^2 - 2(x+1) + 1 \\ &\quad - (x+1) + 1 \\ &\quad - 1 \\ &= (x+1)^3 - 2(x+1)^2 + 0(x+1) + 0 \end{aligned}$$

so the coordinate vector is $[1, -2, 0, 0]$.

15. $[4, 3, -5, -4]$

17. Let $x^3 - 4x^2 + 3x + 7 = b_3(x-2)^3 + b_2(x-2)^2 + b_1(x-2) + b_0$. Setting $x = 2$, we find that $8 - 16 + 6 + 7 = b_0$, so $b_0 = 5$. Differentiating and setting $x = 2$, we obtain $3x^2 - 8x + 3 = 3b_3(x-2)^2 + 2b_2(x-2) + b_1$; $12 - 16 + 3 = b_1$; $b_1 = -1$. Differentiating again and setting $x = 2$, we have $6x - 8 = 6b_3(x-2) + 2b_2$; $12 - 8 = 2b_2$; $b_2 = 2$. Differentiating again, we obtain $6 = 6b_3$, so $b_3 = 1$. The coordinate vector is $[1, 2, -1, 5]$.

19. b. $\{f_1(x), f_2(x)\}$ 21. $2x^2 + 6x + 2$

Section 3.4

1. A linear transformation, $\ker(T) = \{f \in F \mid f(-4) = 0\}$, not invertible

3. A linear transformation, $\ker(T)$ is the zero function, invertible

5. A linear transformation, $\ker(T)$ is the zero function, invertible

7. The zero function is the only function in $\ker(T)$.

9. $\{c_1 e^{2x} + c_2 e^{-2x} - \frac{1}{5}\sin x \mid c_1, c_2 \in \mathbb{R}\}$

11. $\{c_1 x + c_2 + \cos x \mid c_1, c_2 \in \mathbb{R}\}$

13. $\{c_1 \sin 2x + c_2 \cos 2x + \frac{1}{4}x^2 - \frac{1}{8} \mid c_1, c_2 \in \mathbb{R}\}$

15. $\{c_1 e^{2x} + c_2 x + c_3 - \frac{1}{12}x^3 - \frac{1}{8}x^2 \mid c_1, c_2, c_3 \in \mathbb{R}\}$

17. $T(\mathbf{v}) = 4\mathbf{b}_1' + 4\mathbf{b}_2' + 7\mathbf{b}_3' + 6\mathbf{b}_4'$

19. $T(\mathbf{v}) = -17\mathbf{b}_1' + 4\mathbf{b}_2' + 13\mathbf{b}_3' - 3\mathbf{b}_4'$

21. a. $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

b. $A\begin{bmatrix} 4 \\ -5 \\ 10 \\ -13 \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -10 \\ 10 \end{bmatrix}$; $12x^2 - 10x + 10$

c. The second derivative is given by $A^2\begin{bmatrix} -5 \\ 8 \\ -3 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -30 \\ 16 \end{bmatrix}$; $-30x + 16$

23. a. $D(x^2e^x) = x^2e^x + 2xe^x$; $D^2(x^2e^x) = x^2e^x + 4xe^x + 2e^x$; $D(xe^x) = xe^x + e^x$; $D^2(xe^x) = xe^x + 2e^x$; $D(e^x) = e^x$; $D^2(e^x) = e^x$

$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$

b. From the computations in part a, we have $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, and computation shows that $A_1^2 = A$.

25. We obtain $A = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix}$ in both part a and part b.

27. $\begin{bmatrix} 9 & 0 & 0 \\ 0 & 25 & 0 \\ 0 & 0 & 81 \end{bmatrix}$

29. $(a + c)e^{2x} + be^{4x} + (a + c)e^{8x}$ 31. $\begin{bmatrix} -3 & -3 \\ 4 & -4 \end{bmatrix}$

33. $-2b \sin 2x + 2a \cos 2x$

45. If A_1 is the representation of T_1 and A_2 is the representation of T_2 relative to B,B', then $A_1 + A_2$ is the representation of $T_1 + T_2$ relative to B,B'. If A is the representation of T relative to B,B', then rA is the representation of rT relative to B,B'

51. Let $V = D_\infty$, the space of infinitely differentiable functions mapping $\mathbb{R}$ into $\mathbb{R}$. Let $T(f) = f'$ for $f \in D_\infty$. Then range$(T) = D_\infty$ because every infinitely differentiable function is continuous and thus has an antiderivative, but $T(x) = T(x + 1)$ shows that T is not one-to-one.

Section 3.5

1. Not an inner product
3. Not an inner product
5. Not an inner product
7. An inner product
9. Not an inner product

11. a. $\frac{5}{6}$ b. $\frac{1}{\sqrt{3}}$ c. $\frac{1}{\sqrt{30}}$ d. $\frac{1}{\sqrt{2}}$

13. $f(x) = -x + \frac{1}{2}$ and $g(x) = \cos \pi x$. Other answers are possible.

15. 77

CHAPTER 4

Section 4.1

1. -15 3. 15

5. $$\mathbf{b} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} b_1 & b_2 & b_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = b_1(b_2c_3 - b_3c_2) - b_2(b_1c_3 - b_3c_1) + b_3(b_1c_2 - b_2c_1) = 0,$$

$$\mathbf{c} \cdot (\mathbf{b} \times \mathbf{c}) = \begin{vmatrix} c_1 & c_2 & c_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = c_1(b_2c_3 - b_3c_2) - c_2(b_1c_3 - b_3c_1) + c_3(b_1c_2 - b_2c_1) = 0$$

7. 120 9. -9

11. $$\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1 = -(b_1a_2 - b_2a_1) = -\begin{vmatrix} b_1 & b_2 \\ a_1 & a_2 \end{vmatrix}$$

13. $-6\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$ 15. $0\mathbf{i} + 0\mathbf{j} + 0\mathbf{k}$

17. $22\mathbf{i} + 18\mathbf{j} + 2\mathbf{k}$

19. F T T F F T F T T F

21. 38 23. $\sqrt{62}$ 25. $\frac{19}{2}$

27. $\frac{\sqrt{230}}{2}$ 29. 16 31. $\sqrt{390}$

33. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = -6$,
$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = 12\mathbf{i} + 4\mathbf{k}$

35. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 19$,
$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = 3\mathbf{i} - 7\mathbf{j} + \mathbf{k}$

37. 20 39. 9 41. 1 43. $\frac{7}{3}$

45. Not collinear 47. Collinear

49. Not coplanar 51. Not coplanar

53. 0 55. $\|\mathbf{a}\|^2\|\mathbf{b}\|^2$

57. $\mathbf{i} \times (\mathbf{i} \times \mathbf{j}) = \mathbf{i} \times \mathbf{k} = -\mathbf{j}$, but $(\mathbf{i} \times \mathbf{i}) \times \mathbf{j} = \mathbf{0} \times \mathbf{j} = \mathbf{0}$.

59. $$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = (b_2c_3 - b_3c_2)\mathbf{i} - (b_1c_3 - b_3c_1)\mathbf{j} + (b_1c_2 - b_2c_1)\mathbf{k}.$$ Thus,

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = a_1(b_2c_3 - b_3c_2) - a_2(b_1c_3 - b_3c_1) + a_3(b_1c_2 - b_2c_1) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

Equation (4) in the text shows that this determinant is $\pm$(Volume of the box determined by **a**, **b**, and **c**). Similarly,

$$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \mathbf{c} \cdot (\mathbf{a} \times \mathbf{b}) = \begin{vmatrix} c_1 & c_2 & c_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = c_1(a_2b_3 - a_3b_2) - c_2(a_1b_3 - a_3b_1) + c_3(a_1b_2 - a_2b_1),$$

which is the same number.