

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. Find a vector perpendicular to both $[1 - i, 2, 2 + i]$, $[i, 1 + i, -1]$.

Answer: $[-3 - 3i, i, 2 - 2i]$

Solution :

Method 1 Gram-Schmitt process

Method 2 Cross product

2. Find the dot product of $[1 - i, 2, 2 + i]$ and $[i, 1 + i, -1]$.

Answer: $-1 + 4i$

Solution :

$$\langle [1 - i, 2, 2 + i], [i, 1 + i, -1] \rangle = -1 + 4i$$

3. If A is an $n \times n$ normal matrix (i.e. $A^*A = AA^*$), prove that for any $\vec{z} \in \mathbb{C}^n$, $\|A\vec{z}\| = \|A^*\vec{z}\|$.

Solution :

Section 9.2, problem 44.

Hint: $\|A\vec{z}\|^2 = \langle A\vec{z}, A\vec{z} \rangle = (A\vec{z})^*(A\vec{z})$