

Section 6-3

7, 9, 11, 17, 19, 21, 23, 31, 37.

1. 請見課本後的答案。

2. 課本第 21 題

Let  $A$  be an orthogonal matrix. Show that  $A^2$  is an orthogonal matrix, too.

**Answer:**

$A$  be an orthogonal matrix, i.e.  $A^T A = I$ .  $(A^2)^T = (AA)^T = A^T A^T = (A^T)^2$ . Therefore  $(A^2)^T (A^2) = A^T A^T A A = A^T (A^T A) A = A^T I A = A^T A = I$ . We have  $A^2$  is an orthogonal matrix.

3. 課本第 23 題

Find a  $2 \times 2$  matrix with determinant 1 that is not an orthogonal matrix.

**Answer:**

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$\det(A) = 1, \text{ but } A^T A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \neq I$$

4. 課本第 31 題

Let  $A$  and  $C$  be orthogonal  $n \times n$  matrices. Show that  $CAC^{-1}$  is orthogonal.

**Answer:**

$A$  and  $C$  be orthogonal matrices, i.e.  $A^T A = C^T C = I$ . We also know that  $C^{-1} = C^T$ , i.e.  $CAC^{-1} = CAC^T$

$(CAC^T)^T (CAC^T) = (C^T)^T A^T C^T C A C^T = C A^T C^T C A C^T = C A^T (C^T C) A C^T = C A^T I A C^T = C (A^T A) C^T = C I C^T = (C^T C)^T = I^T = I$ . We have  $CAC^T = CAC^{-1}$  is orthogonal