姓名: SOLUTION　　　　　　　　　　　　　　　　葉均承　　應數一線性代數

學號: ________________　　　　**Quiz 6**　　　　考試日期: 2020/11/05

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊! 3. 背面還有題目

1. Consider the set $\mathbb{R}^2$, with the addition defined by $[x,y] \oplus [a,b] = [x+a+1, y+b]$, and with scalar multiplication defined by $r[x,y] = [rx+r-1, ry]$.

   a. Is this set a vector space? *Hint:* Show by verifying the closed under two operations, A1-A4 and S1-S4.

   b. What is the zero vector in this vector space? *Hint:* The zero vector will NOT be the vector $[0,0]$.

   Answer: the zero vector $\vec{0} = [-1,0]$, $-[x,y] = [-x+2, -y]$

   $[x,y] \oplus [a,b] = [x+a+1, y+b]$ and $r[x,y] = [rx+r-1, ry]$ are both in $\mathbb{R}^2$, hence proved the closed.

   A1 $([x,y] \oplus [a,b]) \oplus [p,q] = [x+a+1, y+b] \oplus [p,q] = [x+a+1+p+1, y+b+q] = [x+a+p+1+1, y+b+q] = [x,y] \oplus [a+p+1, b+q] = [x,y] \oplus ([a,b] \oplus [p,q])$

   A2 $[x,y] \oplus [a,b] = [x+a+1, y+b] = [a+x+1, b+y] = [a,b] \oplus [x,y]$

   A3 $\vec{0} = 0[x,y] = [0x+0-1, 0y] = [-1,0]$. $\vec{0} \oplus [x,y] = [-1,0] \oplus [x,y] = [-1+x+1, 0+y] = [x,y]$

   A4 $(-1)[x,y] = [-x+(-1)-1, -y] = [-x-2, -y]$. $[x,y] \oplus [-x-2, -y] = [x-x-2+1, y-y] = [-1,0]$

   S1 $r([x,y] \oplus [a,b) = r([x+a+1, y+b]) = [rx+ra+r+r-1, ry+rb] = [rx+r-1+ra+r-1+1, ry+rb] = [rx+r-1, ry] \oplus [ra+r-1, rb] = r[x,y] \oplus r[a,b]$

   S2 $(r+s)[x,y] = [(r+s)x+(r+s)-1, (r+s)y] = [rx+sx+r+s-1, ry+sy] = [rx+r-1+sx+s-1+1, ry+sy] = r[x,y] \oplus r[a,b]$

   S3 $s(r[x,y]) = s([rx+r-1, ry]) = [srx+sr-s+s-1, sry] = (rs)[x,y]$

   S4 $1[x,y] = [x+1-1, y] = [x,y]$