EXERCISES

1. Let the sequence $a_0, a_1, a_2, \ldots$ be given by $a_0 = 0$, $a_1 = 1$, and $a_k = (a_{k-1} + a_{k-2})/2$ for $k \geq 2$.

   a. Find the matrix A that can be used to generate this sequence as we used a matrix to generate the Fibonacci sequence in Section 5.1.

   b. Classify this generation process as stable, neutrally stable, or unstable.

   c. Compute expression (1) for $x = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix}$ for this process. Check computations with the first few terms of the sequence.

   d. Use the answer to part (c) to estimate a_k for large k.

2. Repeat Exercise 1 if $a_k = a_{k-1} - \left(\frac{3}{16}\right)a_{k-2}$ for $k \geq 2$.

3. Repeat Exercise 1, but change the initial data to $a_0 = 1$, $a_1 = 0$.

4. Repeat Exercise 1 if $a_k = \left(\frac{1}{2}\right)a_{k-1} + \left(\frac{3}{16}\right)a_{k-2}$ for $k \geq 2$.

5. Repeat Exercise 1 if $a_k = a_{k-1} + \left(\frac{3}{4}\right)a_{k-2}$ for $k \geq 2$.

In Exercises 6–13, solve the given system of linear differential equations as outlined in the summary.

6. $\begin{aligned} x_1' &= 3x_1 - 5x_2 \\ x_2' &= \qquad 2x_2 \end{aligned}$

7. $\begin{aligned} x_1' &= x_1 + 4x_2 \\ x_2' &= 3x_1 \end{aligned}$

8. $\begin{aligned} x_1' &= x_1 + 2x_2 \\ x_2' &= 2x_1 + x_2 \end{aligned}$

9. $\begin{aligned} x_1' &= 2x_1 + 2x_2 \\ x_2' &= x_1 + 3x_2 \end{aligned}$

10. $\begin{aligned} x_1' &= 6x_1 + 3x_2 - 3x_3 \\ x_2' &= -2x_1 - x_2 + 2x_3 \\ x_3' &= 16x_1 + 8x_2 - 7x_3 \end{aligned}$

11. $\begin{aligned} x_1' &= -3x_1 + 10x_2 - 6x_3 \\ x_2' &= \qquad 7x_2 - 6x_3 \\ x_3' &= \qquad x_3 \end{aligned}$

12. $\begin{aligned} x_1' &= -3x_1 + 5x_2 - 20x_3 \\ x_2' &= 2x_1 \qquad + 8x_3 \\ x_3' &= 2x_1 + x_2 + 7x_3 \end{aligned}$

13. $\begin{aligned} x_1' &= -2x_1 \qquad - x_3 \\ x_2' &= \qquad 2x_2 \\ x_3' &= 3x_1 \qquad + 2x_3 \end{aligned}$