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葉均承　　化學一微積分

學號: ______

Quiz 6

考試日期: 2020/04/27

不可使用手機、計算器，禁止作弊!
背面還有題目

1. (30%) Evaluate $\int_0^{10} \frac{1}{x-5} dx$ if possible.

$$\int_0^{10} \frac{1}{x-5} dx = \int_0^5 \frac{1}{x-5} dx + \int_5^{10} \frac{1}{x-5} dx = \underbrace{\lim_{t \to 5^-} \int_0^t \frac{1}{x-5} dx}_{①} + \underbrace{\lim_{s \to 5^+} \int_s^{10} \frac{1}{x-5} dx}_{②}$$

$$\lim_{t \to 5^-} \int_0^t \frac{1}{x-5} dx = \lim_{t \to 5^-} \ln|x-5| \Big|_0^t = \lim_{t \to 5^-} (\ln|t-5| - \ln|-5|) = -\infty \quad \therefore div$$

2. (35%) Evaluate $\int_0^1 \ln(x) dx$ if possible. $= \lim_{t \to 0^+} \int_t^1 \ln(x) dx$

$$\int_t^1 \ln x \, dx = x \ln x \Big|_t^1 - \int_t^1 1 \, dx = 1\ln(1) - t\ln(t) - (1-t)$$

$u = \ln x \quad dv = dx$

$$= \underbrace{-t\ln t}_{①} - 1 + t$$

$du = \frac{1}{x} dx \quad v = x$

$$① \lim_{t \to 0^+} t \ln t = \lim_{t \to 0^+} \frac{\ln t}{\frac{1}{t}} \overset{\frac{-\infty}{\infty}}{\underset{L'H}{=}} \lim_{t \to 0^+} \frac{\frac{1}{t}}{-\frac{1}{t^2}} = \lim_{t \to 0^+} -t = 0$$

$$\therefore \int_0^1 \ln(x) dx = \lim_{t \to 0^+} \int_t^1 \ln(x) dx = \lim_{t \to 0^+} [-t\ln(t)] - 1 + t$$

$$= 0 - 1 + 0 = -1$$

3. (35%) Evaluate $\int_{-\infty}^{\infty} \frac{1}{1+x^2} dx$ if possible. $= \int_0^{\infty} \frac{1}{1+x^2} dx + \int_{-\infty}^{0} \frac{1}{1+x^2} dx$

$$= \underbrace{\lim_{t \to \infty} \int_0^t \frac{1}{1+x^2} dx}_{①} + \underbrace{\lim_{s \to -\infty} \int_s^0 \frac{1}{1+x^2} dx}_{②}$$

① $\lim_{t \to \infty} \int_0^t \frac{1}{1+x^2} dx = \lim_{t \to \infty} \arctan(x) \Big|_0^t = \lim_{t \to \infty} \arctan(t) - \arctan(0) = \frac{\pi}{2}$

② $\lim_{s \to -\infty} \int_s^0 \frac{1}{1+x^2} dx = \lim_{s \to -\infty} \arctan(x) \Big|_s^0 = \lim_{s \to -\infty} \arctan(0) - \arctan(s) = \frac{\pi}{2}$

$\therefore \int_{-\infty}^{\infty} \frac{1}{1+x^2} dx = ① + ② = \frac{\pi}{2} + \frac{\pi}{2} = \pi$