

1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!
3. 作答完畢請拍照上傳 Googld Classroom
4. 照片請清晰並轉正

1. Solve the given system of linear equations by Cramer's rule wherever it is possible.

$$\begin{cases} x_1 + x_2 = 1 \\ x_1 + 2x_2 = 2 \end{cases}$$

Answer: $x_1 = 0, x_2 = 1$.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$x_1 = \frac{\det(B_1)}{\det(A)} = \frac{0}{1} = 0, \quad x_2 = \frac{\det(B_2)}{\det(A)} = \frac{1}{1} = 1.$$

2. Find the determinant of the given matrix.

$$A = \begin{bmatrix} 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 1 \\ -1 & 2 & 4 & 0 & 0 \\ 3 & 1 & -2 & 0 & 0 \\ 5 & 1 & 5 & 0 & 0 \end{bmatrix}$$

Answer: $\det(A) = \underline{\underline{-715}}$.

By Exercise 11 in Section 4-3

$$\det \begin{bmatrix} -1 & 2 & 4 & 0 & 0 \\ 3 & 1 & -2 & 0 & 0 \\ 5 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 1 \end{bmatrix} = \det \begin{bmatrix} -1 & 2 & 4 \\ 3 & 1 & -2 \\ 5 & 1 & 5 \end{bmatrix} \det \begin{bmatrix} 3 & -4 \\ 2 & 1 \end{bmatrix} = -65 * 11 = -715$$

Method 1 By Exercise 12 in Section 4-3

$$\text{For } n = r + s, \quad A_{n \times n} = \left[\begin{array}{c|c} 0 & R_{r \times r} \\ \hline S_{s \times s} & 0 \end{array} \right] \Rightarrow \det(A) = (-1)^{rs} \det(R) \det(S)$$

$$\text{Hence } \det(A) = (-1)^{2 \times 3} * (-715) = -715$$

Method 2

$$\begin{aligned}
& \det \begin{pmatrix} 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 1 \\ -1 & 2 & 4 & 0 & 0 \\ 3 & 1 & -2 & 0 & 0 \\ 5 & 1 & 5 & 0 & 0 \end{pmatrix} \\
&= (-1)^2 * \det \begin{pmatrix} -1 & 2 & 4 & 0 & 0 \\ 3 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 1 \\ 5 & 1 & 5 & 0 & 0 \end{pmatrix} & (R_1 \leftrightarrow R_3), (R_2 \leftrightarrow R_4) \\
&= (-1)^4 * \det \begin{pmatrix} -1 & 2 & 4 & 0 & 0 \\ 3 & 1 & -2 & 0 & 0 \\ 5 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix} & (R_4 \leftrightarrow R_5), (R_3 \leftrightarrow R_4) \\
&= \det \begin{pmatrix} -1 & 2 & 4 & 0 & 0 \\ 3 & 1 & -2 & 0 & 0 \\ 5 & 1 & 5 & 0 & 0 \\ 0 & 0 & 0 & 3 & -4 \\ 0 & 0 & 0 & 2 & 1 \end{pmatrix} = -715
\end{aligned}$$