練習(6.2 -6.3) :

Section 6.2

1–18 Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified line. Sketch the region, the solid, and a typical disk or washer.

3. $y = 1/x$, $x = 1$, $x = 2$, $y = 0$;   about the x-axis

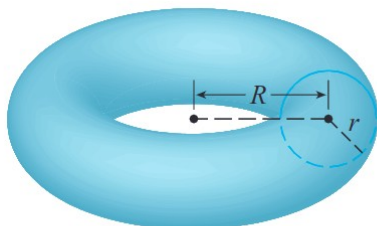**7.** $y = x^3$, $y = x$, $x \geqslant 0$;   about the x-axis

13. $y = 1 + \sec x$, $y = 3$;   about $y = 1$

17. $y = x^2$, $x = y^2$;   about $x = -1$

41–44 Each integral represents the volume of a solid. Describe the solid.

41. $\pi \displaystyle\int_0^{\pi/2} \cos^2 x \, dx$

63. (a) Set up an integral for the volume of a solid *torus* (the donut-shaped solid shown in the figure) with radii r and R.
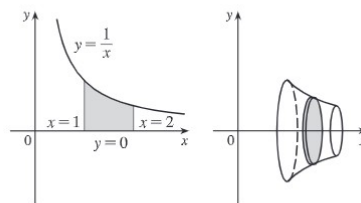(b) By interpreting the integral as an area, find the volume of the torus.

Answers: **41.** Solid obtained by rotating the region $0 \leqslant y \leqslant \cos x$, $0 \leqslant x \leqslant \pi/2$ about the x-axis
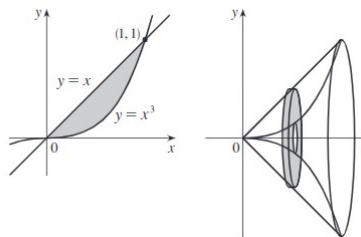
63. (a) $8\pi R \displaystyle\int_0^r \sqrt{r^2 - y^2}\, dy$ (b) $2\pi^2 r^2 R$
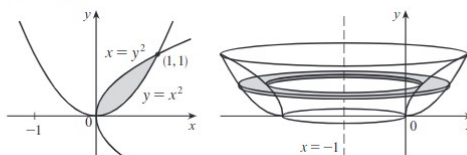
Answers:

3. $\pi/2$

7. $4\pi/21$

13. $2\pi\left(\frac{4}{3}\pi - \sqrt{3}\right)$
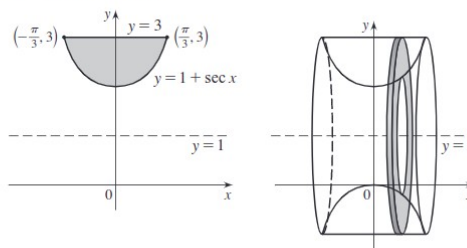
17. $29\pi/30$

Section 6.3

3–7 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis. Sketch the region and a typical shell.

5. $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 1$

7. $y = 4(x - 2)^2$, $y = x^2 - 4x + 7$

9–14 Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis. Sketch the region and a typical shell.

9. $x = 1 + y^2$, $x = 0$, $y = 1$, $y = 2$

13. $x = 1 + (y - 2)^2$, $x = 2$

15–20 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis. Sketch the region and a typical shell.
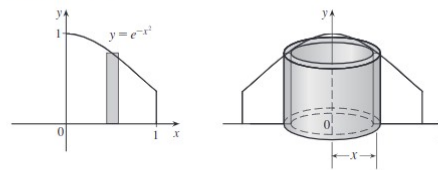
15. $y = x^4$, $y = 0$, $x = 1$; about $x = 2$

19. $y = x^3$, $y = 0$, $x = 1$; about $y = 1$

37–42 The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.
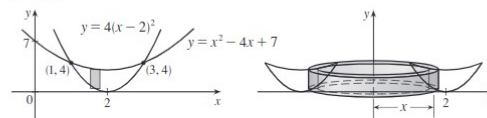
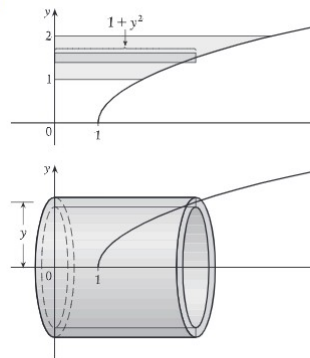**39.** $y = 5$, $y = x + (4/x)$; about $x = -1$

Answers:

5. $\pi(1 - 1/e)$

7. 16π

9. $21\pi/2$

13. $16\pi/3$ **15.** $7\pi/15$ **19.** $5\pi/14$

39. $2\pi(12 - 4 \ln 4)$