

1. 請框出答案. 2. 禁止作弊!

1. Let  $f(x) = x^3 - e^{-x}$ ,  $x_0 = 0.5$ ,  $x_1 = 0.7$ ,  $x_2 = 1.0$ .(a) (60%) Find the Lagrange Polynomial,  $P_2(x)$ , of degree at most 2 for  $f(x)$  using  $x_0, x_1, x_2$ .**Answer:**

$$\begin{aligned}
 & P_2(x) \\
 = & \frac{(x-0.7)(x-1)}{(0.5-0.7)(0.5-1)} * (-0.48153) + \frac{(x-0.5)(x-1)}{(0.7-0.5)(0.7-1)} * (-0.15359) \\
 & + \frac{(x-0.5)(x-0.7)}{(1-0.5)(1-0.7)} * 0.63212
 \end{aligned}$$

(b) (40%) Evaluate  $P_2(0.8)$  and compute the actual error  $|f(0.8) - P_2(0.8)|$ **Answer:**  $|f(0.8) - P_2(0.8)| =$  0.102671035882779

(a)

$$\begin{aligned}
 & P_2(x) \\
 = & \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} * f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} * f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} * f(x_2) \\
 = & \frac{(x-0.7)(x-1)}{(0.5-0.7)(0.5-1)} * (-0.48153) + \frac{(x-0.5)(x-1)}{(0.7-0.5)(0.7-1)} * (-0.15359) \\
 & + \frac{(x-0.5)(x-0.7)}{(1-0.5)(1-0.7)} * 0.63212
 \end{aligned}$$

$$f(x_0) = -0.481530659712633$$

$$f(x_1) = -0.153585303791410$$

$$f(x_2) = 0.632120558828558$$

(b)

$$f(0.8) = 0.06267103588277856$$

$$P_2(0.8) = -0.04$$

$$|f(0.8) - P_2(0.8)| = 0.102671035882779$$