葉均承
Jean Yeh

Homework 4

應數一線性代數

Section 6-1

課本第 25 題

Let W be a subspace of $\mathbb{R}^n$ and let $\vec{b}$ be a vector in $\mathbb{R}^n$. Prove that there is one and only one vector $\vec{p}$ in W such that $\vec{b}-\vec{p}$ is perpendicular to every vector in W. [HINT: Suppose that $\vec{p_1}$ and $\vec{p_2}$ are two such vectors, and show that $\vec{p_1}-\vec{p_2}$ is in $W^{\perp}$]

解答：Assume there're two vectors $\vec{p_1}, \vec{p_2} \in W$ such that $\vec{b}-\vec{p_1}$ and $\vec{b}-\vec{p_2}$ are both perpendicular to every vector in W. *i.e.* $\vec{b}-\vec{p_1}$ and $\vec{b}-\vec{p_2}$ are both in $W^{\perp}$.

For all vector $\vec{v} \in W$

$$
\begin{gathered}
0 = \vec{v}\cdot(\vec{b}-\vec{p_1}) = \vec{v}\cdot\vec{b} - \vec{v}\cdot\vec{p_1} \quad \therefore \vec{v}\cdot\vec{b} = \vec{v}\cdot\vec{p_1} \\
0 = \vec{v}\cdot(\vec{b}-\vec{p_2}) = \vec{v}\cdot\vec{b} - \vec{v}\cdot\vec{p_2} \quad \therefore \vec{v}\cdot\vec{b} = \vec{v}\cdot\vec{p_2} \\
\therefore \vec{v}\cdot(\vec{p_1}-\vec{p_2}) = 0 \\
\therefore \vec{p_1}-\vec{p_2} \in W^{\perp}
\end{gathered}
$$

Note that W is a vector space and $\vec{p_1}, \vec{p_2} \in W$, we will have $\vec{p_1}-\vec{p_2} \in W$. Since $\vec{p_1}-\vec{p_2}$ in both W and W, we can easily checked that $\vec{p_1}-\vec{p_2} = \vec{0}$.