where we obtained 11.45003 for the determinant. We obtained the same result with $m = 20$ as we did with the default roundoff.

With the default roundoff, computed entries of magnitude less than (0.0001)(Smallest nonzero magnitude in A^m) are set equal to zero when A^m is reduced to echelon form. As soon as m is large enough (so that the entries of A^m are large enough), this creates a false zero entry on the diagonal, which produces a false calculation of 0 for the determinant. With roundoff zero, this does not happen. But when m get sufficiently large, roundoff error in the computation of A^m and in its reduction to echelon form creates even greater error in the calculation of the determinant, no matter what roundoff control number is taken. Notice, however, that the value given for the determinant is always small compared with the size of the entries in A^m.

45. $\begin{bmatrix} 12 & -8 & -7 \\ -11 & 13 & 5 \\ 9 & -6 & -1 \end{bmatrix}$

47. $\begin{bmatrix} -4827 & 114 & -2211 & 2409 \\ -3218 & 76 & -1474 & 1606 \\ 8045 & -190 & 3685 & -4015 \\ 1609 & -38 & 737 & -803 \end{bmatrix}$

Section 4.4

1. $\sqrt{213}$ 3. $\sqrt{30}$ 5. 11 7. 38

9. 2 11. $\frac{\sqrt{6}}{2}$ 13. $\frac{2}{3}$

15. Let **a**, **b**, **c**, and **d** be the vectors from the origin to the four points. Let A be the $n \times 3$ matrix with column vectors $\mathbf{b} - \mathbf{a}$, $\mathbf{c} - \mathbf{a}$, and $\mathbf{d} - \mathbf{a}$. The points are coplanar if and only if $\det(A^TA) = 0$.

17. The points are not coplanar. 19. 32

21. 144π 23. 264 25. $\frac{2816\pi}{3}$

27. $5\sqrt{3}$ 29. $25\pi\sqrt{3}$ 31. $16\sqrt{17}$

33. **a.** 0 **b.** 0

35. T F T F T F T T F T

CHAPTER 5

Section 5.1

1. $\mathbf{v}_1$, $\mathbf{v}_3$, and $\mathbf{v}_5$ are eigenvectors of A_1, with corresponding eigenvalues -1, 2, and 2, respectively. $\mathbf{v}_2$, $\mathbf{v}_3$, and $\mathbf{v}_4$ are eigenvectors of A_2, with corresponding eigenvalues 5, 5, and 1, respectively.

3. *Characteristic polynomial*: $\lambda^2 - 4\lambda + 3$
Eigenvalues: $\lambda_1 = 1, \lambda_2 = 3$
Eigenvectors: for $\lambda_1 = 1$: $\mathbf{v}_1 = \begin{bmatrix} -r \\ r \end{bmatrix}, r \neq 0,$
for $\lambda_2 = 3$: $\mathbf{v}_2 = \begin{bmatrix} -s \\ 2s \end{bmatrix}, s \neq 0$

5. *Characteristic polynomial*: $\lambda^2 + 1$
Eigenvalues: There are no real eigenvalues.

7. *Characteristic polynomial*: $-\lambda^3 + 2\lambda^2 + \lambda - 2$
Eigenvalues: $\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 2$
Eigenvectors: for $\lambda_1 = -1$: $\mathbf{v}_1 = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}, r \neq 0,$
for $\lambda_2 = 1$: $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -s \\ s \end{bmatrix}, s \neq 0,$
for $\lambda_3 = 2$: $\mathbf{v}_3 = \begin{bmatrix} -t \\ -t \\ t \end{bmatrix}, t \neq 0$

9. *Characteristic polynomial*: $-\lambda^3 + 8\lambda^2 + \lambda - 8$
Eigenvalues: $\lambda_1 = -1, \lambda_2 = 1, \lambda_3 = 8$
Eigenvectors: for $\lambda_1 = -1$: $\mathbf{v}_1 = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}, r \neq 0,$
for $\lambda_2 = 1$: $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -s \\ s \end{bmatrix}, s \neq 0$
for $\lambda_3 = 8$: $\mathbf{v}_3 = \begin{bmatrix} -t \\ -t \\ t \end{bmatrix}, t \neq 0$

11. *Characteristic polynomial*: $-\lambda^3 - \lambda^2 + 8\lambda + 12$
Eigenvalues: $\lambda_1 = \lambda_2 -2, \lambda_3 = 3$
Eigenvectors: for $\lambda_1, \lambda_2 = -2$: $\mathbf{v}_1 = \begin{bmatrix} -r \\ s \\ r \end{bmatrix},$

r and s not both 0,

for $\lambda_3 = 3$: $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -t \\ t \end{bmatrix}$, $t \neq 0$

13. *Characteristic polynomial*: $-\lambda^3 + 2\lambda^2 + 4\lambda - 8$

Eigenvalues: $\lambda_1 = -2$, $\lambda_2 = \lambda_3 = 2$

Eigenvectors: for $\lambda_1 = -2$: $\mathbf{v}_1 = \begin{bmatrix} -r \\ -3r \\ r \end{bmatrix}$, $r \neq 0$,

for $\lambda_2, \lambda_3 = 2$: $\mathbf{v}_2 = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix}$, $s \neq 0$

15. *Characteristic polynomial*: $-\lambda^3 - 3\lambda^2 + 4$

Eigenvalues: $\lambda_1 = \lambda_2 = -2$, $\lambda_3 = 1$

Eigenvectors: for $\lambda_1, \lambda_2 = -2$: $\mathbf{v}_1 = \begin{bmatrix} 0 \\ r \\ 0 \end{bmatrix}$, $r \neq 0$,

for $\lambda_3 = 1$: $\mathbf{v}_3 = \begin{bmatrix} 3s \\ -s \\ 3s \end{bmatrix}$, $s \neq 0$

17. *Eigenvalues*: $\lambda_1 = -1$, $\lambda_2 = 5$

Eigenvectors: for $\lambda_1 = -1$: $\mathbf{v}_1 = \begin{bmatrix} r \\ r \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = 5$: $\mathbf{v}_2 = \begin{bmatrix} -s \\ s \end{bmatrix}$, $s \neq 0$

19. *Eigenvalues*: $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = 2$

Eigenvectors: for $\lambda_1 = 0$: $\mathbf{v}_1 = \begin{bmatrix} -r \\ 0 \\ r \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = 1$: $\mathbf{v}_2 = \begin{bmatrix} 0 \\ s \\ 0 \end{bmatrix}$, $s \neq 0$,

for $\lambda_3 = 2$: $\mathbf{v}_3 = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix}$, $t \neq 0$

21. *Eigenvalues*: $\lambda_1 = -2$, $\lambda_2 = 1$, $\lambda_3 = 5$

Eigenvectors: for $\lambda_1 = -2$: $\mathbf{v}_1 = \begin{bmatrix} 0 \\ r \\ r \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = 1$: $\mathbf{v}_2 = \begin{bmatrix} -6s \\ 5s \\ 8s \end{bmatrix}$, $s \neq 0$,

for $\lambda_3 = 5$: $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -5t \\ 2t \end{bmatrix}$, $t \neq 0$

23. F F T T F T F T F T

31. *Eigenvalues*: $\lambda_1 = \dfrac{1 + \sqrt{5}}{2}$, $\lambda_2 = \dfrac{1 - \sqrt{5}}{2}$

Eigenvectors: for $\lambda_1 = \dfrac{1 + \sqrt{5}}{2}$:

$\mathbf{v}_1 = r\begin{bmatrix} (1 + \sqrt{5})/2 \\ 1 \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = \dfrac{1 - \sqrt{5}}{2}$:

$\mathbf{v}_2 = s\begin{bmatrix} (1 - \sqrt{5})/2 \\ 1 \end{bmatrix}$, $s \neq 0$

33. a. Work with the matrix $A + 10I$, whose eigenvalues are approximately 30, 12, 7, and -9.5. (Other answers are possible.);

b. Work with the matrix $A - 10I$, whose eigenvalues are approximately -9.5, -8, -13, and -30. (Other answers are possible.)

37. When the determinant of an $n \times n$ matrix A is expanded according to the definition using expansion by minors, a sum of $n!$ (signed) terms is obtained, each containing a product of one element from each row and from each column. (This is readily proved by induction on n.) One of the $n!$ terms obtained by expanding $|A - \lambda I|$ to obtain $p(\lambda)$ is

$$(a_{11} - \lambda)(a_{22} - \lambda)(a_{33} - \lambda) \cdots (a_{nn} - \lambda).$$

We claim that this is the only one of the $n!$ terms that can contribute to the coefficient of λ^{n-1} in $p(\lambda)$. Any term contributing to the coefficient of λ^{n-1} must contain at least $n - 1$ of the factors in the preceding product; the other factor must be from the remaining row and column, and hence it must be the remaining factor from the diagonal of $A - \lambda I$. Computing the coefficient of λ^{n-1} in the preceding product, we find that, when $-\lambda$ is chosen from all but the factor $a_{ii} - \lambda$ in expanding the product, the resulting contribution to the coefficient of λ^{n-1} in $p(\lambda)$ is $(-1)^{n-1}a_{ii}$. Thus, the coefficient of λ^{n-1} in $p(\lambda)$ is

$$(-1)^{n-1}(a_{11} + a_{22} + a_{33} + \cdots + a_{nn}).$$

Now $p(\lambda) = (-1)^n(\lambda - \lambda_1)(\lambda - \lambda_2)(\lambda - \lambda_3) \cdots (\lambda - \lambda_n)$, and computation of this product shows in the same way that the coefficient of λ^{n-1} is also

$(-1)^{n-1}(\lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n)$.

Thus, $\mathrm{tr}(A) = \lambda_1 + \lambda_2 + \lambda_3 + \cdots + \lambda_n$.

39. Because $\begin{vmatrix} 2-\lambda & -1 \\ 1 & 3-\lambda \end{vmatrix} = \lambda^2 - 5\lambda + 7$, we compute

$$A^2 - 5A + 7I = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}^2 - 5\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix} + \begin{bmatrix} -10 & 5 \\ -5 & -15 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

illustrating the Cayley–Hamilton theorem.

43. The desired result is stated as Theorem 5.3 in Section 5.2. Because an $n \times n$ matrix is the standard matrix representation of the linear transformation $T: \mathbb{R}^n \to \mathbb{R}^n$ and the eigenvalues and eigenvectors of the transformation are the same as those of the matrix, proof of Exercise 42 for transformations implies the result for matrices, and vice versa.

45. **a.** $F_8 = 21$;

b. $F_{30} = 832040$;

c. $F_{50} = 12586269025$;

d. $F_{77} = 5527939700884757$;

e. $F_{150} \approx 9.969216677189304 \times 10^{30}$

47. **a.** 0, 1, 2, 1, −3, −7, −4, 10, 25;

b. $\begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$;

c. $a_{30} = -191694$

49. $\lambda_1 = -1$, $\mathbf{v}_1 = \begin{bmatrix} r \\ -r \\ r \\ r \end{bmatrix}$, $r \neq 0$; $\lambda_2 = 3$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ s \end{bmatrix}$, $s \neq 0$; $\lambda_3 = \lambda_4 = 2$, $\mathbf{v}_3 = \begin{bmatrix} 0 \\ -t \\ u \\ t \end{bmatrix}$, t and u not both zero

51. $\lambda_1 = 16$, $\mathbf{v}_1 = \begin{bmatrix} 0 \\ r \\ 0 \\ 0 \end{bmatrix}$, $r \neq 0$; $\lambda_2 = 36$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 0 \\ s \\ s \end{bmatrix}$, $s \neq 0$; $\lambda_3 = \lambda_4 = 4$, $\mathbf{v}_3 = \begin{bmatrix} t+u \\ t \\ t \\ u \end{bmatrix}$, t and u not both zero

53. *Characteristic polynomial*: $-\lambda^3 - 16\lambda^2 + 48\lambda - 261$

Eigenvalues: $\lambda_1 \approx 1.6033 + 3.3194i$, $\lambda_2 \approx 1.6033 - 3.3194i$, $\lambda_3 \approx -19.2067$

55. *Characteristic polynomial*: $\lambda^4 - 56\lambda^3 + 210\lambda^2 + 22879\lambda + 678658$

Eigenvalues: $\lambda_1 \approx -12.8959 + 13.3087i$,
$\lambda_2 \approx -12.8959 - 13.3087i$,
$\lambda_3 \approx 40.8959 + 17.4259i$,
$\lambda_4 \approx 40.8959 - 17.4259i$

M1. **a.** $\lambda^3 + 16\lambda^2 - 48\lambda + 261$;
$\lambda_1 \approx -19.2067$,
$\lambda_2 \approx 1.6033 + 3.3194i$,
$\lambda_3 \approx 1.6033 - 3.3194i$

b. $\lambda^4 + 14\lambda^3 - 131\lambda^2 + 739\lambda - 21533$;
$\lambda_1 \approx -22.9142$,
$\lambda_2 \approx 10.4799$,
$\lambda_3 \approx -0.7828 + 9.4370i$,
$\lambda_4 \approx -0.7828 - 9.4370i$

M3. λ of minimum magnitude $\approx -2.68877026277667$

M7. λ of maximum magnitude ≈ 19.2077
λ of minimum magnitude ≈ 8.0147

Section 5.2

1. *Eigenvalues*: $\lambda_1 = -5$, $\lambda_2 = 5$

Eigenvectors: for $\lambda_1 = -5$: $\mathbf{v}_1 = \begin{bmatrix} -2r \\ r \end{bmatrix}$, r

for $\lambda_2 = 5$: $\mathbf{v}_2 = \begin{bmatrix} s \\ 2s \end{bmatrix}$, $s \neq 0$

$C = \begin{bmatrix} -2 & 1 \\ 1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} -5 & 0 \\ 0 & 5 \end{bmatrix}$

3. *Eigenvalues*: $\lambda_1 = -1$, $\lambda_2 = 3$

Eigenvectors: for $\lambda_1 = -1$: $\mathbf{v}_1 = \begin{bmatrix} -r \\ r \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = 3$: $\mathbf{v}_2 = \begin{bmatrix} -2s \\ s \end{bmatrix}$, $s \neq 0$

$C = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

5. *Eigenvalues*: $\lambda_1 = -3$, $\lambda_2 = 1$, $\lambda_3 = 7$

Eigenvectors: for $\lambda_1 = -3$: $\mathbf{v}_1 = \begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = 1$: $\mathbf{v}_2 = \begin{bmatrix} s \\ s \\ s \end{bmatrix}$, $s \neq 0$,

for $\lambda_3 = 7$: $\mathbf{v}_3 = \begin{bmatrix} t \\ t \\ 0 \end{bmatrix}$, $t \neq 0$

$C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 7 \end{bmatrix}$

7. *Eigenvalues*: $\lambda_1 = -1$, $\lambda_2 = 1$, $\lambda_3 = 2$

Eigenvectors: for $\lambda_1 = -1$: $\mathbf{v}_1 = \begin{bmatrix} -r \\ 0 \\ r \end{bmatrix}$, $r \neq 0$,

for $\lambda_2 = 1$: $\mathbf{v}_2 = \begin{bmatrix} -s \\ 0 \\ 3s \end{bmatrix}$, $s \neq 0$,

for $\lambda_3 = 2$: $\mathbf{v}_3 = \begin{bmatrix} 0 \\ t \\ 0 \end{bmatrix}$, $t \neq 0$

$C = \begin{bmatrix} -1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & 0 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

9. Yes, the matrix is symmetric.

11. Yes, the eigenvalues are distinct.

13. F T T F T F F T F T

15. Assume that A is a square matrix such that $D = C^{-1}AC$ is a diagonal matrix for some invertible matrix C. Because $CC^{-1} = I$, we have $(CC^{-1})^T = (C^{-1})^T C^T = I^T = I$, so $(C^T)^{-1} = (C^{-1})^T$. Then $D^T = (C^{-1}AC)^T = C^T A^T (C^T)^{-1}$, so A^T is similar to the diagonal matrix D^T.

19. b. $A = O$ c. $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

21. $T([x, y]) = \dfrac{1}{1+m^2}[(1 - m^2)x + 2my,$ $2mx + (m^2 - 1)y]$

23. If A and B are similar square matrices, then $B = C^{-1}AC$ for some invertible matrix C. Let λ be an eigenvalue of A, and let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be a basis for E_λ. Then Exercise 22 shows that $C^{-1}\mathbf{v}_1, C^{-1}\mathbf{v}_2, \ldots, C^{-1}\mathbf{v}_k$ are eigenvectors of B corresponding to λ, and Exercise 37 in Section 2.1 shows that these vectors are independent. Thus the eigenspace of λ relative to B has dimension at least as great as the eigenspace of λ relative to A. By the symmetry of the similarity relation (see Exercise 16), the eigenspace of λ relative to A has dimension at least as great as the eigenspace of λ relative to B. Thus the dimensions of these eigenspaces are equal—that is, the geometric multiplicity of λ is the same relative to A as relative to B.

31. Diagonalizable

33. Complex (but not real) diagonalizable

35. Complex (but not real) diagonalizable

37. Not diagonalizable

39. Real diagonalizable

41. Complex (but not real) diagonalizable

Section 5.3

1. a. $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$;

b. Neutrally stable;

c. $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A^k \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{2}{3}(1)^k \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{3}\left(-\frac{1}{2}\right)^k \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. The sequence starts $0, 1, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}$ and $A^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{2}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{24}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{5}{8} \\ \frac{3}{4} \end{bmatrix}$, which checks.

d. For large k, we have $\begin{bmatrix} a_{k-1} \\ a_k \end{bmatrix} \approx \frac{2}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$, so $a_k \approx \frac{2}{3}$.

3. a. $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$; b. Neutrally stable;

c. $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A^k \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{3}(1)^k \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{3}\left(-\frac{1}{2}\right)^k \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. The sequence starts $1, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{8}$ and $A^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{24}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} \\ \frac{1}{4} \end{bmatrix}$, which checks.

d. For large k, we have $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} \approx \frac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$, so $a_k \approx \frac{1}{3}$.

5. a. $A = \begin{bmatrix} 1 & \frac{3}{4} \\ 1 & 0 \end{bmatrix}$; b. Unstable;

c. $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A^k \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{4}\left(\frac{3}{2}\right)^k \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \frac{1}{4}\left(-\frac{1}{2}\right)^k \cdot \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. The sequence starts $0, 1, 1, \frac{7}{4}, \frac{5}{2}$ and $A^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{27}{32}\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \frac{1}{32}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{80}{32} \\ \frac{56}{32} \end{bmatrix} = \begin{bmatrix} \frac{5}{2} \\ \frac{7}{4} \end{bmatrix}$, which checks.

d. For large k, we have $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} \approx \frac{1}{4}\left(\frac{3}{2}\right)^k \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, so $a_k \approx \frac{3^k}{2^{k+1}}$; and a_k approaches ∞ as k approaches ∞.

7. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -k_1e^{-3t} + 4k_2e^{4t} \\ k_1e^{-3t} + 3k_2e^{4t} \end{bmatrix}$

9. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2k_1e^{t} + k_2e^{4t} \\ k_1e^{t} + k_2e^{4t} \end{bmatrix}$

11. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k_1e^{-3t} + k_2e^{t} + k_3e^{7t} \\ k_2e^{t} + k_3e^{7t} \\ k_2e^{t} \end{bmatrix}$

13. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -k_1e^{-t} - k_2e^{t} \\ k_3e^{2t} \\ k_1e^{-t} + 3k_2e^{t} \end{bmatrix}$

CHAPTER 6

Section 6.1

1. $\frac{2}{5}[3, 4]$

3. $\mathbf{p}_1 = [1, 0, 0]$, $\mathbf{p}_2 = [0, 2, 0]$, $\mathbf{p}_3 = [0, 0, 1$

5. $\mathbf{p} = -\frac{1}{3}[2, -3, 1, 2]$

7. sp([1, 0, 1], [−2, 1, 0])

9. sp([−12, 4, 5])

11. sp([2, −7, 1, 0], [−1, −2, 0, 1])

13. a. $-5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ b. $-5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

15. $\frac{1}{3}[5, 4, 1]$ **17.** $\frac{1}{7}[5, 3, 1]$

19. $\frac{1}{6}[2, -1, 5]$ **21.** $\frac{1}{3}[3, -2, -1, 1]$

23. F T T T F T T F F T **29.** $\frac{4}{5}$

31. $\sqrt{\frac{14}{5}}$ **33.** $\frac{\sqrt{161}}{3\sqrt{3}}$ **35.** $\sqrt{10}$ **37.**

Section 6.2

1. $[2, 3, 1] \cdot [-1, 1, -1] = -2 + 3 - 1 = ($ so the generating set is orthogonal. $\mathbf{b}_W = \frac{1}{42}[136, 29, 103]$.

3. $[1, -1, -1, 1] \cdot [1, 1, 1, 1] = 1 - 1 - 1 + 1 = 0$, $[1, -1, -1, 1] \cdot [-1, 0, 0, 1] = -1 + 0 + 0 + 1 = 0$, and $[1, 1, 1, 1] \cdot [-1, 0, 0, 1] = -1 + 0 + 0 + 1 = 0$, so the generating set is orthogonal; $\mathbf{b}_W = [2, 2, 2, 1]$.

5. $\left\{\frac{1}{\sqrt{5}}[1, 0, -2], \frac{1}{\sqrt{70}}[6, -5, 3]\right\}$

7. $\left\{[0, 1, 0], \frac{1}{\sqrt{2}}[1, 0, 1]\right\}$

9. $\left\{\frac{1}{\sqrt{2}}[1, 0, 1], \frac{1}{\sqrt{3}}[-1, 1, 1], \frac{1}{\sqrt{6}}[1, 2, -\right.$

11. $\left\{\frac{1}{\sqrt{2}}[1, 0, 1, 0], [0, 1, 0, 0], \frac{1}{\sqrt{6}}[1, 0, -1\right.$