姓名: Solution　　葉均承　化學一微積分

學號: _____　　**Quiz 4**　　考試日期: 2020/04/13

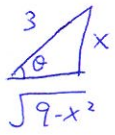
不可使用手機、計算器，禁止作弊!

60%

1. (30%) Evaluate the integral.

$$\int \sqrt{9-x^2}\,dx$$

$\int \sqrt{9-x^2}\,dx$　$x = 3\sin\theta$, $dx = 3\cos\theta\,d\theta$ → $\sin\theta = \frac{x}{3}$



$\therefore \cos\theta = \frac{\sqrt{9-x^2}}{3}$

$\therefore \theta = \arcsin\left(\frac{x}{3}\right)$

$= \int \sqrt{9-9\sin^2\theta}\cdot 3\cos\theta\,d\theta$

$= \int 9\cos^2\theta\,d\theta = \frac{9}{2}\int 1+\cos(2\theta)\,d\theta$

$= \frac{9}{2}(\theta + \frac{1}{2}\sin(2\theta)) + C = \frac{9}{2}(\theta + \sin\theta\cos\theta) + C$

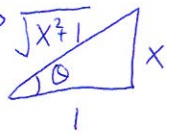
$= \frac{9}{2}\left(\arcsin\left(\frac{x}{3}\right) + \left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right)\right) + C$

40%

2. (30%) Evaluate the integral.

$$\int \frac{1}{(x^2+1)^{3/2}}\,dx$$

$\int \frac{1}{(x^2+1)^{3/2}}\,dx$　$x = \tan\theta$, $dx = \sec^2\theta\,d\theta$



$\therefore \sin\theta = \frac{x}{\sqrt{x^2+1}}$

$= \int \frac{1}{\sec^3\theta}\cdot \sec^2\theta\,d\theta$

$= \int \cos\theta\,d\theta = \sin\theta + C$

$= \frac{x}{\sqrt{x^2+1}} + C$