1. 請框出答案. 2. 不可使用手機、計算器，禁止作弊!

1. (40 points) Let $\vec{u} = [2,\ 3,\ 1], \vec{v} = [1,\ 2,\ 3]$. Compute the indicated quantity (量), if it is defined.

$\|\vec{v} - 2\vec{u}\| = \underline{\quad \|[-3,\ -4,\ 1]\| = \sqrt{26} \quad}$

2. (30 points) Prove or disprove that every vector of nonzero magnitude in $\mathbb{R}^n$ is nonzero.

Solution：

This statement is True!

Let $\vec{v} = [v_1, v_2, ..., v_n]$ be a nonzero vector in $\mathbb{R}^n$.

Since $\vec{v}$ is not zero, at least one v_i is not zero.

Then each $v_i^2 \geq 0$ and the sum $v_1^2 + v_2^2 + ... + v_n^2 > 0$.

So the length of $\vec{v}$ is

$$\|\vec{v}\| = \sqrt{v_1^2 + v_2^2 + ... + v_n^2} > 0,$$

which means the vector is not zero.

3. (30 points) Prove or disprove that there are exactly two unit vectors perpendicular (垂直) to any given nonzero vectors in $\mathbb{R}^n$.

Solution：

The statement is False!

For $n = 3$, $\vec{e_1} = [1, 0, 0]$ is perpendicular to every vector in $S = sp(\vec{e_2}, \vec{e_3}) = sp([0, 1, 0], [0, 0, 1])$.