

姓名: _____

葉均承

應數一線性代數

學號: _____

Quiz 7

考試日期: 2025/10/29

不可使用手機、計算器，禁止作弊!

1. Given $A \sim H$, please answer the following questions.

$$A = \begin{bmatrix} 9 & 4 & 0 & 6 & 1 \\ 9 & 0 & 2 & -2 & 5 \\ -6 & 4 & 2 & 4 & -2 \\ -3 & 6 & 1 & 8 & -3 \\ 3 & -4 & 3 & -9 & 6 \end{bmatrix}, H = \begin{bmatrix} 3 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 3 & -1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) the **rank** of matrix A, is _____.

(b) Is A invertible? _____.

(c) a basis for the **row space** of A is _____.

(d) a basis for the **column space** of A is _____.

(e) a basis for the **nullspace** of A is _____.

2. Prove or disprove (反證) the following statement.

(a) The column space of AC is contained in the column space of A .

(b) $\text{rank}(AC) \leq \text{rank}(A)$.

(c) The column space of AC is contained in the column space of C .

(d) $\text{rank}(AC) \leq \text{rank}(C)$.

(e) Let \vec{v}, \vec{w} be column vectors in \mathbb{R}^n and let A be an $n \times n$ matrix. If $A\vec{v}$ and $A\vec{w}$ are linearly independent, then \vec{v} and \vec{w} are linearly independent

(f) Let \vec{v}, \vec{w} be column vectors in \mathbb{R}^n and let A be an $n \times n$ matrix. If \vec{v} and \vec{w} are linearly independent, then $A\vec{v}$ and $A\vec{w}$ are linearly independent

3. Find all scalars s if any exist, such that $[1, 0, 1], [2, s, 3], [1, -2s, 0]$ are linearly independent.