姓名: sol.

葉均承　化學一微積分

學號: ______

Quiz 9

考試日期: 2020/06/01

不可使用手機、計算器，禁止作弊!

1. (30%) Determin whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$ is absolutely convergent, conditionally convergent, or divergent.

1. Check abs. conv. , i.e. $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

$\because \ln(n) < n$ for $n \geq 2$ $\therefore \frac{1}{\ln n} > \frac{1}{n}$

$\therefore \sum_{n=2}^{\infty} \frac{1}{\ln n} > \sum_{n=2}^{\infty} \frac{1}{n}$: div. by p-test $\therefore \sum_{n=2}^{\infty} \frac{1}{\ln n}$: div. by I.T.

2. check conv., i.e. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

① $\lim_{n\to\infty} \frac{1}{\ln n} = 0$ ② $\ln(n+1) > \ln(n)$ $\therefore \frac{1}{\ln(n+1)} < \frac{1}{\ln(n)}$ i.e. $\{\frac{1}{\ln n}\} \downarrow$

$\therefore$ by AST, $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$: conv.

$\therefore \sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$: conditionally convergent.

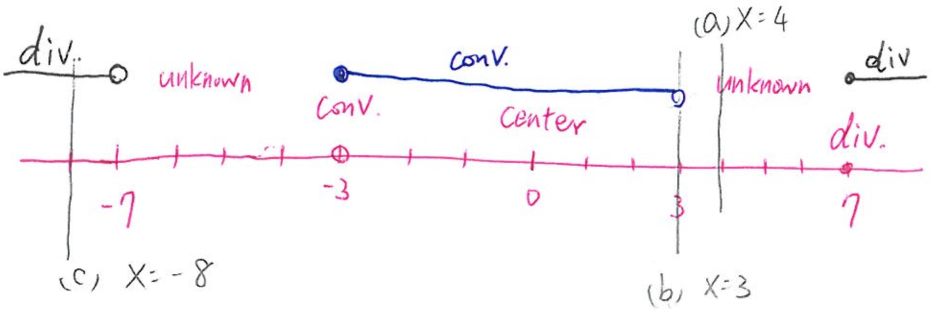
2. (30%) Given that the power series $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -3$ and diverges when $x = 7$, what can be said about the following series (converge/diverge/inconclusive)?

→ center at $x=0$

(a) $\sum_{n=0}^{\infty} c_n 4^n$ inconclusive

(b) $\sum_{n=0}^{\infty} c_n 3^n$ inconclusive

(c) $\sum_{n=0}^{\infty} c_n (-8)^n$ diverge.



姓名: ________________ 葉均承 化學一微積分

學號: ________________ **Quiz 9** 考試日期: 2020/06/01

3. (40%) Determine the radius and interval of convergence for the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+3)^n}{n(-2)^n}$.
(Make sure you check the endpoints.)

$$a_n = \frac{(x+3)^n}{n(-2)^n}$$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(x+3)^{n+1}\, n(-2)^n}{(n+1)(-2)^{n+1}(x+3)^n}\right| = \lim_{n\to\infty} \left|\frac{(x+3)\,n}{(-2)(n+1)}\right| = \left|\frac{x+3}{-2}\right| = \frac{1}{2}|x+3|$$

need $\frac{1}{2}|x+3| < 1$

$\therefore |x+3| < 2 \qquad \therefore \boxed{R=2}$

$-5 < x < -1$

Check:

$\boxed{x=-5}$ $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n(-2)^n} = \sum_{n=1}^{\infty} \frac{1}{n}$ $\rightarrow$ div by P-test

$\boxed{x=-1}$ $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n(-2)^n} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$ $\because b_n \downarrow$ and $\lim_{n\to\infty} b_n = 0$ $\therefore$ conv. by AST

$$\therefore \boxed{I = (-5, -1]}$$