

應數一線性代數 2024 秋, 期末考

學號: _____, 姓名: _____

本次考試共有 8 頁 (包含封面), 有 13 題。如有缺頁或漏題, 請立刻告知監考人員。

考試須知:

- 請在第一頁及最後一頁填上姓名學號。
- 不可翻閱課本或筆記。
- 1-7 題為填空題。
- 8-13 題為計算證明題。請寫出計算過程, 閱卷人員會視情況給予部份分數。沒有計算過程, 就算回答正確答案也不會得到滿分。答卷請清楚乾淨, 儘可能標記或是框出最終答案。
- 書寫空間不夠時, 可利用試卷背面, 但須標記清楚。

高師大校訓: 誠敬宏遠

誠: 一生動念都是誠實端正的。 敬: 就是對知識的認真尊重。
宏: 開拓視界, 恢宏心胸。 遠: 任重致遠, 不畏艱難。

請簽名保證以下答題都是由你自己作答的, 並沒有得到任何的外部幫助。

簽名: _____

1. (10 points) Linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ satisfy $T([1, 3]) = [2, 2, a]$, and $T([2, 1]) = [3, b, 6]$. If T is NOT one-to-one, then $a + b =$ (1)

2. (10 points) Given B and the inverse matrix of B are below , then $a =$ (2)

$$B = \begin{bmatrix} 0 & -2 & 1 \\ 3 & 2 & 1 \\ 1 & 5 & -1 \end{bmatrix}, \quad A^{-1} = \begin{bmatrix} * & * & * \\ a & * & * \\ * & * & * \end{bmatrix}$$

3. (10 points) Suppose that T is a linear transformation with standard matrix representation A , and that A is a 9×15 matrix such that the nullspace of A has dimension 5.

(a) The dimension of the range of T is (3) . (b) The dimension of the kernel of T is (4) .

4. (10 points) Find the area of the parallelogram(平行四邊形) in \mathbb{R}^3 determined by the vectors $[2, 1, 3]$ and $[4, -3, 1]$. The area is (5) .

5. (10 points) Let P_3 be the vector space of polynomials with degree at most 3 with real coefficients. The coordinate vector of $7x^3 + 3x^2 - 2x + 3$ relative to the ordered basis $(x^2 + x, x^3, x^3 + x, 2x^2 + 1)$ is (6) .

6. (10 points) Suppose that C is a 6×6 matrix with determinant 4. The $\det(7C^{-1})$ is (7) .

7. (10 points)

$$D = \begin{bmatrix} 2 & 4 & -2 & 0 & 5 & -1 & 9 \\ 1 & 2 & -1 & 3 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 & 3 & 1 & 2 \\ 0 & 5 & 0 & 0 & 1 & 0 & 0 \\ 1 & 2 & -1 & 0 & 4 & 0 & 3 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}, \text{ The determinant of } D \text{ is } \underline{(8)} .$$

8. (10 points) Let P_3 be the vector space of polynomials with degree at most 3 with real coefficients. $T : P_3 \rightarrow P_3$ be defined by $T(p(x)) = 2p(x) - 3\frac{d}{dx}p(x)$

(a) Prove that T is a linear transformation.

(b) Let the ordered basis for P_3 is $B = (1, x + 1, x^2, x^3 - 1)$. Find the matrix representation A of T relative to the ordered bases B .

Answer: (b) $A =$ _____

9. (10 points) Let F is the vector space of all functions mapping \mathbb{R} into \mathbb{R} and $S = \{\sin(-x), 1, \sin(x), \sin(2x)\}$.

Is S linear independent in F ? (Yes / No) . If not, find a basis of $\text{sp}(S)$ _____.

10. (10 points) (a) Build a linear transformation that is one-to-one but not onto.

(b) Build a linear transformation that is onto but not one-to-one.

11. (10 points) Determine the set S_1 of all functions f such that $f(0) = 0$ is a subspace in the vector space F of all functions mapping \mathbb{R} into \mathbb{R} .

Answer: Is S_1 a subspace of F ? (Yes / No)

12. (10 points) Consider the set \mathbb{R}^2 , with the addition defined by $[x, y] \oplus [a, b] = [x + a + 2, y + b]$, and with scalar multiplication defined by $r \otimes [x, y] = [r(x + 2) - 2, ry]$.

a. Is this set a vector space? (Yes / No)

Hint: Show by verifying the closed under two operations, A1-A4 and S1-S4.

b. If the set is a vector space, then find the zero vector and the additive inverse (加法反元素) in this vector space. *Hint:* The zero vector may NOT be the vector $[0, 0]$.

Answer: the zero vector is _____, for any vectors $[x, y]$, the $-[x, y]$ is _____

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