3. a. $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ 1 & 0 \end{bmatrix}$; **b.** Neutrally stable;

c. $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A^k \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{3}(1)^k \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \frac{1}{3}\left(-\frac{1}{2}\right)^k \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. The sequence starts $1, 0, \frac{1}{2}, \frac{1}{4}, \frac{3}{8}$

and $A^3 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - \frac{1}{24}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{3}{8} \\ \frac{1}{4} \end{bmatrix}$,

which checks.

d. For large k, we have $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} \approx \frac{1}{3}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$,

so $a_k \approx \frac{1}{3}$.

5. a. $A = \begin{bmatrix} 1 & \frac{3}{4} \\ 1 & 0 \end{bmatrix}$; **b.** Unstable;

c. $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} = A^k \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{1}{4}\left(\frac{3}{2}\right)^k \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \frac{1}{4}\left(-\frac{1}{2}\right)^k \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. The sequence starts $0, 1, 1, \frac{7}{4}, \frac{5}{2}$

and $A^3 \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{27}{32}\begin{bmatrix} 3 \\ 2 \end{bmatrix} + \frac{1}{32}\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{80}{32} \\ \frac{56}{32} \end{bmatrix}$

$= \begin{bmatrix} \frac{5}{2} \\ \frac{7}{4} \end{bmatrix}$, which checks.

d. For large k, we have $\begin{bmatrix} a_{k+1} \\ a_k \end{bmatrix} \approx \frac{1}{4}\left(\frac{3}{2}\right)^k \begin{bmatrix} 3 \\ 2 \end{bmatrix}$,

so $a_k \approx \frac{3^k}{2^{k+1}}$; and a_k approaches ∞ as k approaches ∞.

7. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -k_1e^{-3t} + 4k_2e^{4t} \\ k_1e^{-3t} + 3k_2e^{4t} \end{bmatrix}$

9. $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2k_1e^{t} + k_2e^{4t} \\ k_1e^{t} + k_2e^{4t} \end{bmatrix}$

11. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k_1e^{-3t} + k_2e^{t} + k_3e^{7t} \\ k_2e^{t} + k_3e^{7t} \\ k_2e^{t} \end{bmatrix}$

13. $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -k_1e^{-t} - k_2e^{t} \\ k_3e^{2t} \\ k_1e^{-t} + 3k_2e^{t} \end{bmatrix}$

CHAPTER 6

Section 6.1

1. $\frac{2}{5}[3, 4]$

3. $\mathbf{p}_1 = [1, 0, 0]$, $\mathbf{p}_2 = [0, 2, 0]$, $\mathbf{p}_3 = [0, 0, 1$

5. $\mathbf{p} = -\frac{1}{3}[2, -3, 1, 2]$

7. $\text{sp}([1, 0, 1], [-2, 1, 0])$

9. $\text{sp}([-12, 4, 5])$

11. $\text{sp}([2, -7, 1, 0], [-1, -2, 0, 1])$

13. a. $-5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ **b.** $-5\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

15. $\frac{1}{3}[5, 4, 1]$ **17.** $\frac{1}{7}[5, 3, 1]$

19. $\frac{1}{6}[2, -1, 5]$ **21.** $\frac{1}{3}[3, -2, -1, 1]$

23. F T T T F T T F F T **29.** $\frac{4}{5}$

31. $\sqrt{\frac{14}{5}}$ **33.** $\frac{\sqrt{161}}{3\sqrt{3}}$ **35.** $\sqrt{10}$ **37.**

Section 6.2

1. $[2, 3, 1] \cdot [-1, 1, -1] = -2 + 3 - 1 = ($
so the generating set is orthogonal.
$\mathbf{b}_W = \frac{1}{42}[136, 29, 103]$.

3. $[1, -1, -1, 1] \cdot [1, 1, 1, 1] = 1 - 1 - 1 + 1 = 0$,
$[1, -1, -1, 1] \cdot [-1, 0, 0, 1] = -1 + 0 + 0 + 1 = 0$, and
$[1, 1, 1, 1] \cdot [-1, 0, 0, 1] = -1 + 0 + 0 + 1 = 0$,
so the generating set is orthogonal; $\mathbf{b}_W = [2, 2, 2, 1]$.

5. $\left\{\frac{1}{\sqrt{5}}[1, 0, -2], \frac{1}{\sqrt{70}}[6, -5, 3]\right\}$

7. $\left\{[0, 1, 0], \frac{1}{\sqrt{2}}[1, 0, 1]\right\}$

9. $\left\{\frac{1}{\sqrt{2}}[1, 0, 1], \frac{1}{\sqrt{3}}[-1, 1, 1], \frac{1}{\sqrt{6}}[1, 2, -\right.$

11. $\left\{\frac{1}{\sqrt{2}}[1, 0, 1, 0], [0, 1, 0, 0], \frac{1}{\sqrt{6}}[1, 0, -1\right.$

13. $\left[\frac{9}{2}, -3, \frac{9}{2}\right]$

15. $\left[\frac{4}{3}, 0, -\frac{1}{3}, \frac{5}{3}\right]$

17. $\left\{\frac{1}{\sqrt{2}}[1, 0, 1, 0], \frac{1}{\sqrt{6}}[-1, 2, 1, 0], \frac{1}{\sqrt{3}}[1, 1, -1, 0], [0, 0, 0, 1]\right\}$

19. $\{[3, -2, 0, 1], [-9, -8, 14, 11]\}$

21. $\left\{\frac{1}{\sqrt{6}}[2, 1, 1], \frac{1}{\sqrt{2}}[0, -1, 1]\right\}$

23. $\{[2, 1, -1, 1], [1, 1, 3, 0], [-24, 9, 5, 44]\}$

25. F T T F F T F T T T

27. $Q = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & 2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & -1/\sqrt{6} \end{bmatrix}$, $R = \begin{bmatrix} \sqrt{2} & \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} & -1/\sqrt{3} \\ 0 & 0 & 4/\sqrt{6} \end{bmatrix}$

33. $\left\{\sqrt{\frac{2}{\pi}}\sin x, \sqrt{\frac{2}{\pi}}\cos x\right\}$

35. $\left\{1, \sqrt{\frac{2}{4e - e^2 - 3}}\,(e^x - e + 1)\right\}$

Section 6.3

1. Let A be the given matrix. Then
$$A^TA = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
so A is orthogonal and $A^{-1} = A^T = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$.

3. Let A be the given matrix. Then
$$A^TA = \frac{1}{7}\begin{bmatrix} 2 & 3 & -6 \\ -3 & 6 & 2 \\ 6 & 2 & 3 \end{bmatrix}\frac{1}{7}\begin{bmatrix} 2 & -3 & 6 \\ 3 & 6 & 2 \\ -6 & 2 & 3 \end{bmatrix} = \frac{1}{49}\begin{bmatrix} 49 & 0 & 0 \\ 0 & 49 & 0 \\ 0 & 0 & 49 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$
so A is orthogonal and $A^{-1} = A^T = \frac{1}{7}\begin{bmatrix} 2 & 3 & -6 \\ -3 & 6 & 2 \\ 6 & 2 & 3 \end{bmatrix}$.

5. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{6} & \frac{1}{6} \end{bmatrix}$

7. $\frac{1}{49}\begin{bmatrix} 1 & \frac{3}{2} & -3 \\ -3 & 6 & 2 \\ 6 & 2 & 3 \end{bmatrix}$

9. $\pm\frac{1}{\sqrt{6}}\begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$

11. $\pm\frac{\sqrt{23}}{6}$

13. $\frac{1}{\sqrt{2}}\begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

15. $\frac{1}{2}\begin{bmatrix} 1 & -2 & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ 1 & 2 & 1 \end{bmatrix}$

17. $\begin{bmatrix} 0 & \frac{1}{2} & -1\sqrt{2} & \frac{1}{2} \\ -1/\sqrt{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 1/\sqrt{2} & -\frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & 1/\sqrt{2} & \frac{1}{2} \end{bmatrix}$

19. F T T T T T T T F T

23. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

27. An orthogonal matrix A gives rise to an orthogonal linear transformation $T(\mathbf{x}) = A\mathbf{x}$ that preserves the magnitude of vectors. Thus, if $A\mathbf{v} = \lambda\mathbf{v}$, so that $T(\mathbf{v}) = \lambda\mathbf{v}$, we must have $\|\mathbf{v}\| = \|\lambda\mathbf{v}\| = |\lambda|\,\|\mathbf{v}\|$. If $\mathbf{v}$ is an eigenvector, so that $\mathbf{v} \neq \mathbf{0}$, it follows that $|\lambda| = 1$; so $\lambda = \pm 1$.

33. No

35. Yes

37. Yes

Section 6.4

1. $P = \frac{1}{6}\begin{bmatrix} 4 & 2 & -2 \\ 2 & 1 & -1 \\ -2 & -1 & 1 \end{bmatrix}$, projection $= \frac{1}{2}\begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$

3. $P = \frac{1}{35}\begin{bmatrix} 34 & -3 & 5 \\ -3 & 26 & 15 \\ 5 & 15 & 10 \end{bmatrix}$, projection $= \frac{1}{35}\begin{bmatrix} 86 \\ 13 \\ 25 \end{bmatrix}$

5. $P = \frac{1}{6}\begin{bmatrix} 5 & -1 & 2 \\ -1 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$, projection $= \frac{1}{3}\begin{bmatrix} 2 \\ 8 \\ 5 \end{bmatrix}$

7. $P = \frac{1}{21}\begin{bmatrix} 10 & -1 & 3 & 10 \\ -1 & 19 & 6 & -1 \\ 3 & 6 & 3 & 3 \\ 10 & -1 & 3 & 10 \end{bmatrix}$, projection $= \frac{1}{21}\begin{bmatrix} 41 \\ 40 \\ 27 \\ 41 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

13. If P is the projection matrix for a subspace W of $\mathbb{R}^n$ and if $\mathbf{b} \in \mathbb{R}^n$ is a column vector, then the projection of $\mathbf{b}$ on W is $P\mathbf{b}$. Because $P\mathbf{b}$ is in W, geometry indicates that the projection of $P\mathbf{b}$ on W is again $P\mathbf{b}$. Thus, $P(P\mathbf{b}) = P\mathbf{b}$, so $P^2\mathbf{b} = P\mathbf{b}$ and $(P^2 - P)\mathbf{b} = \mathbf{0}$ for all $\mathbf{b} \in \mathbb{R}^n$. It follows from Exercise 41 in Section 1.3 that $P^2 - P = O$, so $P^2 = P$.

15. F T T F F T F F T T

17. I

19. a. 0, 1

b. 0 has geometric and algebraic multiplicity $n - k$,

1 has geometric and algebraic multiplicity k.

c. Because the algebraic and geometric multiplicities of each eigenvalue are equal, P is a diagonalizable matrix.

21. The $n \times n$ identity matrix I for each positive integer n.

23. $\begin{bmatrix} \frac{9}{25} & \frac{12}{25} & 0 \\ \frac{12}{25} & \frac{16}{25} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

25. $\frac{1}{49}\begin{bmatrix} 13 & -18 & 0 & -12 \\ -18 & 36 & 12 & 0 \\ 0 & 12 & 13 & -18 \\ -12 & 0 & -18 & 36 \end{bmatrix}$

27. $\frac{1}{3}\begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 \\ 0 & -1 & 2 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$

29. $\begin{bmatrix} 10 \\ -4 \\ -2 \end{bmatrix}$

31. $\begin{bmatrix} 14 \\ 0 \\ -7 \\ 14 \end{bmatrix}$

33. Referring to Figure 6.11, we see that, for $\mathbf{p} = P\mathbf{b}$, the vector from the tip of $\mathbf{b}$ to the tip of $\mathbf{p}$ is $\mathbf{p} - \mathbf{b}$, which is also the vector from the tip of $\mathbf{p}$ to the tip of $\mathbf{b}_r$. Thus, the vector $\mathbf{b}_r = \mathbf{b} + 2(\mathbf{p} - \mathbf{b}) = 2\mathbf{p} - \mathbf{b} = 2(P\mathbf{b}) - \mathbf{b} = (2P - I)\mathbf{b}$.

35. The projections are approximately $\begin{bmatrix} 1.151261 \\ -1.184874 \\ 3.89916 \end{bmatrix}$, $\begin{bmatrix} 3 \\ 3 \\ -1 \end{bmatrix}$, and $\begin{bmatrix} 1.932773 \\ -.806723 \\ 4.378151 \end{bmatrix}$, respectively.

37. The projections are approximately $\begin{bmatrix} 1.864516 \\ 1.496774 \\ -.135484 \\ 2.819355 \end{bmatrix}$, $\begin{bmatrix} 1.058064 \\ .787097 \\ -.941936 \\ 2.077419 \end{bmatrix}$, and $\begin{bmatrix} 4.116129 \\ 2.574194 \\ 1.116129 \\ 3.154839 \end{bmatrix}$ respectively.

Section 6.5

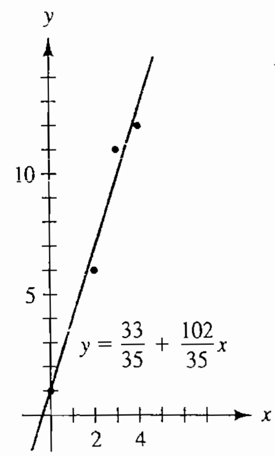
1. a. $y = \frac{116.4}{59} + \frac{60.4}{59}x$

b. $\approx$ 7.092 inches

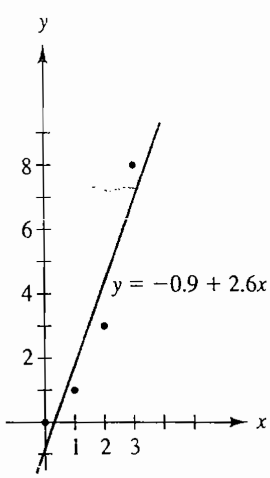
3. a. $y = .528e^{.274x}$

b. $\approx$ \$27,

5. $y = \frac{33}{35} + \frac{102}{35}x$



7. $y = -0.9 + 2.6x$



9. $y = 0.1 - 0.4x + x^2$

11. $y = 1.6 + 2x$

13. 4.5 min

15. Let $t = x - c$, where $c = (\Sigma_{i=1}^{m} a_i)/m$. The data points $(a_1 - c, b_1), (a_2 - c, b_2), \ldots, (a_m - c, b_m)$ have the property that $\Sigma_{i=1}^{m} (a_i - c) = 0$. Exercise 14 then shows that these data points have least-squares linear fit given by $y = r_0 + r_1 t$, where r_0 and r_1 have the values given in Exercise 14. Making the substitution $t = x - c$, we see that the data points $(a_1, b_1), (a_2, b_2), \ldots, (a_m, b_m)$ have the least-squares linear fit given by $y = r_0 + r_1(x - c)$.

17. $\bar{x} = \begin{bmatrix} -\frac{1}{5} \\ \frac{3}{5} \end{bmatrix}$

19. $\bar{x} = \begin{bmatrix} 0 \\ 2 \\ -\frac{1}{4} \end{bmatrix}$

21. F F T T F F T T F F

23. See answer to Exercise 17.

25. See answer to Exercise 19.

27. The computer gave the fit $y = 0.7587548 + 1.311284x$ with a least-squares sum of 0.03891051.

29. We achieved a least-squares sum of 5.838961 with the expoential fit $y = 0.8e^{0.2x}$. The computer achieved a least-squares sum of 6.34004 with the exponential fit $y = 0.8874836e^{0.1960377x}$. The fit using logarithms tries to fit the smaller y-value data accurately at the expense of the larger y-value data, so that the *percent* accuracy of fit to the y-coordinates is as good as possible.

31. The computer gave the fit $y = 12.03846 - 1.526374x$ with a least-squares sum of 0.204176.

33. $y \approx 5.476 - 0.75x + 0.2738x^2$

35. $y \approx 5.632 - 1.139x + 0.1288x^2 + 0.05556x^3 + 0.01512x^4$

37. $y = -5 - 8x + 9x^2 - x^3$

HAPTER 7

ction 7.1

1. $[-1, 1]$

3. $[-4, -2, 1, 5]$

5. $[3, 5, 1, 1]$

7. $2x^2 + 6x + 2$

9. $\begin{bmatrix} -1 \\ -4 \\ -2 \end{bmatrix}$

11. a. $C_{B,B'} = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 0 \\ 1 & 0 & 3 \end{bmatrix}$;

b. $C_{B',B} = \begin{bmatrix} -6 & 3 & 4 \\ 9 & -4 & -6 \\ 2 & -1 & -1 \end{bmatrix}$

13. a. $C_{B,B'} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$;

b. $C_{B',B} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix}$

15. $C_{B',B} = \begin{bmatrix} 1 & 2 & 1 \\ -1 & -2 & -2 \\ 0 & 1 & 0 \end{bmatrix}$

17. $C_{B',B} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}$

19. $C_{B,B'} = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 1 & -1 & 0 & 1 \\ 0 & 1 & -1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

21. $C_{B,B'} = \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$

23. T F T F T F T F T T

25. $C_{B,B''} = C_{B',B}C_{B,B'}$

Section 7.2

1. $R_B = \begin{bmatrix} 6 & 7 \\ -3 & -3 \end{bmatrix}$, $R_{B'} = \begin{bmatrix} 1 & -1 \\ 1 & 2 \end{bmatrix}$,
$C = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

3. $R_B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, $R_{B'} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$,
$C = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{bmatrix}$