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葉均承 化學一微積分

學號: ______________

Quiz 10

考試日期: 2020/06/08

不可使用手機、計算器，禁止作弊!
背面還有題目

以下為參考公式:

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \qquad , x \in (-1,1)$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \qquad , x \in \mathbb{R}$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \qquad , x \in \mathbb{R}$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!} \qquad , x \in \mathbb{R}$$

$$\ln(1+x) = \sum_{n=0}^{\infty} (-1)^{n+1} \frac{x^n}{n} \qquad , x \in (-1,1]$$

$$\tan^{-1}(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{2n+1} \qquad , x \in [-1,1]$$

D

d

1. (25 points) Use the Maclaurin polynomial for $x\cos(x^2)$ to approximate $\int x\cos(x^2)\,dx$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(2n)!}$

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n}}{(2n)!}$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+1}}{(4n+1)(2n)!}$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(4n+2)(2n)!}$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{4n+2}}{(2n)!}$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\therefore \cos(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{(x^2)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n}}{(2n)!}$$

$$\therefore x\cos(x^2) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+1}}{(2n)!}$$

$$\therefore \int x\cos(x^2)\,dx = \sum_{n=0}^{\infty} (-1)^n \int \frac{x^{4n+1}}{(2n)!}\,dx$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{4n+2}}{(2n)!\,(4n+2)}$$

C 2. (25 points) Find the sum of the series. $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$ $= \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{\pi}{6}\right)^{2n}}{(2n)!} = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

(a) 0
(b) 1/2
(c) $\sqrt{3}/2$
(d) $e^{\pi/6}$
(e) $e^{\pi^2/36}$
(f) 1

3. (50 points) Find the Taylor series for $f(x) = \frac{6}{x}$ centered at $x = -4$

$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$ where $a = -4$

10%

$f^{(0)}(x) = \frac{6}{x} = 6x^{-1} \Rightarrow f^{(0)}(-4) = \frac{-6}{4}$

$f^{(1)}(x) = -6x^{-2} \Rightarrow f^{(1)}(-4) = \frac{-6}{4^2}$

$f^{(2)}(x) = 2 \times 6x^{-3}$

$f^{(3)}(x) = -3 \cdot 2 \times 6x^{-4}$

$f^{(4)}(x) = (-4)(-3)(-2)(-1) \cdot 6x^{-5}$

$\vdots$

10% $f^{(n)}(x) = (-n)(-(n-1)) \cdots (-3)(-2)(-1) \times 6x^{-(n+1)}$

$= (-1)^n (n!) 6 x^{-(n+1)}$

10% $\therefore f^{(n)}(-4) = \frac{(-1)^n (n!) 6}{(-4)^{n+1}} = -\frac{6(n!)}{4^{n+1}}$

20% $\therefore T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(-4)}{n!}(x+4)^n = \sum_{n=0}^{\infty} \frac{-6(n!)}{n!\,4^{n+1}}(x+4)^n = \sum_{n=0}^{\infty} \frac{-6}{4^{n+1}}(x+4)^n$