不可使用手機、計算器，禁止作弊!
背面還有題目

1. (30%) Evaluate the integral.

$$\int x^5 \cos(x^2)\,dx$$

$$\int x^5 \cos(x^2)\,dx = \int (x^2)^2 \cos(x^2)\, x\,dx = \int \tfrac{1}{2} w^2 \cos(w)\,dw = \tfrac{1}{2}w^2 \sin(w) - w(-\cos(w))$$
$$+ (-\sin(w)) + C$$

$w = x^2$

$dw = 2x\,dx$

$\tfrac{1}{2}dw = x\,dx$

u	dv
w^2	$\cos(w)$
$2w$	$\sin(w)$
2	$-\cos(w)$
0	$-\sin(w)$

$$= \tfrac{1}{2}(x^2)^2 \sin(x^2) + x^2 \cos(x^2)$$
$$- \sin(x^2) + C$$

$$= \tfrac{1}{2}x^4 \sin(x^2) + x^2 \cos(x^2) - \sin(x^2) + C$$

2. (30%) Evaluate the integral.

$$\int \sin^6(x) \cos^5(x)\,dx$$

$$\int \sin^6(x) \cos^5(x)\,dx = \int \sin^6(x)(\cos^2(x))^2 \cos(x)\,dx = \int \sin^6(x)\left[1 - \sin^2(x)\right]^2 \cos(x)\,dx$$

$u = \sin(x)$

$du = \cos(x)\,dx$

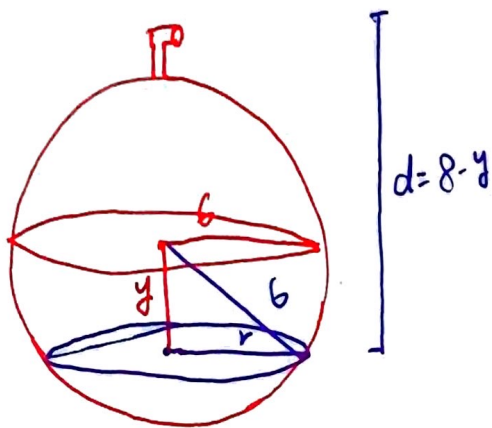$$= \int u^6(1-u^2)^2\,du = \int u^6(1 - 2u^2 + u^4)\,du$$

$$= \int u^6 - 2u^8 + u^{10}\,du = \tfrac{1}{7}u^6 - \tfrac{2}{9}u^9 + \tfrac{1}{11}u^{11} + C$$

$$= \tfrac{1}{7}\sin^7(x) - \tfrac{2}{9}\sin^9(x) + \tfrac{1}{11}\sin^{11}(x) + C$$

3. (40%) A spherical(球狀) tank(水箱) with radius 6 m is half-filled with a liquid that has a density of $\rho = 200 kg/m^3$. The tank has a 2 m spout(噴管) at the top. Find the work needed to pump all of the liquid out of the tank.

$y=8$

$y=6$

$d=8-y$

$y=0$

6

y

6

r

$y=-6$

$y^2 + r^2 = 36$

$r^2 = 36 - y^2$

$r = \sqrt{36 - y^2}$

$A = \pi r^2 = \pi(36 - y^2)$

$d = 8 - y$

$$W = \int_{-6}^{0} \pi(36 - y^2)(8-y)\rho g \, dy$$

$$= \pi \rho g \int_{-6}^{0} 288 - 36y - 8y^2 + y^3 \, dy$$

$$= \pi \rho g \left[288y - 18y^2 - \frac{8}{3}y^3 + \frac{1}{4}y^4 \right] \Big|_{-6}^{0}$$

$$= \pi \rho g \left[-\left(288(-6) - 18 \times 36 + \frac{8}{3} \times 6^3 + \frac{1}{4} \times 6^4 \right) \right]$$

$$= \pi \times 200 \times (9.8) \times 1476$$

$$= 2892960\pi \quad (J) \text{ or } (N \cdot m)$$