

H/

$1 + 1$

5

$1 +$

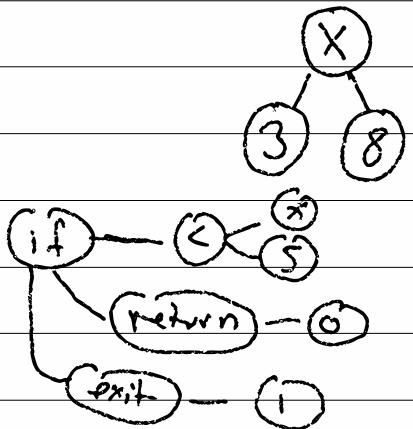
1×3

" 3×8 "

```

if (x < 5) {
    return 0;
} else {
    exit(1);
}

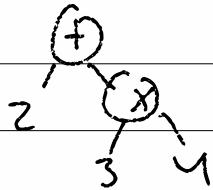
```



$(+ 1 1)$
↑
children



$(+ 2 (* 3 4))$



$\vdash \text{J} \Rightarrow e := v \mid (+ e e)$
 $v := \text{number} \mid (* e e)$

$(+ 1 (* 2 3)) \in J$

interface J Σ \exists

class $JNumber$ implements J Σ

int n ; $JNumber(m + -n) \{ n = -n; \}$

class $JPlus$ imp J Σ

J left, right; $JPlus(\dots)$

class $JMult$ imp J Σ

J l, r; $JMult(\dots)$

$(+ 1 (* 2 3)) \xrightarrow{\text{Expr}}$

$\text{new } JPlus($

$\text{new } JNum(1),$

$\text{new } JMult(\quad = SP(JN(1), JM(JN(2), JN(3))))$

$\text{new } JNum(2))$

$\text{new } JNum(3)))$

class $JPlus$:

def __init__(l, r):

this.l = l;

this.r = r;

BST $n = m \mid (\text{or num}$

$n \ n)$

1-3/6 pp : J₀ \Rightarrow string

③ pp n = itos(n)

④ pp (+ e_L e_R) = "(#pp(e_L) ++ "+" ++ pp(e_R)
++ ")"

⑤ pp (* e_L e_R) = "(" ++ pp(e_L) ++ ">*" ++
pp(e_R) ++ ")"

① interface J₀ { public String pp(); }

② class JNum { ... }

public String pp() {

return intToStr(n); }

③ class JPlus { }

public String pp() {

return this.left.pp() + " + " + this.right.pp(); }

I-9) big-step interpreter

interp : e → v

interp n = n

interp (t e_L e_R) = interp e_L + interp e_R

interp (* e_L e_R) = interp e_L * interp e_R

→ class JMult {

public ^{int} interp () {

return this.left.interp() * this.right.interp(); }

$$(+ \ 1 \ 2 \ 3) = (+ \ 1 \ (+ \ 2 \ 3))$$

desugar →

se = empty | (cons ^{min} se se) | string

(a b c) = (pair "a" (pair "b" (pair "c" null)))

(+ 1 2) = (p "+" (p "1" (p "2" mt)))

(+ 1 (+ 2 3)) = (p "+" (p "1" (p ("+" (p "2" mt)) "3" mt)))

mt)))

I-5) desugar for \mathcal{J}_0

$$(" - " \ e) \Rightarrow (* \ -1 \ (\text{desugar } e))$$

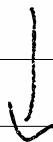
$$(" - " \ e_1 \ e_2) \Rightarrow (+ \ (\text{d } e_1) \ (\text{de } (" - " \ e_2)))$$

$$(" + ") \Rightarrow 0$$

$$(" + " \ e_1 \ \text{more} \dots) \Rightarrow$$

$$(+ \ (\text{d } e_1) \ (\text{dd } (" + " \ \text{more} \dots)))$$

"*



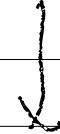
= 1



"x"

"x"

(x



$$\text{se} \Rightarrow \mathcal{J}_0 \Rightarrow v$$

desugar

interp

$$\xrightarrow{\text{compile}} \text{bc} \xrightarrow{\text{vm}} v$$

$\Sigma \cup \{ \}$ b
 desugarer $(- e_1) \Rightarrow (\widehat{*} -1 e'_1)$
 $(- e_1 e_2) \Rightarrow (\widehat{*} e'_1 (-e_2))$
 $(+) \Rightarrow 0$
 $(+ e_1 e_2 \dots) \Rightarrow (\widehat{*} e'_1 (+ e_2 \dots))$
 def
 desugar(se) :
 if isList(se) & length(se) = 2 &&
 first(se) == "-" then
 > def length(se) :
 > if isNull(se) : return 0
 > else if Cons(se) : return 1 + length(right)
 > else false
 new JMut(new JNum(-1), desugar(
 second(se)))

first \leftarrow left \rightarrow right

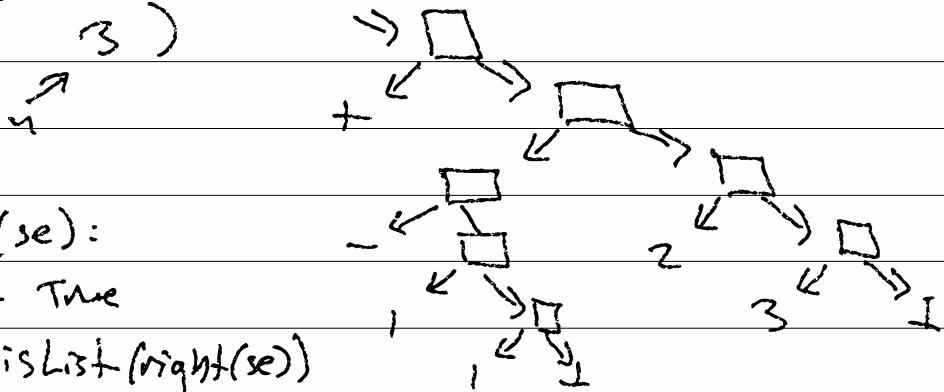
J second/left \rightarrow right

first

if isList(se) & len(se) = 3 && first(se) == "-"
 then : return new JAdd(desugar(second(se)),
 desugar(new Cons("-", ~~new Cons(first(se), null)~~, null)))

22] $\text{Cons}(a, \text{Cons}(b, \text{Cons}(c, \text{null})))$
 $\text{len} = 3$

$(+ (- 1 1) \text{len} = 4$
 $1 \xrightarrow{1} 2 \xrightarrow{2} 2 \leftarrow 3$
 $3)$



isList(se) :

$\text{Null} = \text{True}$

$(\text{cons} : \text{isList}(\text{right}(se))$

$\text{or} : \text{False}$

$\text{Len}(se)$:

$\text{Null} : 0$

$(\text{cons} : 1 + \text{len}(\text{right}(se))$

2-3 / $J_0 \Rightarrow J_1$

\downarrow fun \downarrow args

$$e := v \quad | \quad (e \ e \ \dots)$$
$$| \quad (\text{if } e \ e \ e)$$
$$\begin{matrix} \nearrow & \nearrow & \nearrow \\ c & f & f \end{matrix}$$

$v := b$

b = some set of constants

/l in J_0 , $b = \text{num} \mid + \mid *$
numbers | bools | prim

prim = $+, -, *, /, \leq, <, =, >, \geq, \dots$

interp $v = v$

interp (if e_c e_t e_f) = interp e_k

where $e_k = \text{if interp } e_c \text{ then}$
 $e_t \text{ o.w. } e_f$

interp (e_f $e_a \dots$) = $\delta(p, v_a \dots)$

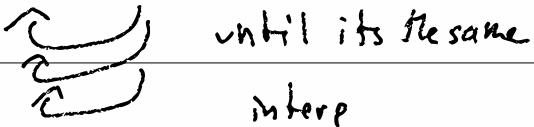
where $p = \text{interp } e_f$

$v_a = \text{interp } e_a \dots$

$\delta : b \dots \rightarrow b$

$\delta(+, 1, 2) = 3 \quad \delta(/, 1, 0) = 1$

$\delta(\leq, 1, 3) = \text{true}$

$\Sigma^4 /$ "small step interp" "big step"
 $e \rightarrow e$ $e \rightarrow v$

 Interp Interp

Interp $e =$

let $e' = \text{interp}(e)$

if $e == e'$ then

ret e

O.V.

Interp (e')

$(+ (+ 1 1) 2) \leftarrow (+ (+ 1 1))$
 $\Rightarrow (+ 2 2) \leftarrow (+ 1 1))$
 $\Rightarrow 4 \downarrow$ \downarrow
 $(+ 2 (+ 1 1))$

int $x = 1;$

$f(x--, x++)$ $(1, 0)$
 $(2, 1)$

(2-3) step : $e \rightarrow e$

step (if true e_1 e_2) = e_1

step (if false e_1 e_2) = e_2

step ($P \vee a \dots$) = $\delta(P, va \dots)$

step $v = v$

step (if $e(\&v)$ $e_1 e_2$) =

(if (step e) $e_1 e_2$) or (if e (step e) e_2)

step ($v_b \dots e(\&v)$ $e_a \dots$) =

($v_b \dots$ (step e) $e_a \dots$)

A context

$C := \text{hole} \quad | \quad \text{if0 } C \ e \ e$

$| \quad \text{if1 } e \ C \ e$

$| \quad \text{if2 } e \ e \ C$

$| \quad (e \dots C \ e \dots)$

plug $C \ e \ (C[e])$

plug hole $x = x$

plug (if0 $C \ e_1 \ e_2$) $x = \text{if } x \ e_1 \ e_2$

plug (if1 $e_1 \ C \ e_2$) $x = \text{if } e_1 \ x \ e_2$

plug ($e_1 \dots C \ e_2 \dots$) $x = (e_1 \dots x \ e_2 \dots)$

2-6

$$\text{step } C[\text{if true } e_1 \text{ et } e_2] = \\ C[e_1]$$

$$\text{step } C[\text{if false } e_1 \text{ et } e_2] = \\ C[e_2]$$

$$\text{step } C[p \text{ va } \dots] = C[S(p, \text{va } \dots)]$$

~~step~~ "parse" : $e \Rightarrow C \times e$

step $\xrightarrow{\quad\quad\quad} e$

$$\text{interp } e = \text{if } e \in V \text{ then } e$$

$$C, e' = \text{parse } e \quad e$$

$$e'' = \text{step } e'$$

$$\text{plug } C \ e''$$

$$\text{parse} : e \Rightarrow C \times e \quad \leftarrow \text{redex}$$

$$\text{parse "(if } e_1 \text{ et } e_2) =$$

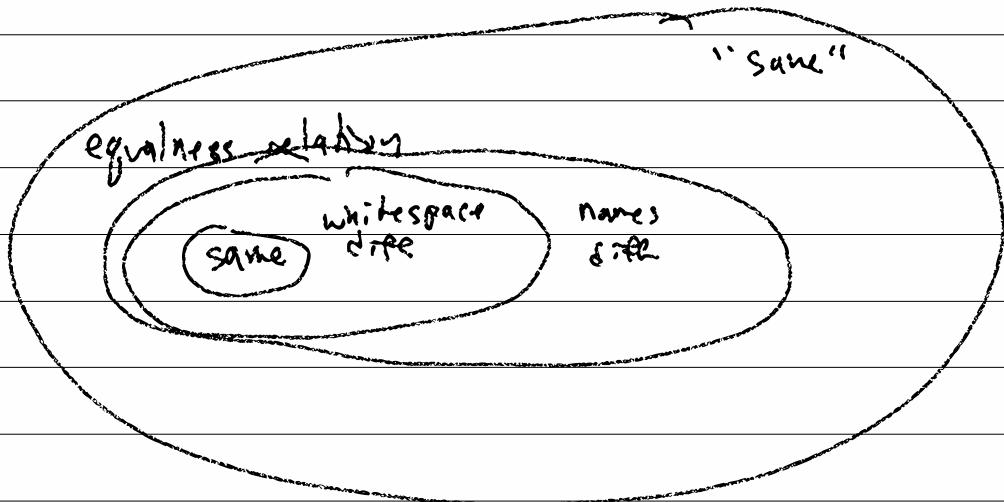
$$\text{if } e_1 \in V \text{ then } (\text{hole}, e)$$

$$\text{o.w. let } C', e' = \text{parse } e_1$$

$$(\text{ifO } C' \text{ et } e_2, e')$$

2-7) Answer: Contexts

Question: How do I know when
two programs do the same thing?



$$x = y$$

$$\forall x, f_x = g_x$$

$$\forall c, C[x] = C[y]$$

$$C[\text{hole } x] = y$$

$$C[+ \text{hole } z] = x + z = y + z$$

$$C[\text{map hole } (l; r + z)]$$

....

Observational Equivalence

$\rightarrow C := \text{hole} \mid \text{if } C \text{ e e}$
 $\quad \mid \text{if } e C e$
 $\quad \mid \text{if } e e C$
 $\mid (e \dots C \dots e \dots)$

$E := \text{hole} \mid \text{if } E \text{ e e}$
 $\mid (\vee \dots E \dots e \dots)$

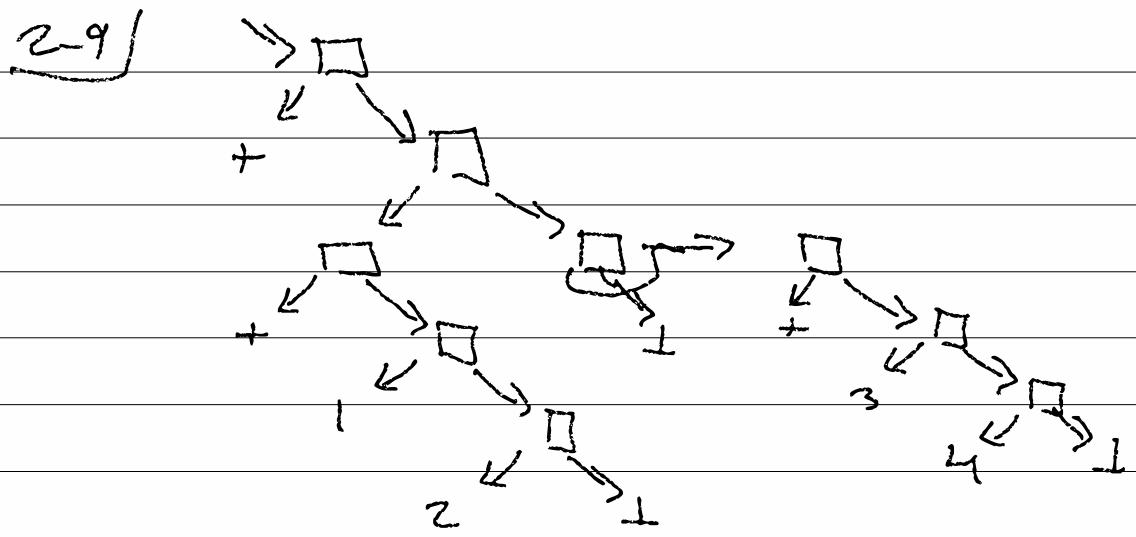
"mique decomp: \vdash_{mique} " $\downarrow^{\text{mique } E}$
 & e. $e \in \vee$ or $e = E[e']$ where
 $e' \in \vee$

$$\delta(1, 2) = 1 \quad \delta(1, 1, 0) = 1$$

$(+ (+ 1 2) (+ 3 4))$ $\stackrel{\text{tree}}{\sim} \text{java tree}$

new JPlus (new JPlus (new JItem (1),
 new JItem (2)))

new JPlus (new JItem (3),
 new JItem (4)))



(2 (+ 1 2)
 (+ 3 4))

3-1) E = hole | (if E e e)
| (v... E e...)

step $E[\text{if true } e + ef] \rightarrow f[e]$

step $E[\text{if false } e + ef] \rightarrow f[ef]$

step $E[p \ v_a \ ...] \rightarrow E[\delta(p, v_a \ ...)]$

interp $e = \text{case (parse } e) \text{ of}$
false $\rightarrow e$

$(E, e) \rightarrow_e^{\leftarrow} \text{step } e$

$E[e']$

gigantic program $\left[(+ \ (+ 1 \ 1) \right. \\ \left. (+ 2 \ 3)) \right]$

3-2/ Sy "language"

C_0 "machine"

$e \rightarrow e$

$st \mapsto st$

lang e $\xrightarrow{\text{inject}}$ machine st

\downarrow step

e'

\leftarrow extract

\downarrow step (3)

st'

done?
fv

$st = \langle e, E \rangle$

done? $\langle v, \text{hole} \rangle$

inject $e = \langle e, \text{hole} \rangle$

extract $\langle e, E \rangle = E[e]$

$\langle \text{if } ec \text{ et } ef, E \rangle \mapsto \langle ec, E[\text{if hole et } ef] \rangle$

$\langle \text{true}, E[\text{if hole et } ef] \rangle \mapsto \langle \text{et}, E \rangle.$

$\langle \text{false}, E[\text{if hole et } ef] \rangle \mapsto \langle ef, E \rangle$

$\langle e_0 e_1 \dots, E \rangle \mapsto \langle e_0, E[\text{hole } e_1 \dots] \rangle$

$\langle v, E[v_0 \dots \text{hole } e_1 e_2 \dots] \rangle \mapsto \langle e_1, E[v_0 \dots v_1 \text{hole } e_2 \dots] \rangle$

$\langle v_n, E[v_0 \dots \text{hole}] \rangle \mapsto \langle \delta(v_0 \dots v_n), E \rangle$

33) $E = \text{hole} \mid \text{if } E \ e \ e \mid (\& \dots E \ e \dots)$

interface context Σ

Expr plug (Expr); }

Hole : Context + Σ

plug (e) = e; }

If C : Context + Σ

Context c; Expr t, f;

plug (e) = $\text{new If}(C \cdot \text{plug}(e), t, f);$ }

AppC : Context Σ

List <V> vs; Context q; List <Expr> es;

plug (e) = new App(vs ++ [C · plug (e)] ++ es); }

< (+ (+ (+ 0 1) 2) 3) , hole >

< (+ (+ 0 1) 2) , AppC
[+] hole [3] >

< (+ 0 1) , AppC
[+] hole [3] >

{ 1 } AppC
[+] AppC
[+] hole [3] > \mapsto < , AppC
[+] AppC
[+] hole [3] >

$\underline{3-4}/$ $E = \text{hole} \quad | \text{if } E \in \{ v \dots E \dots \}$
 $= \text{top} \quad | \text{if } ee \quad \square \quad | \quad (v \dots)(e \sim) \square$
 $K = \text{kre} : \quad | \text{kif } ee \ K \quad | \ K_{\text{app}} \xrightarrow{\vec{v}} \vec{e} \vec{k}$

CK₀ machine $st = \langle e, k \rangle$

inject $e = \langle e, k_{\text{ret}} \rangle$

extract $\langle e, k_{\text{ret}} \rangle = e$

$\langle e, \text{kif } ee \text{ et } ef \ K \rangle = \text{extract}$

$\langle \text{if } e \text{ et } ef, k \rangle$

$\langle e, k_{\text{app}} (v \dots)(e \dots) k \rangle =$

$\text{extract } \langle (v \dots e e_i \dots), k \rangle$

done $\langle v, k_{\text{ret}} \rangle$

0 $\langle \text{if } ee \text{ et } ef, k \rangle \mapsto \langle ee, \text{kif } ee \text{ et } ef \ K \rangle$

1 $\langle \text{true}, \text{kif } ee \text{ et } ef \ K \rangle \mapsto \langle ee, k \rangle$

2 $\langle \text{false}, \text{kif } ee \text{ et } ef \ K \rangle \mapsto \langle ef, k \rangle$

3 $\langle e_0 e_1 \dots, k \rangle \mapsto \langle e_0, k_{\text{app}} () (e_1 \dots) k \rangle$

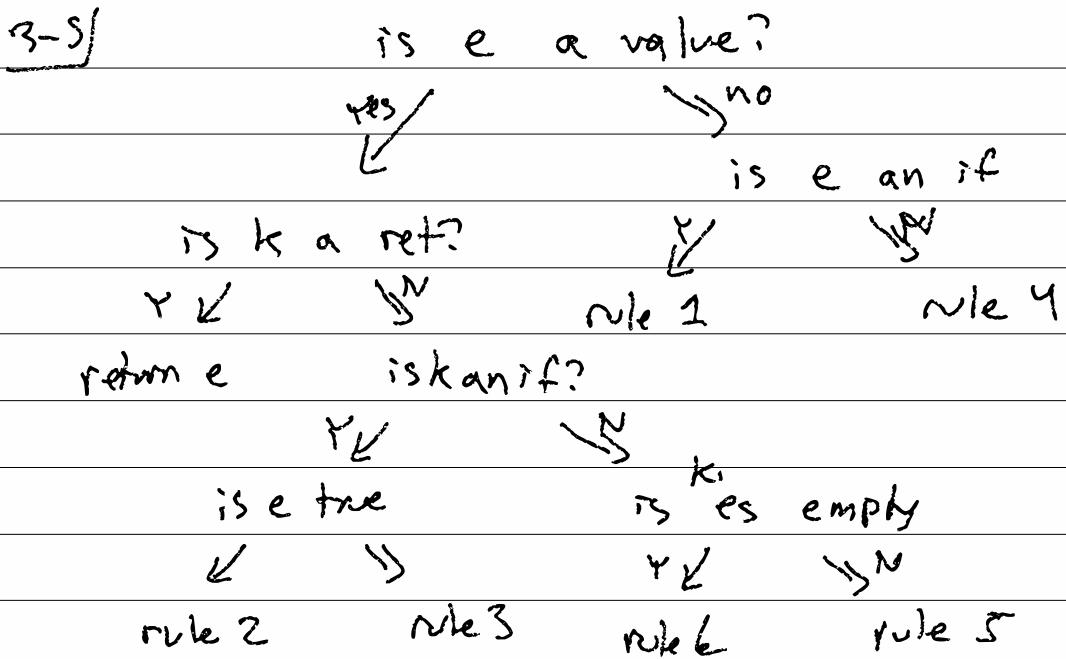
4 $\langle v_1, k_{\text{app}} (v_0 \dots) (e_0 e_1 \dots) k \rangle$

$\mapsto \langle e_0, k_{\text{app}} (v_0 \dots v_1) (e_1 \dots) k \rangle$

5 $\langle v_n, k_{\text{app}} (v_0 \dots) () k \rangle$

$\mapsto \langle \delta(v_0 \dots v_n), k \rangle$

while(1) {



rule 1:

$K = \text{new_if } (e, \text{true}, e, \text{false}, k)$

$e = e.\text{cond};$

~~jump $R + PC$~~

K is a stack and the stack (of c)

= Kontinuation

continuation

3-6/ struct if {
 expr * c, t, f; }
 struct num {
 int n; }
 struct app {
 expr * f, * args; }
 expr * make_if(expr * t, f) {
 if * p = malloc(sizeof ...))
 p->h.tag = IF;
 p->c = c; ...
 return p; }

struct expr {
 enum tag; }
 enum tag {
 IF, NUM, APP,
 BOOL, PRIM,
 RET, KIF,

KAPP, CONS, NIL};

(+ (+ 1 1 7 2)

make-add (make-add (make-num(1),
 make-num(1)),
 make-num(2)));

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4-1 / Σ_0 $e = n \mid (+ \ e \ e)$
 $\mid (\neq \ e \ e)$

Σ_1 $p = \text{unary} - (\text{neg})$, not (λ)

+ , * , / between
 $e = n \mid p \mid^{(2)} (\text{if } e \in e)$
 $\Gamma(e \dots)$

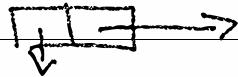
class TApp impl TExpr {
List<TExpr> contents }

App

$(+ \ 1 \ 2) \Rightarrow \text{List}(+, 1, 2)$
 $\downarrow \quad \downarrow \quad \downarrow$
 p n n

desugar

$\rightarrow \text{App}([\text{Prim(PLUS)}, \text{Num}(1), \text{Num}(2)])$
(cons "+ " (cons "1" (cons "2" null)))



$(+ \ 1 \ (+ \ 2 \ 3))$

$\text{App}([\text{Prim(PLUS)}, \text{Num}(1), \text{Num}(2)])$

$\text{App}([\text{Prim(PLUS)}, \text{Num}(2), \text{Num}(3)])$

$(+ \ 1 \ 3 \ (+ \ 2 \ 3))$

$\delta(+ \ 1 \ 3 \ 5) = 1$

4-2) Expr * Delta (List < Expr >) args) {
 if (len(args) == 3
 && args[0] == prim(PLUS)) {
 ret new Num((Num(args[1]) + num(args[2]))
); } };

$$(+ \rightarrow 0$$

$$\cancel{(+ \rightarrow A)} \rightarrow A$$

$$(+ n \text{ more } \dots) \Rightarrow (+ n (+ \text{ more } \dots))$$

desugar (cons "+" empty) = new Num(0);

delta (args)
 args[0] (args)
 \rightarrow
 args[0].apply (args[1...])

T_0 or T_1 : $\text{prog} = e$

4-3 / T_2 :

$e := v \mid (\overset{e}{\overset{e}{e}} \dots) \mid (\text{if } eee)$
| x

$v := \text{number} \mid \text{bool} \mid \text{prim} \mid f$

$X \in \text{some set of variable names}$

$f \in \text{some set of function names}$

$\text{prog} := d \dots e$

$d := (\text{define } (f \ x \dots) \ e)$

$(\text{define } (\text{Double } x) \ (+ \ x \ x))$

$\rightarrow (\text{Double } (+ \ (\text{Double } 1) \ 3))$

$(\text{define } (\text{Quad } x) \ (\text{Double } (\text{Double } x)))$

$(\text{Quad } (+ \ 1 \ (\text{Double } 3)))$

$$f(x) = 1 + x$$
$$f(3) ? = 1 + 3 = 4$$

$$f(x) = 1 - x$$
$$f(3+4) = 1 - (3+4)$$
$$\begin{matrix} " \\ f(7) \end{matrix} = \cancel{3+4} = 2$$
$$= 1 - 7 = -6$$

$$\text{Double}(1+1) = \text{Double}(2)$$

$$(1+1) + (1+1) = 2+2$$

4-3)

$$E = \text{hole} \quad | \quad \text{if } E \ e \ e \\ | \quad (\vee \dots E \ e \dots)$$

~~$E[x] = \dots$~~ $\Sigma / E[\text{if } f \ t \ e \ e] = E[e]$
 $\Sigma / E[f \ \vee \ \dots] = \dots$

$\text{eval} : e \Rightarrow \Sigma \dots \text{smallstep } e \rightarrow e$
 $\text{eval}' : p \Rightarrow \Sigma$
 $\Sigma^x \vdash \dots \text{smallstep } \Sigma \underset{?}{\overset{!}{\text{c}}} e \Rightarrow e$

$\Sigma : f \rightarrow d$

$\text{eval}' \Sigma \text{ do}(\text{define } (f x \dots) e) : \text{more}$
 $= \text{eval}' \Sigma [f \mapsto d] \text{ more}$

$\text{eval}' \Sigma e = \text{do smallstep}$
 $f(x) = 1 + x$
 $f(y)$

$\Sigma / E[f \ \vee \ \dots] = E[e[x \leftarrow v] \dots]$
 $\text{if } \Sigma(f) = (\text{define } (f x \dots) e)$

4-4] e $[x \leftarrow v]$ is pronounced
e where x_s are replaced with
 v

subst ~~x~~ $v \& \rightarrow e$

subst $x \ v \ v' = v'$

subst $x \ v \ x = v$

subst $x \ v \ x' = x'$

subst $x \ v$ (if $e_c \ e + e_c$) =

(if $e_c [x \leftarrow v] \ e + [x \leftarrow v] \ e_f [x \leftarrow v]$)

subst $x \ v$ (e ...) =

($e [x \leftarrow v]$...)

interface JExpr {

JExpr subst (Variable x, JExpr v); }

class JVar {

subst(x, v) { if ($x == \text{this}, x$)
return v

return this; }}

class JIf {

subst(x, v) {

new JIf (this.c, subst(x, v), this.t, subst
(x, v),
this.f, subst(x, v)); }}

4-5)

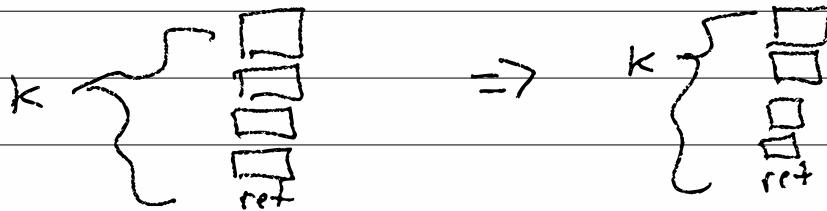
CK

$$6: \langle v_n, k_{app}((v_0 \dots), (), k) \rangle \\ \mapsto \langle \delta_{(v_0 \dots v_n)}, k \rangle$$

$$7: \langle v_n, k_{app}((f v_0 \dots), (), k) \rangle \\ \mapsto \langle e[x_i \leftarrow v_i], k \rangle$$

where $\Sigma(f) = \text{define } (f x_0 \dots x_n) e$

$$c = 2 \quad k_{app}(\text{Expt 7}) \quad c = \dots 7 \dots 2 \dots$$



$$\langle (e_0 e_1 \dots), k \rangle$$

$$\mapsto \langle e_0, k_{app}((), (e_1 \dots), k) \rangle$$

4-6 / (define (F x) (F x))
(F 10)

$\Rightarrow \Sigma = [F \mapsto (\text{define } (F x) (F x))]$

$e = (F 10)$

$k = kret$

$\langle F 10, kret \rangle$

$\langle F, kapp ((), 10), kret \rangle$

$\langle 10, kapp ((F), (), kret) \rangle$

$\Sigma(F) = (\text{define } (F x) (F x))$

$f \leftarrow x \dots \hat{e}$

$\langle e[x \dots \leftarrow v \dots], kret \rangle$

$(F x)[x \leftarrow 10]$

$\langle (F 10), kret \rangle$

error \rightarrow stack trace

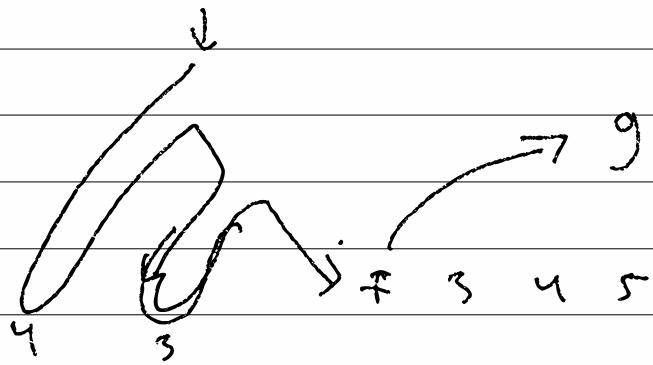
\rightarrow fun "at fault" $\sim F$

fun that called ~~F~~ $\sim f$

"past"

G - H

$y=f$)



$g(\cdot \cdot \cdot)$

$x = f \quad 3 \ 4 \ 5$

$n(x)$

$$\begin{array}{l}
 \underline{6-1} \quad C = \text{hole} \quad | \quad \text{if } C \in e \\
 \qquad\qquad\qquad | \quad \text{if } e \in C \\
 \qquad\qquad\qquad | \quad \text{if } e \in C \\
 \qquad\qquad\qquad | \quad e \dots E e \dots \\
 E = \text{hole} \quad | \quad \text{if } E \in e \\
 \qquad\qquad\qquad | \quad v \dots E e \dots
 \end{array}$$

if true (+ 1 2) 4

$$\begin{array}{ll}
 E = \text{hole} & e = \uparrow \\
 C = \text{hole} & e = \uparrow \\
 C = \text{if true hole} & 4 \\
 e = (+ 1 2)
 \end{array}$$

(+ 1 2)

$E = \text{hole}$ $e = (+ 1 2)$

find-reduce $\underbrace{B}_{, B} = E[e]$

step $e = e'$

6-2] $\text{fr} (\text{if } e + f) =$

$\text{if } (\text{value? } c)$

(hole, e)

O.W. $(E, e) = \text{fr } c$

$(\text{if } (E + f), e)$

$\text{fr} (\text{app } es) =$

for e in es

$\text{if } (\text{value? } e)$

$(+ \mid \times)$

$\langle e_0 \ e \dots, k \rangle$

$\mapsto \langle e_0, \text{kapp} ((), (e \dots), k) \rangle$

$\langle v_n, \text{kapp} ((), (v_0 \dots), (e_{n+1} \dots), k) \rangle$

$\mapsto \langle e_1, \text{kapp} ((v_0 \dots v_n), (e_{n+1} \dots), k) \rangle$

$\langle v_n, \text{kapp} ((p \ v_0 \dots), (), k) \rangle$

$\mapsto \langle \delta(p, v_0 \dots v_n), k \rangle$

6-3 / $\mathcal{I}_2 = \text{PASCAL or C}$

top-level functions

$Ck = \text{have the map } f \rightarrow d$
and we have subst

$\langle v_n, kapp(f v_0 \dots), (), k \rangle$

$\mapsto \langle e [x_0 \leftarrow v_0] \dots [x_n \leftarrow v_n], k \rangle$

where $\Sigma(f) = \text{define } (f x_0 \dots x_n) e$

$$x[x \leftarrow v] = v$$

$$y[x \leftarrow v] = y$$

$$u[x \leftarrow v] = u$$

$$(\text{if } c + f)[x \leftarrow v] = (\text{if } c[x \leftarrow v] + [x \leftarrow v] f)$$

$$(e \dots)[x \leftarrow v] = (e[x \leftarrow v] \dots)$$

$\langle \text{if } c + f, k \rangle$

$\mapsto \langle c, k \text{ if } (+, f, k) \rangle$

OLD: no rule for a variable in the case
 $\langle x, k \rangle \mapsto \dots$

NEW:

$\langle x, k \rangle \mapsto \text{finally do the subst}$

(Σ)

G-γ) C E K st = < e, env, k >

env = Ø | env [x ← v]

k = k_{net} | k_{if} ∈ e k

| k_{app} → e → k

< x, env, k > ↪ < env(x), Ø, k >

< if c t f, env, k >

↪ < c, env, k_{if} t f k >

< true, env, k_{if} t f k >

↪ < t, env, k >

< e₀ e₁ ... , env, k >

↪ < e₀, env, k_{app} () (e₁...) k >

< v₁, , ~~v₀~~, k_{app} (v₀...) (e₀ e₁...) k >

↪ < e₀, env, k_{app} (v₀...v₁) (e₁...) k >

< v_n, env, k_{app} (~~v₀...~~) () k >

↪ < e, ~~v₀~~ [x₀ ← v₀] ... [x_n ← v_n], k >

where Σ(f) = define f x₀ ... x_n ∈

6-5/ define $f(x) = x + z$
define $g(z) = f(y)$
 $g(2)$

define $f(x) = 3$
define $g(z) = (f(1)) + x$
 $g(2)$

6-6]

$\text{CEK} = \langle e, \text{env}, k \rangle$

$\text{env} = \emptyset \mid \text{env}[x \leftarrow v]$

$k = \text{tret} \mid \text{kif env} + f \ k$

$| \ kapp \xrightarrow{\vec{v}} \text{env} \xrightarrow{\vec{e}} k$

$\langle x, \text{env}, k \rangle \mapsto \langle \text{env}(x), \emptyset, k \rangle$

$\langle \text{if } c + f, \text{env}, k \rangle \mapsto \langle c, \text{env}, \text{kif env} + f k \rangle$

$\langle \text{true}, _, \text{kif } (\text{env}, t, f, k) \rangle \mapsto \langle t, \text{env}, k \rangle$

$\langle e_0 e_1 \dots, \text{env}, k \rangle$

$\mapsto \langle e_0, \text{env}, \text{kapp}((_), \text{env}, (e_1 \dots), k) \rangle$

$\langle v_n, _, \text{kapp}((v_0 \dots), \text{env}, (e_0 e_1 \dots), k) \rangle$

$\mapsto \langle e_0, \text{env}, \text{kapp}((v_0 \dots v_n), \text{env}, (e_1 \dots) k) \rangle$

$\langle v_n, _, \text{kapp}((f v_0 \dots), _, (_), k) \rangle$

$\mapsto \langle e, \emptyset[x_0 \leftarrow v_0] \dots [x_n \leftarrow v_n], k \rangle$

where $\Sigma(f)$ define $(f/x_0 \dots x_n) e$

"dynamic scope" ~ equal

- emacs lisp

- JS / Py / Ruby / perl / PHP / etc

specific vars are always dynamic

$\text{new} = \emptyset[A \leftarrow \text{env}(A)] [B \leftarrow \text{env}(B)]$

"this"

Q7) PASCAL/C - all funcs are top-level
 $p = \lambda \dots e$

$\text{JS} = (\lambda) \Rightarrow 1 + x$

$\text{Py} = \text{lambda: } x : 1 + x$

$\text{C++} = [](\text{int } x) \{ \text{return } 1 + x; \}$

J_3

$e = v \mid e \ e \dots \mid \text{if } e \ e \ e \mid x$

$v = b \mid \boxed{(\lambda (x \dots) \ e)} \text{--- new}$

$b = \text{num} \mid \text{bools} \mid \text{prim} \quad // \text{No f's}$

$E = \text{hole} \mid \text{if } E \ e \ e \mid v \dots E \ e \dots$

$E[(\lambda (x_0 \dots x_n) \ e) \ v_0 \dots v_n] =$
 $E[e[x_0 \leftarrow v_0] \dots [x_n \leftarrow v_n]]$

$((\lambda x. \ ((\lambda y. \ (x + y) \ 7)) \ 8) \Rightarrow 15$
let $x = 8$ in
let $y = 7$ in
 $x + y$

let $x = e_1$ in $e_2 \Rightarrow$
 $(\lambda x. \ e_2) \ e_1$

let $\overbrace{x}^{\leftarrow 8} = 8$ in
let $\overbrace{x}^{\leftarrow x + 1} = x + 1$ in
 $x + x$

6-8/

$$(\lambda(x_0 \dots x_n) e)[y \leftarrow v]$$

$$= (\lambda(x_0 \dots x_n)$$

$$e[y \leftarrow v])$$

unless $y \notin x_0 \dots x_n$

$$(\lambda x_1 (\lambda x_1 x+1)) z$$

$$(\lambda x_1 x+1)$$

old
machine $v = \text{theory } v$

$$(v := b \mid \star) \neq (v := b \mid \lambda(x \dots) e)$$

closure $(\lambda(x \dots) e, \text{env})$

$$< \lambda(x \dots) e, \text{env}, k > \mapsto < \text{clo}(\lambda(x \dots) e, \text{env}), \emptyset, k >$$

$$< v_0, \dots, k \text{app}((\text{clo}(\lambda(x_0 \dots x_n) e, \text{env}) v_0 \dots), \dots, (), k) >$$

$$\mapsto < e, \text{env}[x_0 \leftarrow v_0] \dots [x_n \leftarrow v_n], k >$$

7-1 J_3 : $v = \dots | \lambda(x\dots) e$

let $x = e_1$ in e_2

$\Rightarrow (\lambda(x) e_2) e_1$

let $x = 1$ in

$(\lambda(x)$

let $y = 2$ in

$(\lambda(y)$

$x+y$

$(+ x y) 2) 1$

$C_E K_1$: $v = \dots (\lambda(y) (+ 1 y)) 2$

$| \cancel{\lambda(x) e}$

$| \text{clo}(\lambda(x\dots) e, \text{env})$

$\langle \lambda(x\dots) e, \text{env}, k \rangle$

$\mapsto \langle \text{clo}(\lambda(x\dots) e, \text{env}), \emptyset, k \rangle$

$\langle v_n, \dots, k_{app}((c v_0 \dots), \dots, (), k) \rangle$

where $c = \text{clo}(\lambda(x\dots) e, \text{env})$

$\mapsto \langle e, \text{env} [x_0 \mapsto v_0] \dots [x_n \mapsto v_n], k \rangle$

$\text{env} = \perp | \boxed{\quad \quad \quad \quad \quad} \downarrow \downarrow \downarrow \downarrow \downarrow$
 $x \quad v \quad \text{env}$

$\text{clo} = \boxed{\quad \quad \quad} \downarrow \downarrow$
 $\lambda x.e \quad \text{env}$

let $y = 3$ in
let $z = 8$ in [8, 3, 19, 22, 36]

7-2) $\lambda(x)(+x+y) \xleftarrow{\text{clo}} z \rightarrow 8$

\downarrow $y \rightarrow 3$
 $(\lambda. (+ \underset{\text{static}}{\hat{0}} \underset{\text{address}}{\hat{1}}), [\hat{1}]) \quad x \rightarrow 19$
 $a \rightarrow 22$
 $b \rightarrow 36$

...

SA = nat env = vector v

FLAT-CLOSURES $[\hookrightarrow, 3]$

SA = (nat, nat) env = \downarrow , vector v

$(\overset{\wedge}{0}, 0) (\overset{\wedge}{1}, 1) \quad [8, 3, 19, 22, 36] \text{ env}$
 $\cap, [\hookrightarrow, \hookrightarrow]$ NESTED CLOSURES

$$\begin{aligned}
 7-3) &= v \mid (\text{if } e \leq e) \\
 &\quad x \mid (e \leftarrow e) \mid (p \leftarrow e) \\
 v &= b \mid \lambda(x)e \\
 b &= \text{num} \mid \text{bools} \mid \text{prim} \\
 \text{prim} &= + \mid - \mid * \mid \div \mid \lt
 \end{aligned}$$

7-y/ what is a Bool really, man?

$$\text{if True } A \ B = A$$

$$\text{if False } A \ B = B$$

$$\text{True} = \lambda x. \lambda y. x$$

$$\text{False} = \lambda x. \lambda y. y$$

$$\text{if} = \lambda c. \lambda x. \lambda y. c \times y = \lambda c. c$$

$$\underbrace{\text{if}}_{\text{True}} \ A \ B = \text{True} \ A \ B = A$$

$$\text{NOT T} = F$$

$$\text{NOT F} = T$$

$$\text{NOT} = \lambda b. \lambda x. \lambda y. b \ y \ x$$

interface Bool { int choose (int, int); }

class True : Bool { ^{True} int choose (x, y) = x }

class False : Bool { ^{False} int choose (x, y) = y }

class Not : Bool { Not (Bool b) { this. b = b; }

int choose (x, y) {

return b. choose (y, x); }

7-5/ What is a number?

zero := doesn't do something

one := does something once

two := does it twice

$$\text{add } \lambda^n x. \lambda^m y. \text{ ~~λy. y + y~~} \underbrace{\dots}_{n+m}$$

zero := $\lambda f. \lambda x. x$

one := $\lambda f. \lambda x. f x$

two := $\lambda f. \lambda x. f(f x)$

add1 := $\lambda n. \lambda f. \lambda x. f(n f x)$

add := $\lambda n. \lambda m. \lambda f. \lambda x. n f(m f x)$

zero? := $\lambda n. n(\lambda x. \text{FALSE}) \text{ TRUE}$

mult := $\lambda n. \lambda m. \lambda f. \lambda x. n(m f x)$

two two two (two f) x
 $(\lambda x. f f x) ((\lambda x. f f x) x)$

$f f f f x$

7-6/ Pair

$$\text{fst} (\text{pair } A \ B) = A$$

$$\text{snd} (\text{pair } A \ B) = B$$

$$\text{pair} = \lambda a. \lambda b. \lambda c. \text{if } c \ a \ b$$

$$\text{fst} = \lambda p. \ p \ \text{TRUE}$$

$$\text{snd} = \lambda p. \ p \ \text{FALSE}$$

$$\text{subl} := \lambda n. \ \text{fst} (n (\lambda p. \text{pair} (\text{snd} p) (\text{pair} z \ z))) \\ (\text{addl} (\text{snd} p)))$$

λ fac.

$$\text{mkfac} := \lambda n.$$

$$\text{if } (\text{zero? } n)$$

1 = one

$$(\text{addlt } n (\text{fac} (\text{subl } n)))$$

$$g(x) = x \cup \{a, b\} \quad f(x) = 17 \circ x$$

$$\text{fac} := \text{mkfac fac} \quad x = F \ x$$

$$\begin{matrix} \uparrow & \uparrow & \uparrow \\ x & F & x \end{matrix}$$

7-7) Fixed point of a lambda?

$$\text{FIX } F = x \quad F x = x$$

$$F(\text{FIX } F) = \text{FIX } F$$

$\in \mathbb{Z}$ -combinator

$$\begin{aligned} \text{FIX} := & \lambda F. ((\lambda x. F (\lambda v. x x v)) \\ & (\lambda x. F (\lambda v. x x v))) \end{aligned}$$

$$\begin{aligned} \text{FIX } F &:= ((\lambda x. F (\lambda v. x x v)) \\ & (\lambda x. F (\lambda v. x x v))) \end{aligned} \quad A$$

$$= F(\lambda v. A A v)$$

$$= F(A A)$$

$$= F((\lambda x. F(A x x v)) \lambda x. \lambda y. x y \\ (\lambda x. F(A x x v)))$$

$$= F(\text{FIX } F)$$

Lambda-Calculus

$$\begin{array}{c} n, \text{add} \\ \downarrow \\ \lambda x. x + 1 \end{array} \quad 0$$

Church Numeral / Church encoding

8-1/ Lambda - Calculus

tiny (if $T \in PC$) = e

tiny : $e \rightarrow e$ tiny (if $F \in PF$) = eF

tiny ($p, v_0 \dots v_n$) = $\delta(p, v_n)$

small : $e \rightarrow e$

small $e = \text{let } C, e' = \text{find-addr } e$

big : $e \rightarrow v$ let $e'' = \text{tiny } e'$
let $C[e'']$

big $e =$

if $e \in V$: return e

o.w. big (small e)

$CC()$: $C \times e \rightarrow C \times e$
 $CC(C, e) = C' \times e' = \text{move-context } C \ e$
 $e'' \rightarrow \text{tiny } e'$

ret (C', e'')

$CC^* C \ e = \text{if } e \in V \text{ and } (= \text{hole}, \text{ret } C \ e)$

big! : $e \rightarrow e$ o.w. $CC^*(CC^* C \ e)$

extract ($CC^* (\text{inject } e)$)

inject = (hole, e) extract(hole, v) = v

B-3

$\langle \text{if } e_c \text{ et } e_f, E \rangle$

$\mapsto \langle e_c, E[\text{if hole et } e_f] \rangle$

A

$(\frac{+}{8} \ 1 \ c \ 0 \ F)$
 $(\text{if zero? } 1) \quad (+ 2 \ 3) \quad (+ 4 \ 5))$

inject $A = \langle A, \text{hole} \rangle$

$\ll 0 \ \langle A, \text{hole} \rangle$

$\mapsto \langle +, \text{hole}[\text{hole } 1 \ B] \rangle$

$= \langle +, (\text{hole } 1 \ B) \rangle$

$\mapsto \langle 1, (+ \text{ hole } B) \rangle$

$\mapsto \langle B, (+ \cancel{1} \text{ hole}) \rangle = E[\text{if hole et } e_f]$

$\mapsto \langle C, (+ 1 \ (\text{if hole } 0 \ F)) \rangle$

$\mapsto \langle \text{False}, = \rangle$

$\mapsto \langle \text{False}, (+ 1 \ \text{hole}) \rangle$

$\mapsto \langle \text{False}, = \rangle$

$\mapsto \langle 10, \ \text{hole} \rangle$

$\mapsto \text{extract} \rightarrow 10$

8-3) Lambda-calculus

$$e = x \mid e \ e \mid \lambda x. e$$

\mathcal{T}_3 doesn't recursion (except via Ξ)

$$\mathcal{T}_3 \Rightarrow \mathcal{T}_4$$

$$v = \dots \mid \cancel{\lambda(x\dots)e}$$

$$\mid \lambda * (x\dots)e$$

a. map (lambda x : $x+1$)

lambda fib (n): ...
 $\xrightarrow{\text{rec}}$ $\xrightarrow{\text{args}}$

| let fib = $\lambda n. \dots$ (fib (sub1 n))
 \nwarrow unbound

| let $x = xe$ in be
 $\quad := (\lambda x. be) xe$

let fib = λ inner-fib : n

inner-fib (sub1 n)

"(define (f xe) ; b")
 $\xrightarrow{x\dots}$

\Leftrightarrow "let f = $\lambda x. f(x\dots) \cdot xe$ in b"

8-4

$$E[(\lambda v_0 \dots v_n)] = E[b[f \leftarrow \ell][x_0 \leftarrow v_0] \dots]$$

where $\ell = (\lambda f (x_0 \dots x_n) b)$

$\langle \lambda f (x \dots) b, \text{env}, k \rangle$

$\mapsto \langle c, \emptyset, k \rangle$

where $c = \text{clo}(\lambda f (x \dots) b, \text{env}')$

$\text{env}' = \text{env}[f \leftarrow c]$

switch (tag(c)) {

case LAMBDA:

$\text{envp} = \text{make_env}(\text{env}, c \rightarrow \text{fun}, \text{NULL});$

$c = \text{make_clo}(c, \text{envp})$

$\text{envp} \rightarrow \text{val} = c;$

$\text{env} = \text{NULL};$

break;

while(1) {

8-5/ vint x, y;

scanf ("%d", &x);
scanf ("%d", &y);
 $x = \{y\}$



deref (malloc (4), 5) = ⊥

Algebraic data types

dt == O — —

| 1 — void

| dt + dt — interface variants

| dt × dt — pair

| type | constraint | destruct |
|---------|---------------------|-------------------------|
| 1 | void | — |
| O | — | — |
| dt × dt | pair | fst, snd |
| dt + dt | left, right —, — | case / switch / if — |

case (left a) X Y ⇒ X a

case (right a) X Y ⇒ Y a

$$\begin{aligned}
 8-6) \quad \text{Bool} &= 1 + 1 \\
 \text{Nat} &= 1 + \text{Nat} \\
 \text{Bin} &= 1 + \text{Bin} + \text{Bin} \\
 \text{List}(A) &= 1 + (A \times \text{List}(A)) \\
 \text{BMT}(A) &= 1 + (A \times \text{BMT}(A) \times \text{BMT}(A)) \\
 \text{BMT}'(A) &= A + (\text{BMT}'(A), \text{BMT}'(A)) \\
 \text{SE} &= 1 + \text{Atom} + (\text{SE}, \text{SE})
 \end{aligned}$$

$$\begin{aligned}
 d_A 0 &= 1 \\
 d_A 1 &= 0 \\
 d_A A &= 1 \\
 d_A B &= 0 \\
 d_A X + Y &= d_A X + d_A Y \\
 d_A X \times Y &= d_A X \times Y + X * d_A Y
 \end{aligned}$$

$$d_A \text{List}(A) = \text{Zipper}(A)$$

q-1/ (+ 1 2)

$\text{TAAPP} (+, 1, 2) . \text{asc}()$

prim $\downarrow \downarrow$

= "make_japp(make_jprim(PLS),
...) "

writeToFile("x.c", 0, asc())

$J_4 \rightarrow J_5$

$e := x \mid v \mid (e \ e \dots) \mid (\text{if } e \ e)$

case e as $(\text{inl } x) \rightarrow e$ or $(\text{inr } x) \rightarrow e$

$v := \text{num} \mid \text{bool} \mid \text{prim} \mid \lambda x (x \dots) \ e$
 $\text{unit} \mid \text{pair } v \ v \mid \text{inl } v \mid \text{inr } v$

$\text{prim} := \dots \mid \text{pair} \mid \text{inl} \mid \text{inr}$
 $\mid \text{fst} \mid \text{snd}$

$E[\text{fst } (\text{pair } v, v)] = E[v] \quad E[\text{snd } (\text{pair } v, v)] = E[v]$

$E[\text{case } (\text{inl } v) \text{ as } (\text{inl } x_i) \rightarrow e_i \text{ or } (\text{inr } x_r) \rightarrow e_r]$
 $\Rightarrow E[e_i[x_i \leftarrow v]]$

$E[(\text{inr } v)] \Rightarrow E[e_r[x_r \leftarrow v]]$

9-3 // List is either empty
or a cons with a thing
and another list

empty := $\text{inl } \text{unit}$

cons := $\lambda (\text{data rest}) . \text{inn} (\text{pair data rest})$

length := $\lambda \text{rec } (1) .$

case 1 of

$\text{inl } _ \rightarrow 0$

$\text{inn } p \rightarrow 1 + \text{rec } (\text{snd } p)$

map := $\lambda \text{rec } (f 1) .$

case 1 of

$\text{inl } _ \rightarrow 1$

$\text{inn } p \rightarrow \text{cons } (f (\text{fst } p))$

$(\text{rec } f (\text{snd } p))$

reduce := $\lambda \text{rec } (f \ \underline{\underline{z}} \ 1) .$

case 1 of $\text{inl } _ \rightarrow \underline{\underline{z}}$

$\text{inn } p \rightarrow \text{rec } f (f \ \underline{\underline{z}} (\text{fst } p))$
 $(\text{snd } p)$

9-3/ Reduce (+) 0 (cons 1 (cons 2
(cons 3 empty)))

= reduce (+) 1 (cons 2 (cons 3 m+))

= reduce (+) 3 (cons 3 m+)

= reduce (+) 6 m+

= 6

true := int unit

false := int unit

if ec et ef == case ec of int → et
int → ef

int -

int int -

int int -

pair → tuple

fst/snd → π/.proj

$\text{fst} = \text{π}_0$
 $\text{snd} = \text{π}_1$

case^(?) → case (a)

int/int → choice I

obj-+* delta-pair (obj-+* 1, obj-+* r) {

ret make-pair (1, r); }

obj-+* delta-fst (obj-+* o) {

ret (posi-+* o) → fst; }

q-y / $e := \text{obj} ; \{ x : e, \dots \}$

$| e \cdot x$
 $v := \text{obj} ; \{ x = v \dots \}$

$E[\text{obj} ; \{ x_0 : v_0 \dots x_i : v_i \dots x_n : v_n \}]$
 $\cdot x_i] \Rightarrow E[v_i]$

$\{ \dots \}$

$\{ \xrightarrow{\text{empty}} \text{empty} \} = \text{empty}$

$\xrightarrow{\text{Set } o \times e = (\text{cons} (\text{pair } "x" e) o)}$

$\emptyset, x = \text{lookup } "x" e$

$\text{lookup} := \lambda \text{rec} (\text{field obj}) .$

$\text{case obj of int} \rightarrow (\text{rec field obj})$

$\text{inr } p \rightarrow \text{if string=? } \text{field } (\text{fst } p)$

$(\text{snd } p)$

$(\text{rec field } (\text{snd } p))$

$(\text{Ek } K \leftarrow \text{Kcase } (x_1, e_1, x_r, er, env, tc) \text{ Krase}(C, tc))$

$\langle \text{case } e_1 \text{ of int } x_1 \rightarrow e_1 \text{ or inr } x_r \rightarrow er, env, tc \rangle$

$\mapsto \langle e_1, env, \text{Kcase}(x_1, e_1, x_r, er, env, tc) \rangle$

$\langle \text{inr } v, _, \text{Kcase}(x_1, e_1, x_r, er, env, tc) \rangle$

$\mapsto \langle e_1, env[x \leftarrow v], tc \rangle$

9-5 / Mutation

$$A \rightarrow B \supseteq \left\{ \begin{array}{l} (a_1, b) \\ \dots \\ (a_n, b) \end{array} \right\}$$

JS

```
const x = f(3);
```

```
console.log(x); "42"
```

```
const y = f(3); y=x
```

```
console.log(y); "??"
```

```
function f(x){  
    return x + 39; }
```

```
let c = 0; const f = (x) => x + 39 + c++;
```

if p f()

...

if g c()

...

f(3)

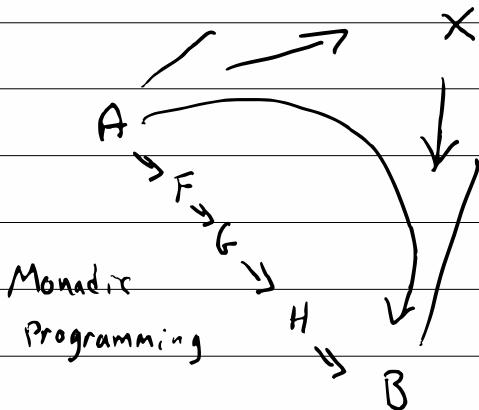
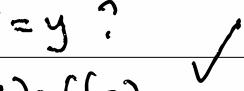
Math

x = f(3)

y = f(3)

x = y ?

f(3) = f(3)



12-1

Goal: add mutation

$$\begin{array}{ll}
 e ::= \dots \mid \text{box } e & E = \dots \mid \text{box } E \\
 \mid \text{unbox } e & \mid \text{unbox } E \\
 \mid \text{set-box! } e \leftarrow e & \mid \text{set-box } E \leftarrow e \\
 & \mid \text{set-box } v E
 \end{array}$$

struct box { void *p; }

box (int x) { ip = malloc (int)

box b = { p = ip }

*ip = x;

ret b; }

let b = box b in

(set-box! b 8;) + (unbox b)

b $\rightarrow \sigma_0$

(set-box! (box b) 8;) + (unbox (box b))

unbox (box b))

ϕ_{fun} $(\text{set-box! } \sigma_0 8;) + (\text{unbox } \sigma_0)$

$\Rightarrow \phi[\sigma_0 \mapsto 8] (\text{unbox } \sigma_0 + \text{ub } \sigma_0) \Rightarrow \sigma[\sigma_0 \mapsto 8] / (8 + 8) = 16$

12-2 / small step : $e \rightarrow e'$
 $\Sigma \times e \rightarrow \Sigma \times e'$
 $(M, S, \rho) \rightarrow (M', S', \rho')$

$\Sigma = \text{store } (\text{memory / heap})$
 $\text{ptrs} \Rightarrow \text{vals}$
 $\sigma \rightarrow v$

$\Sigma / E[\text{if } T \text{ et } e]$
 inject $e = \emptyset / e \rightarrow \Sigma / E[e]$
 extract $\Sigma / v = v$
 $v := \dots | \sigma$

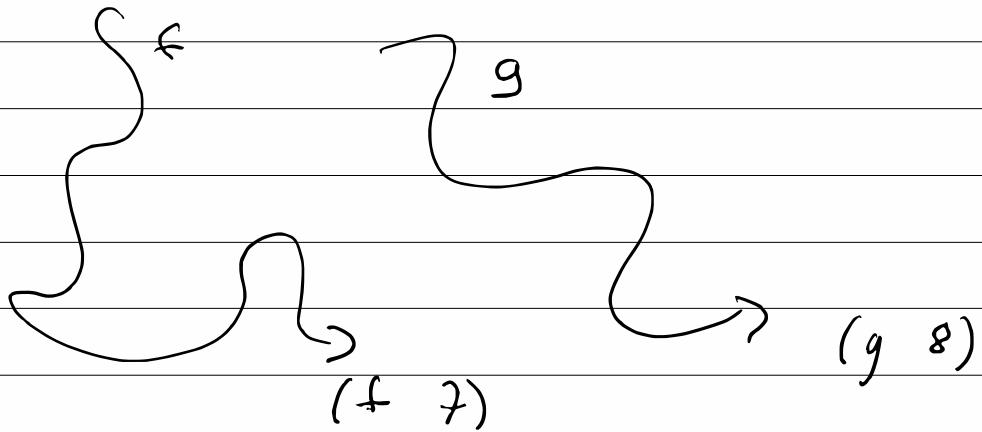
$\Sigma / E[\text{box } v] \rightarrow \Sigma[\sigma \mapsto v] / E[\sigma]$
 where σ d.n.o.. Σ ($\sigma = \text{malloc}$)

$\Sigma / E[\text{unbox } \sigma] \rightarrow \Sigma / E[\Sigma(\sigma)]$
 $\Sigma / E[\text{set-box! } \sigma \ v] \rightarrow \Sigma[\sigma \mapsto v] / E[v]$
 $E[\text{unit}]$
 void

12-3 / let $b = \text{box } 0$ in

let $f = \lambda x, \text{set-box! } b (x+1);$
 $x = 2$ in

let $g = \lambda y, y + \text{unbox } b$ in



$$\begin{array}{c} \text{CEK}_3 \\ \Downarrow \\ \text{CESK}_0 \\ \Downarrow \\ \text{Store} \end{array} \quad \begin{array}{c} \text{CEK}_4 \\ \Downarrow \\ \langle v, \text{env}, \text{sto}, \text{kapp}(\text{clo}(\lambda x, e, \text{env}'), k) \rangle \\ \Downarrow \\ \langle e, \text{env}'[x \mapsto v], \text{sto}, k \rangle \end{array}$$

$v = \dots | \text{ptr } n$
 $p = \dots | \text{box }$
 $| \text{unbox }$
 $| \text{set-box }$

$\langle \text{if } e_c \text{ et } cf, \text{ env}, \text{ sto}, k \rangle$

$\mapsto \langle pc, \text{ env}, \text{ sto}, \text{kif}(\text{env}, \text{et}, \text{ef}, k) \rangle$

$\langle v, \text{ env}, \text{ sto}, \text{kbox}(k, k) \rangle$

$\langle \text{box } e, \text{ env}, \text{ sto}, k \rangle$

$\mapsto \langle \sigma, \text{ env}, \text{ sto}[\sigma \mapsto v], k \rangle$

$\mapsto \langle e, \text{ env}, \text{ sto},$

where $\sigma = \text{malloc(sto)}$

$\text{kbox}(k) \rangle$

(2-y) $\oplus = \dots$ | pair \vee \vee
| ~~box~~ σ

Question : Should we add

set-fst : pair A B \times A \rightarrow ()

Set-snd : pair A B \times B \rightarrow ()

mpair a b = pair (box a) (box b)

mpair-set-fst p a' = setbox (fst p) a'

mpair-fst^L p = unbox (fst p)

let f $\lambda x_i =$
 $\text{let } ! x = \text{box } x_i \text{ in }$
 $\text{set! } x S;$
x ; in

let v = (box 7) ; in

set! v 8

v + v $e := \dots$ | set! x e

(1) \swarrow

\Rightarrow (2)

12-5 / Always store vars as pointers

ULD:

$$\Sigma / E[(\lambda x. e) \ v] \rightarrow \Sigma / E[e[x \leftarrow v]]$$

NEW:

$$\Sigma / E[(\lambda x. e) \ v] \rightarrow \Sigma[\sigma \mapsto v] / \\ \text{where } \sigma = \text{mallc}(\Sigma) \quad E[e[x \leftarrow \text{unbox } \sigma]]$$

$$\Sigma / E[\text{self! } (\text{unbox } \sigma) \ v] \rightarrow \\ \Sigma[\sigma \mapsto v] / E[v]$$

desugar $(e_1; e_2) =$

let $\text{marnihcarinacgnymng}$ = e_1 in

e_2

$$= (\lambda _ . e_2) \ e_1$$

12-6/ Fancy desugar

desugar $(\lambda x. \dots)$

x

$\text{set! } x \leftarrow$

\vdots
)

$= (\lambda x. \dots \text{let } x = \text{box } x_i \text{ in} \dots (\text{unbox } x))$

$\dots \text{set-box! } x \leftarrow$

desugar $M (\lambda x. e)$

if modified $x \in M$ then

$\lambda x. \text{let } x = \text{box } x_i \text{ in desugar } (\overbrace{M \cup \{x\}}^M) e$

o.w.

$\lambda x. \text{desugar } M e$

desugar $M x =$

if $x \in M$ then $\text{unbox } x$

o.w. x

12-7

$\text{eval } e$
= $\text{eval} \left(\text{let } \overbrace{\dots}^{\text{stdlib}} \text{ in } e \right)$

→ prints 11

→ compile

→ nys

desarrollar (map) = $\lambda f. \dots \dots \dots$

$\underline{15-1}$ $1 / 0$ $\delta \xrightarrow{\text{partial}}$
 $(S \quad 1)$ δ is undefined
 set-box! $7 \quad 2$ function app needs a box
 set-box needs a box

$$x \rightarrow y \rightarrow z \rightarrow (1 / 0) \rightarrow$$

$$\text{eval}(p) = v \text{ iff } p \xrightarrow{\text{partial}} v$$

$v = \dots | \text{bad bad bad}$
 $E[(v_0 \ v \dots)] \rightarrow \text{bad bad bad}$
 where $v_0 \in p, \in \lambda \dots$

$$v = \dots | \text{err} | \dots | \text{err} | \dots | \text{ok}$$

$$E[(p \ v \dots)] \rightarrow \text{err} \quad \delta(p, v) = \perp \quad E[a] \rightarrow E[b]$$

$$e = \dots | \text{abort } e \quad E' = E$$

$$E[\text{abort } e] \rightarrow e$$

$$\langle \text{abort } e, \text{env}, k \rangle \mapsto \langle e, \text{env}, \text{kret} \rangle$$

J₁:

15-2) $e = \dots | \text{throw } e$
 $| \text{try } e \text{ with catch } e$

$\text{try } (+ 1 \ (\text{throw } 2))$ $\Rightarrow |$
with catch $(\lambda x. (- x 1))$

$E = \dots | \text{try } \overset{e}{\cancel{\text{try}}} \text{ with catch } E$
 $| \text{try } E \text{ with catch } v$

$E [\text{try } v \text{ with catch } u] \rightarrow E[v]$

$E [\text{try } L[\text{throw } v] \text{ with catch } u]$
 $\rightarrow E[u v]$

$L = E - (\text{try } E \text{ with catch } v)$

$\text{try } (+ 1$
 $(\text{try } (+ 2 \ (\text{throw } 3)))$ $\Rightarrow 7$
with catch $(\lambda(v) (* v 2)))$
with catch $(\lambda(x) (* x 3))$

15-3 / $K = \dots \mid \text{preTry } K \ e \ \text{env} \ k$
 $\mid \cancel{\text{try }} K \ v \ k$

$\langle \text{try } e_b \text{ with catch } e_k, \text{ env}, k \rangle$

$\mapsto \langle e_h, \text{ env}, \text{ preTry } K \ e_b \ \text{env} \ k \rangle$

$\langle v_h, _, \text{ preTry } K \ e_b \ \text{env} \ k \rangle$

$\mapsto \langle e_b, \text{ env}, \text{ Try } K \ v_h \ k \rangle$

$\langle \text{vars}, _, \text{ Try } K \ v_h \ k \rangle$

$\mapsto \langle \text{vars}, _, k \rangle$

$\langle \text{throw } e, \text{ env}, \text{ Try } K \ v_h \ k \rangle$

$\mapsto \langle v_h \ e, \text{ env}, k \rangle$

$\mapsto \langle e, \text{ env}, \text{kapp } (v_h) \ _- \ () \ k \rangle$

$\langle \text{throw } e, \text{ env}, \text{kapp } (v...) \ \text{env}' (e...) k \rangle$

$\mapsto \langle \text{throw } e, \text{ env}, k \rangle$

$\text{preTry } K \ e \ \text{env}' \ k$

$\langle \text{throw } e, \text{ env}, \text{kret} \rangle \mapsto \langle e, \text{ env}, \text{kret} \rangle$

15-y ~~(f a b c)~~ \rightarrow

(let ([av a])

(if (function? av)

(if (= (function-arity av) 2)
^{arity}
(av b c))

(throw "wrong num args"))

(throw "not fun"))

(+ 2
 ^(throw o)))

try ...
 ^L [(throw e)]

with catch

(λ (x tryagain)

(tryagain 8))

$E[\text{try } L[\text{throw } e] \text{ with catch } u]$

$\rightarrow E[u \ e \ (λ (x) \ \text{try } L[x] \text{ with catch } u)]$

15-5/ First-class continuations

$e = \dots | \text{callcc } e$

$E = \dots | \text{callcc } E$

$$E[\text{callcc } v] \rightarrow E[v (\lambda(x) \text{ abort } E[x])]$$

$\text{cek } v = \dots | \text{Kont } K$

$K = \dots | k\text{callcc } K$

$\langle v, -, k\text{callcc } K \rangle$

$\mapsto \langle v^{(\text{kont})}, -, K \rangle$

$\langle \text{callcc } e, \text{env}, K \rangle \mapsto \langle e, \text{env}, k\text{callcc } K \rangle$

$\langle \text{if } c + f, \text{env}, K \rangle \mapsto \langle c, \text{env}, k\text{if env} + f K \rangle$

$\langle v, -, K\text{app } (\text{kont } K) - () - \rangle$

$\mapsto \langle v, -, K \rangle$

15-6 / ~~Ex~~

$f = (\lambda (x) \quad \text{int } f(\text{int } x)\}$
(callee λ (return))
(if ($x == 0$)
 (return 2);
 printf("0x%lx",
 return 2/x);
 ~~return~~
(print x)
(/ 2 x)))

$(+ 1 (f 7))$
 $(+ 1 (f 0))$
return = $\lambda x. \text{abort } (+ 1 x)$

$(\lambda (x \dots) b) \Rightarrow (\lambda (x \dots) \quad \text{callee } (\text{if } (x == 0) \text{return } 2; \text{printf } ("0x%lx", \text{return } 2/x); \text{return } 2/x)))$

15-7 /

(define last-handler
(box (λ (x) (abort x))))

(define throw
(λ (v) ((unbox last-handler) v)))

(esugar (try e₁ with catch e₂)
= (try-catch* (λ () e₁) e₂)

try-catch* := (λ (body new-handler)
(let ([old-handler (unbox last-handler)])
(catch (λ (here)
(set-box! last-handler (λ (x) (set! lh oh)
(here (nh x))))

~~(let ([ans (body)])
(set-box! lh oh)
ans))))))~~

17-1/ (f10) (5 3)

unsafe — just do something

... (f x)

((f10*)f) → code_ptr(x)
↓

jump (f + 8)

(define (f x) (f (scanf)))

unsafe = the language doesn't protect its abstractions

safe = DOES

C no abs., safe
C++ *(scanf()) (void*) o [2]

Java

JS

Py

Racket

] intend to be safe
but loopholes

$(\text{define } L (\text{load} \text{ "libOpenGL"}))$ $\rightarrow \text{load}$
 $(\text{define glDraw} (\text{extract } L \text{ "glDraw}))$ $\rightarrow \text{dlsym}$
 $(\text{glDraw} \dots)$

desugar $(\text{define } (f \ x \ \dots) \text{ body}) ; \text{ more}$
 \Rightarrow

$(\text{let } ([f \ (\lambda f \ (x \ \dots) \text{ body})])$
 (desugar more)

Assume we want safety

| | | |
|--------------------|-------------|------|
| unsafe kernel (vm) | safe kernel | vn |
| safe program | vn | vn |
| unsafe compiler | vn | safe |

safe kernel

$(f \ x) \Rightarrow \text{if } (\text{obj_tag}(f) == \text{CLO}) \{$
 $((\text{CLO}^*) \ f) \rightarrow \text{code_ptr}(x) \}$
 $\} \text{ else } \{ \text{ error } \}$

safe program $\Rightarrow \text{if } (\text{function? } f) \{$
 $(f \ x) \}$
 $\} \text{ else } \{ \text{ error } \}$

17-3] $P = \dots \neq \dots$ unsafe

stdlib =

\dots
(define (+ x y)

(if (and (number? x)

(number? y)))

(unsafe x y)

(error)))

desugar ($f x$) =
(if (fun? f) (if (= (arity f) 1)
 $(f x)$
error) error)

$P = \dots$ unsafe-apply

apply f (list $x y z$) = ($f x y z$)

desugar ($f x$) = safe-apply f (list x)

17-1 safety violation:

- what we wanted ctc
- what we got val
- who gave]- blame pos
- who got neg

(protect cte val pos neg)

(+

(protect num? v p n) ?)

\Rightarrow (if (num? v) v

(error "expected num, got" v "from
pos at neg"))

Desugar (+ x y) \Rightarrow

(msafe+ (protect num? x "line 27" "stdlib")
...)

17-5/ (define (map f l)
 (if (empty? l)
 empty
 (cons (f (first l))
 (map f (rest l)))))

map : (Num → Str) × (List Num) → (List Str)

protect (listof p) v pos neg =>
check all p v pos neg

protect (Num → Str) f pos neg => _{function proxy}
 $\lambda x.$
protect str (f (protect x $\xrightarrow{\text{Num Neg Pos}}$))
pos neg

18-1 / Macro Systems

Wav Macros

C Macros

```
#define DEBUG 1
```

```
#define MAX(x,y)  
((x) > (y) ? (x) : (y))
```

```
#define MAX(z,x,y)  
do {  
    z = (x) > (y) ? (x) : (y); }  
while (0); ;;
```

Excel Macros

```
F2 → Qx ≠ Z1
```

```
(let x+ = (x)  
y+ = (y)  
x+ > y+ ? x+ : y+)
```

C macros are textual not, expression-oriented

MAX(1 ? 2 : 3 , ...)

MAX(a++ , ...)

MAX(Z , x+)

purely substitutional

int ab[32] = { \uparrow b; : f(i) }
known at compile

{ F(0), F(1), F(2) ... }

```
#define F(x) (x) * 2 + 1
```

18-2/ The language kernel should be simple
and flexible...

features should be added on top of
old if possible

call/cc \Rightarrow generator, nondet, threads, try/catch
set-box! \Rightarrow set! and arbitrary descent
 λ \Rightarrow let

(let $x = e$ in b)
 $\Rightarrow (\lambda(x) b) e$)

Great languages have big desugars

(define-desugar-rules
[(let ([x xe]) be)
 (($\lambda(x)$ be) xe)])

[(let () be)
 be])

Syntax

18-3) $(\text{define}-\text{desugar}-\text{rules} \downarrow [\text{(define}-\text{desugar}-\text{rule} \text{ pat tem})$
 $\text{(define}-\text{desugar}-\text{rules}$
 $[\text{pat tem}])])$

dsrs : id \times List (pair (pat, template))

let := "let", $[< (\text{let } ([x \ x_e]) \text{ be}),$
 $((\lambda (x) \text{ be}) \ x_e) >]$

pattern-match : pat \times se \rightarrow env

transcribe : tem \times env \rightarrow se

pm $(\text{let } ([x \ x_e]) \text{ be}) \quad (\text{let } ([\text{foo } (+ 1 2)])$
 $(+ \text{ foo foo}))$

= $[x \mapsto \text{foo}, \quad x_e \mapsto (+ 1 2)$
 $\text{be} \mapsto (+ \text{ foo foo})]$

tr $((\lambda (x) \text{ be}) \ x_e) \quad ? \quad =$
 $((\lambda (\text{foo}) \text{ (+ foo foo)}) \ (+ 1 2))$

$$\begin{aligned} \text{18-4/ } pm \quad '() \quad '() &= \emptyset \\ pm \quad (\text{cons pa pd}) \quad (\text{cons ia id}) &= \\ pm \quad pa \quad ia \quad \uplus \quad pm \quad pd \quad id \\ pm \quad \text{var}(x) \quad in &= [x \mapsto in] \\ pm \quad \text{const}(n) \quad \text{const}(n) &= \emptyset \end{aligned}$$

$$\begin{aligned} tr \quad '() \quad env &= '() \\ tr \quad (\text{cons ta td}) \quad env &= (\text{cons} \quad tr(ta, env) \\ &\quad tr(td, env)) \\ tr \quad \text{var}(x) \quad env &= env[x] \\ tr \quad \text{const}(n) \quad env &= n \end{aligned}$$

18-5 / $(\text{let}^* ([x * 1] [y * 2] [z * x + y] [u * z + 3]))$

~~x~~

u)

$\Rightarrow (\text{let } ([x])$
 $(\text{let } ([y z]))$
 $(\text{let } ([z (+ x y)]))$
 $(\text{let } ([u (+ z 3)]))$
 $u))))$

dsrs

① $(\text{let}^* () \text{ be}) \Rightarrow \text{be}$

② $(\text{let}^* ([x_0 x_{e0}] \text{ more ...}) \text{ be}) \Rightarrow$
 $(\text{let } ([x_0 x_{e0}]) (\text{let}^* (\text{more ...}) \text{ be}))$

pm ② $(\text{let}^* \uparrow) =$

$[x_0 \mapsto x, x_{e0} \mapsto 1, \text{be} \mapsto u]$

$\text{more} \mapsto \text{MANY}([y z], [z (+ x y)], [u (+ z)])$

$\text{fr } (\text{list tmp !...}) \text{ env} = \text{map } (\text{fr tmp}) \text{ (alloutmany env)}$

pm $(\text{list pat !...}) \text{ in} =$

$\text{mergeinto many } (\text{map } (\text{pm pat}) \text{ in})$

$[x \mapsto \text{a}++ , y \mapsto 7]$

18-6/ dsf (or x y) \Rightarrow

(let ([tmp x])
(if *mp tmp
y)) if x x
y}

a--

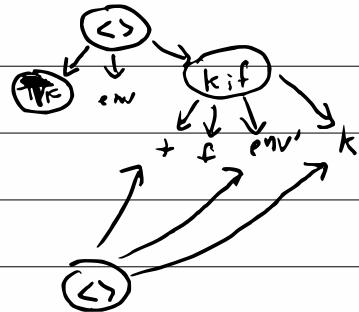
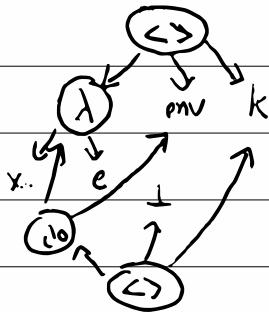
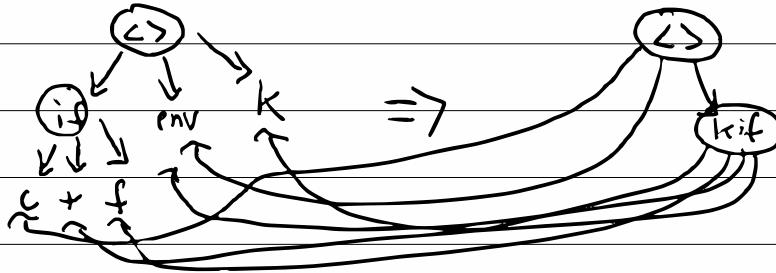
(or ~~tmp~~ 7)

(let ([tmp y]) (let ([tmp y])
(or #false tmp)) \Rightarrow (let ([tmp false])
(if tmp tmp tmp)))

19-1] Memory Management

$\langle \text{if } c + f, \text{ env}, k \rangle$

$\mapsto \langle c, \text{ env}, \text{kif}(+, f, \text{env}, k) \rangle$



19-2) $e = \text{num} \mid (\text{op } e \text{ } e) \{$

$\text{op} = + \mid - \mid \times \mid \div$

$k = \text{ret} \mid L(\text{op } k \text{ } e) \mid R(\text{op num } k)$

app

$\langle \text{op}, e_L, e_R \rangle, k \rangle \mapsto \langle e_L, L(\text{op}, e_R, k) \rangle$

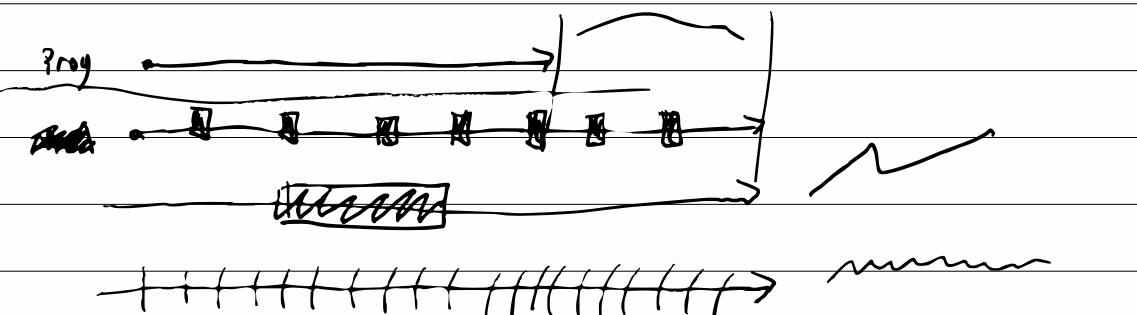
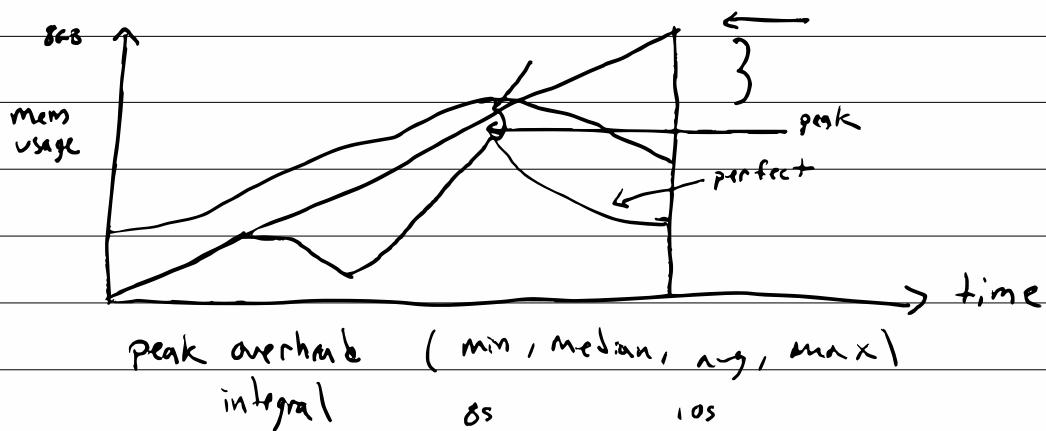
$\langle \text{num}, L(\text{op}, e_R, k) \rangle \mapsto \langle e_R, R(\text{op}, \text{num}, k) \rangle$

$\langle \text{num}_L, R(\text{op}, \text{num}_R, k) \rangle \mapsto \langle \delta(\text{op}, \text{num}_L, \text{num}_R), k \rangle$

19-3/ what should a MM do?

- know when to call free()
- wait to free to end (never free in between)
- freeing ~~next~~ "active" objects
 $f(x) = a$ but $= b$

Soundness = MM preserves same answer



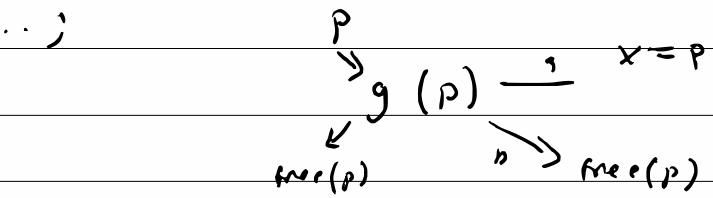
19-y) How do you get ~~some~~ MM in C?

unsafe comes from aliases
one pointer two vars/fields

f() {

char * c = ...;

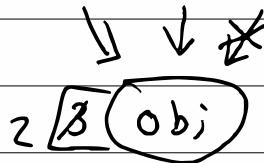
free(c)
return; }



- always pass ownership
- always make copies

(Alias)

19-5 / Reference Counting / Smart Pointers



`mkref (p) :=`

`p.count ++`

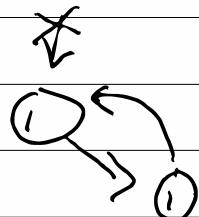
`rmref (p) :=`

`; (! - p.count)`

`free (p);`

64-bit counts

8-bit



`&p`

`*p = NewC`

fails on cycles (leaks)

20-1/ When is an obj free() -able?

scope of var X { int * x = malloc(...)

last use of ~~obj~~ }
ret }
x is unbound

first of x }
x }
not needed }
... }
no xs }
last }

{ int * x = malloc(...)

... x }
free() :
if (f(?)) {

Truth (Does much HALT?) I don't
vs x

use at 3

Probability (Do I know?) I know I
don't x

or never use

: 3

20-2] Reachability

Suppose o is an obj in memory

$\text{reach}(o)$ iff $\text{var}(o)$

∨ $\text{ptr}(\text{p}, o) \wedge \text{reach}(\text{p})$

∨ $\text{field}(\text{o}', o) \wedge (\text{reach}(\text{o}'))$

∨ $\text{reg}(o)$

∨ $\text{stack}(o)$

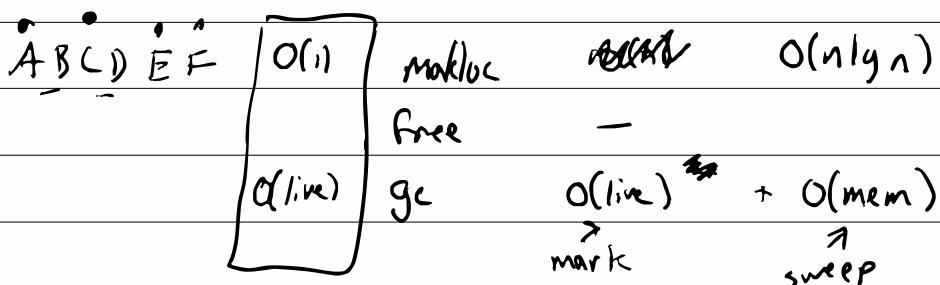
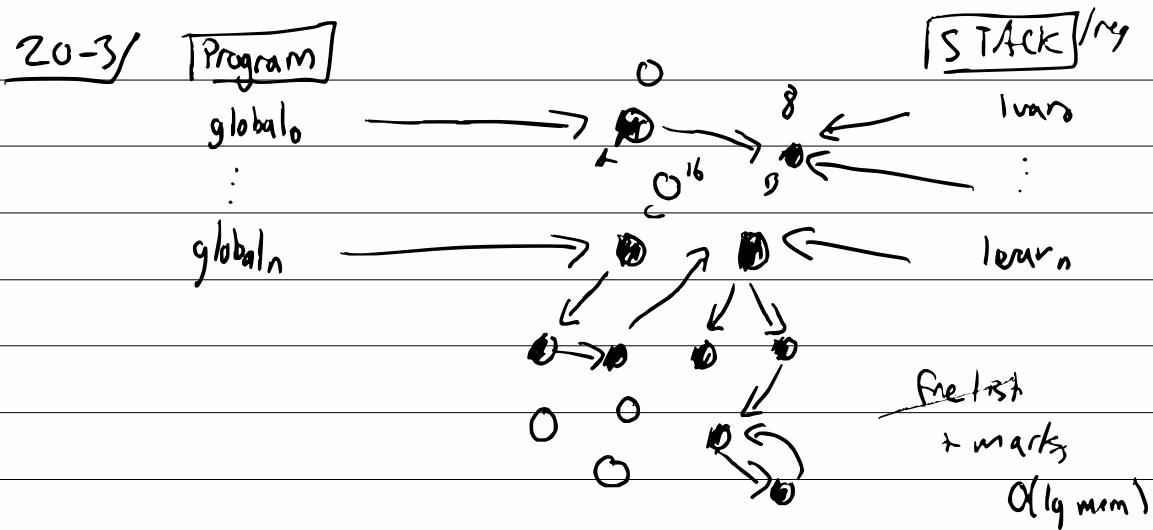


$\checkmark(o.f)[3], m.o.x)$

$o = \text{NULL}$

$\hookrightarrow \perp X \rightarrow \rightarrow \rightarrow$

Unreachable objects may be freed



Constraints:

- know size of objs
 - all known pointers from malloc
 - know obj layout

tricks:

- mark in tag
SB; BoP

John McCarthy 1969
LISP

21-1 <http://p.org>

| | | |
|-----|----------------|----------------|
| BSL | P ₁ | T ₁ |
| ISL | | |

Count * prize = sum ; ASL
not an l-value
cannot cast int to int*

Mark and Sweep

Time - malloc - $O(n \lg n)$
free - X

gc - $O(\text{live}) + O(\text{mem})$

Space - overhead - mark bits = $O(\lg \text{mem})$

latency - could do tri-color - arb small pause

Time - malloc - $O(1)$ Stop

free - X and

gc - $O(\text{live})$ copy

Space - overhead - $\times 2$

latency - long

21-2 malloc

O(m' n)
searching a tree

O(1)

free node

bool sector?

(bump Ptr)

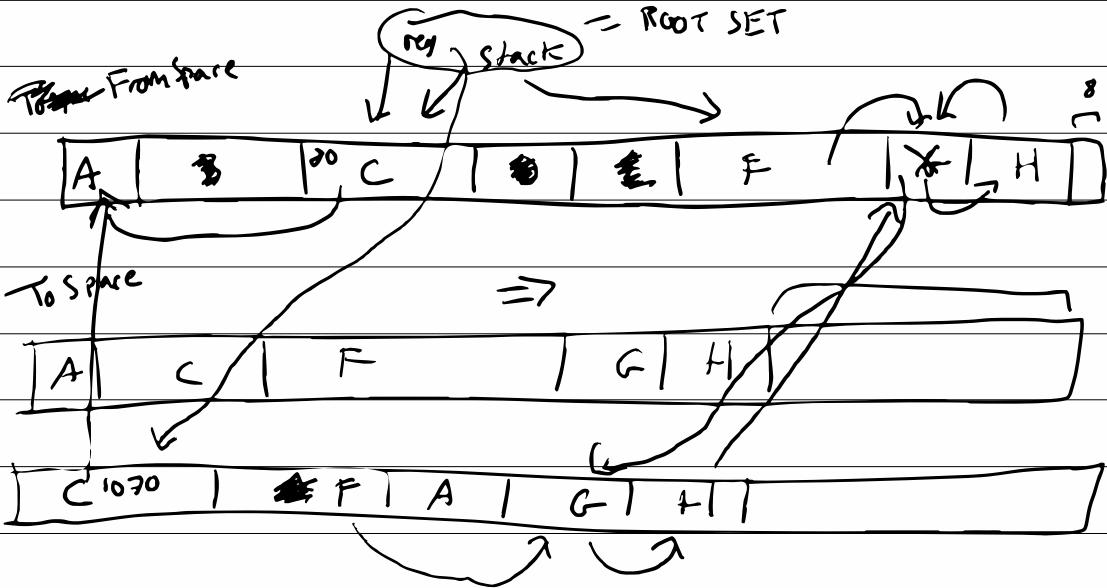
malloc (size -+ sz) {

if (free_p + sz < last_free_spot) {

free_p += sz;

ret free_p - sz; }

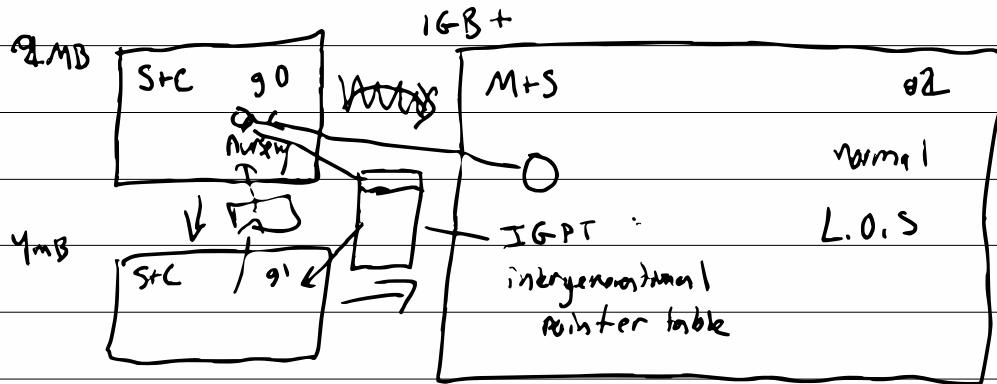
< Isr { gc(); malloc(sz); } }
sz true



forward my pointer : new tag
contains a pointer

| | | | | |
|-------------|-----|---|--|-----------------|
| <u>22-1</u> | M&S | - | $O(\lg n)$ malloc | $O(1/n) + O(n)$ |
| | | | $O(\lg n)$ overhead | time |
| | S&C | - | $O(1)$ malloc | $O(1/n)$ |
| | | | $O(\lg n)$ $\times 2$ overhead | time |

Generational Collection



Generational Hypothesis

Hypothesis: Objects live a very long time or a very short time
 "Most objects die young"

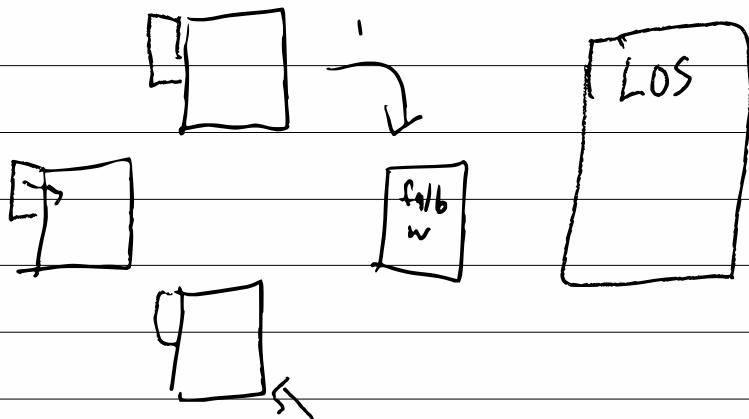
write barrier

Large object can exception

$$O.f = \begin{cases} n & \text{if all} \\ \text{old} & \text{new} \\ \text{if too} \\ \text{large} & \text{IG-PT} \end{cases}$$

22-2/ Radioactive Decay "half life"

Pick any number $1/N$ ($N=4$)



22-3 / Type Systems

Syntaxic

return 1 + ;

logic

return 41;

partial fun

1 / 0;

first (NULL)

type errors

"foo" + 1;

=> "fme" (PHP)

types provide safety

unsafe kernel

< safe kernel

safe compiler (inserts checks)

✓

unsafe k

unsafe compile

Safe type system

$x = \text{rand_a_bool}$

$o = \text{NULL}$

$f(x) :$

$o = \text{new cat}$

else

$o = \text{new Dog}$

...
 $(o \text{ and } x \text{ undefined})$

: if $x :$

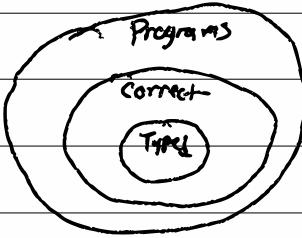
~~$\text{O}, \text{pure!}$~~

else :

$O, \text{bark!}()$

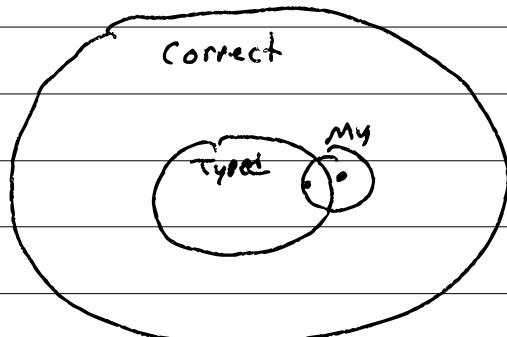
occurrences
typing

/ Typed Racket
TypeScript
Haskell



Cog

23-1/ Gradual Typing



Typed Racket

Type Script

Hack

F#?

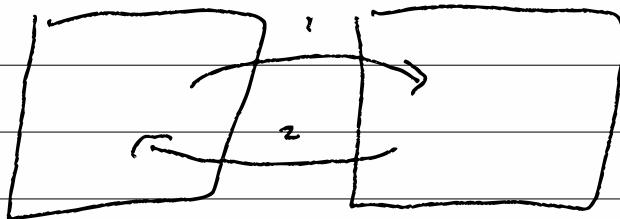
Partialized Python

My programs

Coffee Script

Typed part

Untyped part



1. $T \Rightarrow UT$: Always safe

2. $UT \Rightarrow T$: Unsafe

(typed e) $\Rightarrow e$

(untyped e) \Rightarrow (contract e supposed-to-be
untyped 'typed')

(typed $(\lambda(x).(\text{untyped } (+ x 1)))$) \Rightarrow ok

$(\lambda(x)(\text{typed } (+ x 1)))$ \Rightarrow illegal

$(\lambda(x)(\text{typed } (+ (\text{untyped } x) 1))) \Rightarrow (\lambda x. (+ (\text{constant } x \text{ num}))$

23-2) Type Systems make predictions

Theory — prediction

Model — real phenomenon

" $X \vdash P$ " " X says P "

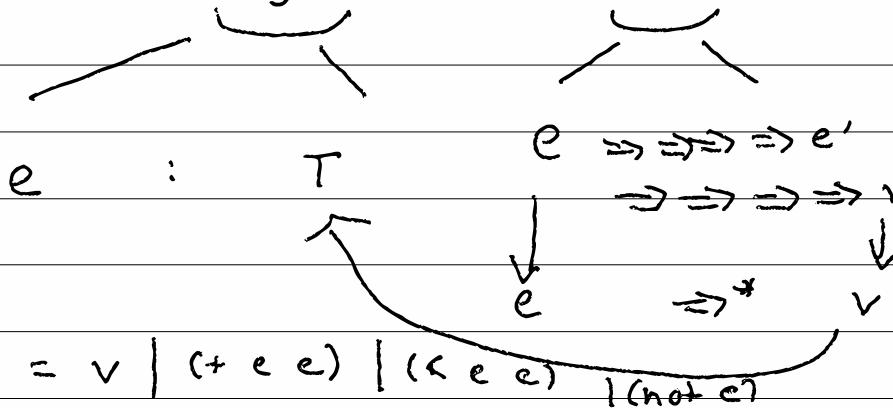
"Gravity + the pen will drop" — Theory stat

"Universe + the pen will drop" — Model stat/exp

O $\forall p. \text{Theory} \vdash P \Rightarrow \text{Model} \vdash P$ soundness

X $\forall p. \text{Model} \vdash P \Rightarrow \text{Theory} \vdash P$ completeness

23-3) HA. Theory + P \Rightarrow Model + P



$$\vdash \text{true} : \text{Bool} \quad \vdash \text{false} : \text{Bool}$$

$$\vdash N : \text{Nat}$$

$$\frac{\vdash e_1 : \text{Bool}}{\vdash (\text{not } e_1) : \text{Bool}}$$

$$\vdash e_1 : \text{Nat} \quad \vdash e_2 : \text{Nat}$$

$$\frac{}{\vdash (e_1 + e_2) : \text{Nat}}$$

$$\frac{\vdash e_1 : \text{Bool} \quad \vdash e_2 : \text{Bool}}{\vdash (e_1 < e_2) : \text{Bool}}$$

$$\frac{H : \text{Nat} \quad \vdash I : \text{Nat}}{\vdash (I + I) : \text{Nat}}$$

$$\frac{\vdash (I + I) : \text{Nat} \quad \vdash I : \text{Nat}}{\vdash ((I + I) < I) : \text{Bool}}$$

$$\vdash (\text{not } ((I + I) < I)) : \text{Bool}$$

$$\frac{\begin{array}{c} \frac{\vdash A \quad \vdash B}{\vdash A \vee B} \\ \frac{\vdash A \quad \vdash B}{\vdash A \wedge B} \end{array}}{\vdash A \vee B}$$

$\boxed{23-y/ \quad \text{Ae}, T.}$ $\boxed{\vdash e : T \quad \Rightarrow_{\exists} e \Rightarrow^* v \quad \vdash v : T}$ $\boxed{\text{Soundness Theorem} \quad \Rightarrow \text{Strong Normalization}}$
 "all programs finish"

$e = \dots | x \quad | \text{ let } x = e \text{ in } e$

$\frac{}{\vdash e : T} \quad \vdash e_2[x \leftarrow e] : T$

$\vdash x : L \quad \frac{}{\vdash \text{let } x = e_1 \text{ in } e_2 : T}$

$\frac{x \rightarrow T}{\vdash e : T}$

$\frac{}{\vdash e : T}$

$\frac{\Gamma(x) = T}{\vdash x : T}$

$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma[x \mapsto T_1] \vdash e_2 : T_2}{\Gamma \vdash \text{let } x = e_1 \text{ in } e_2 : T_2}$

$\frac{}{\vdash x : T}$

$\frac{}{\vdash \text{let } x = e_1 \text{ in } e_2 : T_2}$

$e = \dots | e \quad e \quad | \lambda x. e \quad | \lambda x : T. e$
 $T = \dots | T \rightarrow T \quad \frac{\Gamma[x \mapsto T_1] \vdash e : T_2}{\Gamma \vdash \lambda x : T_1. e : T_1 \rightarrow T_2}$

$\frac{\Gamma \vdash e_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash e_2 : T_1}{\Gamma \vdash (e_1, e_2) : T_2}$

$\frac{\Gamma[x \mapsto T_1] \vdash e : T_2}{\Gamma \vdash \lambda x. e : T_1 \rightarrow T_2}$

23-5 / type of : $\Gamma \vdash \text{Expr} \rightarrow \text{Type}$

$\approx (\text{Var} \rightarrow \text{Type})$

type of g ($\text{Bool } b$) = $T\text{Bool}$

type of g ($\text{Num } n$) = $T\text{Num}$

type of g ($\text{Add } l \ r$) =

if (type of $g \ l$) == $T\text{Num}$

\wedge (type of $g \ r$) == $T\text{Num}$

then $T\text{Num}$

or error

type of g ($\text{Var } x$) = $g \ x$

type of g ($\text{App } e_1 \ e_2$) =

case (type of $g \ e_1$) of

$\text{TArrow } \text{dom } \text{rng} \Rightarrow$

if type of $g \ e_2$ == dom then

rng

or error

$__ \Rightarrow \text{error}$

type of g ($\text{Lam } x^{+1} \ e$) = $\text{TArrow } +_1 \ +_2$

where $+_2 = \text{type of } g' \ e$

$g' = g \ [x^{+1} / x^{+1}]$

$\approx \text{TArrow } +_1 (\text{type of } (\text{ext } g \ x^{+1}) \ e))$

23-6/ Progress:

$\forall e, T, \Pi, \Gamma \vdash e : T$

$\rightarrow \exists e', e \Rightarrow e'$

Or $e \in V$

Preservation

$\forall e, T, \Pi, e' \vdash \Pi \vdash e : T \wedge e \Rightarrow e' \rightarrow \Pi \vdash e' : T$

$(\lambda x. x x) (\lambda x. x x) \Rightarrow \text{itself}$

$(\lambda x : T_x . x x)$ $(\lambda y : T_y . y y)$

$(\lambda x : T_x . x x) : T_x \rightarrow T_x$ $(\lambda y : T_y . y y) : T_y \rightarrow T_y$

$[x : T_x] \vdash x : T_x$ $(T = \text{Bool} \mid \text{Nat} \mid T \rightarrow T)$

$x : T_x \Rightarrow T_x$ $x : T_x$

$T_x = T_x \Rightarrow T_x$

no

$TAF = \epsilon$

\dots

25-1/ struct int + ϵ

int + {

obj tag;

obj tag;

INL

obj* v; }

obj* v; }

INR 3

case e_0 of [inl $x_1 \Rightarrow e_1$]

[inr $x_2 \Rightarrow e_2$]

$= = =$

if (e_0^{tag} == INL)

run e_1 with $x_1 \mapsto e_0 \Rightarrow v$

o.w. run e_2 with $x_2 \mapsto e_0 \Rightarrow v$

(inl e) = pair T e

(inr e) = pair F e

these case s fl fr = if fst s then

fl (snd s)

o.w. fr (snd s)

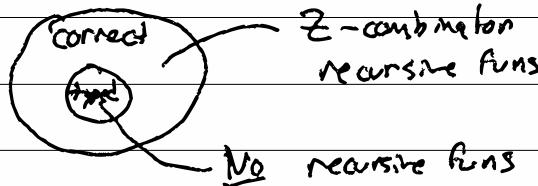
$$25-2 / e = x \mid (e \ e) \mid v$$

$$v = \cancel{x:e} \mid p \mid c$$

$$\tau = T \Rightarrow T \mid B$$

B = "base types" one for every kind of
num & c \rightarrow Num $\in B$

$$\Delta(p) = \tau \quad \Delta(+) = N \Rightarrow N \Rightarrow N$$



$$v = \dots \mid \cancel{\lambda f(x:T). e} \mid f(x:T). C$$

$$(\lambda Q f(x:T). e \quad +: (\Gamma, e))$$

$$\frac{\Gamma [x:T] [f:T \rightarrow Q] \vdash e: Q}{\Gamma \vdash \lambda f(x:T). e : T \rightarrow Q}$$

$\Gamma \vdash e :$

"_ proves _ has type"
"g prove x has type"

25-3) true := $\lambda x. \lambda y. x$

false := $\lambda x. \lambda y. y$

if := $\lambda c. \lambda x. \lambda y. c \times y$

$$\lambda w f(x=w), \lambda w g(y=w), x$$

↓ ↓ ↓ ↓

if $\in L_2$

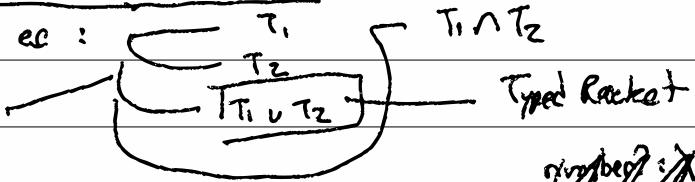
$e = \dots | \text{if } e \ e \ e$

$L_3 = L_2 \cup \{\text{if}\}$

$\Gamma \vdash e_c : \text{Bool} \quad \Gamma \vdash e_t : T_1 \quad \Gamma \vdash e_f : T_2$

$\Gamma \vdash \text{if } e_c \ e_t \ e_f : \boxed{T_1 \quad T_2}$

$T_1 = T_2 \Rightarrow T_1$



new type? ~~new type~~
new term ~~new term~~

$y = \text{if } \cancel{x > 0} \text{ then "four" o.w. } 4$

$\models \{X \text{ is a num}\}$

$\Gamma \vdash e : T ; P_T ; P_F$

$\Gamma \vdash e_c : \text{Bool} ; P_{TC} ; P_{FC}$

$\Gamma \cup P_{TC} \vdash e_t : T_1 ; P_{TT} ; P_{FT}$

$\Gamma \cup P_{FC} \vdash e_f : T_2 ; P_{TF} ; P_{FF}$

$P_T = P_{TT} \cap P_{TF} \quad P_F = P_{FT} \cap P_{FF}$

$\Gamma \vdash \text{if } e_c \ e_t \ e_f : T_1 \cup T_2 ; P_T ; P_F$

| | | | |
|--|-------|-------------|------------------|
| <u>$\Sigma S^{\text{-y}}$</u> | pairs | $e = \dots$ | pair e_1, e_2 |
| | | | fst e_2 snde |
| | | $T = \dots$ | $T \times T$ |

$$\frac{\Gamma \vdash e_1 : T_1 \quad \Gamma \vdash e_2 : T_2}{\Gamma \vdash \text{pair } e_1, e_2 : T_1 \times T_2}$$

$$e = \dots \mid \text{inL } e \mid \text{inr } e \mid \text{case } \dots$$

$$T = \dots \mid T + T$$

$$\frac{\Gamma \vdash e : T_1}{\Gamma \vdash \text{inL}_e : T_1 + T_2} \qquad \frac{\Gamma \vdash e : T_2}{\Gamma \vdash \text{inr}_{T_1} e : T_1 + T_2}$$

$$e = \dots \mid \text{box } e \mid \text{unbox } e \mid \text{setbox } e \ e$$

$$T = \dots \mid \text{Box } (\tau)$$

$$\Gamma \vdash e_1 : \text{Box}(\tau)$$

$$\frac{\Gamma \vdash e : T}{\Gamma \vdash \text{box } e : \text{Box}(T)} \qquad \frac{\Gamma \vdash e \in \text{Box}(C)}{\Gamma \vdash \text{unbox } e : T} \qquad \frac{\Gamma \vdash e_2 : T}{\Gamma \vdash \text{setbox } e_1, e_2 : V}$$

25-5/ template < class X > ArrayList<int>
 size_t f(X x) {
 return 2 * sizeoff(x); } ArrayList<Dog>

f <int> (s)

f < Dog > (Mickey)

$e = \dots | \lambda X. e | e < \cancel{T} >$
 $T = \dots | \forall x. T | x$

untyped typed

$\lambda x. x$ $\lambda \alpha. \lambda \alpha. f(x: \alpha). x = id$
 $: \forall \alpha. \alpha \Rightarrow \alpha$

$\frac{\Gamma, X \vdash e : T}{\Gamma \vdash \lambda x. e : \forall x. T}$

$\frac{\Gamma \vdash e : \forall x. T_2}{\Gamma \vdash e < T_1 > : T_2 [x \leftarrow T_1]}$

id < num > (s) : num
 num \rightarrow num

Java: pretend
 don't exist
 x = object

Fw

$\lambda X. e \Rightarrow$ C++:
 $e < T > \Rightarrow C[(\lambda x. e) < T >] \rightarrow e[x \leftarrow T]$

26-1/ (define length

$(\lambda r. len\ (i))$

(case 1 of

[inl $x \Rightarrow 0$]

[inr $y \Rightarrow (+\ 1\ (\len\ (\snd\ y))))]]))$

true := $\lambda x. \text{dy. } x$

false := $\lambda x. \text{dy. } y$

sharp env $(c\ (\lambda z.\ s)\ (\lambda z.\ f))\ o$

inl := $\lambda x. \text{pair true } x$

inr := $\lambda y. \text{pair false } y$

case¹ := $\lambda s. \lambda lc. \lambda rc.$

let nc := if (fst s) then lc else nc in
nc (snd s)

(case 0 of [inr $x \Rightarrow \text{re}\}]$

[inl $y \Rightarrow \text{le}\}] \Rightarrow$

(case¹ s ($\lambda x. \text{re}\$) ($\lambda y. \text{le}\$))

28-2/

(v-hont k)

$\langle \text{call/cc } e, \text{ env}, k \rangle \mapsto \langle e, \text{env}, \text{kcalle } k \rangle$

$\langle v, \text{env}, \text{kcalle } k \rangle \mapsto \langle v, \underbrace{k}, \text{env}, k \rangle$

$\text{app}(v, [k])$

$\langle v_0, \dots, \text{kapp}((k')), \dots, (), k \rangle \mapsto$

$\mapsto \langle v_0, \dots, k' \rangle$

$\langle \text{call/cc } e, \text{env}, k \rangle \mapsto \langle e \ k, \text{env}, k \rangle$

26-3) $T := \dots \mid x \mid \text{fx. } T$

$e := \dots | e < T >$
 $| \lambda x. e$

C++ Java (if < N.e.
Haskell M N.e.) <^{if}

$\text{id} = \lambda x. \lambda a. x.a \quad : \quad \forall x. x \Rightarrow x \quad \text{false}$
 $(\text{id} < \text{int} > \quad = \quad \text{id} < \text{bool} >) \Rightarrow \text{true} \quad ?$

functional extensibility

$$f = g \quad \text{iff} \quad \forall x, f_x = g_x$$

$$f: \forall x, x \rightarrow x$$

`gdd1 : Num → Num`

$$f := \lambda x. \lambda a. x.a$$

$$\text{add} \models \lambda n. 1 + n$$

$\lambda x. f a : x. \alpha$

~~4-12-2012~~

~~4. f. D~~ ~~f = f + x * b~~ ~~x~~ ~~n := Num~~
 $\lambda x. \lambda y. f(a : X) . \text{ let } r := \lambda z. r(z) . r(\text{add}(x, y))$
 in $r \neq$

$\lambda x. \lambda a z. x$. Let $n = 1 + 2$ in a

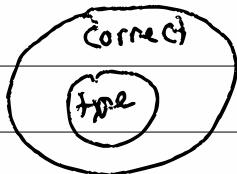
26-4) Template <class A>

A id (A a) {
if (dynamic-cast <dog> (a)) {
return ((dog) a). mate ; }
else {
return a; }
template <class Cat>
(cat id ((cat a) {
return furfield; })

map: ~~Map~~ $\forall A, \forall B, (A \rightarrow B) \rightarrow List(A)$
 $\rightarrow List(B)$

parametric polymorphism

26-5/



↗

dist From Origin (posn p) {
 $\sqrt{p.x^2 + p.y^2}; \}$

~~shifted from~~

posn 2d (int x , int y) {
 $\{ x=x; y=y \}; \}$

posn 3d (x , y , z) {
 $\{ x=x; y=y; z=z \}; \}$

dist From Origin (posn 3d (1 , 2 , 3))
 posn 2d (1 , 2)

T := ... | { f : T , ... }

e := ... | { f = e , ... } | e.f

posn = { x : M , y : M }
 $\{ x : \text{int}, y : \text{int} \}$

$$T_1 = T_2$$

$$T_1 = T_2 [\varphi \leftarrow X]$$

$$\frac{}{\forall x.T_1 = \forall x.T_2}$$

$$\text{Int} = \text{Int}$$

$$\frac{T_1 = T_3 \quad T_2 = T_4}{T_1 \rightarrow T_2 = T_3 \rightarrow T_4}$$

26-6 / A typed program shouldn't crash.

$e.f \Rightarrow e$ must have an f field

$$\frac{\Gamma + e : \{f_0:T_0, \dots, f_i:T_i, \dots f_n:T_n\}}{\Gamma + e.f_i : T_i}$$

$$\frac{\begin{array}{c} X \\ \Gamma + f : \cancel{R} \rightarrow R \\ \Gamma + a : \cancel{R} \end{array}}{\Gamma + f \ a : R} \quad \begin{array}{c} Y \leq X \\ \cancel{\text{compatible}} \end{array} \quad \begin{array}{c} \text{compatible (ie are} \\ \text{"the types match")} \\ \text{subtypes} \end{array}$$

$$\frac{}{\{f_0, \dots, f_n\} \supseteq \{g_0, \dots, g_m\}}$$

$$\frac{T \leq T}{\{f_0 : T_0, \dots, f_n : T_n\} \leq \{g_0 : T'_0, \dots, g_m : T'_m\}}$$

$$T_0 = T'_0$$

$$\text{"subtype relation"} = \leq \quad T_0 \leq T'_0$$

$$\frac{F \leq X \quad G \leq Y}{(F \times G) \leq (X \times Y)}$$

26-7/

$B \subset Y$

$X \subset A$

~~ranges~~

~~X~~

$\underline{X \rightarrow Y \quad \subset \quad A \rightarrow B}$

$\text{Animal} \rightarrow \text{Animal}$

$f : X \rightarrow Y$

$h(f)$

$\Downarrow \text{Cat} \rightarrow \text{Cat}$

$h(Bg(A)) \in$

$w := g(\text{some } : A)$

\vdots

$w.$ do an b thing

$w.\text{pum}()$

Liskov - substitution principle

26-8/ Java / C++

class Posn {

int x; int y;

class Posn3d {

int x; int y; int z;

static int distance(Posn p) {
 p.x, p.y; } }

instance (new Posn3d(1, 2, 3))

~~def~~

$X = T$
 $\underline{X <: Y}$

X inherits from Y
 $X <: Y$

structural
subtyping

nominal
subtyping

Theory

C++ Java

Haskell Python*
ML Go Racket*
Lisp Swift C*

Typed Relation

"duck typing"