Turing Machines

$$\delta : Q \times \text{"}\Sigma\text{"} \longrightarrow Q \times \overset{\text{Left}}{\text{or Right}} \times \text{"}\Sigma\text{"}$$

initial tape = input w/ infinite $\sqcup$ before & after
$\underset{\text{blank}}{\nearrow}$

$\Sigma = \{0, 1\} = $ input
$\text{"}\Sigma\text{"} = \{0, 1, \sqcup, \hat{0}, \hat{1}, EOF, true, false\} = $ tape alphabet

Two special states = ACCEPT and REJECT
↳ TMs may never <u>stop</u> ⟶ "halt"
   ↳ loops on diverges

diverge

loop
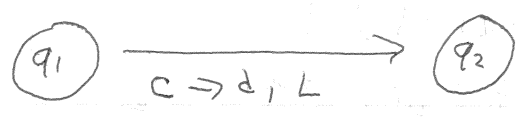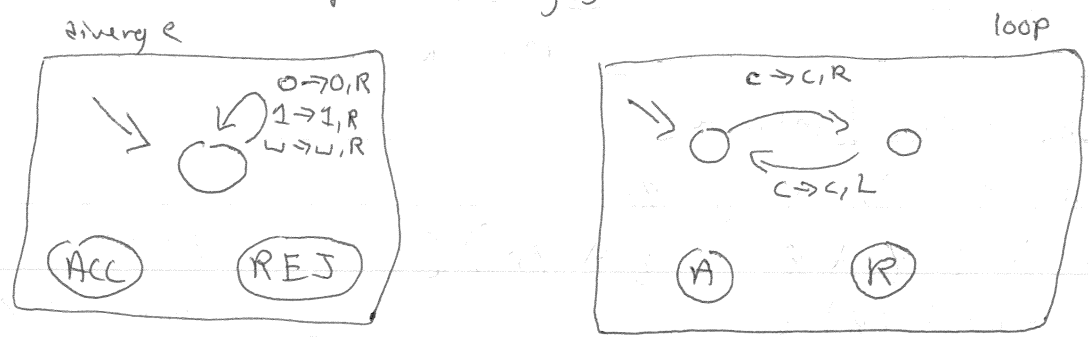
$$\delta(q_1, c) = (q_2, d, L)$$

$$TM \ t = (Q, \Sigma, \Gamma, \underset{\in Q}{q_0}, \delta, \underset{\in Q}{q_a}, \underset{\in Q}{q_r})$$

finite set   alphabet   alphabet

$\sqcup \in \Gamma$ and $\sqcup \notin \Sigma$
$\Sigma \subseteq \Gamma$

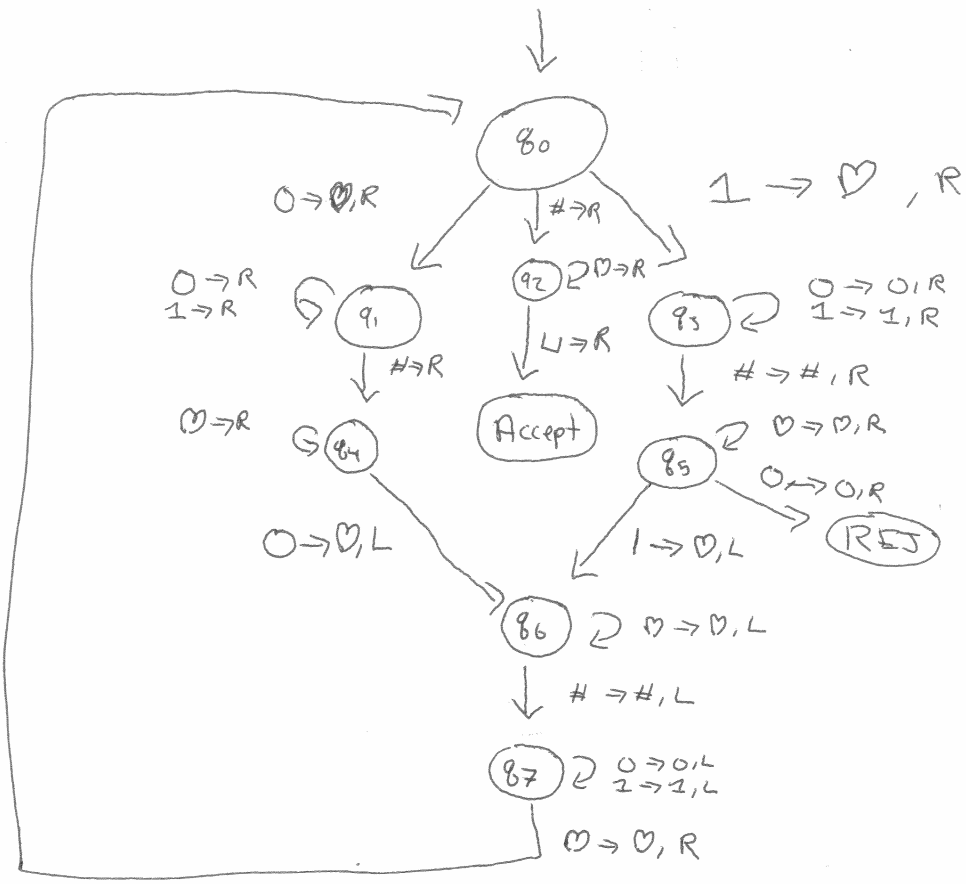$$\delta : (Q - \{q_a, q_r\}) \times \Gamma \longrightarrow Q \times \Gamma \times \{L, R\}$$

(7-2) $\{w\#w \mid w \in \{0,1\}^*\}$

$\Sigma = \{0,1,\#\}$

$\Gamma = \{0,1,\#,\sqcup,\heartsuit\}$

$c \to R$
$=$
$c \to c, R$

$0 \to \heartsuit, R$ q_0 $1 \to \heartsuit, R$

$\# \to R$

$0 \to R$
$1 \to R$ $\circlearrowleft$ q_1

q_2 $\circlearrowleft 0 \to R$

q_3 $\circlearrowleft$ $0 \to 0, R$ $1 \to 1, R$

$\sqcup \to R$

$\# \to R$

$\heartsuit \to R$ $\circlearrowleft$ q_4

Accept

$\# \to \#, R$

$\heartsuit \to \heartsuit, R$

q_5 $\circlearrowleft$ $0 \to 0, R$

$0 \to \heartsuit, L$

$1 \to \heartsuit, L$

REJ

q_6 $\circlearrowleft$ $\heartsuit \to \heartsuit, L$

$\# \to \#, L$

q_7 $\circlearrowleft$ $0 \to 0, L$ $1 \to 1, L$

$\heartsuit \to \heartsuit, R$

```
char * tape [infinite]
int pos = 0;  int q = q0;
while ( q ≠ qa ∧ q ≠ qr ) {
    switch (q) {
        case q0:
            switch ( tape[pos] ) {    ← reads the tape
                case ⊔: q = qr; break;
                case 0: q = q1; tape[pos] = ♡; pos++; break;    ← writes the tape
                case 1: "   q3   "   "
```

$pos{++} \Rightarrow$ right
$pos{--} \Rightarrow$ left

A run of TM t is a sequence of configuration

A config is a state, a head position, and a tape written as

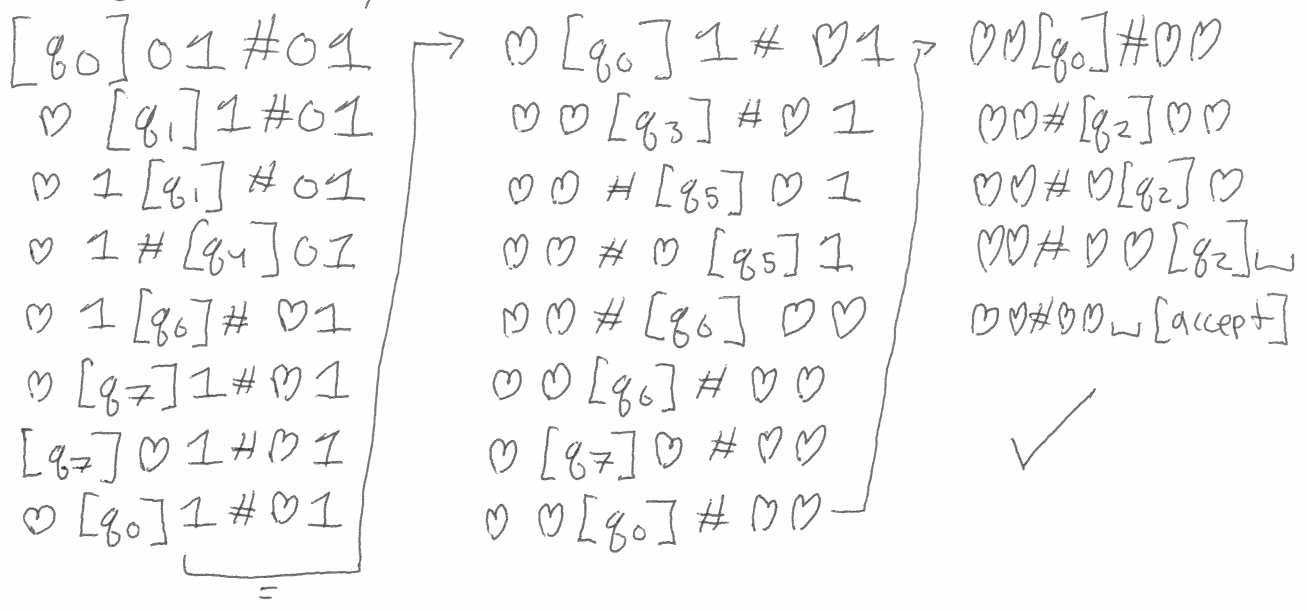$$w [\; q_i \;] v$$

the state is q_i
the tape is $\;\;\;\; \sqcup ... \sqcup v \sqcup ....$
head pos is between w and v (ie on v_0)

The initial config of TM t on input w is

$$\varepsilon [\; q_0 \;] w$$

Run w#w machine on 01#01
(checked 1st char)

$[q_0]$ 01#01 →	♡ $[q_0]$ 1# ♡1 →	♡♡$[q_0]$#♡♡
♡ $[q_1]$ 1#01	♡♡$[q_3]$#♡1	♡♡#$[q_2]$♡♡
♡ 1 $[q_1]$ #01	♡♡ # $[q_5]$♡1	♡♡#♡$[q_2]$♡
♡ 1# $[q_4]$01	♡♡# ♡ $[q_5]$1	♡♡#♡♡$[q_2]$⊔
♡ 1 $[q_6]$# ♡1	♡♡# $[q_6]$ ♡♡	♡♡#♡♡⊔ [accept]
♡ $[q_7]$1# ♡1	♡♡$[q_6]$# ♡♡	
$[q_7]$ ♡1#♡1	♡ $[q_7]$♡ #♡♡	✓
♡$[q_0]$1#♡1	♡ ♡$[q_0]$#♡♡	