Non-deterministic Turing Machines

Are TMs closed under concatenation?

Suppose X recog A and Y recog B, can you Z recog $A \circ B$?

$$[X_{q_0}] a_1 \cdots a_n \Rightarrow^* u_x [X, q_a] v_x$$

and

$$[Y, q_0] b_1 \cdots b_m \Rightarrow^* u_y [Y, q_a] v_y$$

then

$$[Z, q_0] a_1 \cdots a_n b_1 \cdots b_m \Rightarrow^* u_z [Z, q_a] v_z$$

General ND

$$\delta' : Q \times \Gamma \Rightarrow P(Q \times \Gamma \times \{L, R\})$$

$\overset{\nearrow}{\text{next state}}$ $\overset{\nwarrow}{\text{chars write}}$ $\overset{\nwarrow}{\text{direction}}$

$$\delta'(q_i, b) = \{ (q_j, a, L) \quad (q_j, b, R)$$
$$(q_k, a, R) \}$$

Forking

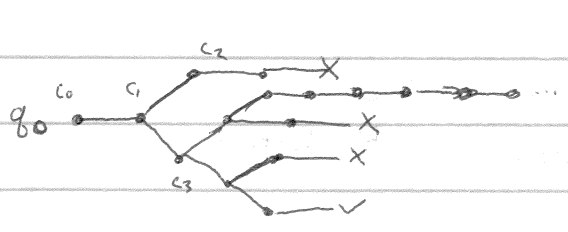$$\delta' : Q \times \Gamma \Rightarrow (Q \times \Gamma \times \{L, R\}) + (Q \times Q)$$

$\qquad\qquad\qquad$ no-nondet $\qquad\qquad\qquad$ fork

$$\delta(q_i, a) = (q_j, q_k) \qquad\qquad \delta(q_i, a) = (q_j, q_k)$$

$$u [q_i] a v \Rightarrow u [q_j] a v \qquad\qquad u [q_i] a v \Rightarrow u [q_k] a v$$

Depth-first search — trace wall

Breadth-first search

$F_{config} = P(config)$

$F_{c_0} = \{c_0\}$

$w \in L(FT)$ iff

$$\{[q_0]w\} \Rightarrow^* FC \cup \{u[q_a]v\}$$

$\delta(q_i, b) = (q_j, c, L)$

$$FC \cup \{ua[q_i]bv\} \Rightarrow FC \cup \{u[q_j]acv\}$$

$\delta(q_i, b) = (q_j, q_k)$

$$FC \cup \{ua[q_i]bv\} \Rightarrow FC \cup \{ua[q_j]bv, \; ua[q_k]bv\}$$

R on xs

then R^* is the refl, trans closure

R^+ is the trans closure

reflexive: $\forall x, \; (x,x) \in R$

transitive: $\forall x,y,z, \; (x,y) \in R$
$\wedge \; (y,z) \in R$
$\Rightarrow (x,z) \in R$

$[,], q \in Q \in \Gamma$

$\{c_0, c_1, c_2, c_3\} \quad \longrightarrow \quad []\ulcorner c_0 \urcorner \# \ulcorner c_1 \urcorner \# \ulcorner c_2 \urcorner \# \ulcorner c_3 \urcorner \qquad \ulcorner u[q_i]av \urcorner = u[q_i]av$

$\Downarrow \qquad\qquad\qquad \Downarrow_x$

$\{c_1, c_2, c_3, c_4, c_5\} \quad \longleftarrow \quad []\ulcorner c_1 \urcorner \# \ulcorner c_2 \urcorner \# \ulcorner c_3 \urcorner \# \ulcorner c_0 \urcorner \qquad (\text{det})$

$\qquad\qquad\qquad\qquad []\ulcorner c_1 \urcorner \# \ulcorner c_2 \urcorner \# \ulcorner c_3 \urcorner \# \ulcorner c_4 \urcorner \# \ulcorner c_5 \urcorner (fork)$

Space inefficient, Time efficient

Directions: Strings of "Left", "Right", "Forward"

$\qquad\qquad$ fork l $\qquad$ Lfork r $\qquad\qquad \hookrightarrow$ determ

Printer: shortest to longest w/o duplicates

$\qquad\qquad\qquad$ tape 1: simulation of current path
$\qquad\qquad\qquad$ tape 2: original input
$\qquad\qquad\qquad$ tape 3: current directions