Eval $\overset{①}{\Rightarrow}$ Apply $\overset{②}{\Rightarrow}$ Eval

① $([(f\ e_1\ ..\ e_n)],\ B,\ ve,\ t)$
  $\Rightarrow (proc,\ \overrightarrow{d},\ ve,\ t')$
where $t' = succ\ (t)$
  $proc = A\ (f,\ B,\ ve)$
  $d_i = A\ (e_i,\ B,\ ve)$

$A\ (v,\ B,\ ve) = ve\ (v,\ B(v))$
  $\underset{Var \to Time}{\nearrow}\quad \underset{Var \times Time \Rightarrow D\ (value)}{\nwarrow}$

$(define\ (f\ x)\ (f\ (+\ x\ 1)))\qquad (f\ 0)$

$A\ (lam,\ B,\ ve) = (lam,\ B)\quad //A\ closure$
② $(([(\lambda\ (v_1\ \cdots\ v_n)\ call)], B),\ \overrightarrow{d},\ ve,\ t)$
  $\Rightarrow (call,\ B',\ ve',\ t')$
where $t' = succ\ (t)$
  $B' = B\ [v_i \mapsto t']$
  $b_i = (v_i,\ t')$
  $ve' = ve\ [b_i \mapsto d_i]$

$$(\lambda \ (halt) \ call)$$

18-2/ $\quad I : PR \Rightarrow State$

$$I(\,[\![(\lambda \ (halt) \ call)]\!]\,) =$$

(Apply) $\quad (\,([\![(\lambda \ (halt) \ call)]\!], \ \emptyset), \ <halt>, \ \emptyset, \ t_0\,)$

$$done \quad = \quad (halt, \ <d>, \ ve, \ t) \quad (Apply)$$

$$V : PR \Rightarrow P(state)$$
$$V(pr) = \{ \ st \ | \ I(pr) \Rightarrow^* st \ \}$$

$$\forall s \in V(pr).$$
$$s.ve \ (v, \ t) = s.ve(v, \ t') \quad \forall t, t'$$

V^* $\qquad V(rr) \subseteq V^*(p\wedge)$

$\qquad\qquad\qquad$ $\underline{\qquad}$ valid safety property

$\qquad\qquad\qquad\qquad$ not liveness

$\qquad\qquad V^*$ should be computable

$$abstract \ state \quad = \quad \widehat{State}$$
$$1 \cdot 1 \ : \ State \Rightarrow \widehat{State}$$
$$\widehat{V} \ : \ PR \Rightarrow P(\widehat{State})$$

18-3/ $\mathbb{Z}$ = integers   0, -1, 1, -2, 2, ...

$\hat{\mathbb{Z}} = P(\{ neg, zero, pos \})$

$$|z| = \begin{cases} \{neg\} & z < 0 \\ \{zero\} & z = 0 \\ \{pos\} & z > 0 \end{cases}$$

$a \hat{+} b =$

$\{pos\} \hat{+} \{pos\} = \{pos\}$

$\{neg\} \hat{+} \{pos\} = \{neg, zero, pos\}$

$\{neg, pos\} \hat{+} \{zero\} = \{neg, pos\}$

$|4 + 3| \subseteq |4| \hat{+} |3|$ $\forall a, b.$

$\{pos\} \subseteq \{pos\}$ $|a+b| \subseteq |a| \hat{+} |b|$

$\forall s, s'. (s \Rightarrow s') \quad \rightarrow \quad |s| \hat{\Rightarrow} A \quad \Rightarrow \quad |s'| \subseteq A.$

$Conc(\hat{st}) \sim \{st : |st| \subseteq \hat{st}\}$

$Conc(\{zero\}) = 0$

$Conc(\{pos, zero\}) = \{z \mid z \geq 0\}$

$V^*(pr) = \bigcup\limits_{\hat{st} \in \hat{V}(pr)} Conc(\hat{st})$

Concrete	Abstract

$st \in Eval = CALL \times BEnv \times VEnv \times Time$ $\hat{st} \in \widehat{Eval} = CALL \times \widehat{BEnv} \times \widehat{VEnv} \times \widehat{Time}$

$+ Apply = Proc \times D^* \times VEnv \times Time$ $+ \widehat{Apply} = \widehat{Proc} \times \widehat{D}^* \times \widehat{VEnv} \times \widehat{Time}$

$d \in D = Proc$ $\hat{d} \in \hat{D} = P(Proc)$

$t \in Time = infinite$ $\hat{t} \in \widehat{Time} = finite$

$A(lam, B, ve) = (lam, B)$ $\hat{A}(lam, B, ve) = \{(lam, B)\}$

$A(v, B, ve) = ve(v, B(v))$ $\hat{A}(v, \hat{B}, \hat{ve}) = \hat{ve}(v, \hat{B}(v))$

$\overset{\nearrow}{one\ thing}$ $\overset{\nearrow}{set\ of\ things}$

$([(f\ e_1 \dots e_n), \hat{B}, \hat{ve}, \hat{t})$ $O \to O \to O \quad f\text{-add}$

$\Rightarrow (\hat{proc}, \vec{\hat{d}}, \hat{ve}, \hat{succ}(\hat{t}))$

where $\hat{proc} \in \hat{A}(f, \hat{B}, \hat{ve})$ $O \overset{\to O}{\underset{\to O}{\nearrow}} \quad f \in \{add$

$\hat{d_i} = \hat{A}(e_i, \hat{B}, \hat{ve})$ $sub\}$

$(\hat{proc}, \vec{\hat{d}}, \hat{ve}, \hat{t})$

$\Rightarrow (call, \hat{B}'[v_i \mapsto \hat{t}], \hat{ve} \sqcup [(v_i, \hat{t}) \to \hat{d_i}], \hat{t})$

where $\hat{proc} = ([(\lambda (v_1 \dots v_n) call)], \hat{B}')$

$\{[x \Rightarrow \{a\}]\} \sqcup \{[x \Rightarrow \{b\}]\} = \{[x \mapsto \{a, b\}]\}$

(define (f x) (f (x+1)) (f0)

$\widehat{Time} = \{NOW\}$ $succ(NOW) = NOW$ O-CFA

(define (f x) (if (x < 0) "Hi" "Bye"))
  (print (f -1)) $x = \{-1, 5\}$
  (print (f 5)) $= \{neg, pos\}$

$\widehat{Time} = CALL$ $(x, 1) = \{neg\}$ 1-CFA
                             $(x, 2) = \{pos\}$
  (define (g x) (f x))
  (g -1) $(x, g) = \{neg, pos\}$
  (g 5)

$\widehat{Time} = CALL^k$ K-CFA