How do we know if a math formula
     is true?

How do we know if an algorithm
     (like Euclid's GCD) works?

                          ✓ ↘

                  correct   effective

Does an algorithm exist?
What is an algorithm?

Does a program exist?   ⟵ problems
What is a program?   ⟵ models

[1-2] A set is "a bunch of stuff"

$\emptyset$ — nothin' in it

$\forall x, x \notin \emptyset$

$\{ \text{pen}, \text{phone} \}$ $\{ \text{phone}, \text{pen} \}$

$\{ \checkmark, ☺ \}$

pen $\in \{ \text{pen}, \text{phone} \}$

$\forall x, \quad x \in \{ y \}$ iff $\quad x = y$

union — $\cup$ $\cup$ $\cup$

$\forall x. \quad x \in A \cup B$ iff $\quad x \in A$ or $x \in B$

$\{ \text{pen}, \text{phone} \} = \{ \text{pen} \} \cup \{ \text{phone} \}$

[-3) "The set of all true math
        formulas"

A set IS its membership

    "$1+1=2$" ∈ TS ↑ ?

    "Is there a god?"
    "Will Buffy be remake?"

All sets "constructed" via $\emptyset$, $\{x\}$, ∪ are
    finite.

    $x \in$ $\{$🐱🐱🐱$\}$ ∪ $\{$🐱$\}$

The <u>Universe</u> (𝒰)

              ↙ subset
    $A \subseteq B$ iff $\forall x,\ x \in A \Rightarrow x \in B$

L-4/ Our universe is made of strings
and strings are sequences of characters
and chars are elements of an alphabet
an alphabet is a finite set

$$\Sigma = \{0, 1\} \qquad \{0, 1, \sqcup, \$, +\}$$

chars   chars

$\iota^0 \iota^1 \iota^2$

"0100001" = a string = s

length = 7 $s(0) = 0$ $s(1) = 1$ $s(2) = 0$

← special notation

$$U = \Sigma^*$$

$$A^* = \{\varepsilon\} \cup \cancel{A} \, A \circ A^*$$

epsilon = " " = the string w/ no
characters

$x \in A \circ B$ iff $x(0) \in A$ and
$$x(1\cdots) \in B$$

$$\{0,1\} \circ \{0,1\} = \{00, 01, 10, 11\}$$
$$\{1\} \circ \{0\} = \{10\}$$

#1. Decide a data type to represent
alphabets and characters.
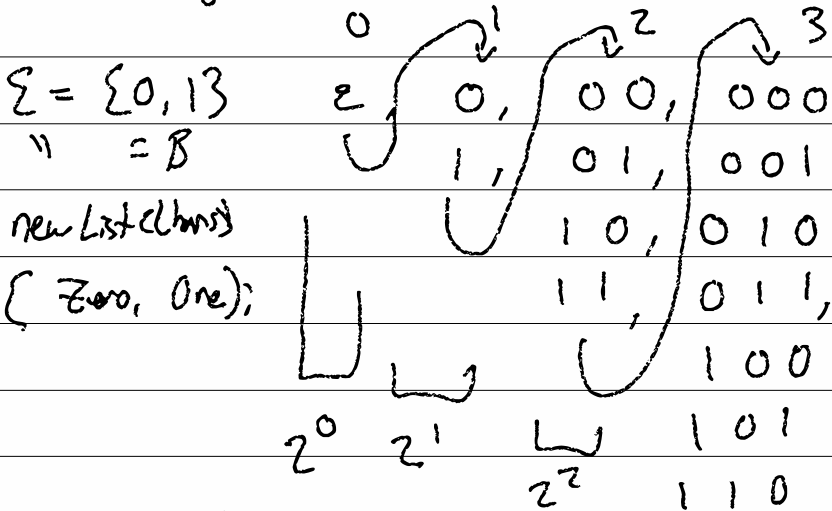
Alphabet = List < Character >
Character = Object   / void *
                    we need   equality

#2. Decide a data type for strings

interface   String   { }
class   Mt String   implements String { .., }
class   One String   impl   String {
    One String (char c, String s) { ... }}
Zero = new   Basic Char('0');   One = new BC('1');
010 = new   One S (Zero, new One S(One, new
            One S (Zero, new MtS())));

(Σ)

1-6/ Every alphabet has a lexicographical
ordering of the strings in Σ^*

$\Sigma = \{0, 1\}$

" $= B$

new List(Chars)

(Zero, One);

$$\begin{array}{cccc} 0 & 1 & 2 & 3 \\ \varepsilon & 0, & 00, & 000 \\ & 1, & 01, & 001 \\ & & 10, & 010 \\ & & 11, & 011, \\ & & & 100 \\ & & & 101 \\ & & & 110 \\ & & & 111 \end{array}$$

$2^0 \quad 2^1 \quad 2^2 \quad \underbrace{}_{2^3}$

$|A|^i$ where i = layer I'm
looking at

$\underbrace{1\ 1\ 1}_{23}$

lexi : $\Sigma \times \mathbb{N} \longrightarrow \Sigma^*$

lexi B $0 = \varepsilon$

lexi B $1 = 0$

lexi B $2 = 1$

lexi B $6 = 10$