Some measurements were made at a mixer temperature of approximately −125°C, where the optimum operating conditions were determined to be: dc bias current 0.4 ma, total current 1.8 ma, at which improvements of 0.4 db in F_x compared to room temperature values were observed.

E. Dependence of Crystal Parameters on RF Circuitry

Using a mixer crystal mount of different design, some of the measurements made previously were repeated. These measurements, which were made first at room temperature, and later at −196°C, showed that the over-all receiver noise factor was approximately 0.7 db lower at all temperatures than the values obtained with first crystal mount, owing, apparently, to superior performance of the new crystal mount. There was, however, no change in the previously determined variations of crystal parameters with temperature. Thus it may be concluded that the failure to observe significant improvements in crystal sensitivity was not due to the performance of the particular crystal mount used.

V. Effect of Temperature Cycling on the Crystal Package

Seventeen 1N263 crystals were used in the tests, not all of them continuously. Of this number, two were ruined by the temperature cycling; *i.e.*, the noise factors became unusably high. The remainder exhibited little change in electrical properties, although small cracks appeared in the glass at the glass-to-metal seal on two of the crystals. This may cause eventual failure of these units.

VI. Conclusions

The following conclusions have been reached concerning the operation of 1N263 crystals in an X-band mixer:

1) No improvement can be made in crystal noise factor by operating the mixer at liquid nitrogen temperatures.

2) A small improvement in crystal noise factor may be possible by operating the mixer at some temperature intermediate between −196°C and room temperature. The improvement is of the order of 0.3 to 0.6 db, with the minimum value of the crystal noise factor occurring in the region between −100°C and −50°C, the exact value depending on the individual crystal, as well as on bias and local oscillator drive.

The failure to obtain significant improvement by mixer cooling is due to an increase in the noise temperature ratio of the crystal as the temperature is lowered. This suggests that flicker noise may be appreciable at 30 mc for germanium as well as for silicon mixer crystals[5] and furthermore that this excess noise is temperature dependent.

The Multiple Branch Waveguide Coupler*

JOHN REED†

Summary—**A multiple branch directional coupler is discussed for rectangular waveguide applications for series junctions. A design method is developed which is valid for any coupling ratio and any number of branch lines with perfect match and directivity. The frequency response of this type coupler is calculated with the aid of a digital computer.**

Introduction

THE multiple branch waveguide directional coupler has been found to be a useful type of coupler, especially for rather tight ratios such as from zero to fifteen decibels. This paper describes the design of such a coupler and the calculation of its frequency response. The coupler is particularly desirable since the design constants are readily found and its frequency response can be calculated.

Fig. 1 shows a typical coupler with five branches. The three center connecting branch lines are made of waveguide of the same width as the main line but of reduced height c. The height of the two end branches is a different value a but the width is the same as the main line. The a and c values are normalized to the height of the main line as unity. The spacing between the center lines of adjacent branch lines is assumed to be identical and the length of all these branch lines is assumed to be the same. At first, each of these two distances will be considered to be a quarter wavelength, but for considering the frequency dependence of the coupler this restriction will be dropped. The junction effects will be disregarded, that is to say, the junctions will be regarded as pure series junctions.[1] This is strictly valid only for the case that a and c are very small, but for a first approximation it is quite valuable.

* Manuscript received by the PGMTT, March 3, 1958; revised manuscript received, July 7, 1958.
† Raytheon Mfg. Co., Wayland, Mass

[1] C. G. Montgomery, R. H. Dicke, and E. M. Purcell, "Principles of Microwave Circuits," McGraw-Hill Book Co., Inc., New York, N. Y., p. 288; 1948.

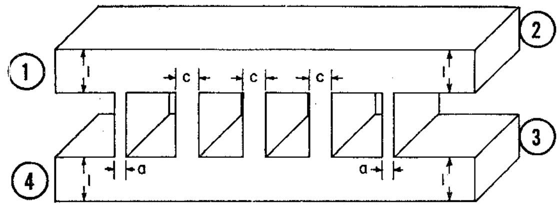

Fig. 1—Typical coupler with five branches.

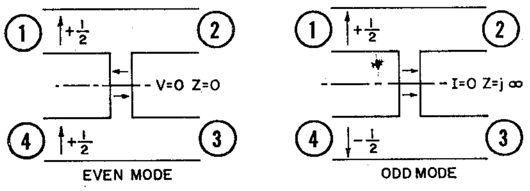

Fig. 2—Even and odd modes.

Symmetry Analysis

A signal of amplitude equal to one is applied to arm (1) of the network of Fig. 2. This figure represents a cross section of one branch of the coupler in Fig. 1. To determine the vector amplitudes out the other arms assuming matched terminations, use is made of the even and odd mode concept.[2–4] In Fig. 2, if two coherent signals, each of amplitude $\frac{1}{2}$ and in phase, are applied at arms (1) and (4) a voltage null will occur at every point on the plane of symmetry. This will be denoted as an even mode. Likewise, if two coherent waves, each of amplitude $\frac{1}{2}$ and out of phase with each other, are applied at arms (1) and (4) there will be a voltage maximum and current null at every point on the plane of symmetry. This will be denoted as an odd mode. In each case the symmetry of the modes will not be disturbed on passing through the network. It will be assumed that the transmitted amplitudes $T_e/2$ and $T_o/2$, and the reflected amplitudes $\Gamma_e/2$ and $\Gamma_o/2$ for the even and odd modes, can be calculated. The sum of these two modes properly phased is equal to a wave of amplitude 1 in arm (1) and 0 in arm (4) incident on the system. The amplitude out arm (2) will be the sum of the transmitted amplitudes for the even and odd modes, and the amplitude out arm (3) will be the difference of these transmitted amplitudes. Likewise, the amplitudes out arms (1) and (4) are the sum and difference of the reflected amplitudes. Putting this in symbols we have

$$\begin{aligned} A_1 &= \Gamma_e/2 + \Gamma_o/2 \qquad & A_2 &= T_e/2 + T_o/2 \\ A_4 &= \Gamma_e/2 - \Gamma_o/2 \qquad & A_3 &= T_e/2 - T_o/2. \end{aligned} \tag{1}$$

Thus the analysis of the directional coupler of the type shown in Fig. 1 can be performed in terms of the results

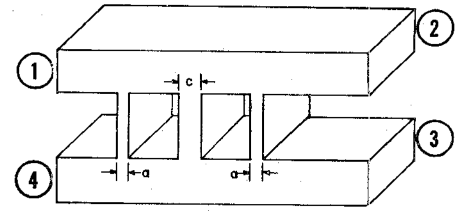

Fig. 3—Three branch coupler.

for two two-terminal-pair networks. The $ABCD$ matrices of the network are determined for both the even and odd mode, and from these matrices the reflected and transmitted amplitudes can be found. In this paper the networks, such as Fig. 1, give rise to matrices of lossless two-terminal-pair circuits so that the A and D terms are pure real quantities while the B and C are pure imaginary quantities. The values of the reflected and transmitted amplitudes of each mode are given in terms of the elements of the $ABCD$ matrix by

$$\Gamma/2 = \frac{A + B - C - D}{2(A + B + C + D)} \quad T/2 = \frac{1}{A + B + C + D}. \tag{2}$$

The quantities T and Γ are the elements S_{12} and S_{11} of the scattering matrix for a two-port network.[5]

Design at Band Center

For the first application of the above method, consider a three branch coupler shown in Fig. 3. The three connecting branch lines are assumed to be a quarter wavelength long, the center line is of height c relative to the main line, and the two end branch lines are of height a. The branch lines are spaced a quarter wavelength apart on uniform height lines. The matrix for the even mode is

$$\begin{aligned} M_{e3} &= \begin{bmatrix} 1 & ja \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \begin{bmatrix} 1 & jc \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \begin{bmatrix} 1 & ja \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} -a(-c) - 1 & -j(a^2(-c) + 2a) \\ +j(-c) & -a(-c) - 1 \end{bmatrix}. \end{aligned}$$

This matrix is the product of five matrices; the first, third, and fifth being those of short circuited waveguide stubs an eighth wavelength long and of characteristic impedances a, c, and a respectively. The second and fourth are of quarter wavelength lines of characteristic impedance unity. The matrix for the odd mode will be similar except that open circuited eighth wavelength stubs are imagined. The matrix product is found quickly by replacing a and c by $-a$ and $-c$.

$$M_{o3} = \begin{bmatrix} ac - 1 & -j(a^2c - 2a) \\ +jc & ac - 1 \end{bmatrix}$$

For perfect match and directivity, there must be no reflection of either the even or odd mode. For these

[2] B. A. Lippmann, "Theory of Directional Couplers," Mass. Inst. Tech., Cambridge, Mass., Rad. Lab. Rep. No. 860; December 28, 1945.

[3] J. Reed and G. J. Wheeler, "A method of analysis of symmetrical four-port networks," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 246–252; October, 1956.

[4] L. Young, "Branch guide couplers," *Proc. Natl. Electronics Conf.*, vol. 12, pp. 723–732; 1956.

[5] H. M. Altschuler and W. K. Kahn, "Nonreciprocal two-ports represented by modified Wheeler networks," IRE Trans. on Microwave Theory and Techniques, vol. MTT-4, pp. 228–233; October, 1956.

matrices this condition is achieved by setting the B term equal to the C term since the A term is already equal to the D term. Once the value of c is chosen, there are two possible values of a to be found from equating the B and C terms of the matrices. The smaller of these should be chosen to reduce junction effects and increase bandwidth.

$$a = \frac{1 - \sqrt{1 - c^2}}{c}$$

The matrices for the three branch couplers, when this relationship of a to c is chosen, can be written in terms of an angle θ as

$$M_{e3} = \begin{bmatrix} \cos\theta & j\sin\theta \\ j\sin\theta & \cos\theta \end{bmatrix} \text{ ayd } M_{o3} = \begin{bmatrix} \cos\theta & -j\sin\theta \\ -j\sin\theta & \cos\theta \end{bmatrix}$$

and so $A_2 = \cos\theta$ and $A_3 = j\sin\theta$. Thus when this coupler is matched and perfectly directive, the voltage amplitude out arm (2) is equal to the magnitude of the A term of the M_{e3} matrix, while the amplitude out arm (3) is equal to the magnitude of the C term.

For a four element coupler (see Fig. 4) a similar procedure may be followed.

$$M_{e4} = \begin{bmatrix} -a(c^2-1)-(-c) & -j(a^2(c^2-1)+2a(-c)+1) \\ j(c^2-1) & -a(c^2-1)-(-c) \end{bmatrix}$$

As before, the matrix for the odd mode may be found by putting $-a$ and $-c$ for a and c. The amplitude out arm (3), when the proper relationship of a to c is chosen, is now equal to the A term in the matrix, and the amplitude out arm (2) is equal to the C term. This is true for all of this class of couplers with an even number of branches while the former case of power division is true for all couplers with an odd number of branch arms.

For a five branch coupler such as shown in Fig. 1 the matrix is

$$M_{e5} = \begin{bmatrix} -a(-c^3+2c) - (c^2-1) & -j(a^2(-c^3+2c) + 2a(c^2-1) - c) \\ j(-c^3+2c) & -a(-c^3+2c) - (c^2-1) \end{bmatrix}.$$

For the general case of $n+2$ branches, with each branch a quarter wavelength long and spaced a quarter wavelength away from adjacent branches on uniform height waveguide, the matrix can be found. If the center n branches all have the same impedance c, and the two end branches have the impedance a, the even mode matrix is

$$M_{e(n+2)} = \begin{bmatrix} -aS_n(-c) - S_{n-1}(-c) & -j(a^2S_n(-c) + 2aS_{n-1}(-c) + S_{n-2}(-c)) \\ +jS_n(-c) & -aS_n(-c) - S_{n-1}(-c) \end{bmatrix}. \quad (3)$$

The elements $S_n(-c)$ are Chebyshev polynomials in c and the first few have the following values.

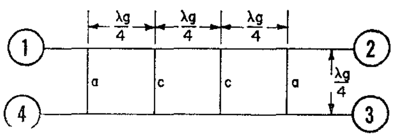

Fig. 4—Four branch coupler schematic.

$$\begin{aligned}
S_0(-c) &= +1 \\
S_1(-c) &= -c \\
S_2(-c) &= c^2 - 1 \\
S_3(-c) &= -c^3 + 2c \\
S_4(-c) &= c^4 - 3c^2 + 1 \\
S_5(-c) &= -c^5 + 4c^3 - 3c \\
S_6(-c) &= c^6 - 5c^4 + 6c^2 - 1 \\
S_{n+1}(-c) &= -cS_n(-c) - S_{n-1}(-c) \\
S_n(2x) &= U_n(x)
\end{aligned}$$

The values of $S_n(x)$ are available[6] to twelve decimal places for x ranging from 0 to 2 in steps of 0.001 and for integral values of n from 2 to 12.

In order to design a matched and perfectly directive coupler of given coupling ratio the value of c, the relative height of the center n waveguide branches, is to be calculated. To do this the number of branches $n+2$ is chosen and the voltage amplitudes out arms (2) and (3) for unity amplitude into arm (1) determined. For an even number of branches $S_n(-c)$, the magnitude of the C term in the Me_{n+2} matrix, is set equal to A_2 and the value of c computed. If the number of branches is odd, the value of $S_n(-c)$ is set equal to A_3, and c is calculated. The value of a which will then give a matched and perfectly directive coupler is

$$a = \left| \frac{|\sqrt{1 - S_n{}^2(-c)}| - |S_{n-1}(-c)|}{S_n(-c)} \right|. \quad (4)$$

This equation is found from the general matrix above by equating the sum of the squares of the A term and the C term in the matrix to unity. The magnitude signs are used in this equation to make sure that the smaller of the two possible values of a is used.

For example an 8.5 decibel coupler was desired with six branches. The amplitudes A_2 and A_3 are therefore

[6] Tables of Chebyshev Polynomials $S_n(x)$ and $C_n(x)$, National Bureau of Standards Applied Mathematics Series No. 9, United States Government Printing Office, Washington 25, D. C., 1952.

TABLE I

EXAMPLES OF 0, 3, AND 10-DB COUPLERS

Number of Branches $(n+2)$	0-db Coupler		3-db Hybrid Even Split		10-db Coupler	
	a	c	a	c	a	c
3	1.000*	1.000*	0.4141	0.7071	0.162	0.3162
4	0.500*	1.000*	0.2346*	0.5412*	0.0945	0.2265
5	0.618	0.618	0.2088	0.3810	0.0811	0.1602
6	0.309	0.618	0.1464*	0.3179*	0.0592	0.1312
7	0.445	0.445	0.1374	0.2583		
8	0.2225	0.445	0.1064	0.2257		
9	0.3473	0.3473	0.1022	0.1948		
10	0.1736	0.3743	0.0837	0.1752		
11	0.2846*	0.2846*				
12	0.1423*	0.2846*				
13	0.2410	0.2410				
14	0.1205	0.2410	0.0587*	0.12104*		
23	0.1365	0.1365				
24	0.0682*	0.1365*				

* Frequency response curves given.

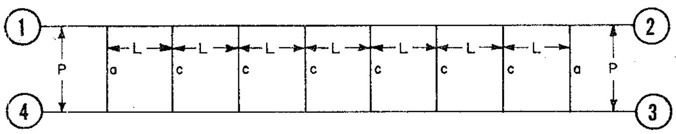

Fig. 5—General coupler schematic.

respectively in magnitude 0.926686 and 0.375837. On setting $S_4(-c)$ equal to 0.926686 the value of c is found to be 0.156972. Then the value of a is found to be 0.070964. The values of a and c are made to be the heights of the branches with respect to that of the main and auxiliary waveguides. A tabulation of values of a and c for couplers of 0 decibels coupling [complete transfer of power from arm (1) to arm (3)], 3 decibels [even split of power between arms (2) and (3)], and 10 decibels [power out arm (3) is 10 decibels below the incident], is given in Table I.

FREQUENCY SENSITIVITY

To determine the frequency response of this type of coupler, we assume that the junction effects do not change with frequency. That is to say, the junctions are regarded as series junctions with reference planes chosen so that the phase between them varies as does the phase in an equivalent piece of uniform waveguide. A schematic diagram of the coupler is shown in Fig. 5.

The coupler is considered as a cascade of series branches each of a length P and spaced along a uniform waveguide a distance L from the next. The impedances of the branches are chosen to be c for the center n branches and a for the two end branches. The basic matrices for the components of this array are as follows.

Even mode stub at ends Even mode stub at center

$$\begin{bmatrix} 1 & jag \\ 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & jcg \\ 0 & 1 \end{bmatrix}$$

$$g = \tan\frac{\pi P}{\lambda_g} \qquad \lambda_g = \text{guide wavelength.}$$

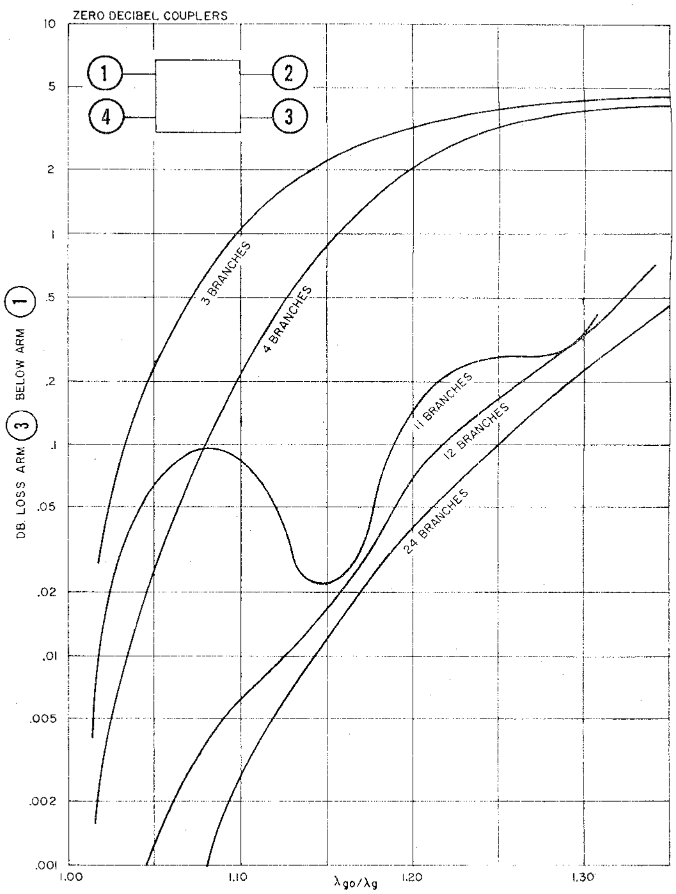

Fig. 6—Zero decibel couplers of 3, 4, 11, 12, and 24 branches.

Length $L/2$ of unity impedance waveguide

$$\begin{bmatrix} \dfrac{1}{\sqrt{1+t^2}} & \dfrac{jt}{\sqrt{1+t^2}} \\ \dfrac{jt}{\sqrt{1+t^2}} & \dfrac{1}{\sqrt{1+t^2}} \end{bmatrix} \qquad t = \tan\frac{\pi L}{\lambda_g}.$$

Note that when $L=P=\lambda_g/4$ then $t=g=1$.

The even mode matrix for the coupler above is therefore

$$M_{e(n+2)} = \frac{1}{1+t^2}\begin{bmatrix} 1-tga & j(t+ag) \\ jt & 1 \end{bmatrix} \times \begin{bmatrix} \dfrac{1-t^2-tcg}{1+t^2} & \dfrac{j(2t+cg)}{1+t^2} \\ \dfrac{j(2t-t^2cg)}{1+t^2} & \dfrac{1-t^2-tcg}{1+t^2} \end{bmatrix}^n \times \begin{bmatrix} 1 & j(t+ag) \\ jt & 1-tga \end{bmatrix}. \tag{5}$$

The first matrix is that of an end even mode stub followed by a length $L/2$ of line. The second matrix is that of a length of line followed by a center even mode stub followed by a length of line. The third matrix is that of a length of line followed by an end even mode

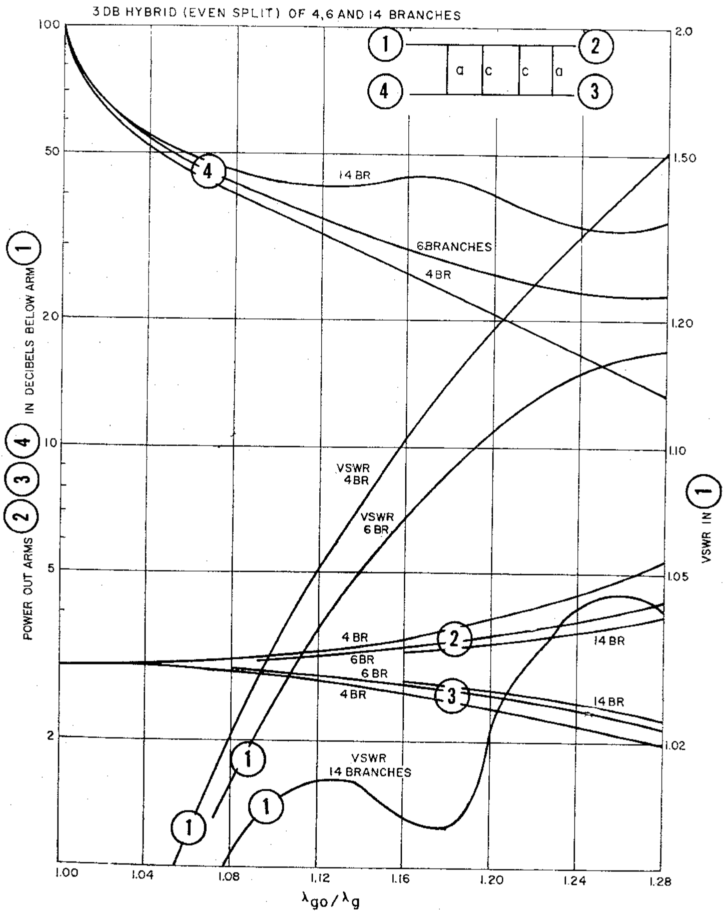

Fig. 7—Three decibel couplers 4, 6, and 14 branches.

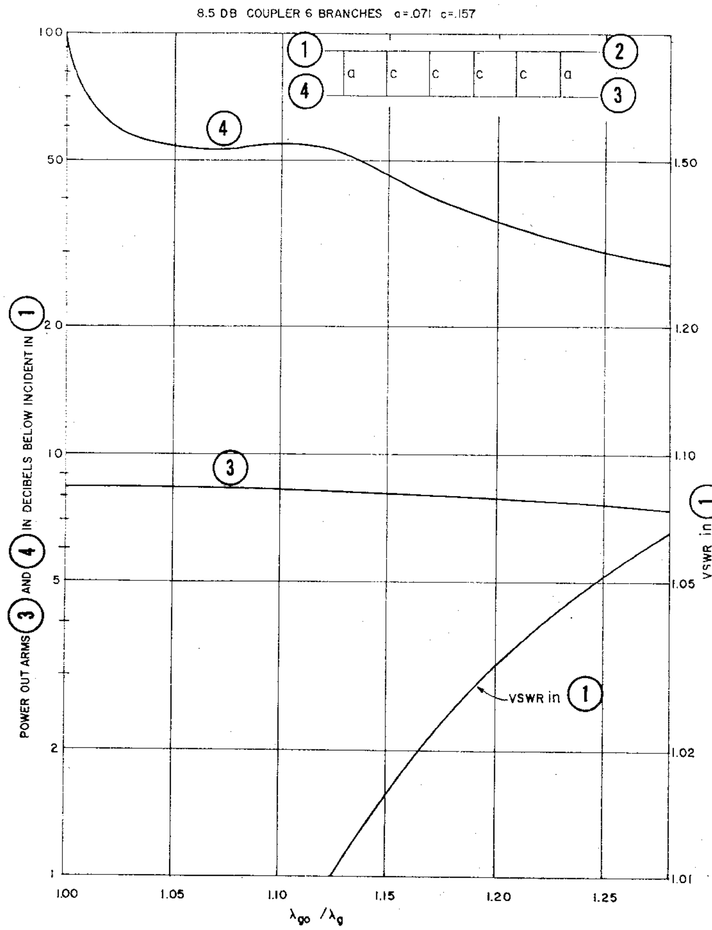

Fig. 8—8.5-db coupler of six branches (see example in text).

stub. The quantity $1/\sqrt{1+t^2}$ is factored out of both the first and third matrix. The matrix product is that of the first matrix times the second matrix raised to the nth power where n is the number of center elements times the third matrix. From this matrix product, given t, g, a, c, and n, the quantities $\Gamma_e/2$ and $T_e/2$ can be computed. The matrix for the odd mode is formed by substituting $-1/g$ for g everywhere in the even mode matrix. From this $\Gamma_0/2$ and $T_0/2$ can be calculated. Thus the amplitudes out each arm can be found.

This computation was programmed for an IBM 650 digital computer. The quantities a, c, n, P/λ_{g0}, and $L/_{g0}$ (where λ_{g0} is the design guide wavelength) are the data fed into the computer. The ouput is the voltage standing-wave ratio to be expected in arm (1) and the power out arms (2), (3), and (4), expressed in decibels below incident power in arm (1), assuming matched loads in arms (2), (3), and (4). In a single run, the values of a, c, and n are kept constant but the values of t and g must be computed over again for each value of λ_{g0}/λ_g. For a given set of input data the program was arranged so that thirty values of λ_{g0}/λ_g would be chosen. The limits on them and distance between them can be varied as desired.

The time required to perform the calculation for each value of λ_{g0}/λ_g varied from about 4 seconds for a value of n equal to 1, to about 11 seconds for n equal to 22. In raising the center matrix to the nth power ordinary matrix multiplication was used to retain flexibility. If a lot of work for very high values of n were to be done, the program could be modified to reduce computer time.

Some calculated results are shown in the next few figures. Fig. 6 shows results to be expected for zero decibel couplers [complete transfer of power from arm (1) to arm (3) at the design frequency]. These plots show the power loss in decibels going from arm (1) to arm (3) as a function of λ_{g0}/λ_g. This abscissa would be f/f_0 if the couplers were made of coaxial lines and a and c the characteristic admittances. For this graph the lengths of line L and P are a quarter wavelength at midband so the response will be arithmetically symmetrical about the value of $\lambda_{g0}/\lambda_g=1$. That is to say, for example, the insertion loss from the input at arm (1) to the output at any arm will be the same at a value of $\lambda_{g0}/\lambda_g=1.20$ as it would be for $\lambda_{g0}/\lambda_g=0.80$. Therefore only one side of the curves need be plotted. The curves shown are for couplers of 3, 4, 11, 12, and 24 branches with the values of a and c as shown in Table I. The results for the three arm coupler agree with those already plotted.[3]

Fig. 7 shows the results calculated for branch guide hybrids which give even power split. In this graph the power out arms (2), (3), and (4), expressed in decibels below incident power, is plotted as a function of λ_{g0}/λ_g. The input VSWR to be expected looking into arm (1) is also plotted. These curves are for couplers of 4, 6, and 14 branches with the values of a and c given in Table I.

Fig. 8 shows the results calculated for the 8.5-db coupler discussed earlier as an example. Fig. 9 shows the results to be expected if the length of the branch guides is reduced to 0.22 λ_{g0} while the distance between them is maintained at 0.25 λ_{g0} and all factors remain the same as in Fig. 8. The curves, of course, will not be symmetrical about the $\lambda_{g0}/\lambda_g=1$ value.

In the practical application of the above calculations it is seen that the longer the coupler is the broader the band that can result. As the sizes of the coupling slots decrease in size the discontinuity effects get less and less so that the performance approaches the theoretical value closer and closer. Thus the effect of more slots and longer length is doubly beneficial. The theoretical performance becomes better and the actual performance approaches the theoretical.

Acknowledgment

The author expresses his appreciation to Thomas A. Weil who programmed and ran the computer.

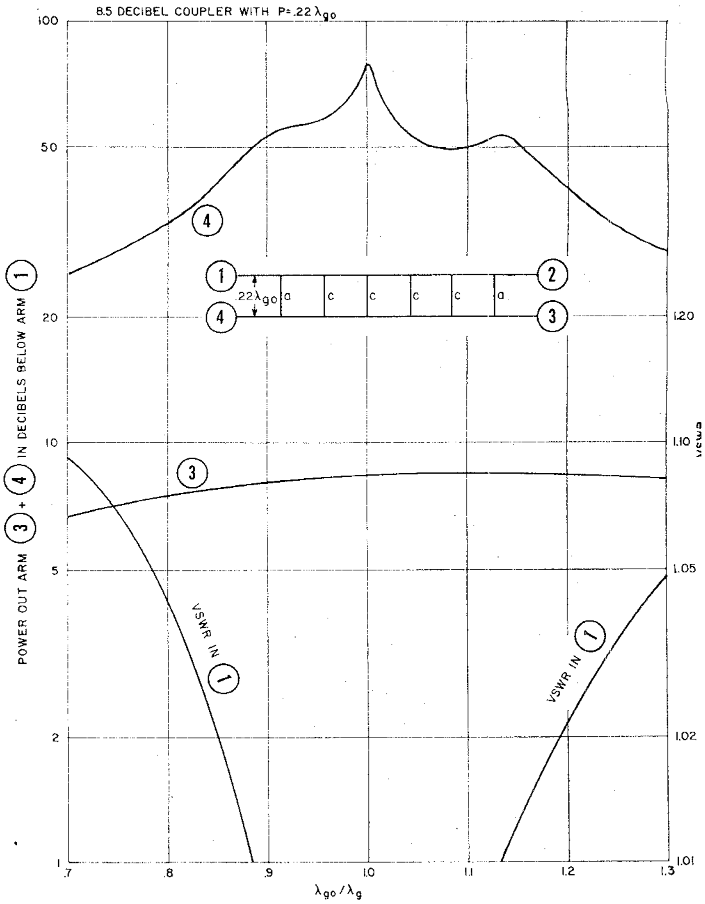

Fig. 9—8.5-db coupler with P reduced.

Coupled-Transmission-Line Directional Couplers*

J. K. SHIMIZU† AND E. M. T. JONES†

Summary—**Formulas are presented for the design of coupled-transmission-line directional couplers that are rigorous for any value of coupling. Two basic types are treated in detail; the simplest is one-quarter wavelength long at the center of its frequency band, while the other is three-quarter wavelength long. The quarter-wavelength type can be used over an octave of frequencies with approximately constant coupling, while the three-quarter-wavelength type can be used equally well over more than two octaves. For example, a −3-db coupler of the first type has a variation of ±0.3 db over a 2:1 band, while the second type has the same variation over a 4.5:1 band. Theoretically both types should have infinite directivity at all frequencies. The experimental results for models of these directional couplers have been found to conform very closely to the theoretical coupling functions, while the directivity, although usually good, is limited by discontinuity effects and constructional tolerances.**

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† Stanford Res. Inst., Menlo Park, Calif.

Introduction

Several investigators have recently pointed out that coupled transmission lines can be made into directional couplers having excellent wide-band performance, with infinite directivity and constant input impedance theoretically available at all frequencies