Chapter 9
Engineering Economic Analysis

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Definitions

- P – Principal or Present Value (of an investment)
- F_n – Future Value (of an investment)
- n – Years (or other time unit) between P and F
- i – Interest Rate (based on time interval of n) per anum

Basic premise: Money when invested earns money

$1 today is worth more than $1 in the future

Interest

- Simple Interest – Annual Basis
  - Interest paid in any year = Pi_s
    - Pi_s – Fraction of investment paid as interest per year
  - After n years total interest paid = Pi_sn
  - Total investment is worth = $P + Pi_sn$
  - Total investment after 1 year (n = 1) = $P\ (1+i_s)$
  - What is the drawback of simple interest?

We can earn interest on earned interest

Interest

- Compound Interest

At time 0 we have P

At the end of Year 1, we have $F_1 = P\,(1 + i\,)$

At the end of Year 2, we have $F_2 = P\,(1 + i\,)^2$

$\downarrow \qquad \downarrow \qquad \downarrow$

At the end of Year n, we have $F_n = P\,(1 + i\,)^n$

or $P = F_n / (1 + i\,)^n$

Example

- How much would I need to invest at 8 % p.a. to yield \$5000 in 10 years?

$$i = 0.08$$

$$n = 10$$

$$F_{10} = 5000$$

$$P = \frac{5000}{(1+0.08)^{10}} = \$2315.97$$

What if Interest Rate Changes with Time?

$$F_n = P\prod_{j=1}^{n}(1+i_j) = P(1+i_1)(1+i_2)\ldots\ldots(1+i_n) \qquad \text{Eq. (7.7)}$$

Different Time Basis for Interest Calculations

- Relates to statement “Your loan is 6 % p.a., compounded monthly”
- Define actual interest rate per compounding period as r
  - i_{nom} = Nominal annual interest rate
  - m = Number of compounding periods per year (12)

Different Time Basis for Interest Calculations cont.

- i_{eff} = effective annual interest rate

$$r = \frac{i_{nom}}{m}$$

- Look at condition after 1 year

$$F_1 = P\left(1 + i_{eff}\right) = P\left(1 + \frac{i_{nom}}{m}\right)^m$$

$$i_{eff} = \left(1 + \frac{i_{nom}}{m}\right)^m - 1$$

Example

- I invest $1000 at 10 % p.a. compounded monthly. How much do I have in 1 year, 10 years?

$$F_1 = P\left(1+\frac{i_{nom}}{m}\right)^m = 1000\left(1+\frac{0.10}{12}\right)^{12} = \$1104.71$$

$$i_{eff} = \left(1+\frac{0.10}{12}\right)^{12} - 1 = 0.1047$$

$$F_{10} = P\left(1+i_{eff}\right)^{10} = \$2707.04$$

Example cont.

- As m decreases i_{eff} increases
- Is there a limit as m goes to infinity
  - Yes – continuously compounded interest
  - Derivation – pp. 265-266
  - i_{eff} (continuous) = $e^{i_{nom}} - 1$

Cash Flow Diagram (CFD)

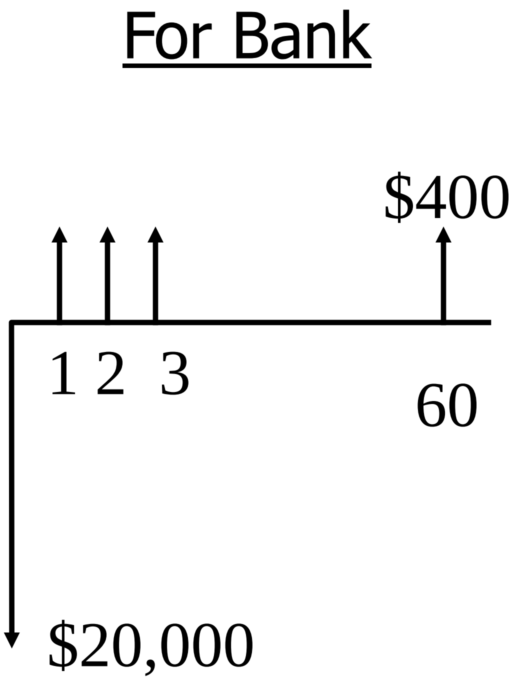
- Represent timings and approximate magnitude of investment on a cfd
  - *x*-axis is time and *y*-axis is magnitude
  - both positive and negative investments are possible.
- In order to determine direction (sign) of cash flows, we must define what system is being considered.

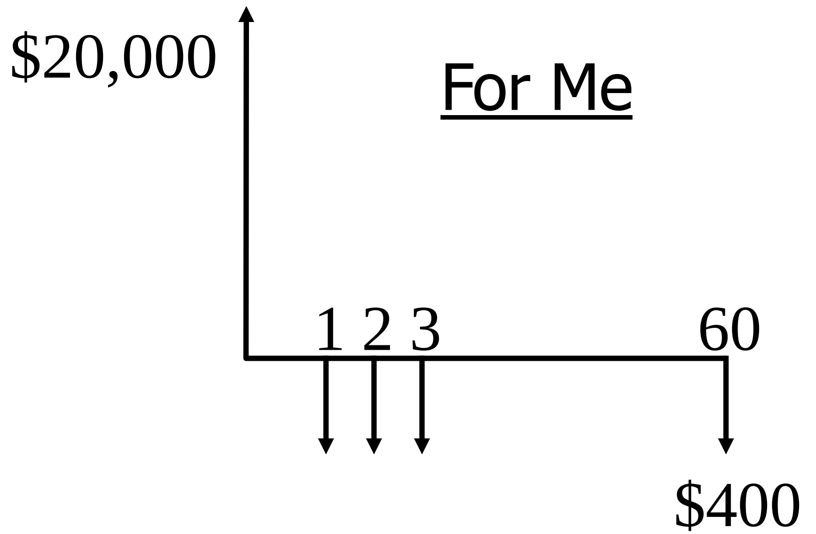
Consider a Discrete Cash Flow Diagram

- Discrete refers to individual CFDs that are plotted

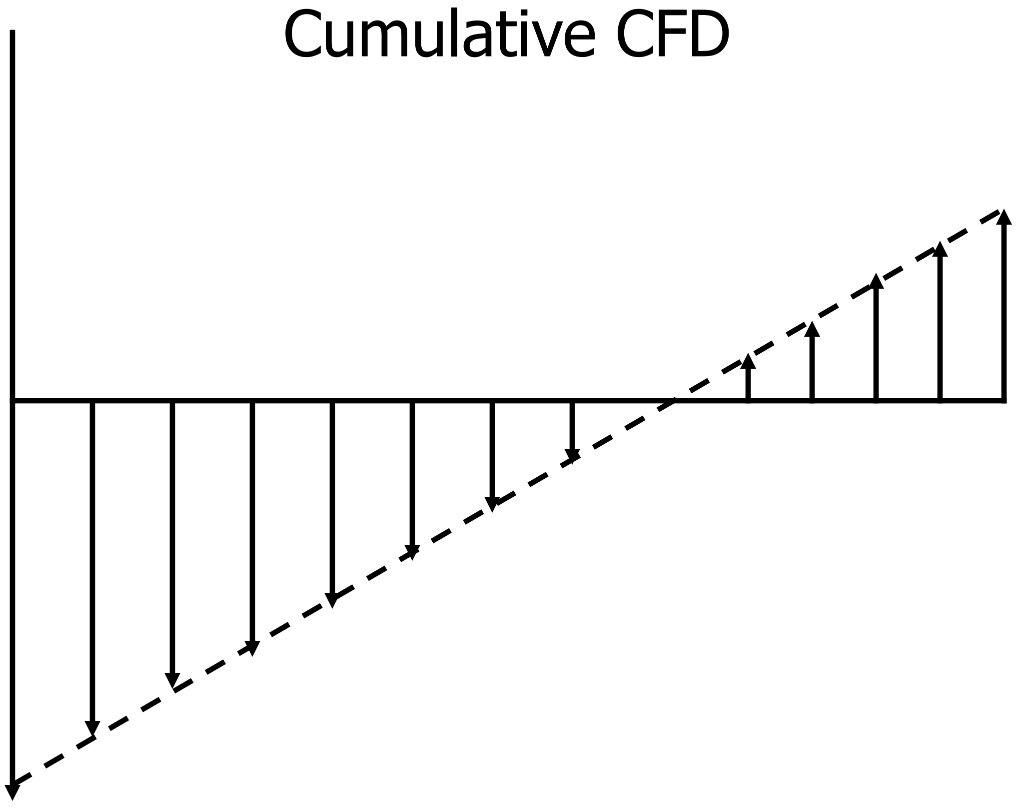
Example

- I borrow $20 K for a car and repay as a $400 monthly payment for 5 years.

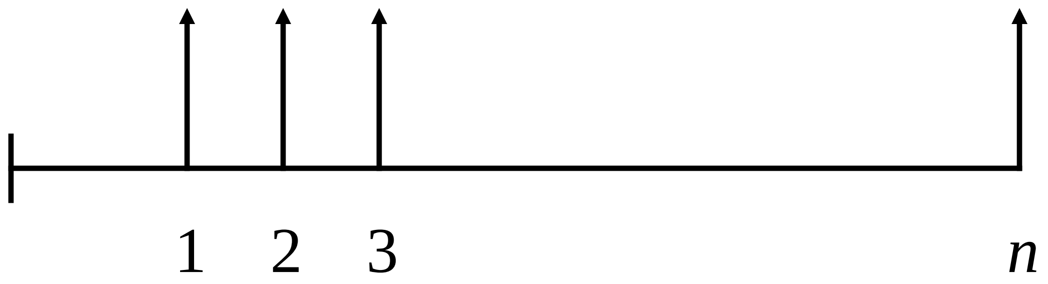




Cumulative CFD



Annuities



Uniform series of equally spaced, equal value cash flows
Note: The first payment is at the beginning of year 1 not at $t = 0$

Annuities

- What is future value F_n?

$$F_n = A(1+i)^{n-1} + A(1+i)^{n-2} + ...A$$

- Geometric progression

$$F_n = S_n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

Discount Factors

- Just a shorthand symbol for a formula in *i* and *n*

$$P = \frac{F}{(1+i)^n} \Rightarrow \left(\frac{P}{F}, i, n\right) = \frac{1}{(1+i)^n}$$

$$\Rightarrow P = F\left(\frac{P}{F}, i, n\right) = F\left(\frac{1}{(1+i)^n}\right)$$

See Table 9.1

$$\Rightarrow A \rightarrow P \Rightarrow \left(\frac{P}{A}, i, n\right) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Discount Factors

$$\left(\frac{F}{A}, i, n\right) = \frac{(1+i)^n - 1}{i}$$

$$\left(\frac{P}{F}, i, n\right) = \frac{1}{(1+i)^n}$$

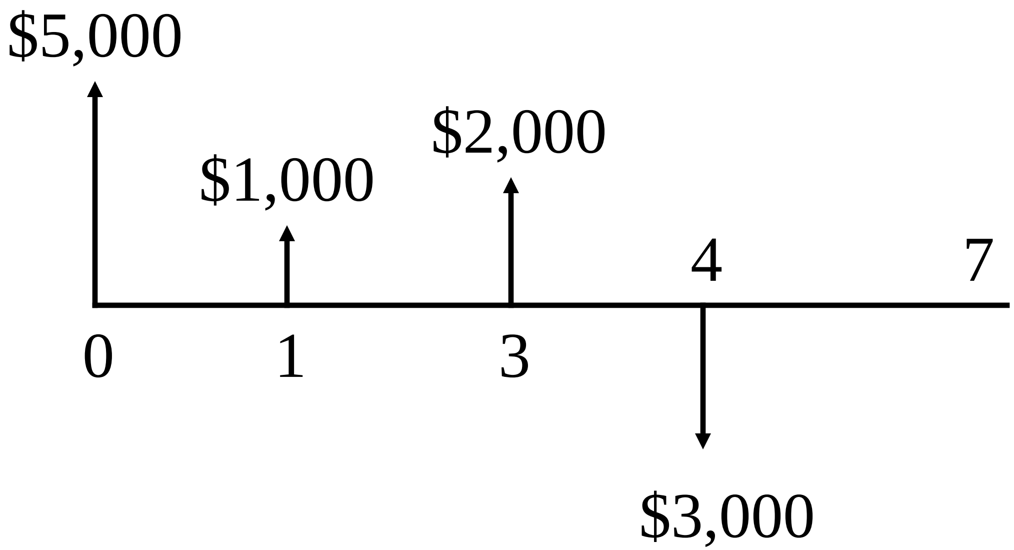
therefore

$$\left(\frac{P}{A}, i, n\right) = \frac{(1+i)^n - 1}{i(1+i)^n}$$

Table 9.1 has six versions of these

Three are reciprocals of the other three

Calculations with Cash Flow Diagrams



- Invest 5K, 1K, 2K at end of Years 0, 1, 3, and take 3K at end of Year 4
- Note that annuity payments are all at the end of the year

Example 1

- How much in account at end of Year 7 if *i* = 8% p.a.?

$$F_7 = 5,000(1+0.08)^7 + 1000(1+0.08)^6 + 2000(1+0.08)^4$$
$$-3000(1+0.08)^3$$
$$F_7 = \$9097.84$$

- What would investment be at Year 0 to get this amount at Year 7?

$$P = \frac{9097.84}{(1.08)^7} = 5308.50$$

Example 2

- What should my annual monthly car payment be if interest rate is 8% p.a. compounded monthly?

Example 2 (cont'd)

- Compare at *n* = 60

$$F_{60} = A\left[\frac{\left(1+\dfrac{0.08}{12}\right)^{60} - 1}{\dfrac{0.08}{12}}\right] = 73.47A$$

$$F_{60} = -20{,}000\left[\left(1+\frac{0.08}{12}\right)^{60}\right] = -29{,}796.90$$

$$73.47A - 29{,}796.90 = 0$$

$$A = \$405.53$$

$\boxed{\text{Interest paid} = \$4{,}331.20}$

Example 2 (cont'd)

- Another method

$$\left(\frac{A}{P},0.08,60\right)=\frac{\frac{i}{m}\left(1+\frac{i}{m}\right)^{mn}}{\left(1+\frac{i}{m}\right)^{mn}-1}=\frac{\frac{0.08}{12}\left(1+\frac{0.08}{12}\right)^{60}}{\left(1+\frac{0.08}{12}\right)^{60}-1}=0.020276$$

$$A=P\left(\frac{A}{P},0.08,60\right)=20{,}000(0.020276)=\$405.52$$

$$\text{interest paid}=60(405.52)-20{,}000=\$4331.20$$

Example 3

- You buy a house where you finance \$200 K at 6% p.a. interest, compounded monthly. What is your monthly payment, and how much interest do you pay over the lifetime of the loan for a 15-year and a 30-year mortgage in current dollars?

Example 3 (cont'd)

$$\frac{A}{P} = \frac{i(1+i)^n}{(1+i)^n - 1} = \frac{\frac{i}{m}\left(1+\frac{i}{m}\right)^{mn}}{\left(1+\frac{i}{m}\right)^{mn} - 1}$$

$$15\text{-year mortgage} \quad m = 12,\ n = 15$$

$$30\text{-year mortgage} \quad m = 12,\ n = 30$$

Example 3 (cont’d)

- For 15-year mortgage
  - $1687.71/month
  - total of $303,788 paid
  - $103,788 interest
- For 30-year mortgage
  - $1199.10/month
  - total of $431,676 paid
  - $231,676 interest

Example 4

- You invest \$5000/year (the maximum, for now) in a Roth IRA, starting at age 25 for 40 years. Assuming a return of 8% p.a., how much will you have at age 65 in future dollars?

Example 4 (cont'd)

$$\frac{F}{A} = \frac{(1+i)^n - 1}{i}$$

$$\frac{F}{A} = \frac{(1+0.08)^{40} - 1}{0.08}$$

$$\frac{F}{A} = 259.06$$

$$A = 5000$$

$$F = \$1{,}295{,}283$$

Example 5

- Repeat the previous calculation, assuming that you do not start investing until age 35 or age 45.

$$\frac{F}{A} = \frac{(1+i)^n - 1}{i}$$

$$\frac{F}{A} = \frac{(1+0.08)^n - 1}{0.08}$$

$$A = 5000$$

$$\text{if } n = 30 \qquad \frac{F}{A} = 113.28 \qquad F = \$566{,}400$$

$$\text{if } n = 20 \qquad \frac{F}{A} = 45.76 \qquad F = \$228{,}800$$

Depreciation

- Total Capital Investment = Fixed Capital + Working Capital
  - Fixed Capital – All costs associated with new construction, but Land cannot be depreciated
  - Working Capital – Float of material to start operations cannot depreciate

$$TCI = FCI_L + Land + WC$$

Definitions

- Salvage Value, S
  - Value of FCI_L at end of project
  - Often = 0
- Life of Equipment
  - n – set by IRS
    - Not related to actual equipment life
- Total Capital for Depreciation
  - $FCI_L - S$

4 Basic Methods for Depreciation

- Straight Line
- Sum of Years Digits (SOYD)
- Double Declining Balance (DDB)
- Modified Accelerated Cost Recovery System (MACRS)

Straight Line

$$d_k^{SL} = \left(\frac{FCI_L - S}{n} \right)$$

n = # of years over which depreciation is taken

Sum of Years Digits (SOYD)

$$d_k^{SOYD} = \frac{\left[(n+1-k)(FCI_L - S)\right]}{\frac{1}{2}n(n+1)}$$

SOYD

Double Declining Balance (DDB)

$$d_k^{DDB} = \frac{2}{n}\left[FCI_L - \sum_{j=0}^{k-1} d_j\right]$$

MACRS

- Current IRS-approved method

Year	Depreciation Percentage
1	20.00
2	32.00
3	19.20
4	11.52
5	11.52
6	5.76

See Chapter 9

Based on combination of DDB and SL

Example 9.21

$$FCI_L = \$150 \times 10^6$$

$$S = \$10 \times 10^6$$

$$n = 7$$

$$1^{st}\ Year - d_{SL} = ?$$

$$d_{SL} = \frac{150 - 10}{7} = 20$$

Same for Years 1-7

Example 9.21 (cont'd)

Sum of Year's Digits

$$d_{SOYD,1} = \frac{(7+1-1)}{\frac{1}{2}(7)(8)}[150-10] = \frac{7}{28}[150-10] = 35$$

$$d_{SOYD,2} = \frac{(7+1-2)}{\frac{1}{2}(7)(8)}[150-10] = \frac{6}{28}[150-10] = 30$$

Example 9.21 (cont'd)

Double Declining Balance

$$d_{DDB,1} = \frac{2}{7}\left(150\right) = 42.9$$

$$d_{DDB,2} = \frac{2}{7}\left(150 - 42.9\right) = 30.6$$

Example 9.21 (cont'd)

MACRS

$$d_{MACRS,yr\,1} = 0.20(150) = 30$$

$$d_{MACRS,yr\,2} = 0.32(150) = 48$$

$$d_{MACRS,yr\,3} = 0.192(150) = 28.8$$

$$d_{MACRS,yr\,4} = 0.1152(150) = 17.28$$

$$d_{MACRS,yr\,5} = 0.1152(150) = 17.28$$

$$d_{MACRS,yr\,6} = 0.0576(150) = 8.64$$

Taxation, Cash Flow, and Profit

- Tables 9.3 – 9.4
- Expenses = $COM_d + d_k$
- Income Tax = $(R - COM_d - d_k)t$
- After Tax (net)Profit =

  $(R - COM_d - d_k)(1 - t)$
- After Tax Cash Flow =

  $(R - COM_d - d_k)(1 - t) + d_k$ (+ other cash flows)
- Other cash flows might include working capital return, salvage value, etc.

Inflation

- $ Net Worth Now vs. $ Next Year

$$CEPCI\left(j+n\right)=\left(1+f\right)^{n}CEPCI\left(j\right)$$

- *f* = Average inflation rate between years *j* and *n*

Inflation

- Example

$$CEPCI\left(1993\right)=359$$

$$CEPCI\left(2003\right)=402$$

$$\left(1+f\right)^{10}=\frac{402}{359}$$

$$f=\left(\frac{402}{359}\right)^{0.1}-1=0.0114 \text{ or } 1.14\%$$

Inflation

- What is inflation rate since 2003? The current CEPCI is 600 (2011).

$$(1+f)^8 = \frac{600}{402} = 1.4925$$

$$f = 1.4925^{0.125} - 1 = 0.0513$$

$$5.13\%$$

Inflation

- Effect of inflation on interest rate

  f affects the purchasing power of the $

- Look at the purchasing power of future worth, then

$$F' = \frac{F}{(1+f)^n}$$

- If this future worth was obtained by investing at a rate *i*, then the inflation adjusted interest rate, *i'* is given by

$$F' = \frac{F}{(1+f)^n} = P\frac{(1+i)^n}{(1+f)^n} = P\left(\frac{1+i}{1+f}\right)^n = P(1+i')^n$$

$$\boxed{i' = \frac{1+i}{1+f} - 1 \approx i - f}$$