1 Introduction

Welcome to Section!

- Starting next week, section will be held in GSPP 105 on Wednesdays from 4 to 5PM
- Office hours: Currently Thursdays 4:45-5:45 in Giannini 304.
  - Can be changed
- Email policy: Please put **[DEVP252]** in the subject line.
  - I will try my best to respond within 48 hours during the week.
- Class websites
  - bCourses: course announcements, files, videos, assignments
  - Gradescope: website for submitting completed assignments
  - Datahub server: host for Jupyter notebooks

2 Inequality

The most common way to measure inequality within a country is with the Gini coefficient

$$G = \frac{1}{2N^2\mu}\sum_{j=1}^{N}\sum_{k=1}^{N} n_j n_k |y_j - y_k|$$

1. What does each variable in the formula for G mean? Can you give a verbal summary of the math in the formula?

The Gini coefficient is a measure of dispersion in income. The formula averages up the absolute distance between income for all possible pairs of households (or groups)

N	Population
μ	Average income
n	Subgroup population
y	Income

2. What would the Gini coefficient be if everyone had equal wealth? If one person had all of the economy's wealth?

The Gini coefficient would be 0 if everyone had equal wealth and 1 if one person had all the economy's wealth

Practice Problem

Let's say the economy of ShortLife has two kinds of jobs, which are the only sources of income for the people. One kind of job pays \$200, the other pays \$100. Individuals in this economy live for two years. In each year, only half the population can manage to get the high-paying jobs. The other half has to be content with the low-paying one. At the end of each year, everybody is fired from existing positions, and those people assigned to the high-paying job next year are chosen randomly. This means that at any date, each person, irrespective of past earnings, has probability 1/2 of being selected for the high-paying job.

1. Calculate the Gini coefficient based on people's incomes in any one particular period. Now calculate each person's average per period lifetime income and compute the Gini coefficient based on these incomes. Which measure suggests more inequality? Explain why.

In any period, $\frac{1}{2}$ of the population will have \$100 and the other $\frac{1}{2}$ will have \$200. Note that the question doesn't mention the total population of ShortLife (we don't actually need to know this as long as we know how income is distributed across groups), so let's set N to 1 meaning that we can think of n as the population share of each group. We also know that the average income, mu, is always going to be \$150. So let's plug into the formula.

$$\begin{aligned}
G &= \frac{1}{2N^2\mu}\sum_{j=1}^{N}\sum_{k=1}^{N} n_j n_k |y_j - y_k| \\
&= \frac{1}{2\times 150}\left(\frac{1}{2}\frac{1}{2}|100-100| + \frac{1}{2}\frac{1}{2}|100-200| + \frac{1}{2}\frac{1}{2}|200-100| + \frac{1}{2}\frac{1}{2}|200-200|\right) \\
&= \frac{1}{2\times 150}(25+25) \\
&= \frac{1}{6} \text{ or about } 0.167
\end{aligned}$$

Now let's turn to expected lifetime income. In expectation, everyone can expect to earn \$150 next year (a 50% chance at \$200 and a 50% chance at \$100). So this means that the group currently earning \$200 can expect a lifetime income of \$350, which is on average \$175 per period. Likewise the group currently earning \$100 can expect lifetime income of \$250, or per-period income of \$125. So all we need to do is replace 100 and 200 with 125 and 175 above (note that average income for the population stays the same).

$$\begin{aligned}
G &= \frac{1}{2N^2\mu}\sum_{j=1}^{N}\sum_{k=1}^{N} n_j n_k |y_j - y_k| \\
&= \frac{1}{2\times 150}\left(\frac{1}{2}\frac{1}{2}|125-125| + \frac{1}{2}\frac{1}{2}|125-175| + \frac{1}{2}\frac{1}{2}|175-125| + \frac{1}{2}\frac{1}{2}|175-175|\right) \\
&= \frac{1}{2\times 150}(12.5+12.5) \\
&= \frac{1}{12} \text{ or about } 0.083
\end{aligned}$$

So the Gini coefficient is about half in this case. There is more inequality at a given point in time because every gets a chance to start fresh the second year.

2. Now change the scenario somewhat. Suppose that a person holding a job of one type has probability 3/4 of having the same kind of job next year. Calculate the expected lifetime income (per year average) of a person who currently has a high-paying job, and do the same for a person with a low-paying job. Compute the Gini coefficient based on these expected per-period incomes and compare it with the measure obtained in Question 1. Explain the difference you observe.

Again, let's first compute expected income for each group.

For people with \$200 jobs in the first period, total lifetime income is

$$y = 200 + \frac{3}{4}200 + \frac{1}{4}100 = 375$$

so average income is \$187.5 For people with \$100 jobs in the first period, total lifetime income is

$$y = 100 + \frac{3}{4}100 + \frac{1}{4}200 = 225$$

so average income is \$112.5.

To compute the Gini coefficient for this scenario, just substitute \$187.5 and \$112.5 for 125 and 175 in the calculation above and we end up with $\frac{1}{8}$ or 0.125.

So there is more inequality in this case. This is because the greater persistence in jobs tends to limit social mobility, compared to the case in which everyone gets an equal chance at a high paying job in the second period, Other useful measures of inequality include the **Kuznets Ratio**

$$\frac{H_{20}}{L_{40}}$$

where H_{20} (L_{40}) is the consumption of the richest 20% (poorest 40%) and **Lorenz Curves**

$$L(p) = \frac{L_p}{Y}$$

which is a function of a percentile of income p where Y is total consumption.

3 Poverty

In class we've discussed a few different poverty measures. What's an intuitive definition for each? Which definition do you think is the most useful and why?

1. Headcount poverty (P_0)[1]

$$P_0 = \frac{1}{N}\sum_{j=1}^{N} \mathbb{1}(y_j < z)$$

2. Poverty index (P_1)

$$P_1 = \frac{1}{N}\sum_{j=1}^{q} \left(\frac{z - y_j}{z}\right)$$

3. Poverty severity index (P_2)

$$P_2 = \frac{1}{N}\sum_{j=1}^{q} \left(\frac{z - y_j}{z}\right)^2$$

Here, N is total population, y is income, z is the poverty threshold and q is the number of people with $y_j < z$.

P_0 is headcount poverty, or the share of people that are poor. P_1 is an index of poverty which gives a measure of how far below the poverty line poor people are on average. P_2 is called a severity index because it places more weight on people that are further below the poverty line. The headcount measure is the easiest to interpret but it doesn't capture anything about *how* poor on average poor people are. The other indices capture this but the choice of whether to measure distance to the poverty line linearly, quadratically, or some other way can be quite arbitrary.

Compute each of these three measures for the economy described in Question 2 using lifetime income[2]. Assume the poverty line is \$120 per year.

All the formula for P_0 is saying is assign a 1 to each person who's income y is below $z = 120$ and sum those up. Let's figure out the distribution of lifetime income.

The proportion of people who will have average income $y = \$200$ is the $\frac{1}{2}$ of the population that starts with \$200 times the $\frac{3}{4}$ of them who keep their jobs which equals $\frac{3}{8}$. Likewise $\frac{3}{8}$ of people will earn $y = \$100$ in both years. The remaining $\frac{1}{4}$ of the population will either go from \$100 to \$200 or \$200 to \$100, giving them each $y = \$150$. Since only the group with $y = 100$ is poor according to the definition of poverty with $z = 120$, P_0 is just their share of the population which is $\frac{3}{8}$ or 0.375.

To compute P_1, the term inside the parentheses is $\frac{120-100}{120} = \frac{1}{6}$ for the poor and 0 otherwise so

$$P_1 = \frac{3}{8} \times \frac{1}{6} = \frac{1}{16}$$

[1] Note the $\mathbb{1}()$ is an indicator function which takes a value of 1 if the argument is true and 0 otherwise

[2] in which exactly $\frac{3}{4}$ of each group keeps their old jobs. Note that we would get a very different if we used individuals' ex-ante expected lifetime income here

Likewise, P_2 would just require squaring the term in the parentheses so

$$P_2 = \frac{3}{8}\left(\frac{1}{6}\right)^2 = \frac{1}{96}$$

4 Further questions

1. What are some of the limitations of these income-based measures of poverty and inequality?
2. What are the tradeoffs between policies that emphasize reducing poverty vs. reducing inequality?

These are for you to think about!