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Spring 2024 Section Handout 2

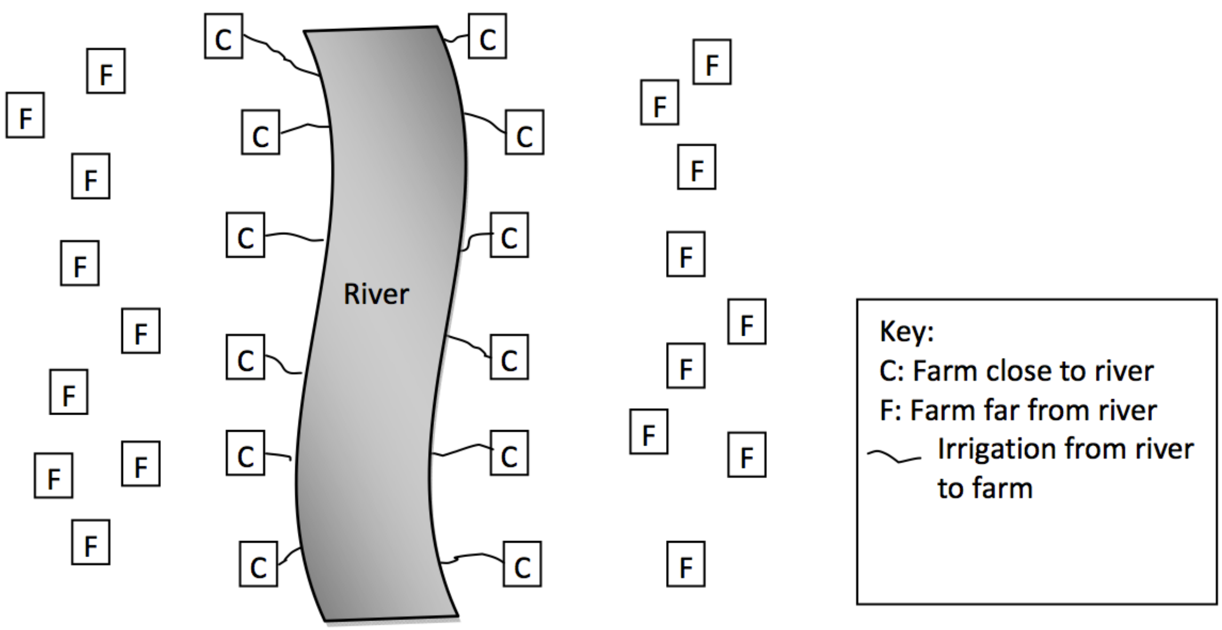
Difference-in-Difference

Motivation

Usually when trying to analyze the impacts of a policy, we don't have the luxury of running an RCT. But if we have two or more periods of data where some units are "treated" by the policy and others are not, we can still estimate its *causal effects* if we're willing to make some assumptions. Difference-in-differences is a way of getting around a non-random assignment of a program or policy. In this method, we exploit the timing of treatment to look at how our outcome of interest *changes* between the treatment and control groups before and after both groups are treated (over time). The idea is that if we believe that the two groups would have evolved similarly if not for the program, then the (observed) *changes* in the control group are a good counterfactual for how the treatment group *would have changed* in the absence of the program.

Example

The World Bank used to think that big infrastructure projects were the key to development in poor countries. For example, building irrigation passages to divert water from a river to nearby farms. Suppose you were asked to evaluate whether this particular irrigation project successfully increased farmers' yields.



Suppose that the World Bank does the project and then collects a season's worth of data on crop yields (metric tons per hectare) for farms in the area, both those close enough to the river to get irrigation and those too far away to be irrigated. So the snapshot of data looks something like this. Suppose you find that average farm yields are 1,000 kg/ha on farms **C**lose to the river and 700 kg/ha on farms **F**ar from the river.

1. What would you "naively" conclude is the effect of irrigation?
2. What is the counterfactual here i.e the comparison group? What's wrong with this strategy?

Instead, suppose that the World Bank collects two waves of data, one before the project was started and one after it was completed. Here's what you now have:

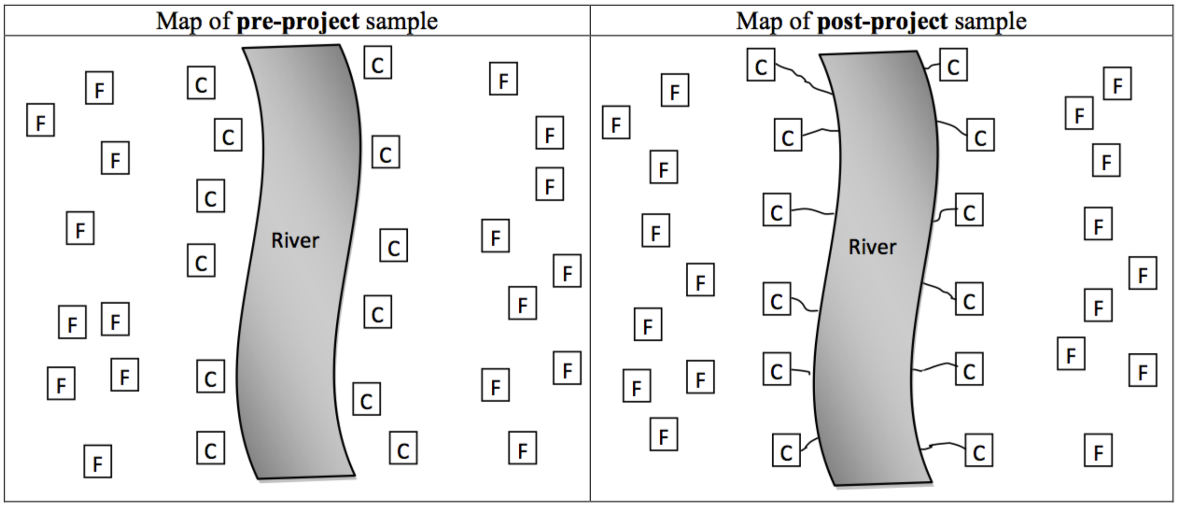


Table 1: Farm yields (kg/ha)

	F	C
Yield (pre-project)	500	625
Yield (post-project)	700	1,000

How do we identify the causal find the effect of irrigation? We can use Differences-in-Differences. We can do this in three ways that should yield identical answers

1. Arithmetically
2. Econometrically
3. Graphically

They key identifying assumption is that the difference between before and after in the comparison group is a good counterfactual for the treatment group. In other words the trend in outcomes of the comparison group is what we would have observered in the treatment group absent the policy/intervention/reform. We refer to this assumption as **parallel trends**.

Arithmetic DiD

In the simple two-by-two set up in this example, we can simply compute the DiD estimate by comparing differences in means.

1. Compute the difference in the outcome variable Y after (period 1) and before (period 0) for the control group (F in this case):
$$\bar{Y}_{F1} - \bar{Y}_{F0} = \Delta\bar{Y}_F$$
$$700 - 500 = 200$$
In order for Differences-in-Differences to be valid, we need to assume that the change in yields between the two periods far from the river would have been the same change we would have observed if it weren't for the irrigation program. In other words, we assume that yields would have also increased by 200kg on farms close to the river.

2. Compute the difference in the outcome variable Y after (period 1) and before (period 0) for the treatment group (C in this case):
$$\bar{Y}_{C1} - \bar{Y}_{C0} = \Delta\bar{Y}_C$$
$$1000 - 625 = 375$$
But yields actually increased by 375kg...

3. The impact of the program is measured by the difference in differences is:
$$(\bar{Y}_{C1} - \bar{Y}_{C0}) - (\bar{Y}_{F1} - \bar{Y}_{F0}) = (\Delta\bar{Y}_C - \Delta\bar{Y}_F)$$
$$(1000 - 625) - (700 - 500) = 375 - 200 = 175$$
So the difference between how much yields actually went up in the treatment group and how much we would have expected them to have gone up otherwise is 150kg.

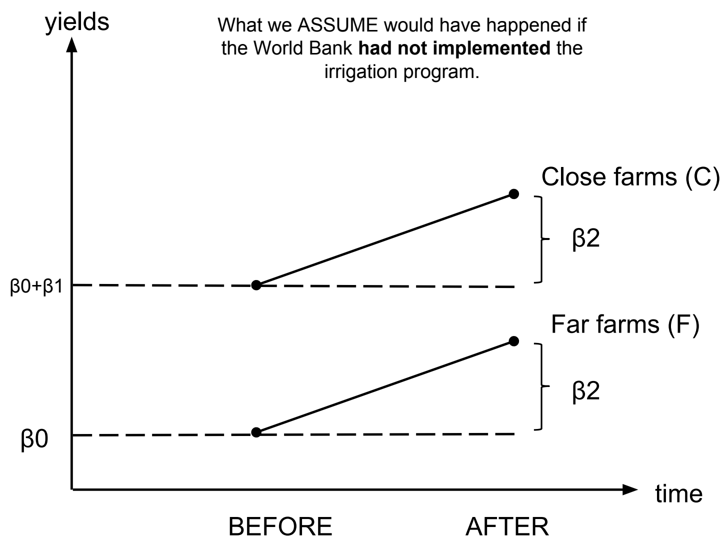
DiD Regression

In a regression framework, we estimate the following:
$$Y_i = \beta_0 + \beta_1 Post_i + \beta_2 Treat_i + \beta_3 Post_i \times Treat_i + u_i$$
The regression framework has the added benefit of providing us with standard errors, and t-statistics/p-value so we can test for significance of our estimators. But in the simple 2×2 case, we can actually directly compute these coefficients from the table above. What are $\beta_0 - \beta_3$?

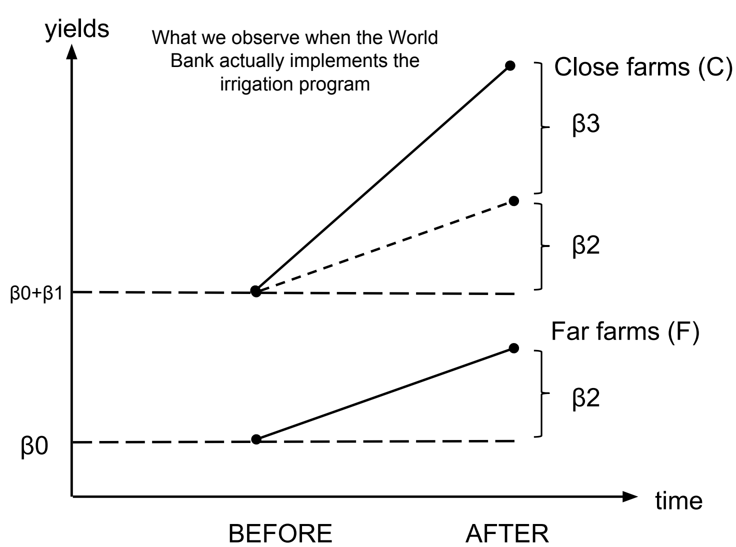
- β_0: $\bar{Y}_{F0} = 500 = \beta_0 + \beta_1 \times 0 + \beta_2 \times 0 + \beta_3 \times 0 \times 0 \implies \beta_0 = 500$
- β_1: $\bar{Y}_{F1} = 700 = \beta_0 + \beta_1 \times 1 + \beta_2 \times 0 + \beta_3 \times 1 \times 0 = 500 + \beta_1 \implies \beta_1 = 200$
- β_2: $\bar{Y}_{C0} = 625 = \beta_0 + \beta_1 \times 0 + \beta_2 \times 1 + \beta_3 \times 0 \times 1 = 500 + \beta_2 \implies \beta_2 = 125$
- β_3: $\bar{Y}_{C1} = 1000 = \beta_0 + \beta_1 \times 1 + \beta_2 \times 1 + \beta_3 \times 1 \times 1 = 500 + 200 + 125 + \beta_3 \implies \beta_3 = 175$

Graphical DiD

We can also draw a picture to understand the diff in diff assumptions and strategy. First, let's think about what might have happened to the close and far farms' yields if the World Bank hadn't done anything:



This picture demonstrates the parallel trends assumption: if not for the irrigation program, the close farms would have had the same *change* in yields over time as the far farms. Now we can think about what actually happened (the irrigation passages were dug, and the close farms were treated by the program), and you should see how the difference-in-differences strategy finds the treatment effect—given that the key assumption holds.



Basically, the diff-in-diff strategy is to conclude that any difference in the slope of these two lines is due to the treatment (because we are assuming that the slopes *would* have been the same without the program).

So, how do we interpret β_3? It's the average treatment effect! It's the additional difference between irrigated farms and non-irrigated farms after the irrigations passages have been dug—in other words, the estimated effect of improved irrigation on farmer yields under our key assumption.

Validation for parallel trends

- We want to show parallel trends hold
- To do this, we can look at yields from farms near the irrigation and for farms far from the irrigation for many periods (days, months, years, etc.) before this data. With this data, we could see whether the slopes, or trends, in yields were the same for both groups leading up to the introduction of the irrigation. If they were pretty similar before, then it sounds more reasonable to assume they would have *continued* to have similar slopes.

Generalizations of DiD

In the wild, it's unlikely we'll encounter simple cases with two groups and two time periods. When we have multiple groups and multiple periods, we need to consider whether the policy affects different groups differently, whether treatment switches on and off or stays on, and what to do when parallel trends doesn't quite hold. See Roth et al. (2023) for an excellent practitioner's guide, which also discusses which different software packages to use in these scenarios.

References

Roth, Jonathan, Pedro HC Sant'Anna, Alyssa Bilinski, and John Poe. 2023. "What's trending in difference-in-differences? A synthesis of the recent econometrics literature." *Journal of Econometrics* .