Modeling and Simulation of Ocean-Groundwater Interactions through the Beach: a Navier-Stokes and Richards Equations Coupling

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The aim of this work is to reproduce accurately hydraulic interactions between oceanic waves and watersheds at a large space and time scales (from meters to kilometers and minutes to hours). This modeling is essential for accurate numerical simulations that can be used for regional forecasts in a context of climate change, particularly concerning water resources and morphodynamics issues. The problem of infiltration in the unsaturated zone, known as the vadose zone, appears to be the main obstacle to the relevance of predictions. Thus, it is at the core of the project, as infiltration, along with changes in the water table and streams, plays a key role in the hydraulic response of a watershed [2].

In this work, we propose a model to simulate the exchanges between the ocean and a groundwater table through the beach. To achieve this, a consistant coupling between surface flows (two-phase Navier-Stokes equations) and subsurface flows (mixed form of the Richards equation [3]) is proposed. It has been implemented in the massively parallel finite volume based code Notus (developed at the Institute of Mechanics and Engineering of Bordeaux), with particular attention given to the boundary conditions at the porous medium/surface interface to ensure mass and energy conservation during transfers, whether the porous medium receives mass (infiltration) or releases it (exfiltration).

Various strategies have been explored to achieve this coupling, including the use of the VANS model [1] to analyze the asymptotic models obtained on either side of the interface (Navier-Stokes for the exterior flow and Richards for the porous medium). Another possible approach involves applying a variable transformation to the Richards equation, facilitating the interaction between the two models.

After validation through several benchmark test cases from the literature [3, 4], the method was used to simulate the wetting of the La Verre dam and compared to results from [4].

References

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