Modelling the permeability of a carbon fibre preform

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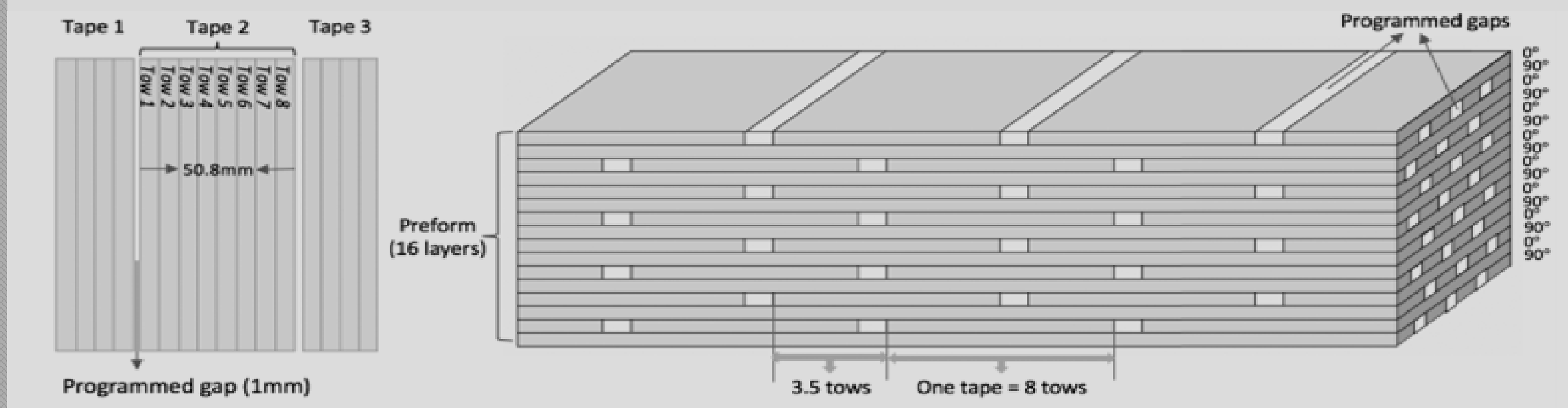
Introduction:

The versatility of composite materials makes them ideal for use in many engineering-based industries. One technique to cut production costs and reduce risk of human error is automated fibre placement (AFP) – the use of robots to construct the material. This project aims to analyse the permeability along the gaps of a carbon fibre preform by considering a Gaussian field model.

Data:

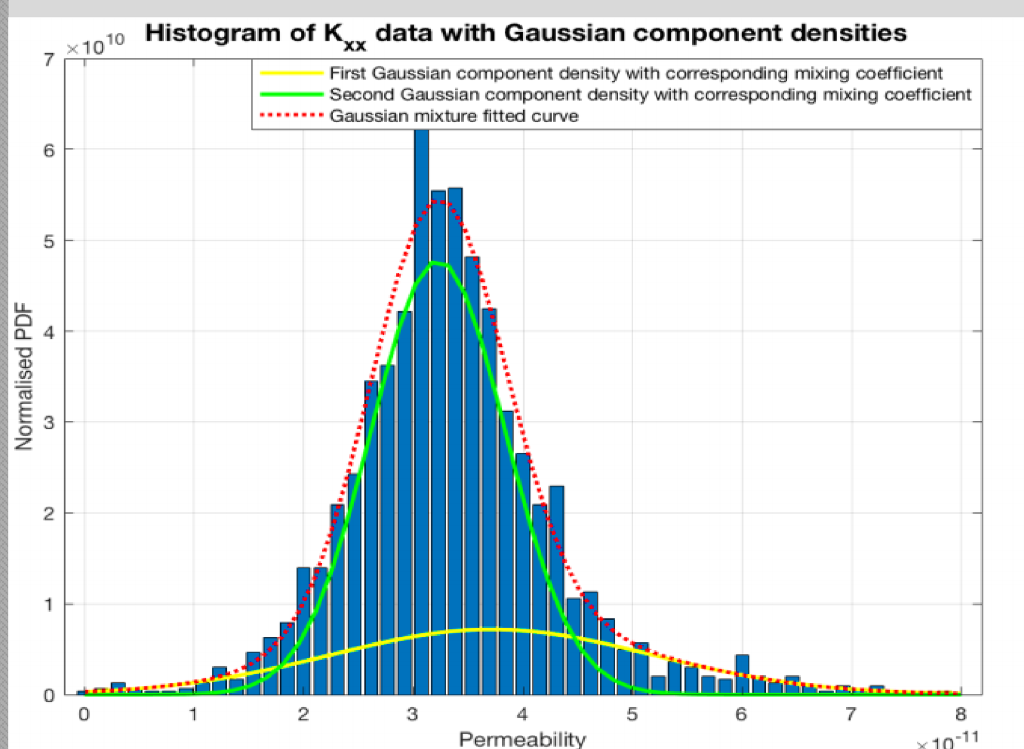
Threads (or tows) of carbon fibre are placed alongside each other filling a $1m^2$ panel, with a programmed 1mm gap between tapes (groups of 8 tows). The preform has 16 layers arranged into a cross-ply design (adjacent layers are perpendicular). Images of each layer are processed into gap data by an automated image analysis system. Each gap is divided into smaller volume elements before conversion into local permeabilities, which allow the evolution of the permeability across the gap to be investigated [1].



Previous findings:

Random independent samples of the local permeabilities were taken to investigate potential models for the marginal distribution (distribution of any individual volume element). Log-normal, normal, and truncated-normal distributions were deemed unsuitable after goodness-of-fit tests.

Stepping back, a positive skewness was calculated which suggested that a Gaussian mixture model might fit the sample. This model can be thought of as a weighted sum of normal distribution components, with latent variables and mixing coefficients which represent the proportion of the sample each component is responsible for. Using the expectation-maximisation algorithm, a two-component mixture was developed and tested, this time successfully. Using the parameters obtained, we hope to fit a model which incorporates the dependencies of points along the gap.



$$p(x) = \sum_{k=1}^{K} \pi_k \, \mathrm{N}(x|\theta_k)$$

Gaussian Mixture Model (above): The marginal has two components (K=2). The optimised parameters with VE length 5 pixels are: π =(0.2649, 0.7351), μ=(0.3714, 0.3226)*10^{-10} and $\sigma^2 = (0.2163, 0.0377) * 10^{-21}$.

Random processes:

Autocovariance measures the linear dependence between two points:

$\gamma_x(s, t) = \mathrm{Cov}(x_s, x_t) = \mathrm{E}[(x_s - \mu_s) * (x_t - \mu_t)]$

Autocorrelation measures linear predictability of two points:

$$\rho_x(s,t) = \frac{\gamma_x(s,t)}{\sqrt{\gamma_x(s,s) * \gamma_x(t,t)}}$$

A random process is an ordered sequence of random variables defined on a set of time points. Let the distances along the gap denote the time points, then the gap widths and permeabilities are discrete random processes. A random process is weakly stationary if it has constant mean and autocovariance depending only on the distance between points.

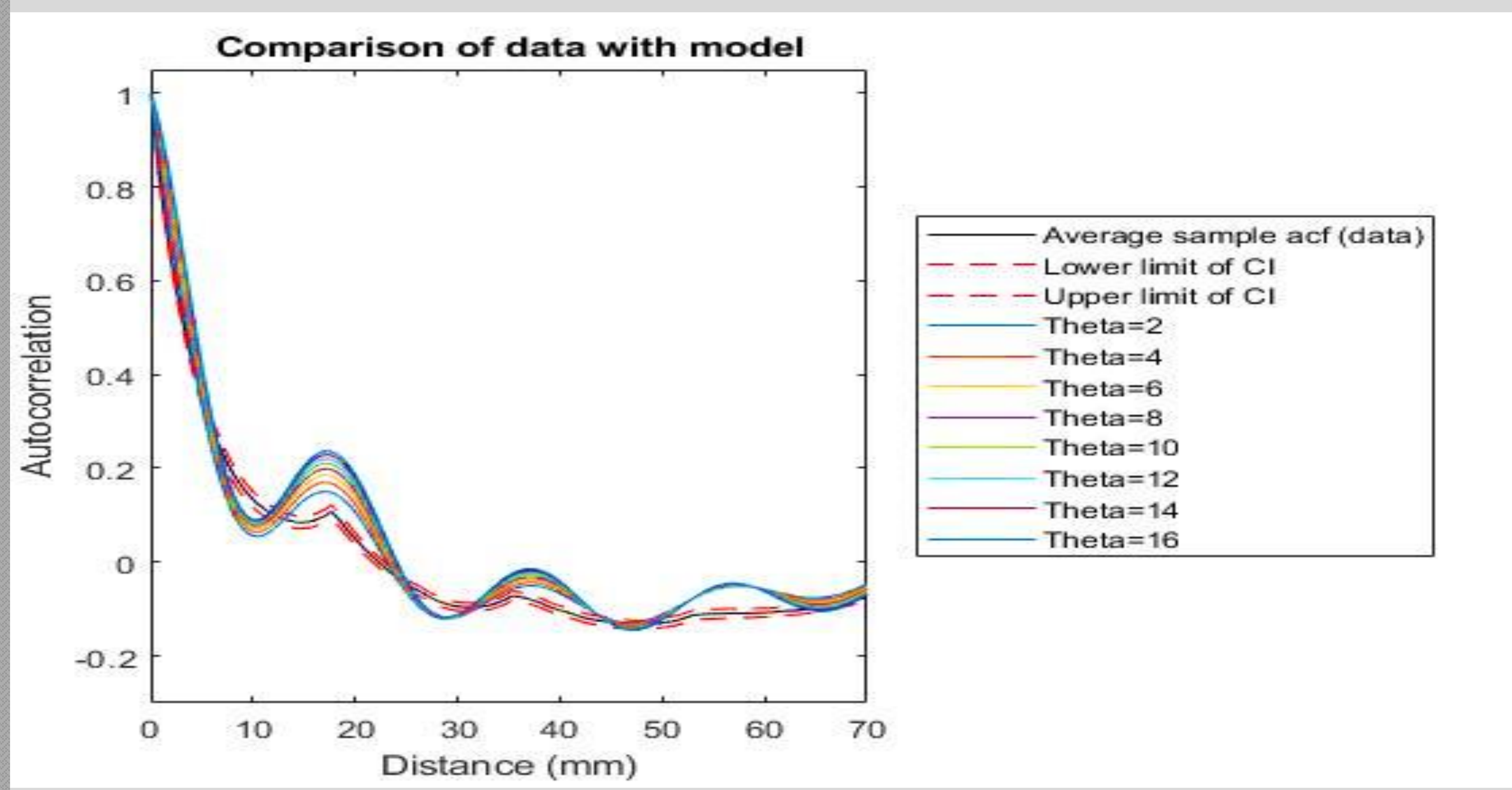
The gap widths appear to be weakly stationary; we can assume the permeabilities are weakly stationary as a function of gap width [3].

Modelling:

Based upon the marginal distribution, a model comprised of three Gaussian processes is proposed, two for the normal components and one for the latent variable within the marginal. The Gaussian processes are assumed to be independent, with parameters transferred where possible from the known marginal and observed power density. Their covariance functions are unknown.

Concentrating on the two processes representing the normal components, different candidates for the covariances were studied with the aim of matching the model and data autocorrelations.

The sample autocorrelation possesses two strange properties: it is negative over a particular interval and has three evenly spaced peaks. This meant the Matern model could be disregarded due to being strictly positive. A product of decay and oscillatory terms [2] seemed more promising, with the two properties attainable when an inverted power term is combined with a sum of trigonometric terms.



$$E[X(0)X(x)] = \frac{1}{(1+\frac{x}{\theta})}[\cos\frac{x}{l_1} + a(\sin\frac{x}{l_2})^2]$$

Decay-Oscillation Product Covariance (above): This is the covariance function that is used when generating the model autocorrelation in the image.

Discussion:

A partly negative, oscillatory model autocorrelation visually resembling the sample autocorrelation can be achieved by multiplying a term of order -1 with a linear trigonometric sum, meaning this product is the most suitable candidate to be included in the model.

To develop this project, the third Gaussian process needs to be investigated. Furthermore, extensions on the analysis of the model autocorrelation should be considered in conjunction with the power spectrum.

Reflection:

What I've gained…

- Confidence when communicating with professional mathematicians
- MATLAB skills: How to write more efficient codes and resilience
- A taste of how research is structured and led

References:

[1] Mikhail Y Matveev, Frank G Ball, I Arthur Jones, Andrew C Long, Peter J Schubel, and MV Tretyakov. Uncertainty in geometry of fibre preforms manufactured with automated dry fibre placement and its effects on permeability. Journal of Composite Materials, 52(16):2255–2269, 2018.

[2] Hongyi Xu. Constructing Oscillating Function-Based Covariance Matrix to Allow Negative Correlations in Gaussian Random Field Models for Uncertainty Quantification. Journal of Mechanical Design, 142(7): 074501, 2020.

[3] A.M Yaglom. Correlation Theory of Stationary and Related Random Functions. Springer-Verlag New York, 1987

Acknowledgements:

This project was made possible by the UoN Composites Research Group. A massive thank you to them and my patient supervisors for their professional support and guidance.