Winning Strategies For Mexico

DAM Project 2 Group 4:

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Mexico: How To Play

STEP ONE: Player 1 rolls both dice and the two values are put together to create the highest possible number.

= 53

STEP TWO: Stick or roll again? They have the choice to stick with their score, or re-roll (up to 3 times) in hopes of achieving a higher score

STEP THREE: Each subsequent player now rolls for their turn – but only have the option of rolling up-to the amount of times that Player 1 did

STEP FOUR: The player with the lowest score is eliminated. Player with the highest score is Player 1 in the next round.

STEP FIVE: The final player remaining in the final round wins

Rank of scores in ascending order: 11,21,22,23,…,64,65,66

The process a player goes through:

| Roll for the first score. | → | Make a decision to stick or re-roll | → | Keep rolling until satisfied, or you've run out of rolls. |

OUR QUESTION:

When should a player take another roll and when shouldn't they?

OUR AIM:

Derive a strategy which maximises the chance of winning.

The Distribution Of Scores

Assumptions
1. Players always maximise their score
2. Dice used are fair
3. Throws independent -> Scores independent
4. Players not assigned a strategy roll until safe (beat min. score)

Possible Scores	Ways of achieving
21	Two ways
31, 32	Two ways
41, 42, 43	Two ways
51, 52, 53, 54	Two ways
61, 62, 63, 64, 65	Two ways
11, 22, 33, 44, 55, 66	One way

$$P(Score = k) = \begin{cases} \dfrac{1}{36} & \text{k palindromic*} \\ \dfrac{1}{18} & \text{k non-palindromic} \\ 0 & \text{Otherwise} \end{cases}$$

Median: 51 (one roll), 61 (two rolls)
Mean: 47.25 (one roll), 53.53 (two rolls)

→ 'Median' strategy:
Stick above 51 on roll one,
stick above 61 on roll two

* where palindromic means both numbers rolled were the same, eg. rolling two 1's

Strategies For Two-Player Game

Median Strategy: (52,62)

Advantages:- Provides >50% chance of winning if threshold achieved

- ≈56.4% games won overall with this strategy

Disadvantages:-Thresholds only achieved in ≈74% of rounds

-Player 1 often takes three rolls (player 2 has more throws)

Simulate many games with different strategies for player 1, find the thresholds that maximises number of games won.

Simulated Strategy born: (44, 52)

win percentage of different strategies after 300,000 games n = 2
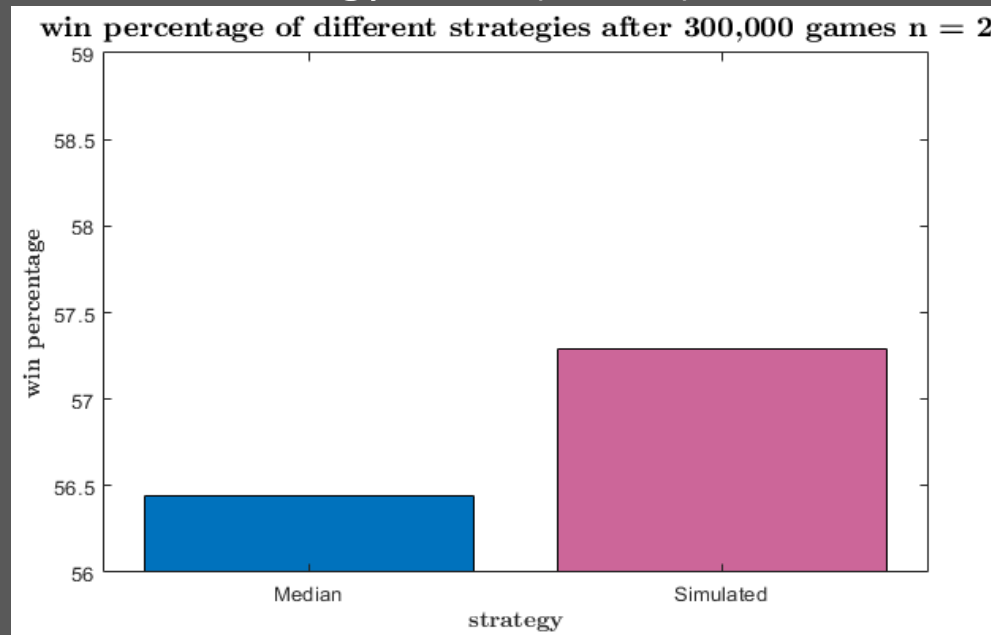
≈57.4% games won? God-tier strategy!

Higher-Player Rounds: Binomial

$p = P(Any\ player\ scores\ \geq k)$

Assume: - All players aim to beat player 1's score

- Player 1 scores exactly k

$N \sim Bin(n-1, (1-p))\ ,\ E[N] = (n-1)*(1-p)\ ,\ Var[N] = p*E[N]$

$E[N] > 1$ ⟹ $p < \frac{n-2}{n-1}$ ⟹ Use $p = \frac{n-2}{n-1}$ for a strategy!
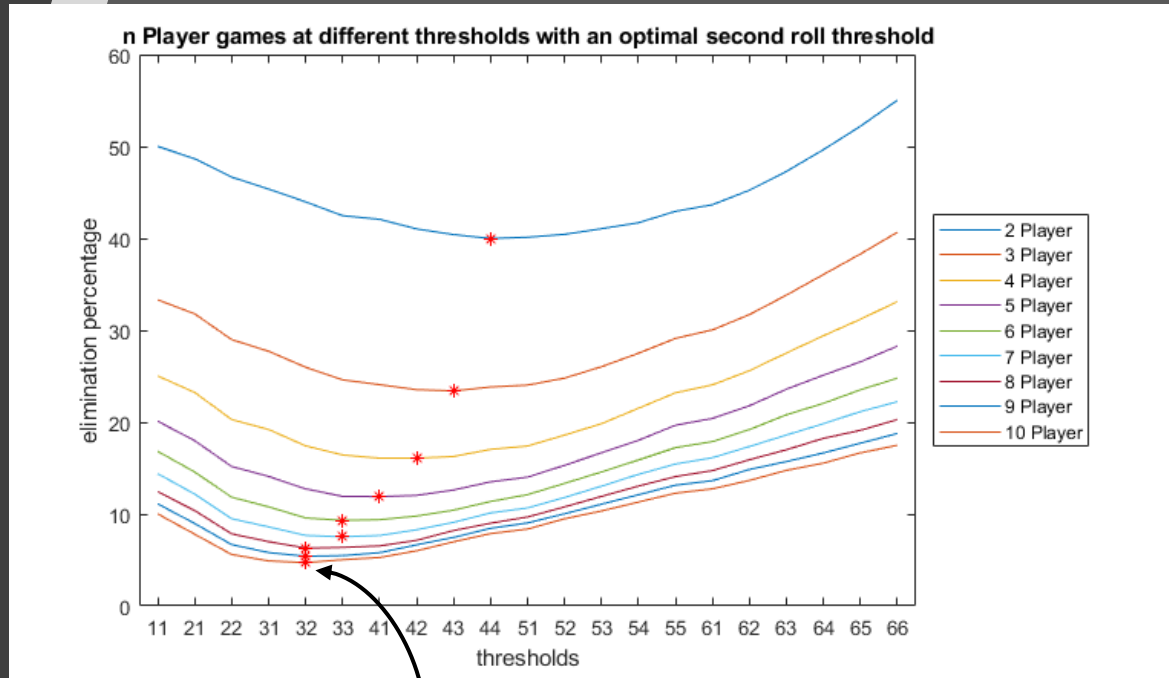
e.g. n=9, p=0.875, thresholds are (31,43)

$P(N = 0) = p^{n-1}$ (Player 1 is eliminated!)

$P(N = n-1) = (1-p)^{n-1}$ (Player 1 first to roll!)

Player 1

Other players

Simulation for Higher Player Rounds

n Player games at different thresholds with an optimal second roll threshold

Set up a round with n number of players

Every other player is trying to knock out player 1

Try every strategy and see if player 1 survives

This one little trick works 95% of the time!

Strategies for Other Players

3 Possible Cases
1. Player 1 sticks on roll 1 (no strategy required. Your result is purely luck based)
2. Player 1 sticks on roll 2
3. Player 1 sticks on roll 3

General Rule:
- If players before you score high, treat game as if you are player 1 and use threshold values for an n – k round
- If players before you score low, set threshold just above minimum score so far if it is easily beatable.
- Otherwise set threshold value to just above minimum score.

Rolls = 3, players = 3, player 1 score = 61

Rolls = 3, players = 3, player 1 score = 51
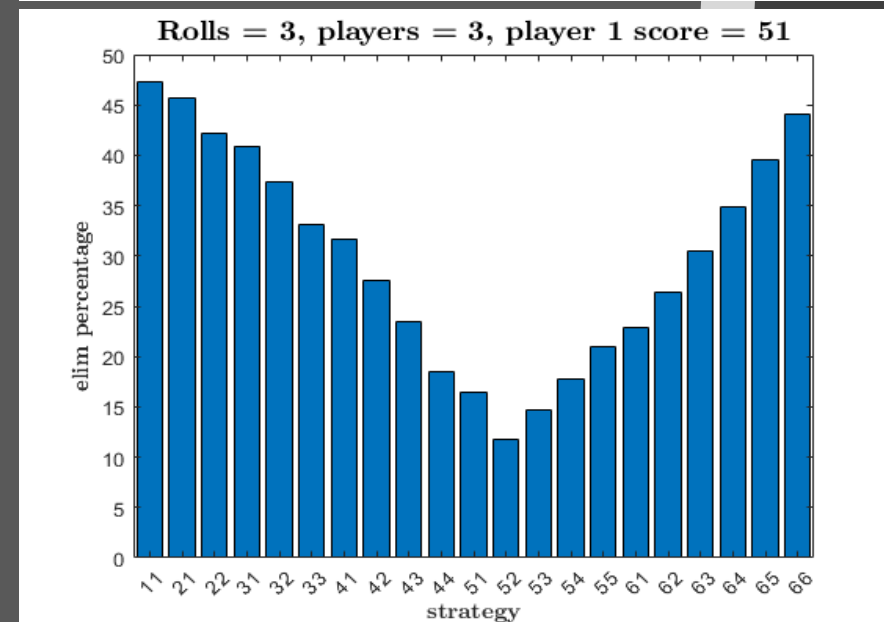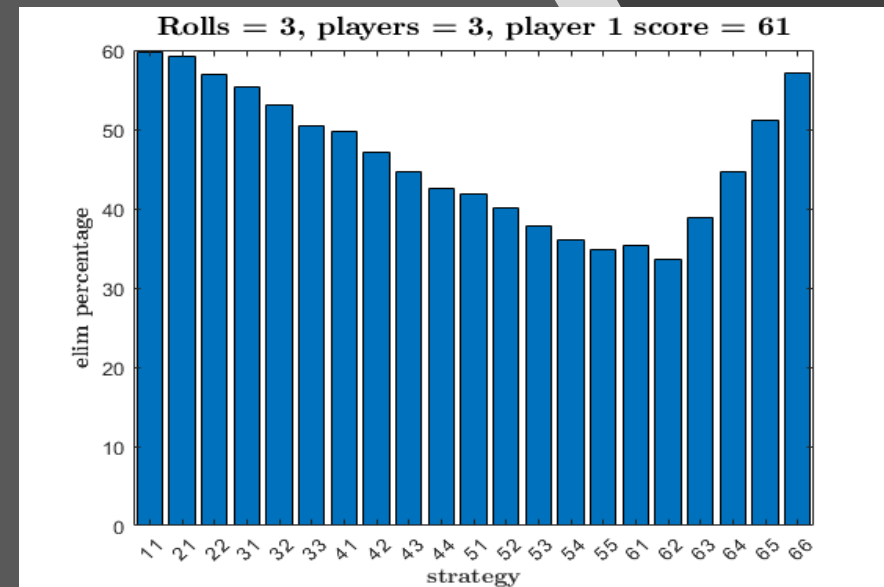
Summary of Strategies

Player 1's strategy:

Number of players (n)	1st Roll Threshold	2nd Roll Threshold
2	44	52
3	43	51
4	41	44
5	41	43
6	33	43
7	33	43
8	33	42
9	32	42

Player 1 'sticks' if they obtain these scores or higher

Strategies for players other than player 1:

- Player n (last player to roll) simply continues to roll until they beat the minimum score within the designated number of rolls
- Player k≠n,1:
  - If the k-1 players before them all have high scores, player k uses the above threshold scores for an n-k round
  - If the minimum score is achievable, player k sets that score as their threshold. Otherwise, player k sets their threshold just below the minimum score

Concluding Remarks

Limitations:

- We only simulated strategies for games with up to 10 players

- Incentive to roll first excluded from our strategies

Areas for Further Analysis:

- We have looked at the three dice game as an extension but there are also many other possible extensions to investigate… E.g. even more dice or increased number of rolls

Ultimately, 'Mexico' is a game of luck… and, the assumptions we made on human behaviour may not hold up in reality.

Thank you for listening! Are there any questions…?