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Contents

§1 My first chapter

Here is some introductory statement.

§1.1 The Riemann hypothesis

In this section, we give a long-awaited proof of the Riemann hypothesis.

§1.1.1 Preliminaries

Definition 1.1.1. The function $\zeta : \{s \in \mathbb{C} : \Re(s) > 1\} \to \mathbb{C}$ such that

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p \text{ prime}} \frac{1}{1 - p^{-s}}.$$

is called the *Riemann zeta function*.

Considering the analytic continuation of ζ, Riemann showed that ζ satisfied a particular meromorphic functional equation:

Theorem 1.1.1. *For* all $s \in \mathbb{C}$, ζ *satisfies*

$$\zeta(s) = 2^s \pi^{s-1} \sin\left(\frac{\pi s}{2}\right) \Gamma(1-s)\zeta(1-s).$$

§1.1.2 Zeroes of the Riemann zeta function: the hypothesis

A consequence of Theorem 1.1.1 is that every negative, even integer is a zero of ζ. (The same is *not* true of the positive, even integers: why?) Each of these zeroes is called a *trivial zero*. A natural question for one to ask is where, if any, the *nontrivial zeroes* of ζ lie. Some of these have been found, and curiously they all lay on the strip $\Re(s) = \frac{1}{2}$. This curiosity led Riemann, in 1859, to conjecture that

Conjecture 1.1.1 (Riemann, 1859)**.** *All nontrivial zeroes of the Riemann zeta function lie on the strip* $Re(s) = \frac{1}{2}$.

We can now prove this conjecture holds, turning it into a theorem:

Proof. Obvious. □