SUBTYPING

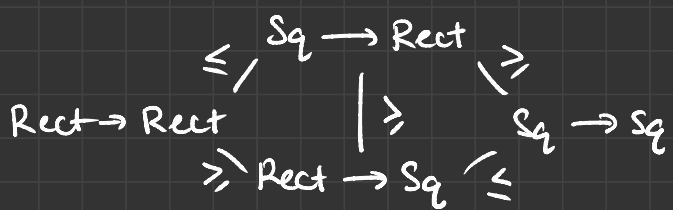
Ex.1: Consider a square and a rectangle type.

① What's the subtype relation b/w them?

$Sq \leq Rect$ | coerce :: $Sq \to Rect$
coerce $(Sq\ n) = Rect\ n\ n$

② Lattice of subtypes for

$\{ Rect \to Rect,\ Rect \to Sq,\ Sq \to Rect,\ Sq \to Sq \}$?



④ For, e.g. $(R \to S) \leq (R \to R)$

$c' :: (R \to S) \to (R \to R)$
$(R \to S) \to R \to R$
$c'\ f\ r = coerce\ (f\ r)$ ——— coerce ∘ f ✓
or f ∘ coercion for some others.

or

$$\frac{\overline{R \leq R}\ \checkmark \qquad \overline{S \leq R}\ \checkmark}{(R \to S) \leq (R \to R)}$$

LCR CONDITIONS, CRITICAL SECTIONS

```
t := x;
x := t+1;
:
```

Ex. 1: `x := 0;`

Thread 1	Thread 2
`x := x + 1;` A	`x := x x 2;` C
`x := x - 1;` B	

① Final values for x if each instruction is atomic:

0: A → B → C

1: A → C → B

② Rewrite to meet LCR:

```
T1: var t;        A      T2: var u;      F
    t := x;       B          u := x;     G
    x := t+1;     C          x := u x 2; H
    t := x        D
    x := t-1;     E
```

Final values for x now? 0, 1 as before

2: A → B → C [x=1] → F → G [u=1] → D → E → H

-1: also possible.

③ use locks to ensure only values from (a) are possible:

```
T1: var t;                T2: var u;              lock ℓ.
    take(ℓ);                  take(ℓ);
    t := x;                   u := x;
    x := t+1;                 x := u x 2;
    release(ℓ);               release(ℓ);
    take(ℓ);
    t := x
    x := t-1;
    release(ℓ)
```

Ex. 2 Prove mutual exclusion of Manna-Pnueli.

var *wantp*, *wantq* := 0, 0	
while *True* **do**	**while** *True* **do**
p_1 : *non-critical section*	q_1 : *non-critical section*
p_2 : **if** $wantq = -1$	q_2 : **if** $wantp = -1$
then $wantp := -1$	**then** $wantq := 1$
else $wantp := 1$	**else** $wantq := -1$
p_3 : **await** $wantp \neq wantq$	q_3 : **await** $wantp \neq -wantq$
p_4 : *critical section*	q_4 : *critical section*
p_5 : $wantp := 0$	q_5 : $wantq := 0$
od	**od**

Note: at p_3, $|wantp| = 1$, sim. at q_3 $|wantq| = 1$

i.e. $|wantp| = |wantq|$

This implies either $wantp = wantq \Rightarrow q$ awaited

or $wantp = -wantq \Rightarrow p$ awaited

So at least one thread is blocked.