Package 'PRIMsrc'

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Type Package

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Title PRIM Survival Regression Classification

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Description Performs a unified treatment of Bump Hunting by Patient Rule Induction Method (PRIM) in Survival, Regression and Classification settings (SRC). The current version is a development release that only implements the case of a survival response.								
Depends R ($>= 3.5.0$)								
Imports parallel, survival, glmnet, superpc, Hmisc, quantreg								
NeedsCompilation yes								
URL https://github.com/jedazard/PRIMsrc, https://www.primsrc.com Repository CRAN, GitHub, Inc. Date/Publication 2015-07-28 License GPL (>= 3) file LICENSE Archs i386, x64 R topics documented: PRIMsrc-package								
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Description

"Bump Hunting" (BH) refers to the procedure of mapping out a local region of the multi-dimensional input space where a target function of interest, usually unknown, assumes smaller or larger values than its average over the entire space. In general, the region R could be any smooth shape (e.g. a convex hull) possibly disjoint.

PRIMsrc implements a unified treatment of "Bump Hunting" (BH) by algorithms derived from the Patient Rule Induction Method (PRIM) (Friedman and Fisher, 1999) for Survival, Regression and Classification outcomes (SRC). To estimate the region, PRIM generates decision rules delineating hyperdimensional boxes (hyperrectangles) of the input space, not necessarily contiguous, where the outcome is smaller or larger than its average over the entire space.

Assumptions are that the multivariate input variables can be discrete or continuous, and the univariate outcome variable can be discrete (Classification), continuous (Regression), or a time-to-event, possibly censored (Survival). It is intended to handle low and high-dimensional multivariate datasets, including the paradigm where the number of covariates p exceeds or dominates that of samples $(p > n \text{ or } p \gg n)$.

Please note that the current version is a development release, that only implements the case of a survival outcome. At this point, this version of 'PRIMsrc' is also restricted to a directed peeling search of the first box covered by the recursive coverage (outer) loop of our Patient Recursive Survival Peeling (PRSP) algorithm (Dazard and Rao (2014, 2015, 2016, 2021a)), or our Patient Recursive Group Survival Peeling (PRGSP) algorithm (Dazard and Rao (2021b)). New features will be added as soon as available.

Details

In a direct application, "Bump Hunting" (BH) can identify subgroups of observations for which their outcome is as extreme as possible. Similarly to this traditional goal of subgroup finding, **PRIMsrc** also implements the alternative goal of mapping out a region (possibly disjointed) of the input space where the outcome *difference* between existing (fixed) groups of observations is as extreme as possible. We refer to the later goal as "Group Bump Hunting" (GBH).

In the case of a time-to event, possibly censored (Survival) outcome, "Survival Bump Hunting" (SBH) is done by our Patient Recursive Survival Peeling (PRSP) algorithm. See Dazard and Rao (2014, 2015, 2016, 2021a) for details, as well as Dazard et al. (2021c) for an application in Patient Survival Subtyping. Alternatively, "Group Survival Bump Hunting" (GSBH) is done by using specifc peeling and cross-validation criterion by derivation of PRSP, which we call Patient Recursive Group Survival Peeling (PRGSP). See Dazard and Rao (2021b) for details, as well as Rao et al. (2021d) for an application in Survival Disparity Subtyping.

The package relies on an optional variable screening (pre-selection) procedure that is run before the PRSP algorithm and final variable usage (selection) procedure is done. This is done by four

possible cross-validated variable screening (pre-selection) procedures offered to the user from the main end-user survival Bump Hunting function sbh. See Dazard and Rao (2021a) as well as Yi and Huang (2017) for details, and Dazard et al. (2021c) for an application in Patient Survival Subtyping.

The following describes the end-user functions that are needed to run a complete procedure. The other internal subroutines are not documented in the manual and are not to be called by the end-user at any time. For computational efficiency, some end-user functions offer a parallelization option that is done by passing a few parameters needed to configure a cluster. This is indicated by an asterisk (* = optionally involving cluster usage). The R features are categorized as follows:

1. END-USER FUNCTION FOR PACKAGE NEWS

PRIMsrc.news Display the PRIMsrc Package News

Function to display the log file NEWS of updates of the **PRIMsrc** package.

2. END-USER S3-METHOD FUNCTIONS FOR SUMMARY, DISPLAY, PLOT AND PREDICTION

summary **Summary Function**

S3-method summary function to summarize the main parameters used to generate the sbh object.

print Print Function

S3-method print function to display the cross-validated estimated values of the sbh object.

plot 2D Visualization of Data Scatter and Encapsulating Box

S3-method plot function for two-dimensional visualization of scatter of data points and cross-validated encapsulating box of a sbh object for the highest risk (inbox) versus lower-risk (outbox) groups (PRSP), and between the two specified fixed groups (PRGSP), if this option is used. The scatter plot is done for a given peeling step (or number of steps) of the peeling sequence (inner loop of our PRSP or PRGSP) and in a given plane of the used covariates of the sbh object, both specified by the user.

predict Predict Function

S3-method predict function to predict the box membership and box vertices on an independent set.

3. END-USER SURVIVAL BUMP HUNTING FUNCTION

sbh * Cross-Validated Survival Bump Hunting

Main end-user function for fitting a Survival Bump Hunting (SBH) model. It returns an object of class sbh, as generated by our Patient Recursive Survival Peeling (PRSP) algorithm (or Patient Recursive Group Survival Peeling (PRGSP)), containing cross-validated estimates of the target region of the input space with end-points statistics of interest. The main function relies on an optional internal variable screening (pre-selection) procedure that is run before the actual variable usage (selection) is done at the time of fitting the Survival Bump Hunting (SBH) or Group Survival Bump Hunting (GSBH) model model using our PRSP or PRGSP algorithm. At this point, the user can choose between four possible variable screening (pre-selection) procedures:

- (a) Univariate Patient Recursive Survival Peeling algorithm (default of package PRIMsrc)
- (b) Penalized Censored Quantile Regression (by Semismooth Newton Coordinate Descent algorithm adapted from package hqreg)
- (c) Penalized Partial Likelihood (adapted from package glmnet)
- (d) Supervised Principal Component Analysis (adapted from package superpc)

In this version, the Cross-Validation (CV) that controls model size (#covariates) and model complexity (#peeling steps), respectively, to fit the Survival Bump Hunting model, are carried out internally by two consecutive tasks within the single main function sbh(). The returned S3-class sbh object contains cross-validated estimates of all the decision-rules of used covariates and all other statistical quantities of interest at each iteration of the peeling sequence (inner loop of the PRSP algorithm). This enables the graphical display of results of profiling curves for model selection/tuning, peeling trajectories, covariate traces and survival distributions (see plotting functions for more details). The function offers a number of options for the number of replications of the fitting procedure to be performed: B; the type of K-fold cross-validation desired: (replicated)-averaged or-combined; as well as the peeling and cross-validation critera for model selection/tuning, and a few more parameters for the PRSP algorithm. The function takes advantage of the R packages parallel and snow, which allows users to create a parallel backend within an R session, enabling access to a cluster of compute cores and/or nodes on a local and/or remote machine(s) with either. **PRIMsrc** supports two types of communication mechanisms between master and worker processes: 'Socket' or 'Message-Passing Interface' ('MPI').

4. END-USER PLOTTING FUNCTIONS FOR MODEL VALIDATION AND VISUALIZATION OF RESULTS

plot_profile Visualization for Model Selection/Validation

Function for plotting the cross-validated model selection/tuning profiles of a sbh object. It uses the user's choice of cross-validation criterion statistics among the Log Hazard Ratio (LHR), Log-Rank Test (LRT) or Concordance Error Rate (CER). The function plots (as it applies) both profiles of cross-validation criterion as a function of variables screening size (cardinal subset of top-screened variables in the PRSP variable screening procedure), and peeling length (number of peeling steps of the peeling sequence in the inner loop of the PRSP or PRGSP algorithm).

plot_traj Visualization of Peeling Trajectories/Profiles

Function for plotting the cross-validated peeling trajectories/profiles of a sbh object. Applies to the user-specified covariates among the pre-selected ones and all other statistical quantities of interest at each iteration of the peeling sequence (inner loop of our PRSP or PRGSP algorithm).

plot_trace Visualization of Covariates Traces

Function for plotting the cross-validated covariates traces of a sbh object. Plot the cross-validated modal trace curves of covariate importance and covariate usage of the user-specified covariates among the pre-selected ones at each iteration of the peeling sequence (inner loop of our PRSP or PRGSP algorithm).

plot_km Visualization of Survival Distributions

Function for plotting the cross-validated survival distributions of a sbh object. It plots the cross-validated Kaplan-Meir estimates of survival distributions between the highest risk (in-box) versus lower-risk (outbox) groups of observations (PRSP), or between the two specified fixed groups (PRGSP), if this option is used. The plot is done for a given peeling step (or number of steps) of the peeling sequence (inner loop of our PRSP or PRGSP) algorithm) of the sbh object, as specified by the user.

5. END-USER DATASETS

Synthetic.1, Synthetic.1b, Synthetic.2, Synthetic.3, Synthetic.4 **Five Datasets From Simulated Regression Survival Models**

Five datasets from simulated regression survival models #1-4 as described in Dazard et al. (2015), representing low- and high-dimensional situations, and where regression parameters represent various types of relationship between survival times and covariates including saturated and noisy situations. In three datasets where non-informative noisy covariates were

used, these covariates were not part of the design matrix (models #2-3 and #4). In one dataset, the signal is limited to a box-shaped region R of the predictor space (model #1b). In the last dataset, the signal is limited to 10% of the predictors in a p>n situation (model #4). See each dataset for more details.

Real.1 Clinical Dataset

Publicly available HIV clinical data from the Women's Interagency HIV cohort Study (WIHS). The entire study enrolled 1164 women. Inclusion criteria of the study are: women at enrolment must be (i) alive, (ii) HIV-1 infected, and (iii) free of clinical AIDS symptoms. Women were followed until the first of the following occurred: (i) treatment initiation (HAART), (ii) AIDS diagnosis, (iii) death, or administrative censoring. The studied outcomes were the competing risks "AIDS/Death (before HAART)" and "Treatment Initiation (HAART)". However, for simplification purposes, only the first of the two competing events (i.e. the time to AIDS/Death), was used. Likewise, for simplification in this clinical dataset example, only n=485 complete cases were used. Variables included history of Injection Drug Use ("IDU") at enrollment, African American ethnicity ('Race'), age ('Age'), and baseline CD4 count ('CD4'). The question in this dataset example was whether it is possible to achieve a prognostication of patients for AIDS and HAART. See dataset documentation for more details.

Real. 2 Genomic Dataset

Publicly available lung cancer genomic data from the Chemores Cohort Study. This data is part of an integrated study of mRNA, miRNA and clinical variables to characterize the molecular distinctions between squamous cell carcinoma (SCC) and adenocarcinoma (AC) in Non Small Cell Lung Cancer (NSCLC) aside large cell lung carcinoma (LCC). Tissue samples were analysed from a cohort of 123 patients, who underwent complete surgical resection at the Institut Mutualiste Montsouris (Paris, France) between 30 January 2002 and 26 June 2006. The studied outcome was the "Disease-Free Survival Time". Patients were followed until the first relapse occurred or administrative censoring. In this genomic dataset, the expression levels of Agilent miRNA probes (p=939) were included from the n=123 cohort samples. In addition to the genomic data, five clinical variables, also evaluated on the cohort samples, are included as continuous variable ('Age') and nominal variables ('Type', 'KRAS.status', 'EGFR.status', 'P53.status'). This dataset represents a situation where the number of covariates dominates the number of complete observations, or p>>n case. See dataset documentation for more details.

Known Bugs/Problems: None at this time.

Acknowledgments

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See Also

- · R package parallel
- · R package glmnet
- R package hqreg
- R package superpc

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plot.sbh

2D Visualization of Data Scatter and Encapsulating Box

Description

S3-method plot function for two-dimensional visualization of scatter of data points and cross-validated encapsulating box of a sbh object for the highest risk (inbox) versus lower-risk (outbox) groups (PRSP), and between the two specified fixed groups (PRGSP), if this option is used. The scatter plot is done for a given peeling step (or number of steps) of the peeling sequence (inner loop of our PRSP or PRGSP), and in a given projection plane of the used covariates of the sbh object, both specified by the user.

Usage

```
## S3 method for class 'sbh'
plot(x,
     main = NULL,
     proj = x$cvfit$cv.used[c(1,2)],
     steps = x$cvfit$cv.nsteps,
     pch = 16,
     cex = 0.5,
     col = c(1,2),
     boxes = TRUE,
     asp = NA,
     col.box = rep(2,length(steps)),
     lty.box = rep(2,length(steps)),
     lwd.box = rep(1,length(steps)),
     add.caption.box = boxes,
     text.caption.box = paste("Step: ", steps, sep=""),
     pch.group = c(1,1),
     cex.group = c(1,1),
     col.group = c(3,4),
     add.caption.group = ifelse(test = x$cvarg$peelcriterion == "grp",
                                  yes = TRUE,
                                  no = FALSE),
     text.caption.group = levels(x$groups),
     device = NULL,
     file = "Scatter Plot",
     path = getwd(),
     horizontal = FALSE,
     width = 5,
     height = 5, \ldots)
```

Arguments

x Object of class sbh as generated by the main function sbh.

main Character vector. Main Title. Defaults to NULL.

proj Integer vector of length two, specifying the two used (selected) covariates in which the scatter plot is to be plotted. See details. Defaults to first two used (selected) covariates.

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steps	Integer vector. Vector of peeling steps at which to plot the inbox samples and box vertices. Defaults to the last peeling step of sbh object x.
pch	Integer scalar specifying the symbol for the outbox and inbox data points (Defaults to 16 for both).
cex	Numeric scalar specifying the symbol expansion for the outbox and inbox data points (Defaults to 0.5 for both).
col	Integer vector specifying the symbol color for the outbox and inbox data points (Defaults to "black" and "red", respectively).
boxes	Logical scalar. Shall the encapsulating box(es) be plotted as well? Default to TRUE.
asp	Numeric scalar giving the y/x aspect ratio. Default to asp=NA i.e. a regular plot. See details.
col.box	Integer vector of line color of box vertices for each step. Defaults to vector of 2's (red) of length the number of steps. The vector is reused cyclically if it is shorter than the number of steps.
lty.box	Integer vector of line type of box vertices for each step. Defaults to vector of 2's of length the number of steps. The vector is reused cyclically if it is shorter than the number of steps.
lwd.box	Integer vector of line width of box vertices for each step. Defaults to vector of 1's of length the number of steps. The vector is reused cyclically if it is shorter than the number of steps.
add.caption.box	
text.caption.b	Logical scalar. Shall the caption be plotted? Defaults to boxes value.
text.Caption.b	Character vector of caption content. Defaults to paste("Step: ",steps,sep="").
pch.group	Integer vector specifying the symbol for the two groups data points (Defaults to 0.5 for both).
cex.group	Numeric vector specifying the symbol expansion for the two groups data points (Defaults to 0.5 for both).
col.group	Integer vector specifying the symbol color for the two groups data points (Defaults to "green" and "blue").
add.caption.gr	
	Logical scalar. Shall the caption be plotted? Defaults to TRUE or FALSE, depending on x\$cvarg\$peelcriterion.
text.caption.g	roup Character vector of caption content. Defaults to levels(x\$groups).
device	Graphic display device in {NULL, "PS", "PDF"}. Defaults to NULL (standard output screen). Currently implemented graphic display devices are "PS" (Postscript) or "PDF" (Portable Document Format).
file	File name for output graphic. Defaults to "Scatter Plot".
path	Absolute path (without final (back)slash separator). Defaults to working directory path.
horizontal	Logical scalar. Orientation of the printed image. Defaults to FALSE, that is potrait orientation.
width	Numeric scalar. Width of the graphics region in inches. Defaults to 5.
height	Numeric scalar. Height of the graphics region in inches. Defaults to 5.
	Generic arguments passed to other plotting functions.

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Details

Use graphical parameter asp=1 for a plotting a proportional scatter plot on the graphical device with geometrically equal scales on the x and y axes. In that case, it produces a proportional scatter plot where distances between points are represented accurately on screen. The window is set up so that one data unit in the x direction is equal in length to one data unit in the y direction.

The two dimensions (proj) of the projection plane in which the scatter plot is to be plotted, must be a subset (in the large sense) of the used (selected) covariates of sbh object x. If the number of used covariates in the sbh object is zero, the scatterplot will not be plotted. If the number of used covariates is one, the scatterplot will be plotted using the specified covariate and an arbitrary dimension, both specified by the user.

Value

Invisible. None. Displays the plot(s) on the specified device.

Acknowledgments

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Note

End-user plotting function.

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plot_km

Visualization of Survival Distributions

Description

Function for plotting the cross-validated survival distributions of a sbh object. It plots the cross-validated Kaplan-Meir estimates of survival distributions between the highest risk (inbox) versus lower-risk (outbox) groups of observations (PRSP), or between the two specified fixed groups (PRGSP), if this option is used. The plot is done for a given peeling step (or number of steps) of the peeling sequence (inner loop of our PRSP or PRGSP) algorithm) of the sbh object, as specified by the user.

Usage

```
plot_km(object,
        main = NULL,
        xlab = "Time",
        ylab = "Probability",
        ci = TRUE,
        precision = 1e-3,
        mark = 3,
        col = ifelse(test = object$cvarg$peelcriterion != "grp",
                     yes = c(1,2),
                     no = c(3,4)),
        1ty = 1,
        1wd = 0.5,
        cex = 0.5,
        steps = 1:object$cvfit$cv.nsteps,
        add.caption = TRUE,
        text.caption = ifelse(test = object$cvarg$peelcriterion != "grp",
                               yes = c("outbox", "inbox"),
                               no = levels(object$groups)),
        nr = 3,
        nc = 4,
        device = NULL,
        file = "Survival Plots",
        path = getwd(),
        horizontal = TRUE,
        width = 11,
        height = 8.5, ...)
```

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Arguments

object	Object of class sbh as generated by the main function sbh.
main	Character vector. Main Title. Defaults to NULL.
xlab	Character vector. X-axis label. Defaults to "Time".
ylab	Character vector. Y-axis label. Defaults to "Probability".
ci	Logical scalar. Shall the 95% confidence interval be plotted? Defaults to TRUE.
precision	Precision of log-rank p -values of separation between two survival curves. Defaults to 1e-3.
mark	Integer scalar of mark parameter, which will be used to label the inbox and outbox curves. Defaults to 3.
col	Integer scalar specifying the color of the inbox and outbox curves (Defaults to c(1,2)), or of the two groups (Defaults to c(3,4)), depending on object\$cvarg\$peelcriterion.
lty	Integer scalar. Line type for the survival curve. Defaults to 1.
lwd	Numeric scalar. Line width for the survival curve. Defaults to 0.5.
cex	Numeric scalar specifying the size of the marks, symbol expansion used for titles, captions, and axis labels. Defaults to 0.5.
steps	Integer vector. Vector of peeling steps at which to plot the survival curves. Defaults to all the peeling steps of sbh object object.
add.caption	Logical scalar. Shall the caption be plotted? Defaults to TRUE.
text.caption	Character vector of caption content. Defaults to {"outbox","inbox"}, or levels(object\$groups), depending on object\$cvarg\$peelcriterion.
nr	Integer scalar of the number of rows in the plot. Defaults to 3.
nc	Integer scalar of the number of columns in the plot. Defaults to 4.
device	Graphic display device in {NULL, "PS", "PDF"}. Defaults to NULL (standard output screen). Currently implemented graphic display devices are "PS" (Postscript) or "PDF" (Portable Document Format).
file	File name for output graphic. Defaults to "Survival Plots".
path	Absolute path (without final (back)slash separator). Defaults to the working directory path.
horizontal	Logical scalar. Orientation of the printed image. Defaults to TRUE, that is potrait orientation.
width	Numeric scalar. Width of the graphics region in inches. Defaults to 11.
height	Numeric scalar. Height of the graphics region in inches. Defaults to 8.5.
•••	Generic arguments passed to other plotting functions, including plot.survfit (R package survival).

Details

Some of the plotting parameters are further defined in the function plot.survfit (R package survival). Step #0 always corresponds to the situation where the starting box covers the entire test-set data before peeling. Cross-validated LRT, LHR of inbox samples and log-rank p-values of separation are shown at the bottom of the plot with the corresponding peeling step. P-values are lower-bounded by the precision limit given by 1/A, where A is the number of permutations.

Value

Invisible. None. Displays the plot(s) on the specified device.

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See Also

• plot.survfit (R package survival)

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plot_profile	Visualization for Model Selection/Tuning	

Description

Function for plotting the cross-validated model selection/tuning profiles of a sbh object. It uses the user's choice of cross-validation criterion statistics among the Log Hazard Ratio (LHR), Log-Rank Test (LRT) or Concordance Error Rate (CER). The function plots (as it applies) both profiles of cross-validation criterion as a function of variables screening size (cardinal subset of top-screened variables in the PRSP variable screening procedure), and peeling length (number of peeling steps of the peeling sequence in the inner loop of the PRSP or PRGSP algorithm).

Usage

```
plot_profile(object,
             main = NULL,
             xlim = NULL,
             ylim = NULL,
             add.sd = TRUE,
             add.profiles = TRUE,
             add.caption = TRUE,
             text.caption = c("Mean", "Std. Error"),
             pch = 20,
             col = 1,
             1ty = 1,
             1wd = 0.5,
             cex = 0.5,
             device = NULL,
             file = "Profile Plots",
             path = getwd(),
             horizontal = FALSE,
             width = 8.5,
             height = 5.0, ...)
```

Arguments

object	Object of class sbh as generated by the main function sbh.
main	Character vector. Main Title. Defaults to NULL.
xlim	Numeric vector of length 2. The x limits $[x1, x2]$ of the plot. Defaults to NULL.
ylim	Numeric vector of length 2. The y limits $[y1, y2]$ of the plot. Defaults to NULL.
add.sd	Logical scalar. Shall the standard error bars be plotted? Defaults to TRUE.
add.profiles	Logical scalar. Shall the individual profiles (for all replicates) be plotted? Defaults to TRUE.
add.caption	Logical scalar. Should the caption be plotted? Defaults to TRUE.
text.caption	Character vector of caption content. Defaults to {"Mean", "Std. Error"}.
pch	Integer scalar of symbol number for all the profiles. Defaults to 20.
col	Integer scalar of line color of the mean profile. Defaults to 1.
lty	Integer scalar of line type of the mean profile. Defaults to 1.

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lwd	Numeric scalar of line width of the mean profile. Defaults to 0.5.
cex	Numeric scalar of symbol expansion for all the profiles. Defaults to 0.5.
device	Graphic display device in {NULL, "PS", "PDF"}. Defaults to NULL (standard output screen). Currently implemented graphic display devices are "PS" (Postscript) or "PDF" (Portable Document Format).
file	File name for output graphic. Defaults to "Profile Plot".
path	Absolute path (without final (back)slash separator). Defaults to working directory path.
horizontal	Logical scalar. Orientation of the printed image. Defaults to FALSE, that is potrait orientation.
width	Numeric scalar. Width of the graphics region in inches. Defaults to 8.5.
height	Numeric scalar. Height of the graphics region in inches. Defaults to 5.0.
	Generic arguments passed to other plotting functions.

Details

Model tuning is done by applying the cross-validation criterion defined by the user's choice of specific statistic. The goal is to find the optimal value of model parameters by maximization of LHR or LRT, or minimization of CER. The parameters to optimize are (i) the cardinal of top-ranked variables subsets (if the "prsp" variable screening is chosen), and (ii) the number of peeling steps of the peeling sequence (inner loop of our PRSP algorithm) in any case of variable screening method.

Currently, this is done internally for visualization purposes, but it will ultimately offer the option to be done interactively with the end-user as well for parameter choosing/model selection.

Value

Invisible. None. Displays the plot(s) on the specified device.

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Note

End-user plotting function.

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- "J. Sunil Rao, Ph.D." <Rao@biostat.med.miami.edu>

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plot_trace

Visualization of Covariates Traces

Description

Function for plotting the cross-validated covariates traces of a sbh object. Plot the cross-validated modal trace curves of covariate importance and covariate usage of the pre-selected covariates specified by user at each iteration of the peeling sequence (inner loop of our PRSP or PRGSP algorithm).

Usage

```
plot_trace(object,
    main = NULL,
    xlab = "Box Mass",
    ylab = "Covariate Range (centered)",
    toplot = object$cvfit$cv.used,
    center = TRUE,
    scale = FALSE,
    col.cov,
    lty.cov,
    lwd.cov,
    col = 1,
    lty = 1,
    lwd = 0.5,
```

plot_trace

cex = 0.5,
add.caption = FALSE,
text.caption = NULL,
device = NULL,
file = "Covariate Trace Plots",
path = getwd(),
horizontal = FALSE,
width = 8.5,
height = 8.5, ...)

Arguments

horizontal

potrait orientation.

object	Object of class sbh as generated by the main function sbh.
main	Character vector. Main Title. Defaults to NULL.
xlab	Character vector. X-axis label. Defaults to "Box Mass". NULL
ylab	Character vector. Y-axis label. Defaults to "Covariate Range (centered)".
toplot	Numeric vector. Which of the pre-selected covariates to plot (in reference to the original index of covariates). Defaults to covariates used for peeling.
center	Logical scalar. Shall the data be centered?. Defaults to TRUE.
scale	Logical scalar. Shall the data be scaled? Defaults to FALSE.
col.cov	Integer vector. Line color for the covariate importance curve of each selected covariate. Defaults to vector of colors of length the number of selected covariates. The vector is reused cyclically if it is shorter than the number of selected covariates.
lty.cov	Integer vector. Line type for the covariate importance curve of each selected covariate. Defaults to vector of 1's of length the number of selected covariates. The vector is reused cyclically if it is shorter than the number of selected covariates.
lwd.cov	Integer vector. Line width for the covariate importance curve of each selected covariate. Defaults to vector of 1's of length the number of selected covariates. The vector is reused cyclically if it is shorter than the number of selected covariates.
col	Integer scalar. Line color for the covariate trace curve. Defaults to 1.
lty	Integer scalar. Line type for the covariate trace curve. Defaults to 1.
lwd	Numeric scalar. Line width for the covariate trace curve. Defaults to 0.5.
cex	Numeric scalar. Symbol expansion used for titles, captions, and axis labels. Defaults to 0.5.
add.caption	Logical scalar. Should the caption be plotted?. Defaults to FALSE.
text.caption	Character vector of caption content. Defaults to NULL.
device	Graphic display device in {NULL, "PS", "PDF"}. Defaults to NULL (standard output screen). Currently implemented graphic display devices are "PS" (Postscript) or "PDF" (Portable Document Format).
file	File name for output graphic. Defaults to "Covariate Trace Plots".
path	Absolute path (without final (back)slash separator). Defaults to working direc-

Logical scalar. Orientation of the printed image. Defaults to FALSE, that is

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width	Numeric scalar. Width of the graphics region in inches. Defaults to 8.5.
height	Numeric scalar. Height of the graphics region in inches. Defaults to 8.5.
	Generic arguments passed to other plotting functions

Details

The trace plots limit the display of traces to those only covariates that are used for peeling. If centered, an horizontal black dotted line about 0 is added to the plot.

Due to the variability induced by cross-validation and replication, it is possible that more than one covariate be used for peeling at a given step. So, for simplicity of the trace plots, only the modal or majority vote trace value (over the folds and replications of the cross-validation) is plotted.

The top plot shows the overlay of covariate importance curves for each covariate. The bottom plot shows the overlay of covariate usage curves for each covariate. It is a dicretized view of covariate importance.

Both point to the magnitude and order with which covariates are used along the peeling sequence.

Value

Invisible. None. Displays the plot(s) on the specified device.

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This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

Note

End-user plotting function.

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plot_traj

Visualization of Peeling Trajectories/Profiles

Description

Function for plotting the cross-validated peeling trajectories/profiles of a sbh object. Applies to the pre-selected covariates specified by user and all other statistical quantities of interest at each iteration of the peeling sequence (inner loop of our PRSP or PRGSP algorithm).

Usage

```
plot_traj(object,
          main = NULL,
          toplot = object$cvfit$cv.used,
          col.cov,
          lty.cov,
          lwd.cov,
          col = 1,
          lty = 1,
          1wd = 0.5,
          cex = 0.5,
          add.caption = FALSE,
          text.caption = NULL,
          nr = NULL,
          nc = NULL
          device = NULL,
          file = "Trajectory Plots",
          path = getwd(),
          horizontal = FALSE,
          width = 8.5,
          height = 11, \ldots)
```

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Arguments

object	Object of class sbh as generated by the main function sbh.
main	Character vector. Main Title. Defaults to NULL.
toplot	Numeric vector. Which of the pre-selected covariates to plot (in reference to the original index of covariates). Defaults to covariates used for peeling.
col.cov	Integer vector. Line color for the covariate trajectory curve of each selected covariate. Defaults to vector of colors of length the number of selected covariates. The vector is reused cyclically if it is shorter than the number of selected covariates.
lty.cov	Integer vector. Line type for the covariate trajectory curve of each selected covariate. Defaults to vector of 1's of length the number of selected covariates. The vector is reused cyclically if it is shorter than the number of selected covariates.
lwd.cov	Integer vector. Line width for the covariate trajectory curve of each selected covariate. Defaults to vector of 1's of length the number of selected covariates. The vector is reused cyclically if it is shorter than the number of selected covariates.
col	Integer scalar. Line color for the trajectory curve of each statistical quantity of interest. Defaults to 1.
lty	Integer scalar. Line type for the trajectory curve of each statistical quantity of interest. Defaults to 1.
lwd	Numeric scalar. Line width for the trajectory curve of each statistical quantity of interest. Defaults to 0.5.
cex	Numeric scalar. Symbol expansion used for titles, captions, and axis labels. Defaults to 0.5.
add.caption	Logical scalar. Should the caption be plotted? Defaults to FALSE.
text.caption	Character vector of caption content. Defaults to NULL.
nr	Integer scalar of the number of rows in the plot. If NULL, defaults to 3.
nc	Integer scalar of the number of columns in the plot. If NULL, defaults to 3.
device	Graphic display device in {NULL, "PS", "PDF"}. Defaults to NULL (standard output screen). Currently implemented graphic display devices are "PS" (Postscript) or "PDF" (Portable Document Format).
file	File name for output graphic. Defaults to "Trajectory Plots".
path	Absolute path (without final (back)slash separator). Defaults to working directory path.
horizontal	Logical scalar. Orientation of the printed image. Defaults to FALSE, that is potrait orientation.
width	Numeric scalar. Width of the graphics region in inches. Defaults to 8.5.
height	Numeric scalar. Height of the graphics region in inches. Defaults to 11.
	Generic arguments passed to other plotting functions.

Details

The plot limits the display of trajectories to those only covariates that are used for peeling.

The plot includes box descriptive statistics (such as support), survival endpoint statistics (such as Maximum Event-Free Time (MEFT), Minimum Event-Free Probability (MEVP), LHR, LRT) and prediction performance (such as CER).

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Value

Invisible. None. Displays the plot(s) on the specified device.

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This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

Note

End-user plotting function.

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	predict.sbh	Predict Function	
--	-------------	------------------	--

Description

S3-method predict function to predict the box membership and box vertices on an independent set, using a cross-validated sbh fitted object.

Usage

Arguments

object	Object of class sbh as generated by the main function sbh.
newdata	A numeric matrix containing the new input data of same format as input data object\$X. If not a matrix, newdata will be coerced to a matrix.
steps	Integer vector. Vector of peeling steps at which to predict the box memberships and box vertices. Defaults to all the peeling steps of sbh object object.
na.action	A function to specify the action to be taken if NAs are found. The default action is na.omit, which leads to rejection of incomplete cases.
	Further generic arguments passed to the predict function.

Details

Only the used covariates of the final sbh object will be retained for the covariates of newdata. So, the used covariates of sbh object must be equal or a subset of the the covariates of newdata.

Value

List containing the following 5 fields:

boxind	Logical matrix of predicted box membership indicator (columns) by peeling steps (rows). TRUE = inbox, FALSE = outbox.
vertices	List of size the number of chosen peeling steps, where each entry is a numeric matrix of predicted box vertices: lower and upper bounds (rows) by covariate (columns).
rules	List of size the number of chosen peeling steps, where each entry is a numeric matrix of decision rules on the covariates (columns) for all peeling steps (rows).
sign	numeric vector in $\{-1,+1\}$ of directions of peeling for all used (selected) covariates.
used	numeric vector of covariates used (selected) for peeling, indexed in reference to original index.

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This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

Note

End-user predict function.

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PRIMsrc.news

Display the PRIMsrc Package News

Description

Function to display the log file NEWS of updates of the PRIMsrc package.

Usage

```
PRIMsrc.news(...)
```

Arguments

... Further arguments passed to or from other methods.

Value

None.

Acknowledgments

This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

Note

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print.sbh

Print Function

Description

S3-method print function to display the cross-validated estimated values of the sbh object.

Usage

```
## S3 method for class 'sbh'
print(x, ...)
```

Arguments

x Object of class sbh as generated by the main function sbh.

... Further generic arguments passed to the print function.

Value

Display of the cross-validated fitted values of its argument.

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This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

Note

End-user print function.

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Real.1 Real Dataset #1: Clinical Dataset (p < n case)

Description

Publicly available HIV clinical data from the Women's Interagency HIV cohort Study (WIHS). The entire study enrolled 1164 women. Inclusion criteria of the study are: women at enrolment must be (i) alive, (ii) HIV-1 infected, and (iii) free of clinical AIDS symptoms. Women were followed until the first of the following occurred: (i) treatment initiation (HAART), (ii) AIDS diagnosis, (iii) death, or administrative censoring. The studied outcomes were the competing risks "AIDS/Death (before HAART)" and "Treatment Initiation (HAART)". However, for simplification purposes, only the first of the two competing events (i.e. the time to AIDS/Death), was used. Likewise, for simplification in this clinical dataset example, only complete cases were used. Variables included history of Injection Drug Use ("IDU") at enrollment, African American ethnicity ('Race'), age ('Age'), and baseline CD4 count ('CD4') for a total of p=4 clinical covariates. The question in this dataset example was whether it is possible to achieve a prognostication of patients for AIDS and HAART. See Bacon et al. (2005) and the WIHS website for more details.

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Usage

```
data("Real.1", package="PRIMsrc")
```

Format

Dataset consists of a numeric data. frame containing n=485 complete observations (samples) by rows and p=4 clinical covariates by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

Acknowledgments

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Source

See real data application in Dazard et al., 2015.

References

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Real.2 27

 Dazard J-E., Choe M., LeBlanc M., and Rao J.S. (2014). "Cross-Validation of Survival Bump Hunting by Recursive Peeling Methods." In JSM Proceedings, Survival Methods for Risk Estimation/Prediction Section. Boston, MA, USA. American Statistical Association IMS - JSM, p. 3366-3380.

• Bacon M.C., von Wyl V., Alden C. et al. (2005). "The Women's Interagency HIV Study: an observational cohort brings clinical sciences to the bench." Clin. Diagn. Lab. Immunol., 12(9):1013-1019.

See Also

Women's Interagency HIV cohort Study website: https://statepi.jhsph.edu/wihs/wordpress/

Real.2

Real Dataset #2: Genomic Dataset (p >> n case)

Description

Publicly available lung cancer genomic data from the Chemores Cohort Study. This data is part of an integrated study of mRNA, miRNA and clinical variables to characterize the molecular distinctions between squamous cell carcinoma (SCC) and adenocarcinoma (AC) in Non Small Cell Lung Cancer (NSCLC) aside large cell lung carcinoma (LCC). Tissue samples were analysed from a cohort of 123 patients, who underwent complete surgical resection at the Institut Mutualiste Montsouris (Paris, France) between 30 January 2002 and 26 June 2006. The studied outcome was the "Disease-Free Survival Time". Patients were followed until the first relapse occurred or administrative censoring. In this genomic dataset, the expression levels of Agilent miRNA probes (p = 939) were included from the n = 123 cohort samples. The miRNA data contains normalized expression levels. See below the paper by Lazar et al. (2013) and Array Express data repository for complete description of the samples, tissue preparation, Agilent array technology, and data normalization. In addition to the genomic data, five clinical variables, also evaluated on the cohort samples, are included as continuous variable ('Age') and nominal variables ('Type', 'KRAS.status', 'EGFR.status', 'P53.status'). This dataset represents a situation where the number of covariates dominates the number of complete observations, or p >> n case. See Lazar et al. (2013) and the CHEMORES Consortium website for more details.

Usage

```
data("Real.2", package="PRIMsrc")
```

Format

Dataset consists of a numeric data.frame containing n=123 complete observations (samples) by rows and p=939 genomic covariates by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

Acknowledgments

This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

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Source

See real data application in Dazard et al., 2015.

References

- Dazard J-E. and Rao J.S. (2021a). "Variable Selection Strategies for High-Dimensional Recursive Peeling-Based Survival Bump Hunting Models." (in prep).
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- Lazar V. et al. (2013). "Integrated molecular portrait of non-small cell lung cancers." BMC Medical Genomics 6:53-65.

See Also

Array Express data repository at the European Bioinformatics Institute. Accession number: #E-MTAB-1134 (MIR): www.ebi.ac.uk/arrayexpress/

CHEMORES Consortium website: http://www.chemores.ki.se/index.html

sbh

Cross-Validated Survival Bump Hunting

Description

Main end-user function for fitting a Survival Bump Hunting (SBH) model (or Group Survival Bump Hunting (GSBH)). It returns an object of class sbh, as generated by our Patient Recursive Survival Peeling (PRSP) algorithm (or Patient Recursive Group Survival Peeling (PRGSP)), containing cross-validated estimates of the target region of the input space with end-points statistics of interest.

Usage

```
sbh(X,
   у,
   delta,
   B = 30,
   K = 5,
   A = 1000
    vs = TRUE,
   vstype = "ppl",
   vsarg = "alpha=1,
             nalpha=1,
             nlambda=100",
   vscons = 0.5,
   cv = TRUE,
   cvtype = "combined",
   cvarg = "alpha=0.01,
             beta=0.10,
             peelcriterion=\"lrt\",
             cvcriterion=\"cer\"",
   groups = NULL,
   pv = FALSE,
   decimals = 2,
   onese = FALSE,
   probval = NULL,
   timeval = NULL,
   parallel.vs = FALSE,
   parallel.rep = FALSE,
   parallel.pv = FALSE,
   conf = NULL,
    verbose = TRUE,
   seed = NULL)
```

Arguments

Χ

 $(n \times p)$ data.frame or numeric matrix of n observations and p input covariates. If a data.frame is provided, it will be coerced to a numeric matrix. Discrete nominal covariates will be treated as ordinal variables. NA missing values are not allowed.

У

n-numeric vector of observed times to event. NA missing values are not allowed.

delta

n-numeric vector of observed status (censoring) indicator variable.

В

Postitive integer of the number of replications of the cross-validation procedure. Defaults to 30.

Κ

Postitive integer of the number of cross-validation folds (partitions) into which the total number of observations (n) should be randomly split. K must be bigger than 2 for a regular K-fold cross-validation procedure to work and should be greater than 3 for a regular procedure to make sense; $K \in \{5,...,10\}$ is recommended; defaults to K=5. Setting K also specifies the type of cross-validation to be done:

- K = 1 carries no cross-validation out, or set-value when cv = FALSE (see below).
- $K \in \{2,...,n-1\}$ carries out K-fold cross-validation.
- K = n carries out leave-one-out cross-validation.

Α

Positive integer of the number of permutations for the computation of log-rank permutation *p*-values. Defaults to 1000. Ignored if pv=FALSE or cv=FALSE.

٧S

logical scalar. Flag for optional variable (covariate) screening (pre-selection). Defaults to TRUE.

vstype

character vector in {"ppl", "pcqr", "spca", "prsp"} of one of the four possible variable screening (pre-selection) procedure. See details below. Defaults to "ppl". Ignored if vs is FALSE.

vsarg

Character vector of parameters of the cross-validated variable screening (preselection) procedure. Defaults to parameters values of default variable screening (pre-selection) procedure "ppl": vsarg="alpha=1,nalpha=1,nlambda=100". Note that vsarg comes as a character string between double quotes, with comas separated values, and without white spaces. All the following parameters are ignored if vs is FALSE.

PRSP:

- alpha = numeric data quantile in (0,1) to peel off at each step of the peeling sequence of the PRSP algorithm. Suggests 0.01.
- beta = numeric scalar of minimum box support at the end of the peeling sequence. Suggests 0.10.
- msize = positive integer or NULL to control the model size, i.e the number of screened variables used for fitting the Survival Bump Hunting model.
 Use a single non-NULL value as the maximum model size (cardinal of subset of top-screened variables) within the allowable range [1,floor(p)]. Alternatively, use msize=NULL to allow the optimal model size to be determined by cross-validation. See below for details. Suggests NULL.
- peelcriterion in {"Ihr", "Irt", "chs", "grp"} stands for the peeling criterion Log-Hazard Ratio (LHR), Log-Rank Test (LRT), Cumulative Hazard Summary (CHS), or Group (GRP), respectively, that is used in the PRSP algorithm (LHR, LRT, CHS) or the PRGSP algorithm (GRP). Suggests "Irt".
- cvcriterion in {"lhr", "lrt", "cer"} stands for the cross-validation criterion Log-Hazard Ratio (LHR), Log-Rank Test (LRT), or Concordance Error Rate (CER), respectively, that is used for optimizing the model size (cardinal of subset of top-screened variables) and the optimal number of peeling steps (optimal peeling sequence length) in the PRSP variable screening procedure. Suggests "cer".

PCQR:

• tau = numeric quantile in [0, 0.5] used in the censored quantile regression model. It is the tuning parameter of the censored quantile loss. It represents the conditional censored quantile of the survival response to be estimated. It includes the absolute loss when tau = 0.5. Suggests 0.5.

- alpha = numeric elasticnet mixing parameter in [0, 1] that controls the relative contribution from the lasso and the ridge penalty. The penalty is defined as (1-alpha)/2||beta||_2^2+alpha||beta||_1. alpha = 1 is the lasso penalty, and alpha = 0 the ridge penalty. If alpha is set to NULL, a vector of values of length nalpha is used, else alpha value is used and nalpha is set to 1. Suggests alpha=1 (lasso).
- nalpha = positive integer of number of alpha values to consider in the grid search. Suggests 1 (see above: lasso).
- nlambda = positive integer of number of elasticnet penalization lambda values to consider in the grid search. Suggests 100.

PPL:

- alpha = numeric elasticnet mixing parameter in [0, 1] that controls the relative contribution from the lasso and the ridge penalty. See R package **glm-net**. The penalty is defined as (1-alpha)/2||beta||_2^2+alpha||beta||_1. alpha = 1 is the lasso penalty, and alpha = 0 the ridge penalty. If alpha is set to NULL, a vector of values of length nalpha is used, else alpha value is used and nalpha is set to 1. Suggests alpha=1 (lasso).
- nalpha = positive integer of number of alpha values to consider in the grid search. Suggests 1 (see above: lasso).
- nlambda = positive integer of number of elasticnet penalization lambda values to consider in the grid search. Suggests 100.

SPCA:

- n. thres = number of thresholds to consider in the grid search. It cannot be less than *n* (sample size). Suggests 20.
- n.pcs = number of cross-validation principal components to use in {1,2,3}.
 It cannot be less than n (sample size) and greater than p (dimensionality of covariates); otherwise, it will be reset to n.pcs = p 1. Suggests 3.
- n.var = minimum number of variables to include in determining range for threshold. If cannot be greater than p (dimensionality of covariates); otherwise, it will be reset to n.var = p 1. Suggests 5.

numeric scalar in [1/K, 1], specifying the conservativeness of the variable screening (pre-selection) procedure, where 1/K is the least conservative and 1 is the most. Suggests 0.5

logical scalar. Flag for optional cross-validation (CV) of variable screening (pre-selection) parameters and Survival Bump Hunting fitting by PRSP algorithm. See below for details. Defaults to TRUE.

character vector in {"combined", "averaged"} specifying the cross-validation technique. Defaults to "combined". Ignored if cv is FALSE.

character vector describing the parameters used in the PRSP algorithm for fitting the Survival Bump Hunting model. Defaults to: cvarg="alpha=0.01,beta=0.10,peelcriterion=\"lrt\",cvcriterion=\"". Note that cvarg comes as a character string between double quotes, with comas separated values, and without white spaces.

• alpha = numeric data quantile in (0,1) to peel off at each step of the peeling sequence of the PRSP algorithm. Defaults to 0.01.

vscons

С٧

cvarg

cvtype

• beta = numeric scalar of minimum box support at the end of the peeling sequence. Defaults to 0.10.

- peelcriterion in {"Irt", "lhr", "chs", "grp"} stands for the peeling criterion Log-Hazard Ratio (LHR), Log-Rank Test (LRT), Cumulative Hazard Summary (CHS), or Group (GRP), respectively, that is used in the PRSP or PRGSP algorithm. Defaults to "Irt".
- cvcriterion in {"Irt", "Ihr", "cer"} stands for the cross-validation criterion Log-Hazard Ratio (LHR), Log-Rank Test (LRT), or Concordance Error Rate (CER), respectively, that is used for tuning/optimizing the optimal number of peeling steps (i.e. optimal peeling sequence length) in the PRSP algorithm. Defaults to "cer". Ignored if cv is FALSE.

if peelcriterion is in {"lrt", "lhr", "chs"}, groups is automatically set to NULL.

groups

character or numeric vector, or factor of group membership indicator variable of length the data sample size with as many different values or levels as the number of data sample groups. To be specified if algorithm Patient Recursive Group Survival Peeling (PRGSP) is to be used, i.e. with option peelcriterion = "grp". Only two groups are allowed at this point. Defaults to NULL, i.e. when regular PRSP is to be used.

pν

logical scalar. Flag for computation of log-rank *p*-values. Defaults to FALSE.

decimals

Positive integer of the number of user-specified significant decimals to output results. Defaults to 2.

onese

logical scalar. Flag for using the 1-standard error rule instead of extremum value of the cross-validation criterion when tuning/optimizing model parameters. Defaults to FALSE.

probval

numeric scalar of the survival probability at which we want to get the endpoint box survival time. Defaults to NULL (i.e. maximal survival probability value).

timeval

numeric scalar of the survival time at which we want to get the endpoint box survival probability. Defaults to NULL (i.e. maximal survival time value).

parallel.vs

logical. Is parallelization to be performed for variable screening? Defaults to FALSE, because it is not implemented yet.

parallel.rep

logical. Is parallelization to be performed for replications? Defaults to FALSE. logical. Is parallelization to be performed for computation of log-rank *p*-

parallel.pv

values? Defaults to FALSE.

list of 5 fields containing the parameters values needed for creating the parallel

conf

backend (cluster configuration). See details below for usage. Optional, defaults to NULL, but all fields are required if used:

- type: character vector specifying the cluster type ("SOCKET", "MPI").
- spec : A specification (character vector or integer scalar) appropriate to the type of cluster.
- homogeneous: logical scalar to be set to FALSE for inhomogeneous clusters
- verbose : logical scalar to be set to FALSE for quiet mode.
- outfile: character vector of an output log file name to direct the stdout and stderr connection output from the workernodes. "" indicates no redirection.

verbose

logical scalar. Is the output to be verbose? Optional, defaults to TRUE.

seed

Positive integer scalar of the user seed to reproduce all the results. Defaults to NULL.

Details

The main function sbh relies on an optional variable screening (pre-selection) procedure that is run before the actual variable usage (selection) is done at the time of fitting the Survival Bump Hunting (SBH) or Group Survival Bump Hunting (GSBH) model using our PRSP or PRGSP algorithm, respectively. The user can choose between four possible variable screening (pre-selection) procedures. See Dazard and Rao (2014, 2015, 2016, 2021a) as well as Yi and Huang (2017) for details, and as Dazard et al. (2021c) for an application in Patient Survival Subtyping:

- Patient Recursive Survival Peeling (PRSP) (by univariate screening of our algorithm)
- Penalized Censored Quantile Regression (PCQR) (by Semismooth Newton Coordinate Descent fiting algorithm adapted from package hqreg)
- Penalized Partial Likelihood (PPL) (by Elasticnet Regularization adapted from package glm-net)
- Supervised Principal Component Analysis (SPCA) (by Supervised Principal Component adapted from package superpc)

NA missing values are not allowed in **PRIMsrc**, because it depends on R package **glmnet**, which doesn't handle missing values. In case of high-dimensional data (p >> n), the recommendation is to use PPL or SPCA because of computational efficiency. Variable screening (pre-selection) is done by computing occurrence frequencies of top-ranking variables over the cross-validation folds and replicates. The conservativeness of the procedure is controlled by the argument vscons.

Example of vscons values for pre-selection are as follows:

- '1.0' represents a presence in all the folds (unanimity vote)
- '0.5' represents a presence in at least half of the folds (majority vote)
- '1/K' represents a presence in at least one of the folds (minority vote)

Although any value in the interval [1/K,1] is accepted, we recommand using the interval [1/K, 1/2] to avoid excessive conservativeness. Final variable usage (selection) is done at the time of fitting the Survival Bump Hunting (SBH) model itself using our PRSP algorithm on previously screened variables by collecting those variables that have the maximum occurrence frequency in each peeling step over cross-validation folds and replicates.

If cross-validation is done (cv = TRUE, the optimal number of peeling steps (optimal peeling sequence length), and the optimal model size (cardinal of subset of top-screened variables) will be determined by cross-validation. If cv = FALSE, no cross-validation at all will be performed, and the values of K and vscons will both be reset to 1, and traditional log-rank Mantel-Haenszel p-values will be computed (using the Chi-Squared distribution with 1 df for the null distribution) instead of log-rank permutation p-values (using the permutation distribution for the null distribution).

The argument groups is to be specified if the Patient Recursive Group Survival Peeling (PRGSP) algorithm is used. The PRGSP algorithm is a derivation of our original Patient Recursive Survival Peeling (PRSP) algorithm to search for (or find an extreme of) outcome difference within existing (fixed) groups of observations. See Dazard and Rao (2021b) for details, as well as Rao et al. (2021d) for an application in Survival Disparity Subtyping.

In the PRSP variable screening procedure (vsarg of "prsp"), setting option msize to a single non-NULL value within the allowable range [1,floor(p)] will override the cross-validation setting within the variable screening procedure. This could be recommended for high-dimensional data (p >> n) to reduce the computational burden. In this situation, we suggest an arbitrary value of msize within [1,floor(p/5)]. Conversely, setting msize=NULL will force the cross-validation within the variable screening procedure by automatically generating a vector of model sizes (cardinals of subset of top-screened variables) within the restricted range [1,floor(p/5)], which will be used to determine the optimal value of model size.

In fitting the Survival Bump Hunting (SBH) model itself, note that the result contains initial step #0, which corresponds to the entire set of the (training) data. Also, the number of peeling steps that is within the allowable range $[1,\text{ceiling}(\log(1/n) / \log(1 - (1/n)))]$ is further restricted when either of the metaparameter alpha or beta takes on values other than the smallest possible fraction of the (training) data, i.e. $\frac{1}{n^t}$, where n^t is the training sample size:

- ceiling(log(beta) / log(1 alpha)): alpha and beta fixed by user
- ceiling($\log(1/n^t)/\log(1 \text{alpha})$): alpha fixed by user and beta fixed by data
- $\operatorname{ceiling}(\log(\operatorname{beta})/\log(1-(1/n^t)))$: alpha fixed by data and beta fixed by user
- $\operatorname{ceiling}(\log(1/n^t) / \log(1 (1/n^t)))$: alpha and beta fixed by data

When cross-validation is requested (cv=TRUE), the function performs a supervised (stratified) random splitting of the observations accounting for the classes/strata provided by delta (censoring). This is because it is desireable that the data splitting balances the class distributions of the outcome within the cross-validation splits. For each screening method and for building the final Survival Bump Hunting (SBH) model, all model tuning parameters are simultaneously estimated by cross-validation. The function offers a number of options for the cross-validation to be performed: the number of replications B; the type of technique; the peeling criterion; and the optimization criterion.

The returned S3-class sbh object contains cross-validated estimates of all the decision-rules of used (selected) covariates and all other statistical quantities of interest at each iteration of the peeling sequence (inner loop of the PRSP algorithm). This enables the graphical display of results of profiling curves for model tuning, peeling trajectories, covariate traces and survival distributions (see plotting functions for more details).

In case replicated cross-validations are performed, a "summary report" of the outputs is done over the B replicates as follows:

- Even thought the PRSP algorithm uses only one covariate at a time at each peeling step, the reported matrix of "Replicated CV" box decision rules may show more than one covariate being used in a given step depending on the replication. In the end, the reported "Replicated CV" trace values are computed (at each peeling step) as a *single* modal trace value of covariate usage over the B replicates. This is also reflected in the "Replicated CV" importance and usage plots of covariate traces.
- Similarly, the reported "Replicated CV" box membership indicators are computed (at each peeling step) as the point-wise modal membership value, that is majority vote, over the B replicates (right-hand side of equation #22 in Dazard et al. 2016). The reported "Replicated CV" box support and corresponding box sample size are computed (at each peeling step) based on the above "Replicated CV" box membership indicators (i.e. *not* as equation #23 in Dazard et al. 2016).
- All other reported "Replicated CV" box estimates are computed (at each peeling step) as average statistics over the B replicates (i.e. as equation #21 in Dazard et al. 2016), that is, *not* as a single box estimate computed from the "Replicated CV" box membership indicators. This includes the decision rules, the p-values, and all other box statistics. This may result in some apparent discordance if these estimates are re-computed directly from the reported "Replicated CV" box membership indicators.

If the computation of log-rank p-values is desired, then running with the parallelization option is strongly advised. In case of large (p > n) or very large (p >> n) datasets, it is also highly recommended to use the parallelization option.

The function sbh relies on the R package **parallel** to create a parallel backend within an R session. This enables access to a cluster of compute cores and/or nodes on a local and/or remote machine(s)

and scaling-up with the number of CPU cores available and efficient parallel execution. To run a procedure in parallel (with parallel RNG), argument parallel is to be set to TRUE and argument conf is to be specified (i.e. non NULL). Argument conf uses the options described in function makeCluster of the R packages **parallel** and **snow**. **PRIMsrc** supports two types of communication mechanisms between master and worker processes: 'Socket' or 'Message-Passing Interface' ('MPI'). In **PRIMsrc**, parallel 'Socket' clusters use sockets communication mechanisms only (no forking) and are therefore available on all platforms, including Windows, while parallel 'MPI' clusters use high-speed interconnects mechanism in networks of computers (with distributed memory) and are therefore available only in these architectures. A parallel 'MPI' cluster also requires R package **Rmpi** to be installed. Value type is used to setup a cluster of type 'Socket' ("SOCKET") or 'MPI' ("MPI"), respectively. Depending on this type, values of spec are to be used alternatively:

- For 'Socket' clusters (conf\$type="SOCKET"), spec should be a character vector naming the hosts on which to run the job; it can default to a unique local machine, in which case, one may use the unique host name "localhost". Each host name can potentially be repeated to the number of CPU cores available on the local machine. It can also be an integer scalar specifying the number of processes to spawn on the local machine; or a list of machine specifications if you have ssh installed (a character value named host specifying the name or address of the host to use).
- For 'MPI' clusters (conf\$type="MPI"), spec should be an integer scalar specifying the total number of processes to be spawned across the network of available nodes, counting the workernodes and masternode.

The actual creation of the cluster, its initialization, and closing are all done internally. For more details, see the reference manual of R package **snow** and examples below.

When random number generation is needed, the creation of separate streams of parallel RNG per node is done internally by distributing the stream states to the nodes. For more details, see the vignette of R package **parallel**. The use of a seed allows to reproduce the results within the same type of session: the same seed will reproduce the same results within a non-parallel session or within a parallel session, but it will not necessarily give the exact same results (up to sampling variability) between a non-parallelized and parallelized session due to the difference of management of the seed between the two (see parallel RNG and value of returned seed below).

Value

Object of class sbh (Patient Recursive Survival Peeling) list containing the following 23 fields:

Χ	numeric matrix of original dataset.
у	numeric vector of observed failure / survival times.
delta	numeric vector of observed event indicator in $\{1,0\}$.
В	positive integer of the number of replications used in the cross-validation procedure. $ \\$
K	positive integer of the number of folds used in the cross-validation procedure.
A	positive integer of the number of permutations used for the computation of log-rank p -values.
VS	logical scalar of returned flag of optional variable pre-selection.
vstype	character vector of the optional variable pre-selection procdure used.
vsarg	list of parameters used in the pre-selection procedure.
vscons	numeric scalar of conservativeness of the variable screening (pre-selection) procedure.

cv logical scalar of returned flag of optional cross-validation.

cvtype character vector of the cross-validation technique used.

cvarg list of parameters used in the Survival Bump Hunting procedure.

groups vector of group membership if algorithm Patient Recursive Group Survival

Peeling (PRGSP) is used.

pv logical scalar of returned flag of optional computation of log-rank p-values.

onese logical scalar of returned flag of 1-standard error rule.

decimals integer of the number of user-specified significant decimals.

probval Numeric scalar of survival probability used.

timeval Numeric scalar of survival time used.

cvprofiles list of 10 fields of cross-validated tuning profiles and estimates, each of length B (one for each replicate):

- cv.varprofiles: numeric matrix of cross-validation criterion used for tuning/optimizing the variable screening size in the PRSP variable screening (pre-selection) procedure (NULL otherwise). Values are by columns (peeling steps) and replicates (rows).
- cv.varprofiles.mean: numeric vector of means (across replicates) of the above cross-validation criterion by peeling steps.
- cv.varprofiles.se: numeric vector of standard errors (across replicates) of the above cross-validation criterion by peeling steps.
- cv.varset.opt: numeric scalar of optimal variable screening size according to the extremum.
- cv.varset.1se: numeric scalar of optimal variable screening size according to 1SE rule.
- cv.stepprofiles: numeric matrix of cross-validation criterion used for tuning/optimizing the peeling sequence length (i.e. number of peeling steps) in the PRSP algorithm. Values are by columns (peeling steps) and replicates (rows).
- cv.stepprofiles.mean: numeric vector of means (across replicates) of the above cross-validation criterion by peeling steps.
- cv.stepprofiles.se: numeric vector of standard errors (across replicates) of the above cross-validation criterion by peeling steps.
- cv.nsteps.opt: numeric scalar of optimal number of peeling steps according to the extremum.
- cv.nsteps.1se: numeric scalar of optimal number of peeling steps according to 1SE rule.

list with 12 fields of cross-validated SBH output estimates, each of length B (one for each replicate):

- cv.maxsteps: numeric scalar of maximal number of peeling steps (counting step #0 see Details section).
- cv.nsteps: numeric scalar of optimal number of peeling steps (counting step #0 see Details section).
- cv.boxind: logical matrix in TRUE, FALSE of individual observation box membership indicator (columns) for all peeling steps (rows).
- cv.boxind.size: numeric vector of box sample size for all peeling steps.
- cv.boxind.support: numeric vector of box support for all peeling steps.

cvfit

• cv.rules: data.frame of decision rules on the covariates (columns) for all peeling steps (rows).

- cv.screened: numeric vector of screened (pre-selected) covariates, indexed in reference to original index.
- cv.trace: numeric vector of the modal trace values of covariate usage for all peeling steps.
- cv.sign: numeric vector in {-1,+1} of directions of peeling for all used (selected) covariates.
- cv.used: numeric vector of covariates used (selected) for peeling, indexed in reference to original index.
- cv.stats: numeric matrix of box endpoint quantities of interest (columns) for all peeling steps (rows).
- cv.pval: list with 2 fields of two vectors. The first cvfit\$pval is a numeric vector for log-rank p-values of separation of survival distributions, The second cvfit\$seed is an integer scalar if parallelization is used, or an integer vector of A values, one for each permutation, if parallelization is not used.

success

logical scalar of the returned flag of success at fitting the SBH model.

seed

User seed. An integer scalar if parallelization is used, or an integer vector of B values, one for each replication, if parallelization is not used.

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This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

Note

Unique end-user function for fitting the Survival Bump Hunting model.

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- Dazard J-E., Choe M., Pawitan Y., and Rao J.S. (2021c). "Identification and Characterization of Informative Prognostic Subgroups by Survival Bump Hunting." (in prep).

• Rao J.S., Huilin Y., and Dazard J-E. (2021d). "Disparity Subtyping: Bringing Precision Medicine Closer to Disparity Science." (in prep).

- Yi C. and Huang J. (2017). "Semismooth Newton Coordinate Descent Algorithm for Elastic-Net Penalized Huber Loss Regression and Quantile Regression." J. Comp Graph. Statistics, 26(3):547-557.
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- Dazard J-E., Choe M., LeBlanc M., and Rao J.S. (2015). "R package PRIMsrc: Bump Hunting by Patient Rule Induction Method for Survival, Regression and Classification." In JSM Proceedings, Statistical Programmers and Analysts Section. Seattle, WA, USA. American Statistical Association IMS JSM, p. 650-664.
- Dazard J-E., Choe M., LeBlanc M., and Rao J.S. (2014). "Cross-Validation of Survival Bump Hunting by Recursive Peeling Methods." In JSM Proceedings, Survival Methods for Risk Estimation/Prediction Section. Boston, MA, USA. American Statistical Association IMS - JSM, p. 3366-3380.

See Also

- makeCluster (R package parallel)
- glmnet, cv.glmnet (R package glmnet)
- hqreg, cv. hqreg (R package hqreg)
- superpc.cv (R package superpc)

Examples

```
# Loading the library and its dependencies
library("PRIMsrc")
## Not run:
 # PRIMsrc Package news
 PRIMsrc.news()
 # PRIMsrc Package citation
 citation("PRIMsrc")
 # Demo with a synthetic dataset
 # Use help for descriptions
 data("Synthetic.1", package="PRIMsrc")
 ?Synthetic.1
## End(Not run)
# Simulated dataset #1 (n=250, p=3)
```

```
# Peeling criterion = LRT
# Cross-Validation criterion = LRT
# With Combined Cross-Validation (RCCV)
# Without Replications (B = 1)
# Without variable screening (pre-selection)
# Without computation of log-rank \eqn{p}-values
# Without parallelization
data("Synthetic.1", package="PRIMsrc")
synt1 <- sbh(X = Synthetic.1[, -c(1,2), drop=FALSE],
             y = Synthetic.1[ ,1, drop=TRUE],
             delta = Synthetic.1[ ,2, drop=TRUE],
             B = 1,
             K = 3,
             vs = FALSE,
             cv = TRUE,
             cvtype = "combined",
             cvarg = "alpha=0.05,
                      beta=0.10,
                      peelcriterion=\"lrt\",
                      cvcriterion=\"lrt\"",
             groups = NULL,
             pv = FALSE,
             decimals = 2,
             onese = FALSE,
             probval = 0.5,
             timeval = NULL,
             parallel.vs = FALSE,
             parallel.rep = FALSE,
             parallel.pv = FALSE,
             conf = NULL,
             verbose = TRUE,
             seed = 123)
summary(object = synt1)
print(x = synt1)
n <- 100
p <- length(synt1$cvfit$cv.used)</pre>
x \leftarrow matrix(data = runif(n = n*p, min = 0, max = 1),
            nrow = n, ncol = p, byrow = FALSE,
            dimnames = list(1:n, paste("X", 1:p, sep="")))
synt1.pred <- predict(object = synt1,</pre>
                      newdata = x,
                      steps = synt1$cvfit$cv.nsteps)
plot(x = synt1,
     main = paste("Scatter plot for model #1", sep=""),
     proj = synt1$cvfit$cv.used[c(1,2)],
     steps = synt1$cvfit$cv.nsteps,
     pch = 16, cex = 0.5, col = c(1,2),
     boxes = TRUE,
     col.box = 2, lty.box = 2, lwd.box = 1,
     add.caption.box = TRUE,
     text.caption.box = paste("Step: ", synt1$cvfit$cv.nsteps, sep=""),
     device = NULL)
```

```
plot_profile(object = synt1,
           main = "Cross-validated tuning profiles for model #1",
           pch = 20, col = 1, lty = 1, lwd = 0.5, cex = 0.5,
           add.sd = TRUE,
           add.profiles = TRUE,
           add.caption = TRUE,
           text.caption = c("Mean", "Std. Error"),
           device = NULL)
plot_traj(object = synt1,
        main = paste("Cross-validated peeling trajectories for model #1", sep=""),
        col = 1, lty = 1, lwd = 0.5, cex = 0.5,
        toplot = synt1$cvfit$cv.used,
        device = NULL)
plot_trace(object = synt1,
         main = paste("Cross-validated trace plots for model #1", sep=""),
         xlab = "Box Mass", ylab = "Covariate Range (centered)",
         col = 1, lty = 1, lwd = 0.5, cex = 0.5,
         toplot = synt1$cvfit$cv.used,
         center = TRUE, scale = FALSE,
         device = NULL)
plot_km(object = synt1,
       main = paste("Cross-validated probability curves for model #1", sep=""),
       xlab = "Time", ylab = "Probability",
       ci = TRUE,
       steps = 1:synt1$cvfit$cv.nsteps,
       col = c(1,2), lty = 1, lwd = 0.5, cex = 0.5,
       add.caption = TRUE,
       text.caption = c("outbox", "inbox"),
       device = NULL)
## Not run:
   # Examples of parallel backend parametrization
   #-----
   if (require("parallel")) {
      cat("'parallel' is attached correctly \n")
   } else {
      stop("'parallel' must be attached first \n")
   # Ex. #1 - Multicore PC
   # Running WINDOWS
   # SOCKET communication cluster
   # Shared memory parallelization
   #-----
   cpus <- parallel::detectCores(logical = TRUE)</pre>
   conf <- list("spec" = rep("localhost", cpus),</pre>
               "type" = "SOCKET",
               "homo" = TRUE,
               "verbose" = TRUE,
               "outfile" = "")
   # Ex. #2 - Master node + 3 Worker nodes cluster
   # All nodes equipped with identical setups of multicores
```

```
# (8 core CPUs per machine for a total of 32)
# SOCKET communication cluster
# Distributed memory parallelization
masterhost <- Sys.getenv("HOSTNAME")</pre>
slavehosts <- c("compute-0-0", "compute-0-1", "compute-0-2")</pre>
nodes <- length(slavehosts) + 1
cpus <- 8
conf <- list("spec" = c(rep(masterhost, cpus),</pre>
                    rep(slavehosts, cpus)),
           "type" = "SOCKET",
           "homo" = TRUE,
           "verbose" = TRUE,
           "outfile" = "")
# Ex. #3 - Enterprise Multinode Cluster w/ multicore/node
# Running LINUX with SLURM scheduler
# MPI communication cluster
# Distributed memory parallelization
# Below, variable 'cpus' is the total number of requested
# taks (threads/CPUs), which is specified from within a
# SLURM script.
#-----
if (require("Rmpi")) {
   print("Rmpi is loaded correctly \n")
} else {
   stop("Rmpi must be installed first to use MPI\n")
cpus <- as.numeric(Sys.getenv("SLURM_NTASKS"))</pre>
conf <- list("spec" = cpus,</pre>
           "type" = "MPI",
           "homo" = TRUE,
           "verbose" = TRUE,
           "outfile" = "")
# Simulated dataset #1 (n=250, p=3)
# Peeling criterion = LRT
# Cross-Validation criterion = LRT
# With Combined Cross-Validation (RCCV)
# With Replications (B = 30)
# With PPL variable screening (pre-selection)
# With computation of log-rank \eqn{p}-values
# With parallelization
#========
data("Synthetic.1", package="PRIMsrc")
delta = Synthetic.1[ ,2, drop=TRUE],
           B = 30.
           K = 5,
           A = 1000
           vs = TRUE,
           vstype = "ppl",
           vsarg = "alpha=1,
                   nalpha=1,
                   nlambda=100",
           vscons = 0.5,
```

```
cv = TRUE,
             cvtype = "combined",
             cvarg = "alpha=0.01,
                      beta=0.10,
                      peelcriterion=\"lrt\"
                      cvcriterion=\"lrt\"",
             groups = NULL,
             pv = TRUE,
             decimals = 2,
             onese = FALSE,
             probval = 0.5,
             timeval = NULL,
             parallel.vs = FALSE,
             parallel.rep = TRUE,
             parallel.pv = TRUE,
             conf = conf,
             verbose = TRUE,
             seed = 123)
# Simulated dataset #4 (n=100, p=1000)
# Peeling criterion = LRT
# Cross-Validation criterion = CER
# With Combined Cross-Validation (RCCV)
# With Replications (B = 30)
# With PRSP variable screening (pre-selection)
# With computation of log-rank eqn{p}-values
# With parallelization
data("Synthetic.4", package="PRIMsrc")
synt4 \leftarrow sbh(X = Synthetic.4[, -c(1,2), drop=FALSE],
             y = Synthetic.4[ ,1, drop=TRUE],
             delta = Synthetic.4[ ,2, drop=TRUE],
             B = 30,
             K = 5,
             A = 1000
             vs = TRUE,
             vstype = "prsp",
             vsarg = "alpha=0.01,
                      beta=0.10,
                      msize=NULL,
                      peelcriterion=\"lrt\",
                      cvcriterion=\"cer\"",
             vscons = 0.5,
             cv = TRUE,
             cvtype = "combined",
             cvarg = "alpha=0.01,
                      beta=0.10,
                      peelcriterion=\"lrt\",
                      cvcriterion=\"cer\"",
             groups = NULL,
             pv = TRUE,
             decimals = 2,
             onese = FALSE,
             probval = 0.5,
             timeval = NULL,
             parallel.vs = FALSE,
             parallel.rep = TRUE,
```

summary.sbh 43

```
parallel.pv = TRUE,
    conf = conf,
    verbose = TRUE,
    seed = 123)
## End(Not run)
```

summary.sbh

Summary Function

Description

S3-method summary function to summarize the main parameters used to generate the sbh object.

Usage

```
## S3 method for class 'sbh'
summary(object, ...)
```

Arguments

object Object of class sbh as generated by the main function sbh.

... Further generic arguments passed to the summary function.

Value

Summarizes the main parameters used to generate its argument.

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Note

End-user summary function.

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- Dazard J-E., Choe M., Pawitan Y., and Rao J.S. (2021c). "*Identification and Characterization of Informative Prognostic Subgroups by Survival Bump Hunting.*" (in prep).
- Rao J.S., Huilin Y., and Dazard J-E. (2021d). "Disparity Subtyping: Bringing Precision Medicine Closer to Disparity Science." (in prep).
- Yi C. and Huang J. (2017). "Semismooth Newton Coordinate Descent Algorithm for Elastic-Net Penalized Huber Loss Regression and Quantile Regression." J. Comp Graph. Statistics, 26(3):547-557.
- Dazard J-E., Choe M., LeBlanc M., and Rao J.S. (2016). "Cross-validation and Peeling Strategies for Survival Bump Hunting using Recursive Peeling Methods." Statistical Analysis and Data Mining, 9(1):12-42.
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- Dazard J-E., Choe M., LeBlanc M., and Rao J.S. (2014). "Cross-Validation of Survival Bump Hunting by Recursive Peeling Methods." In JSM Proceedings, Survival Methods for Risk Estimation/Prediction Section. Boston, MA, USA. American Statistical Association IMS - JSM, p. 3366-3380.

Synthetic.1

Synthetic Dataset #1: p < n *case*

Description

Dataset from simulated regression survival model #1 as described in Dazard et al. (2015). Here, the regression function uses all of the predictors, which are also part of the design matrix. Survival time was generated from an exponential model with rate parameter λ (and mean $1/\lambda$) according to a Cox-PH model with hazard exp(eta), where eta(.) is the regression function. Censoring indicator were generated from a uniform distribution on [0, 3]. In this synthetic example, all covariates are continuous, i.i.d. from a multivariate uniform distribution on [0, 1].

Usage

data("Synthetic.1", package="PRIMsrc")

Format

Each dataset consists of a numeric matrix containing n=250 observations (samples) by rows and p=3 variables by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

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This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

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Source

See simulated survival model #1 in Dazard et al., 2015.

- Dazard J-E. and Rao J.S. (2021a). "Variable Selection Strategies for High-Dimensional Recursive Peeling-Based Survival Bump Hunting Models." (in prep).
- Dazard J-E. and Rao J.S. (2021b). "Group Bump Hunting by Recursive Peeling-Based Methods: Application to Survival/Risk Predictive Models." (in prep).
- Dazard J-E., Choe M., Pawitan Y., and Rao J.S. (2021c). "*Identification and Characterization of Informative Prognostic Subgroups by Survival Bump Hunting.*" (in prep).
- Rao J.S., Huilin Y., and Dazard J-E. (2021d). "Disparity Subtyping: Bringing Precision Medicine Closer to Disparity Science." (in prep).
- Yi C. and Huang J. (2017). "Semismooth Newton Coordinate Descent Algorithm for Elastic-Net Penalized Huber Loss Regression and Quantile Regression." J. Comp Graph. Statistics, 26(3):547-557.
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Synthetic.1b

Synthetic Dataset #1b: p < n case

Description

Dataset from simulated regression survival model #1b as described in Dazard et al. (2015). Here, the regression function uses all of the predictors, which are also part of the design matrix. In this example, the signal is limited to a box-shaped region R of the predictor space. Survival time was generated from an exponential model with rate parameter λ (and mean $1/\lambda$) according to a Cox-PH model with hazard exp(eta), where eta(.) is the regression function. Censoring indicator were generated from a uniform distribution on [0, 3]. In this synthetic example, all covariates are continuous, i.i.d. from a multivariate uniform distribution on [0, 1].

Usage

```
data("Synthetic.1b", package="PRIMsrc")
```

Format

Each dataset consists of a numeric matrix containing n=250 observations (samples) by rows and p=3 variables by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

Acknowledgments

This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

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Source

See simulated survival model #1b in Dazard et al., 2015.

- Dazard J-E. and Rao J.S. (2021a). "Variable Selection Strategies for High-Dimensional Recursive Peeling-Based Survival Bump Hunting Models." (in prep).
- Dazard J-E. and Rao J.S. (2021b). "Group Bump Hunting by Recursive Peeling-Based Methods: Application to Survival/Risk Predictive Models." (in prep).

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• Dazard J-E., Choe M., Pawitan Y., and Rao J.S. (2021c). "*Identification and Characterization of Informative Prognostic Subgroups by Survival Bump Hunting.*" (in prep).

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- Yi C. and Huang J. (2017). "Semismooth Newton Coordinate Descent Algorithm for Elastic-Net Penalized Huber Loss Regression and Quantile Regression." J. Comp Graph. Statistics, 26(3):547-557.
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Synthetic.2

Synthetic Dataset #2: p < n case

Description

Dataset from simulated regression survival model #2 as described in Dazard et al. (2015). Here, the regression function uses some informative predictors. The rest represent un-informative noisy covariates, which are not part of the design matrix. Survival time was generated from an exponential model with rate parameter λ (and mean $1/\lambda$) according to a Cox-PH model with hazard exp(eta), where eta(.) is the regression function. Censoring indicator were generated from a uniform distribution on [0, 3]. In this synthetic example, all covariates are continuous, i.i.d. from a multivariate uniform distribution on [0, 1].

Usage

data("Synthetic.2", package="PRIMsrc")

Format

Each dataset consists of a numeric matrix containing n=250 observations (samples) by rows and p=3 variables by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

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Source

See simulated survival model #2 in Dazard et al., 2015.

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Synthetic.3

Synthetic Dataset #3: p < n case

Description

Dataset from simulated regression survival model #3 as described in Dazard et al. (2015). Here, the regression function does not include any of the predictors. This means that none of the covariates is informative (noisy), and are not part of the design matrix. Survival time was generated from an exponential model with rate parameter λ (and mean $1/\lambda$) according to a Cox-PH model with hazard exp(eta), where eta(.) is the regression function. Censoring indicator were generated from a uniform distribution on [0, 3]. In this synthetic example, all covariates are continuous, i.i.d. from a multivariate uniform distribution on [0, 1].

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Usage

```
data("Synthetic.3", package="PRIMsrc")
```

Format

Each dataset consists of a numeric matrix containing n=250 observations (samples) by rows and p=3 variables by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

Acknowledgments

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Source

See simulated survival model #3 in Dazard et al., 2015.

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50 Synthetic.4

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Synthetic.4

Synthetic Dataset #4: p > n case

Description

Dataset from simulated regression survival model #4 as described in Dazard et al. (2015). Here, the regression function uses 1/10 of informative predictors in a p>n situation with p=1000 and n=100. The rest represents non-informative noisy covariates, which are not part of the design matrix. Survival time was generated from an exponential model with rate parameter λ (and mean $1/\lambda$) according to a Cox-PH model with hazard exp(eta), where eta(.) is the regression function. Censoring indicator were generated from a uniform distribution on [0, 2]. In this synthetic example, all covariates are continuous, i.i.d. from a multivariate standard normal distribution.

Usage

data("Synthetic.4", package="PRIMsrc")

Format

Each dataset consists of a numeric matrix containing n=100 observations (samples) by rows and p=1000 variables by columns, not including the censoring indicator and (censored) time-to-event variables. It comes as a compressed Rda data file.

Acknowledgments

This work made use of the High Performance Computing Resource in the Core Facility for Advanced Research Computing at Case Western Reserve University. This project was partially funded by the National Institutes of Health NIH - National Cancer Institute (R01-CA160593) to J-E. Dazard and J.S. Rao.

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Source

See simulated survival model #4 in Dazard et al., 2015.

Synthetic.4 51

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