CSE 468 FFT

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- $\tilde{\mathbf{x}} = \tilde{\mathbf{x8}} = \tilde{\mathbf{x4}}_{\mathbf{ev}} \pm \omega_{\mathbf{8}}^{\mathbf{n}} \cdot \tilde{\mathbf{x4}}_{\mathbf{od}}$

$\rightarrow \tilde{x8}[n] = \tilde{x4}_{ev}[n] + w_8^n \cdot \tilde{x4}_{od}[n], \quad n = 0, 1, 2, 3$

$\rightarrow \tilde{x8}[4+n] = \tilde{x4}_{ev}[n] - w_8^n \cdot \tilde{x4}_{od}[n], \quad n = 0, 1, 2, 3$

- $\tilde{\mathbf{x4}}_{\mathbf{ev}} = \tilde{\mathbf{x2}}_{\mathbf{ev,ev}} \pm \omega_{\mathbf{4}}^{\mathbf{n}} \cdot \tilde{\mathbf{x2}}_{\mathbf{ev,od}}$

$\rightarrow \tilde{x4}_{ev}[n] = \tilde{x2}_{ev,ev}[n] + w_4^n \cdot \tilde{x2}_{ev,od}[n], \quad n = 0, 1$

$\rightarrow \tilde{x4}_{ev}[2+n] = \tilde{x2}_{ev,ev}[n] - w_4^n \cdot \tilde{x2}_{ev,od}[n], \quad n = 0, 1$

- $\tilde{\mathbf{x4}}_{\mathbf{od}} = \tilde{\mathbf{x2}}_{\mathbf{od,ev}} \pm \omega_{\mathbf{4}}^{\mathbf{n}} \cdot \tilde{\mathbf{x2}}_{\mathbf{od,od}}$

$\rightarrow \tilde{x4}_{od}[n] = \tilde{x2}_{od,ev}[n] + w_4^n \cdot \tilde{x2}_{od,od}[n], \quad n = 0, 1$

$\rightarrow \tilde{x4}_{od}[2+n] = \tilde{x2}_{od,ev}[n] - w_4^n \cdot \tilde{x2}_{od,od}[n], \quad n = 0, 1$

- $\tilde{\mathbf{x2}}_{\mathbf{ev,ev}} = \tilde{\mathbf{x1}}_{\mathbf{ev,ev,ev}} \pm \omega_{\mathbf{2}}^{\mathbf{n}} \cdot \tilde{\mathbf{x1}}_{\mathbf{ev,ev,od}}$

$\rightarrow \tilde{x2}_{ev,ev}[n] = \tilde{x1}_{ev,ev,ev}[n] + w_2^n \cdot \tilde{x1}_{ev,ev,od}[n], \quad n = 0$

$\rightarrow \tilde{x2}_{ev,ev}[1+n] = \tilde{x1}_{ev,ev,ev}[n] - w_2^n \cdot \tilde{x1}_{ev,ev,od}[n], \quad n = 0$

We can write similar equations for the following FFT terms,
$\tilde{\mathbf{x2}}_{\mathbf{ev,od}} = \tilde{\mathbf{x1}}_{\mathbf{ev,od,ev}} \pm \omega_{\mathbf{2}}^{\mathbf{n}} \cdot \tilde{\mathbf{x1}}_{\mathbf{ev,od,od}}$
$\tilde{\mathbf{x2}}_{\mathbf{od,ev}} = \tilde{\mathbf{x1}}_{\mathbf{od,ev,ev}} \pm \omega_{\mathbf{2}}^{\mathbf{n}} \cdot \tilde{\mathbf{x1}}_{\mathbf{od,ev,od}}$
$\tilde{\mathbf{x2}}_{\mathbf{od,od}} = \tilde{\mathbf{x1}}_{\mathbf{od,od,ev}} \pm \omega_{\mathbf{2}}^{\mathbf{n}} \cdot \tilde{\mathbf{x1}}_{\mathbf{od,od,od}}$

Time Domain Signal $\Rightarrow x[n] = [-12+9i, -11-2i, 8-14i, -5-12i, 11+1i, 13+9i, 13-2i, -10+4i]$

Step-1 (Divide)

$$
\begin{tikzcd}
 & & & {[-12+9i, -11-2i, 8-14i, -5-12i, 11+1i, 13+9i, 13-2i, -10+4i]} \arrow[dll] \arrow[drr] & & & \\
 & {[-12+9i, 8-14i, 11+1i, 13-2i]} \arrow[dl] \arrow[dr] & & & & {[-11-2i, -5-12i, 13+9i, -10+4i]} \arrow[dl] \arrow[dr] & \\
{[-12+9i, 11+1i]} \arrow[d] & & {[8-14i, 13-2i]} \arrow[d] & & {[-11-2i, 13+9i]} \arrow[d] & & {[-5-12i, -10+4i]} \arrow[d] \\
{[\tilde{x}1_{ev,ev,ev}, \tilde{x}1_{ev,ev,od}]} & & {[\tilde{x}1_{ev,od,ev}, \tilde{x}1_{ev,od,od}]} & & {[\tilde{x}1_{od,ev,ev}, \tilde{x}1_{od,ev,od}]} & & {[\tilde{x}1_{od,od,ev}, \tilde{x}1_{od,od,od}]}
\end{tikzcd}
$$

$[\tilde{x}1_{ev,ev,ev},\ \tilde{x}1_{ev,ev,od}] = [-12+9i,\ 11+1i]$,
$[\tilde{x}1_{ev,od,ev}\ \tilde{x}1_{ev,od,od}] = [8-14i, 13-2i]$,
$[\tilde{x}1_{od,ev,ev}\ \tilde{x}1_{od,ev,od}] = [-11-2i, 13+9i]$,
$[\tilde{x}1_{od,od,ev}\ \tilde{x}1_{od,od,od}] = [-5-12i, -10+4i]$

Step-2 (Compute $\tilde{x}4_{ev}$)

$w_2 = e^{-2\pi i/2} = -1$

$$\tilde{x}2_{ev,ev}[0] = \tilde{x}1_{ev,ev,ev}[0] + w_2^0 \cdot \tilde{x}1_{ev,ev,od}[0] = -12+9i + (-1)^0 \cdot (11+1i) = -1+10i$$
$$\tilde{x}2_{ev,ev}[1] = \tilde{x}1_{ev,ev,ev}[0] - w_2^0 \cdot \tilde{x}1_{ev,ev,od}[0] = -12+9i - (-1)^0 \cdot (11+1i) = -23+8i$$

$$\tilde{x}2_{ev,od}[0] = \tilde{x}1_{ev,od,ev}[0] + w_2^0 \cdot \tilde{x}1_{ev,od,od}[0] = 8-14i + (-1)^0 \cdot (13-2i) = 21-16i$$
$$\tilde{x}2_{ev,od}[1] = \tilde{x}1_{ev,od,ev}[0] - w_2^0 \cdot \tilde{x}1_{ev,od,od}[0] = 8-14i - (-1)^0 \cdot (13-2i) = -5-12i$$

$w_4 = e^{-2\pi i/4} = -i$

$$\tilde{x}4_{ev}[0] = \tilde{x}2_{ev,ev}[0] + w_4^0 \cdot \tilde{x}2_{ev,od}[0] = -1+10 + (-i)^0 \cdot (21-16i) = 20-6i$$

$$\tilde{x}4_{ev}[1] = \tilde{x}2_{ev,ev}[1] + w_4^1 \cdot \tilde{x}2_{ev,od}[1] = -23+8i + (-i)^1 \cdot (-5-12i) = -35+13i$$

$$\tilde{x}4_{ev}[2] = \tilde{x}2_{ev,ev}[0] - w_4^0 \cdot \tilde{x}2_{ev,od}[0] = -1+10i - (-i)^0 \cdot (21-16i) = -22+26i$$

$$\tilde{x}4_{ev}[3] = \tilde{x}2_{ev,ev}[1] - w_4^1 \cdot \tilde{x}2_{ev,od}[1] = -23+8i - (-i)^1 \cdot (-5-12i) = -11+3i$$

Step-3 (Compute $\tilde{x}4_{od}$)

$$\tilde{x}2_{od,ev}[0] = \tilde{x}1_{od,ev,ev}[0] + w_2^0 \cdot \tilde{x}1_{od,ev,od}[0] = -11-2i + (-1)^0 \cdot (13+9i) = 2+7i$$
$$\tilde{x}2_{od,ev}[1] = \tilde{x}1_{od,ev,ev}[0] - w_2^0 \cdot \tilde{x}1_{od,ev,od}[0] = -11-2i - (-1)^0 \cdot (13+9i) = -24-11i$$

$$\tilde{x}2_{od,od}[0] = \tilde{x}1_{od,od,ev}[0] + w_2^0 \cdot \tilde{x}1_{od,od,od}[0] = -5-12i + (-1)^0 \cdot (-10+4i) = -15-8i$$
$$\tilde{x}2_{od,od}[1] = \tilde{x}1_{od,od,ev}[0] - w_2^0 \cdot \tilde{x}1_{od,od,od}[0] = -5-12i - (-1)^0 \cdot (-10+4i) = 5-16i$$

$$\tilde{x}4_{od}[0] = \tilde{x}2_{od,ev}[0] + w_4^0 \cdot \tilde{x}2_{od,od}[0] = 2+7i + (-i)^0 \cdot (-15-8i) = -13-1i$$

$\tilde{x4}_{od}[1] = \tilde{x2}_{od,ev}[1] + w_4^1 \cdot \tilde{x2}_{od,od}[1] = -24 - 11i + (-i)^1 \cdot (5 - 16i) = -40 - 16i$

$\tilde{x4}_{od}[2] = \tilde{x2}_{od,ev}[0] - w_4^0 \cdot \tilde{x2}_{od,od}[0] = 2 + 7i - (-i)^0 \cdot (-15 - 8i) = 17 + 15i$

$\tilde{x4}_{od}[3] = \tilde{x2}_{od,ev}[1] - w_4^1 \cdot \tilde{x2}_{od,od}[1] = -24 - 11i - (-i)^1 \cdot (5 - 16i) = -8 - 6i$

Step-4 (Compute $\tilde{x8}$)

$w_8 = e^{-2i\pi/8} = (1/\sqrt{2} - i/\sqrt{2})$
$w_8^2 = e^{-2i\pi \cdot 2/8} = (1/\sqrt{2} - i/\sqrt{2})^2 = -i$
$w_8^3 = e^{-2i\pi \cdot 3/8} = (1/\sqrt{2} - i/\sqrt{2})^3 = (-1/\sqrt{2} - i/\sqrt{2})$

$\tilde{x8}[0] = \tilde{x4}_{ev}[0] + w_8^0 \cdot \tilde{x4}_{od}[0] = 20 - 6i + (1/\sqrt{2} - i/\sqrt{2})^0 \cdot (-13 - 1i) = 7 - 7i$

$\tilde{x8}[1] = \tilde{x4}_{ev}[1] + w_8^1 \cdot \tilde{x4}_{od}[1] = -35 + 13i + (1/\sqrt{2} - i/\sqrt{2})^1 \cdot (-40 - 16i) = -74.59797975 + 29.97056275i$

$\tilde{x8}[2] = \tilde{x4}_{ev}[2] + w_8^2 \cdot \tilde{x4}_{od}[2] = -22 + 26i + (1/\sqrt{2} - 1/\sqrt{2})^2 \cdot (17 + 15i) = -7 + 9i$

$\tilde{x8}[3] = \tilde{x4}_{ev}[3] + w_8^2 \cdot \tilde{x4}_{od}[3] = -11 + 3i + (1/\sqrt{2} - i/\sqrt{2})^3 \cdot (-8 - 6i) = -9.58578644 + 12.89949494i$

$\tilde{x8}[4] = \tilde{x4}_{ev}[0] - w_8^0 \cdot \tilde{x4}_{od}[0] = 20 - 6i - (1/\sqrt{2} - i/\sqrt{2})^0 \cdot (-13 - 1i) = 33 - 5i$

$\tilde{x8}[5] = \tilde{x4}_{ev}[1] - w_8^1 \cdot \tilde{x4}_{od}[1] = -35 + 13i - (1/\sqrt{2} - i/\sqrt{2})^1 \cdot (-40 - 16i) = 4.59797975 - 3.97056275i$

$\tilde{x8}[6] = \tilde{x4}_{ev}[2] - w_8^2 \cdot \tilde{x4}_{od}[2] = -22 + 26i - (1/\sqrt{2} - i/\sqrt{2})^2 \cdot (17 + 15i) = -37 + 43i$

$\tilde{x8}[7] = \tilde{x4}_{ev}[3] - w_8^2 \cdot \tilde{x4}_{od}[3] = -11 + 3i - (1/\sqrt{2} - i/\sqrt{2})^3 \cdot (-8 - 6i) = -12.41421356 - 6.89949494i$