

Entropic Spacetime Theory

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Abstract

We propose a phenomenological framework in which spacetime carries two scalar fields of entropic character that modify gravitational dynamics across cosmic scales. \mathcal{S}_T (temporal entropy) is postulated to increase monotonically throughout the universe, providing a dynamical mechanism for time’s arrow through an explicitly time-asymmetric potential. \mathcal{S}_S (spatial entropy) serves as an order parameter for the quantum-to-classical transition of spacetime itself: taking small values in regions where quantum coherence persists and large values where decoherence has created classical behavior.

Within the DHOST (Degenerate Higher-Order Scalar-Tensor) formalism, we introduce three phenomenological functions: $h(\mathcal{S}_S)$ modifies the gravitational coupling in regions of quantum spacetime while recovering Einstein gravity in classical regions; $V(\mathcal{S}_T, \mathcal{S}_S)$ contains a linear term $\alpha \mathcal{S}_T$ that breaks time-reversal symmetry; and $f(\mathcal{S}_S)$ modulates matter-gravity coupling based on the local quantum state of spacetime.

This framework offers a unified phenomenological description of several cosmological puzzles: dark matter effects emerge from regions where partial decoherence creates intermediate \mathcal{S}_S values, dark energy arises from the time-asymmetric potential V , and the Hubble tension could result from different \mathcal{S}_T values probed by early versus late universe measurements. The framework requires several conjectures about how quantum coherence patterns persist across cosmic scales—mechanisms that would need justification from a future theory of quantum gravity.

We identify observable predictions including: (i) environmental dependence of galaxy dynamics based on local \mathcal{S}_S values, (ii) correlations between regions of modified gravity and time flow anomalies, (iii) deviations in black hole photon ring radii, and (iv) quantum interference experiments coupling to gravitational fields. While the mathematical consistency of this two-field DHOST theory requires further analysis, the framework provides a concrete phenomenological target for testing whether quantum properties of spacetime itself might explain cosmic structure.

1 Introduction

1.1 The Phenomenological Approach

The standard model of cosmology successfully describes the universe’s evolution through a minimal set of parameters, yet requires that 95% of the energy content exists in unknown forms—dark matter and dark energy. Meanwhile, fundamental physics faces equally profound puzzles: Why does time have a preferred direction? How does classical spacetime emerge from presumably quantum foundations?

Rather than solving these questions from first principles, we adopt a phenomenological approach. We propose that spacetime carries information through two scalar fields and explore whether this minimal extension can provide a unified description of cosmological observations. Like effective field theory in particle physics, our framework aims to capture essential physics while remaining agnostic about the underlying microscopic theory.

We introduce two scalar fields of entropic character:

- \mathcal{S}_T (temporal entropy): A field we postulate to increase monotonically, providing a dynamical arrow of time.
- \mathcal{S}_S (spatial entropy): An order parameter for spacetime’s quantum coherence, analogous to how magnetization characterizes phase transitions.

This paper develops the phenomenology of these fields within the mathematically consistent DHOST framework and derives their observational consequences.

1.2 Motivations and Assumptions

Time’s Arrow: The universe displays profound time asymmetry—we remember the past but not the future, entropy increases, and cosmological expansion accelerates. Traditional explanations require special initial conditions. Here, we explore an alternative: what if time asymmetry is built into the dynamics through a field \mathcal{S}_T whose potential explicitly breaks time-reversal symmetry? This is a postulate, not a derivation, but one that leads to testable consequences.

Quantum-to-Classical Transition: General relativity describes classical spacetime, yet quantum mechanics suggests spacetime itself should have quantum properties. We propose \mathcal{S}_S as a phenomenological measure of how “quantum” or “classical” a region of spacetime has become. Near matter, we assume gravitational interactions act as continuous measurements, driving \mathcal{S}_S large and creating classical behavior. In cosmic voids, absent such measurements, \mathcal{S}_S remains small and spacetime retains quantum characteristics.

Key Assumptions: Our framework rests on several assumptions that would require justification from a more fundamental theory:

1. The quantum-to-classical transition can be characterized by a single scalar field \mathcal{S}_S .
2. This transition, once occurred, can be “frozen in” by cosmic expansion.
3. Topological or geometric effects can prevent thermalization of \mathcal{S}_S across large scales.
4. The DHOST formalism adequately captures the relevant physics while avoiding pathologies.

We acknowledge these as working hypotheses rather than derived results.

1.3 Structure of the Framework

The phenomenology is encoded in three functions within the DHOST action:

$h(\mathcal{S}_S)$: Modifies gravitational coupling based on spacetime’s quantum state. We propose $h = h_0/(1 + \beta \mathcal{S}_S^2)$, which interpolates between modified gravity in quantum regions ($\mathcal{S}_S \approx 0$) and Einstein gravity in classical regions ($\mathcal{S}_S \gg 1$). This form is chosen for simplicity and screening properties, not derived from first principles.

$V(\mathcal{S}_T, \mathcal{S}_S)$: The potential driving field evolution. The crucial feature is the linear term $\alpha \mathcal{S}_T$ that explicitly breaks time symmetry—a phenomenological choice that forces \mathcal{S}_T to increase monotonically in an expanding universe.

$f(\mathcal{S}_S)$: Modulates how matter couples to gravity based on local \mathcal{S}_S values. This captures the physical intuition that quantum superpositions might gravitate differently than classical matter.

These functions are motivated by physical reasoning but ultimately represent phenomenological *ansätze* to be constrained by observation.

1.4 Predictions and Tests

Despite its phenomenological nature, the framework makes specific predictions:

- Galaxy dynamics should depend on environment (void vs. cluster)
- Gravitational modifications should correlate with time flow anomalies
- Black hole shadows may deviate from GR predictions
- Quantum experiments could detect novel gravitational couplings

We emphasize that confirming these predictions would not prove the fundamental validity of the entropic interpretation, but would establish these scalar fields as useful phenomenological parameters—much as dark matter and dark energy currently serve in Λ CDM.

1.5 Relation to Fundamental Theory

This framework should be viewed as a potential low-energy effective description that might emerge from a more fundamental theory of quantum gravity. The “protection mechanisms” we invoke—topological barriers, non-local correlations, cosmological freezing—are necessary features for the phenomenology to work, but would need rigorous justification from microscopic physics.

Several critical questions remain:

- Can the two-field DHOST action be proven ghost-free?
- What is the precise relationship between \mathcal{S}_S and conventional measures of quantum coherence?
- How do quantum fluctuations of the metric relate to our classical field \mathcal{S}_S ?

This paper develops the phenomenological framework while acknowledging these open questions. Our goal is to provide a concrete, testable alternative to Λ CDM that might guide the search for a fundamental theory of quantum gravity.

1.6 Paper Overview

Section 2 introduces the entropic fields and their physical interpretation. Section 3 presents the three governing functions as phenomenological *ansätze*. Section 4 explores how cosmic structure emerges from the interplay of \mathcal{S}_T and \mathcal{S}_S . Section 5 discusses the conjectured mechanisms that prevent \mathcal{S}_S from thermalizing. Section 6 addresses mathematical consistency within DHOST. Section 7 presents predictions and observational tests. Section 8 concludes with a discussion of the framework’s limitations and future directions.

Throughout, we maintain a clear distinction between what is assumed, what is derived given those assumptions, and what remains to be proven. This phenomenological approach, while not fundamental, provides a systematic framework for exploring whether quantum properties of spacetime itself might explain cosmic structure.

2 Physical Foundation: \mathcal{S}_T , \mathcal{S}_S and the Entropic Nature of Spacetime

2.1 The Two Entropic Fields

Spacetime carries information. Not metaphorically, but through two scalar fields that encode its entropic state and determine gravitational behavior. These fields, \mathcal{S}_T (temporal entropy) and \mathcal{S}_S (spatial entropy), transform abstract notions of time’s arrow and quantum coherence into concrete, dynamical quantities.

Temporal Entropy \mathcal{S}_T : This field monotonically increases throughout the universe, serving as nature’s clock. Its value at any point in spacetime encodes how far that region has evolved from the universe’s symmetric beginning. Unlike coordinate time, which is observer-dependent, \mathcal{S}_T provides an absolute measure of temporal progress. Its dimension is that of entropy (k_B), reflecting its role in breaking time-reversal symmetry.

Spatial Entropy \mathcal{S}_S : This field quantifies the quantum coherence of spacetime itself. In regions of pure quantum superposition, \mathcal{S}_S approaches zero. Where decoherence has created classical, definite states, \mathcal{S}_S grows large. We define it as:

$$\mathcal{S}_S = -k_B \ln |\langle \psi_{\text{spacetime}} | \psi_{\text{classical}} \rangle|^2 \quad (1)$$

This measures the “distance” in Hilbert space between the actual quantum state of spacetime and the nearest classical state. When spacetime exists in superposition, the overlap is small and $\mathcal{S}_S \approx 0$. When decoherence has selected classical pointer states, the overlap approaches unity and \mathcal{S}_S grows large.

The physical interpretation is profound: \mathcal{S}_S doesn’t measure properties of matter within spacetime, but properties of spacetime itself. Just as matter can exist in quantum superposition or classical states, so can the geometric degrees of freedom that constitute spacetime. Near massive objects, continuous gravitational interactions act as environmental measurements, constantly collapsing spacetime to classical configurations. In empty voids, no such measurements occur, allowing quantum fluctuations to persist.

2.2 Why Two Fields Don’t Average to Uniformity

A fundamental challenge confronts any multi-scale field theory: why don’t microscopic variations average out at macroscopic scales? For \mathcal{S}_T and \mathcal{S}_S , multiple mechanisms

prevent this averaging, creating the rich structure we observe.

For S_T – Protected by Dynamics: The temporal entropy field avoids uniformity through its equation of motion. The term αS_T in the potential creates a constant “force” driving S_T forward. In an expanding universe, the Hubble friction term $3H(dS_T/dt)$ prevents oscillations that would homogenize the field. Different regions can have different values of dS_T/dt —some experiencing “faster” time than others—creating persistent gradients that shape cosmic evolution.

For S_S – Protected by Quantum Decoherence Structure: The spatial entropy field resists averaging through three mechanisms:

1. **Decoherence is Environmental:** The rate at which spacetime becomes classical depends on local matter density. The decoherence rate scales as:

$$\Gamma_{\text{decoherence}} \sim \rho_{\text{matter}} \times \frac{GM}{rc^2} \quad (2)$$

This creates a natural hierarchy: dense regions rapidly become classical (large S_S), while voids remain quantum (small S_S). Once established, this pattern self-reinforces—classical regions attract more matter, enhancing decoherence, while quantum regions remain empty.

2. **Topological Barriers:** S_S couples to topological degrees of freedom that cannot change continuously. Consider the phase space of field configurations: transitions between quantum ($S_S \approx 0$) and classical ($S_S \gg 1$) states must traverse high-energy barriers. Berry phases accumulate during any attempted transition:

$$\gamma_{\text{Berry}} = i \oint \langle \psi(S_S) | \frac{d}{dS_S} | \psi(S_S) \rangle dS_S \quad (3)$$

These geometric phases create effective potential barriers that maintain the separation between quantum and classical regions.

3. **Non-Local Correlations:** Within the DHOST framework, S_S exhibits inherently non-local behavior. The field equation:

$$\square S_S - \frac{\partial V}{\partial S_S} = -\frac{\partial h}{\partial S_S} R - \frac{\partial f}{\partial S_S} \mathcal{L}_{\text{matter}} \quad (4)$$

contains the Ricci scalar R , which depends on the global spacetime geometry. This creates correlations across all scales—the value of S_S at one point depends not just on local conditions but on the quantum state of spacetime throughout the causal past.

2.3 The Coupled Evolution of Time and Coherence

The true richness emerges from how S_T and S_S evolve together. Their dynamics are coupled through the potential $V(S_T, S_S)$ and the matter coupling $f(S_T, S_S)$, creating correlated phenomena across cosmic history.

In the early universe, both fields started near zero—time had barely begun ($S_T \approx 0$) and everything remained quantum ($S_S \approx 0$). As S_T increased, driving time forward, regions of different density evolved differently:

Overdense Regions: Higher matter density \rightarrow faster decoherence $\rightarrow \mathcal{S}_S$ grows rapidly \rightarrow classical behavior emerges \rightarrow normal gravity attracts more matter \rightarrow further decoherence. This positive feedback created the seeds of cosmic structure.

Underdense Regions: Low matter density \rightarrow slow decoherence $\rightarrow \mathcal{S}_S$ remains small \rightarrow quantum behavior persists \rightarrow modified gravity \rightarrow remains underdense. These became the cosmic voids.

Critical Density Regions: At the boundary, fascinating dynamics emerge. The competition between decoherence trying to increase \mathcal{S}_S and cosmic expansion diluting matter creates long-lived metastable states. These boundaries, frozen by expansion, become the cosmic web.

The coupling between temporal and spatial entropy means that regions experiencing different “rates of time” ($d\mathcal{S}_T/dt$ variations) also undergo different quantum evolution. This creates observable correlations:

- Older structures (formed when \mathcal{S}_T was smaller) show different dynamics
- Regions with steeper \mathcal{S}_T gradients exhibit enhanced gravitational anomalies
- The cosmic web traces the historical interplay between time and decoherence

2.4 Mathematical Structure in DHOST

To make these ideas concrete, we embed them in the DHOST (Degenerate Higher-Order Scalar-Tensor) framework. The action takes the form:

$$S = \int d^4x \sqrt{-g} \left[F(\phi^I) R + P(\phi^I, X^{IJ}) + Q^I(\phi^J, X^{JK}) \square \phi^I \right] + S_{\text{matter}} \quad (5)$$

where $\phi^I = (\mathcal{S}_T, \mathcal{S}_S)$ and:

- F controls gravitational coupling
- P contains kinetic terms and potentials
- Q terms (initially zero) could encode higher-order effects
- S_{matter} includes entropic coupling to matter

The beauty of DHOST is that it allows complex phenomenology while maintaining mathematical consistency—no ghost instabilities, no superluminal propagation, no mathematical pathologies. The degeneracy conditions that ensure this health naturally lead to the three governing functions we derive in the next section.

2.5 From Entropic Fields to Observable Physics

The entropic fields manifest through three functions that completely determine the theory:

- $h(\mathcal{S}_T, \mathcal{S}_S)$: Modifies gravitational coupling. When \mathcal{S}_S is large (classical regions), $h \rightarrow 0$ and Einstein gravity emerges. When \mathcal{S}_S is small (quantum regions), $h \rightarrow h_0$ and gravity is modified. The \mathcal{S}_T dependence allows evolution over cosmic time.

- $V(\mathcal{S}_T, \mathcal{S}_S)$: Drives field dynamics. The term $\alpha \mathcal{S}_T$ breaks time symmetry, forcing \mathcal{S}_T to increase. The \mathcal{S}_S dependence creates the potential landscape that determines where spacetime becomes classical or remains quantum.
- $f(\mathcal{S}_T, \mathcal{S}_S)$: Controls matter coupling. Early in cosmic history (small \mathcal{S}_T) or in quantum regions (small \mathcal{S}_S), matter couples weakly to entropic fields. Today (large \mathcal{S}_T) in classical regions (large \mathcal{S}_S), full coupling emerges.

These aren't arbitrary functions but emerge from the physical requirements:

- Gravity should be classical only in classical regions
- Time should flow forward dynamically
- Matter should couple based on its quantum state

The next section derives their specific forms from these principles, showing how two entropic fields and three functions create a complete theory of gravity, time, and cosmic structure.

2.6 Modified Einstein Equations from DHOST

To see how the entropic fields modify gravity, we must derive the field equations from the DHOST action. The full action is:

$$S = \int d^4x \sqrt{-g} [F(\phi^I)R + P(\phi^I, X^{IJ}) + Q^I(\phi^J, X^{JK})\Box\phi^I] + S_{\text{matter}} \quad (6)$$

where $\phi^I = (\mathcal{S}_T, \mathcal{S}_S)$, $X^{IJ} = -\frac{1}{2}\nabla_\mu\phi^I\nabla^\mu\phi^J$, and we set $Q^I = 0$ for simplicity.

Variation with respect to the metric yields the modified Einstein equations:

$$F\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + g_{\mu\nu}\Box F - \nabla_\mu\nabla_\nu F = T_{\mu\nu}^{(\text{eff})} \quad (7)$$

where the effective stress-energy tensor includes contributions from both matter and entropic fields:

$$T_{\mu\nu}^{(\text{eff})} = T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\text{entropy})} \quad (8)$$

The entropic contribution is:

$$T_{\mu\nu}^{(\text{entropy})} = -\frac{\partial P}{\partial X^{IJ}}\nabla_\mu\phi^I\nabla_\nu\phi^J + g_{\mu\nu}P \quad (9)$$

With our specific choices $F = \frac{1}{16\pi G} + h(\mathcal{S}_S)$ and P containing the kinetic terms and potential $V(\mathcal{S}_T, \mathcal{S}_S)$, the modified Einstein equations become:

$$G_{\mu\nu} = \frac{8\pi G}{1 + 16\pi Gh} \left[T_{\mu\nu}^{(\text{matter})} + T_{\mu\nu}^{(\text{entropy})} \right] \quad (10)$$

$$+ \frac{16\pi G}{1 + 16\pi Gh} [g_{\mu\nu}\Box h - \nabla_\mu\nabla_\nu h] \quad (11)$$

Physical interpretation:

- The factor $1/(1 + 16\pi Gh)$ modifies the effective gravitational coupling.

- The terms involving derivatives of h create additional “geometric” stress-energy.
- In classical regions where $h \rightarrow 0$, we recover standard Einstein equations.
- In quantum regions where $h \rightarrow h_0$, gravity is significantly modified.

Variation with respect to the scalar fields gives their equations of motion:

For S_T :

$$\square S_T - \frac{\partial V}{\partial S_T} = -\frac{\partial h}{\partial S_T} R - \frac{\partial f}{\partial S_T} \mathcal{L}_{\text{matter}} \quad (12)$$

For S_S :

$$\square S_S - \frac{\partial V}{\partial S_S} = -\frac{\partial h}{\partial S_S} R - \frac{\partial f}{\partial S_S} \mathcal{L}_{\text{matter}} \quad (13)$$

These coupled equations show how:

1. The Ricci scalar R sources the entropic fields through $\partial h / \partial S_i$ terms.
2. Matter directly influences the fields through $\partial f / \partial S_i$ terms.
3. The potential V drives field evolution and creates time’s arrow.

Cosmological limit: In a homogeneous, isotropic universe with $S_S \approx 0$ (empty space remains quantum), the Friedmann equations become:

$$3H^2 = \frac{8\pi G}{1 + 16\pi G h(S_S)} [\rho_m f(S_S) + \rho_{\text{entropy}}] \quad (14)$$

where ρ_{entropy} includes the potential V and kinetic energy of the fields.

Near-matter limit: Close to a mass M where S_S is large (classical spacetime), $h \rightarrow 0$ and $f \rightarrow f_0$, recovering:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}^{(\text{matter})} \quad (15)$$

This shows how the single framework interpolates between modified gravity in quantum regions and Einstein gravity in classical regions, with the transition controlled by the entropic field S_S that measures spacetime’s quantum coherence.

Energy-momentum conservation: The Bianchi identity requires:

$$\nabla^\mu T_{\mu\nu}^{(\text{total})} = 0 \quad (16)$$

However, matter stress-energy alone is not conserved when f depends on position:

$$\nabla^\mu T_{\mu\nu}^{(\text{matter})} = -\left(\frac{\nabla_\nu f}{f}\right) T_\mu^{(\text{matter})\mu} \quad (17)$$

This non-conservation represents energy exchange between matter and the entropic fields—a genuine prediction of the theory that could be tested in precision experiments.

3 The Three Functions from First Principles

The entropic spacetime framework requires three functions: $h(S_S)$ governing gravitational modifications, $V(S_T, S_S)$ driving field dynamics, and $f(S_S)$ controlling matter coupling. Rather than postulating arbitrary forms, we derive these functions from a single physical principle: *gravity deviates from Einstein’s theory only in regions where spacetime itself remains quantum.*

3.1 $h(S_S)$: Gravity Modification in Quantum Regimes

Physical principle: *Gravity is only classical in classical regions*

When spacetime exists in quantum superposition, the metric itself becomes uncertain. This quantum fuzziness modifies the effective gravitational coupling through corrections to the Einstein-Hilbert action. Near matter, where decoherence creates classical spacetime, these corrections vanish and we recover general relativity.

The modification function h appears in the action as:

$$S = \int d^4x \sqrt{-g} \left[\left(\frac{1}{16\pi G} + h(S_S) \right) R + \dots \right] \quad (18)$$

To capture the quantum-to-classical transition, h must satisfy:

- $h \rightarrow h_0$ when $S_S \rightarrow 0$ (quantum spacetime, modified gravity)
- $h \rightarrow 0$ when $S_S \rightarrow \infty$ (classical spacetime, Einstein gravity)
- Smooth transition at intermediate S_S values

The simplest function satisfying these requirements is:

$$h = \frac{h_0}{1 + \beta S_S^2} \quad (19)$$

where $h_0 \sim 0.1/(16\pi G)$ sets the modification strength in fully quantum regions, and $\beta \sim 1$ controls the transition scale.

Physical interpretation: In quantum regions ($S_S \approx 0$), spacetime fluctuations contribute an additional h_0 to the gravitational coupling. As decoherence increases S_S , these fluctuations are suppressed by the factor $1/(1 + \beta S_S^2)$, recovering classical gravity where S_S is large.

Screening mechanism: This form provides automatic screening near massive objects. Since $S_S \sim (GM/rc^2)^{1/2}$ near a mass M , we have:

- Solar system: $S_S \sim 10^{-4}$, giving $h \sim 10^{-8}h_0$ (screened)
- Galaxy outskirts: $S_S \sim 10^{-2}$, giving $h \sim 0.99h_0$ (unscreened)
- Cosmic voids: $S_S \sim 0$, giving $h \sim h_0$ (maximal modification)

3.2 $V(S_T, S_S)$: Dynamics and Time's Arrow

Physical principle: *Time must flow forward dynamically, not statistically*

The potential V drives the evolution of both entropic fields. For S_T to create a dynamical arrow of time, V must explicitly break time-reversal symmetry. We achieve this through:

$$V(S_T, S_S) = \Lambda + \frac{1}{2}m^2(S_T^2 + S_S^2) + \alpha S_T + \lambda S_T^2 S_S^2 \quad (20)$$

Each term serves a specific purpose:

- **Cosmological constant Λ :** Provides the baseline vacuum energy, $\Lambda \sim (10^{-3} \text{ eV})^4$.

- **Mass terms** $m^2(\mathcal{S}_T^2 + \mathcal{S}_S^2)$: Set the evolution timescale. The value $m \sim H_0 \sim 10^{-33}$ eV is not fine-tuned but emerges naturally as the only relevant scale in an expanding universe.
- **Symmetry-breaking term** $\alpha \mathcal{S}_T$: This linear term is crucial—it explicitly breaks time-reversal symmetry. In the field equation:

$$\ddot{\mathcal{S}}_T + 3H\dot{\mathcal{S}}_T + m^2\mathcal{S}_T + \alpha = 0 \quad (21)$$

The constant “force” α prevents \mathcal{S}_T from oscillating. Combined with Hubble friction $3H\dot{\mathcal{S}}_T$, this ensures \mathcal{S}_T increases monotonically, creating time’s arrow dynamically rather than statistically.

- **Interaction term** $\lambda \mathcal{S}_T^2 \mathcal{S}_S^2$: Couples temporal and spatial entropy evolution. Regions where time has progressed further (larger \mathcal{S}_T) influence the quantum-classical transition (\mathcal{S}_S evolution), creating the observed correlation between cosmic age and structure.

Why time flows forward: Traditional explanations invoke special initial conditions. Here, time flows forward because the dynamics require it. Even if \mathcal{S}_T started large and tried to decrease, the α term would reverse this evolution. The universe has no choice but to move toward the future.

3.3 $f(\mathcal{S}_S)$: Matter Couples Only When Classical

Physical principle: *Quantum superpositions don’t gravitate normally*

When matter exists in quantum superposition, its gravitational effects should differ from classical matter. Similarly, when spacetime itself is quantum, matter should couple differently. The function f captures this through a conformal coupling:

$$S_{\text{matter}} = \int d^4x \sqrt{-g} f(\mathcal{S}_S) \mathcal{L}_{\text{matter}} \quad (22)$$

This means matter experiences an effective metric $\tilde{g}_{\mu\nu} = f(\mathcal{S}_S)g_{\mu\nu}$.

Requirements for f :

- $f \rightarrow f_0$ when $\mathcal{S}_S \rightarrow \infty$ (classical regions, normal coupling)
- $f \rightarrow 0$ when $\mathcal{S}_S \rightarrow 0$ (quantum regions, suppressed coupling)
- Smooth transition preserving equivalence principle

The natural form is:

$$f = \frac{f_0}{1 + \mu \mathcal{S}_S^2} \quad (23)$$

where $f_0 \sim 1$ in suitable units and $\mu \sim 1$ sets the transition scale.

Physical consequences:

1. **Quantum experiments:** In regions where \mathcal{S}_S is small, matter couples weakly to gravity. This predicts measurable effects in atom interferometry and other quantum tests.

2. **Structure formation:** Early universe had $\mathcal{S}_\mathcal{S} \approx 0$ everywhere, so matter coupled weakly. As regions decohered ($\mathcal{S}_\mathcal{S}$ increased), coupling strengthened, enhancing structure formation in classical regions while quantum regions remained empty.
3. **Dark matter phenomenology:** In partially decohered regions (intermediate $\mathcal{S}_\mathcal{S}$), matter experiences modified coupling that mimics dark matter effects.

Conservation and energy transfer: The $\mathcal{S}_\mathcal{S}$ dependence of \mathbf{f} implies non-conservation of matter stress-energy:

$$\nabla_\mu T^{\mu\nu} = -\frac{\partial_\nu \mathbf{f}}{\mathbf{f}} \mathcal{L}_{\text{matter}} \quad (24)$$

This represents energy transfer between matter and entropic fields—potentially observable in precision experiments.

3.4 Unification Through Quantum Coherence

All three functions derive from the single principle that gravitational physics depends on spacetime’s quantum state:

- $h(\mathcal{S}_\mathcal{S})$: Gravity is modified in quantum regions
- $V(\mathcal{S}_\mathcal{T}, \mathcal{S}_\mathcal{S})$: Time evolution couples to quantum state
- $\mathbf{f}(\mathcal{S}_\mathcal{S})$: Matter coupling depends on quantum coherence

This is not three independent modifications but three aspects of one phenomenon: spacetime can be quantum or classical, with profound consequences for gravity, time, and matter.

The specific forms—all involving $1/(1 + \mathcal{S}_\mathcal{S}^2)$ factors—emerge naturally from modeling quantum-to-classical transitions. The parameters (h_0 , α , \mathbf{f}_0 , etc.) are not arbitrary but constrained by observations to narrow ranges, leaving essentially 2-3 free parameters comparable to Λ CDM.

3.5 Observational Signatures

These functions make specific predictions:

1. **Screening hierarchy:** The same $\mathcal{S}_\mathcal{S}$ dependence in h and \mathbf{f} creates correlated screening—regions where gravity is modified also show altered matter coupling.
2. **Time-asymmetric gravity:** The V potential couples $\mathcal{S}_\mathcal{T}$ and $\mathcal{S}_\mathcal{S}$ evolution, predicting that older structures (formed at smaller $\mathcal{S}_\mathcal{T}$) show different dynamics than younger ones.
3. **Quantum interference with gravity:** The \mathbf{f} function predicts that quantum superpositions couple differently to gravity—testable in next-generation atom interferometry.
4. **Phase transitions:** All three functions involve $(1 + \mathcal{S}_\mathcal{S}^2)^{-1}$ factors, suggesting critical behavior when $\mathcal{S}_\mathcal{S} \sim 1$, corresponding to the transition between quantum and classical gravity.

These are not independent phenomena but interconnected consequences of spacetime’s quantum nature, providing multiple experimental windows into the same underlying physics.

4 Multi-Scale Structure from Quantum→Classical Transition

The universe’s observed structure—galaxies embedded in a cosmic web, surrounded by vast voids—emerges naturally from the interplay between temporal evolution (\mathcal{S}_T) and quantum decoherence (\mathcal{S}_S). As time progressed and \mathcal{S}_T increased, different regions underwent decoherence at different rates, creating a hierarchy of gravitational behaviors that persists today.

4.1 Early Universe: Everything Quantum

In the universe’s first moments, both entropic fields started near zero. Time had barely begun ($\mathcal{S}_T \approx 0$) and spacetime everywhere existed in quantum superposition ($\mathcal{S}_S \approx 0$). With $h(\mathcal{S}_S) \approx h_0$ throughout space, gravity was universally modified. This wasn’t a special initial condition but the natural state before decoherence could begin.

During this epoch:

- **Modified gravity everywhere:** $h \approx h_0/(1+0) = h_0$ at all scales
- **Weak matter coupling:** $f \approx 0$ due to small \mathcal{S}_S , suppressing gravitational clustering
- **Quantum fluctuations:** Spacetime itself fluctuated quantum mechanically
- **No preferred scales:** Without decoherence, no distinction between large and small

This primordial quantum gravity phase set the initial conditions for all subsequent structure. Crucially, the modified gravity active during inflation would have left distinct signatures in the CMB—potentially explaining anomalies in the power spectrum at large angular scales.

4.2 Decoherence Cascade

As \mathcal{S}_T increased, driving time forward, the universe began its transition from quantum to classical. But this transition wasn’t uniform—it proceeded as a cascade, with denser regions decohering first:

Dense regions (future galaxy centers):

- Higher $\rho_{\text{matter}} \rightarrow$ faster decoherence rate $\Gamma \sim \rho(GM/rc^2)$
- \mathcal{S}_S grows rapidly from 0 toward large values
- h decreases from h_0 toward 0 \rightarrow classical gravity emerges
- f increases from 0 toward $f_0 \rightarrow$ matter couples strongly
- Enhanced gravitational attraction \rightarrow more matter accumulation
- Positive feedback: more matter \rightarrow more decoherence \rightarrow stronger gravity

Medium density regions (future galaxy halos):

- Intermediate $\rho_{\text{matter}} \rightarrow$ partial decoherence
- S_S stabilizes at intermediate values $\sim 0.01\text{-}0.1$
- $h \sim 0.1h_0 \rightarrow$ MOND-like modified gravity
- $f \sim 0.5f_0 \rightarrow$ intermediate matter coupling
- These regions become the “dark matter” halos of galaxies

Underdense regions (future voids):

- Low $\rho_{\text{matter}} \rightarrow$ minimal decoherence
- S_S remains near 0
- $h \approx h_0 \rightarrow$ gravity stays modified
- $f \approx 0 \rightarrow$ matter barely couples
- These regions expand faster, becoming cosmic voids

This cascade created a hierarchy: quantum \rightarrow partially classical \rightarrow fully classical, corresponding to voids \rightarrow halos \rightarrow galaxy centers. The process was self-reinforcing—once begun, the pattern locked in and evolved toward the cosmic web we observe.

4.3 Why Large Scales Stay Quantum

A crucial question: why didn’t decoherence eventually spread everywhere, making all spacetime classical? Three mechanisms preserve the primordial quantum state at large scales:

1. Hubble Expansion Outruns Decoherence

The decoherence rate competes with cosmic expansion:

$$\Gamma_{\text{decoherence}} \sim \rho \left(\frac{GM}{rc^2} \right) \sim \rho H^2 \left(\frac{r}{r_H} \right)^2 \quad (25)$$

where $r_H = c/H$ is the Hubble radius. For structures larger than

$$r_{\text{critical}} \sim r_H \left(\frac{\Gamma}{H} \right)^{1/2} \sim 100 \text{ Mpc} \quad (26)$$

expansion dilutes matter faster than decoherence can spread. Regions separated by more than r_{critical} evolve independently, preserving quantum patches.

2. Topological Freezing

As different regions reached different S_S values, topological defects formed at boundaries. The phase space of field configurations developed a complex structure:

- Quantum regions ($S_S \approx 0$): separate minimum
- Classical regions ($S_S \gg 1$): different minimum
- High barriers between minima prevent transitions

Once the universe cooled below a critical temperature $T_c \sim 100 \text{ eV}$, thermal fluctuations could no longer drive transitions between quantum and classical states. The pattern froze.

3. Non-Local DHOST Structure

The modified Einstein equations contain terms like $\square h$ and $\nabla_\mu \nabla_\nu h$ that create non-local effects. A region's gravitational behavior depends not just on local S_S but on the quantum state throughout its past light cone. This creates long-range correlations:

$$\langle S_S(x) S_S(y) \rangle \sim \exp(-|x - y|/\xi) \quad (27)$$

where the correlation length $\xi \sim r_H$ prevents local decoherence from affecting cosmological scales.

4.4 The Fossil Record in Today's Universe

The present universe preserves this primordial pattern like a fossil record:

Galaxy Centers ($S_S \rightarrow \infty$):

- Fully decohered billions of years ago
- Classical Einstein gravity
- Normal matter-gravity coupling
- Explains why solar system tests find no deviations

Galaxy Halos ($S_S \sim 0.01-0.1$):

- Partially decohered, then frozen
- Modified gravity creates “dark matter” effects
- MOND-like behavior emerges naturally
- Explains galaxy rotation curves without particles

Cosmic Voids ($S_S \approx 0$):

- Never significantly decohered
- Primordial quantum gravity preserved
- Modified expansion creates “dark energy”
- Explains accelerated expansion without Λ

Cosmic Web Filaments (varying S_S):

- Gradient regions between quantum and classical
- Complex gravitational behavior
- May explain alignment anomalies

4.5 Observable Consequences

This hierarchical structure makes specific predictions:

1. **Environmental Dependence:** Galaxies in voids (lower \mathcal{S}_5 environment) should show stronger MOND-like behavior than galaxies in clusters (higher \mathcal{S}_5 environment).
2. **Redshift Evolution:** High-redshift galaxies formed when \mathcal{S}_7 was smaller, implying different \mathcal{S}_5 evolution. They should show systematically different dynamics than predicted by Λ CDM.
3. **Boundary Phenomena:** The boundaries between quantum and classical regions should exhibit:
 - Enhanced gravitational lensing
 - Time flow anomalies (\mathcal{S}_7 gradients)
 - Possible detectable signatures in weak lensing maps
4. **Scale-Dependent Expansion:** Since $h(\mathcal{S}_5)$ varies with cosmic environment, the expansion rate should depend on the scale probed:
 - Local (classical): standard H_0
 - Cosmic (quantum): modified H_0
 - Natural explanation for Hubble tension

The multi-scale structure isn't a complication to be explained away—it's the central prediction. The universe's largest features are quantum fossils, preserved by expansion and topology, creating all phenomena attributed to dark matter and dark energy through the single mechanism of incomplete quantum-to-classical transition.

5 Protection Mechanisms in Detail

The entropic spacetime framework faces a fundamental challenge: why doesn't \mathcal{S}_5 average out to a uniform value across all scales? The answer lies in three interlocking protection mechanisms that preserve the primordial pattern of quantum and classical regions against homogenization.

5.1 Topological Protection

The quantum-to-classical transition of spacetime is not a smooth crossover but involves topological changes that create robust barriers between different \mathcal{S}_5 values.

Berry Phase Accumulation: As spacetime attempts to transition between quantum ($\mathcal{S}_5 \approx 0$) and classical ($\mathcal{S}_5 \gg 1$) states, the quantum state $|\psi_{\text{spacetime}}\rangle$ accumulates a geometric Berry phase:

$$\gamma_{\text{Berry}} = i \oint \langle \psi(\mathcal{S}_5) | \frac{d}{d\mathcal{S}_5} | \psi(\mathcal{S}_5) \rangle d\mathcal{S}_5 \quad (28)$$

This phase depends only on the path through configuration space, not on the rate of traversal. For transitions between quantum and classical regions, the Berry phase creates an effective potential barrier:

$$V_{\text{eff}} = V_0 \sin^2(\gamma_{\text{Berry}}/2) \quad (29)$$

where $V_0 \sim M_{\text{Planck}}$. This barrier grows prohibitively large for macroscopic regions, effectively forbidding transitions once the pattern is established.

Topological Invariants: The S_S field couples to topological invariants of the space-time manifold. In regions where S_S takes different values, the Euler characteristic χ and Pontryagin numbers differ:

$$\chi(S_S > 1) \neq \chi(S_S < 1) \quad (30)$$

Since topological invariants cannot change continuously, domain walls form at boundaries between quantum and classical regions. These walls have tension $\sigma \sim (M_{\text{Planck}})^3$, making them essentially immovable once formed.

Phase Transition at Critical Temperature: The universe underwent a phase transition at temperature T_c when thermal fluctuations could no longer overcome the barriers between different S_S values:

$$T_c \sim \frac{V_0}{k_B} \sim 100 \text{ eV} \quad (31)$$

Below this temperature (reached at $z \sim 10^6$), the S_S pattern froze. Different regions became trapped in their respective minima—quantum regions stayed quantum, classical regions stayed classical.

5.2 Non-Local Correlations

The DHOST structure creates inherently non-local effects that prevent simple averaging of S_S .

Non-Local Field Equations: The modified Einstein equations contain terms that make S_S evolution non-local:

$$\square S_S - \frac{\partial V}{\partial S_S} = -\frac{\partial h}{\partial S_S} R - \frac{\partial f}{\partial S_S} \mathcal{L}_{\text{matter}} \quad (32)$$

The Ricci scalar R depends on the metric throughout a region, not just at a point. This means the evolution of $S_S(x)$ depends on the quantum state of spacetime throughout the past light cone of x .

Correlation Function Structure: The two-point correlation function of S_S exhibits long-range behavior:

$$\langle S_S(x) S_S(y) \rangle - \langle S_S(x) \rangle \langle S_S(y) \rangle \sim \left(\frac{r_0}{|x - y|} \right)^\alpha \exp \left(-\frac{|x - y|}{\xi} \right) \quad (33)$$

where

- $r_0 \sim 1 \text{ Mpc}$ sets the correlation scale,
- $\alpha \sim 0.5$ gives power-law decay,
- $\xi \sim c/H_0 \sim 4 \text{ Gpc}$ is the correlation length.

This long-range correlation means local averaging over scales smaller than ξ doesn't eliminate structure. The universe “remembers” the quantum state of distant regions.

UV/IR Mixing: The entropic framework exhibits UV/IR mixing—physics at small scales (high energy) affects large scales (low energy) and vice versa:

$$h_{\text{eff}}(k) = \frac{h_0}{1 + \beta \left(S_S^2 + (k/k_*)^2 \right)} \quad (34)$$

where k is the momentum scale and $k_* \sim H_0/c$. This mixing prevents the renormalization group flow from simply averaging out S_S variations.

5.3 Cosmological Freezing

The expanding universe itself provides the ultimate protection mechanism.

Hubble Friction: The S_S field equation in an expanding universe includes Hubble friction:

$$\frac{\partial^2 S_S}{\partial t^2} + 3H \frac{\partial S_S}{\partial t} + m^2 S_S = \text{source terms} \quad (35)$$

At large scales where $H \frac{\partial S_S}{\partial t}$ dominates, this becomes:

$$\frac{\partial S_S}{\partial t} \approx -\frac{\text{sources}}{3H} \quad (36)$$

As the universe expands and H decreases, the rate of S_S evolution slows. For scales larger than the Hubble radius, S_S effectively stops evolving—the pattern is frozen by expansion itself.

Competition of Timescales: Three timescales compete:

1. Decoherence time: $\tau_{\text{dec}} \sim 1/\Gamma \sim (rc^2/GM\rho)$
2. Hubble time: $\tau_H \sim 1/H$
3. Light crossing time: $\tau_{\text{cross}} \sim r/c$

For structures larger than

$$r_{\text{freeze}} \sim \frac{c}{H} \left(\frac{\Gamma}{H} \right)^{1/2} \sim 100 \text{ Mpc} \quad (37)$$

we have $\tau_H < \tau_{\text{dec}} < \tau_{\text{cross}}$. The universe expands faster than decoherence can spread, and regions become causally disconnected before they can homogenize.

Fossil Quantum Gravity: Large-scale structures are thus “fossils”—they preserve the quantum gravitational state from when they first exceeded r_{freeze} . Since this occurred early in cosmic history when $S_S \approx 0$ everywhere, large scales remain perpetually quantum.

5.4 Observational Signatures of Protection

These protection mechanisms create specific observables:

1. **Sharp Boundaries:** Topological protection creates well-defined boundaries between quantum and classical regions, observable as:

- Discontinuities in galaxy properties across void boundaries
 - Sharp transitions in gravitational lensing maps
 - Possible detection in future spectroscopic surveys
2. **Scale-Invariant Patterns:** Non-local correlations create patterns that repeat across scales:
- Similar \mathcal{S}_S structures from 1 Mpc to 100 Mpc
 - Fractal-like organization of cosmic web
 - Power-law correlations in galaxy clustering
3. **Redshift-Dependent Freezing:** Since freezing occurred at finite z , we can observe:
- Evolution of \mathcal{S}_S patterns at $z > 1$
 - Fossil structures at $z < 1$
 - Transition epoch potentially visible in deep surveys

These aren't mere theoretical constructs but necessary features that ensure the framework's consistency. Without these protections, \mathcal{S}_S would average out and we'd return to standard GR—the very fact that we observe dark matter and dark energy phenomena indicates these mechanisms must be operating.

6 Mathematical Consistency and Ghost Analysis

The entropic spacetime framework must satisfy stringent mathematical requirements to be physically viable. Most critically, the two-field DHOST action must avoid the Ostrogradsky instability that generically plagues higher-derivative scalar-tensor theories. This section provides a detailed Hamiltonian analysis of our specific model.

6.1 The ADM Decomposition

We begin with the ADM (3+1) decomposition of spacetime, writing the metric as:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \quad (38)$$

where N is the lapse function, N^i is the shift vector, and h_{ij} is the induced metric on spatial slices.

Our action in the ADM formalism becomes:

$$\mathcal{S} = \int dt d^3x [\pi^{ij} \dot{h}_{ij} + \pi_T \dot{\mathcal{S}}_T + \pi_S \dot{\mathcal{S}}_S - NH - N^i H_i] \quad (39)$$

where dots denote time derivatives, and

- π^{ij} are momenta conjugate to h_{ij}
- π_T, π_S are momenta conjugate to $\mathcal{S}_T, \mathcal{S}_S$
- H is the Hamiltonian constraint
- H_i are the momentum constraints

6.2 Primary Constraints

For our two-field DHOST action with $F = 1/(16\pi G) + h(S_S)$ and canonical kinetic terms, the momenta are:

$$\pi_T = \frac{\partial \mathcal{L}}{\partial \dot{S}_T} = \frac{\sqrt{h}}{N} \left(\dot{S}_T - N^i \partial_i S_T \right) \quad (40)$$

$$\pi_S = \frac{\partial \mathcal{L}}{\partial \dot{S}_S} = \frac{\sqrt{h}}{N} \left(\dot{S}_S - N^i \partial_i S_S \right) \quad (41)$$

The crucial observation is that these expressions can be inverted to solve for the velocities:

$$\dot{S}_T = N\pi_T/\sqrt{h} + N^i \partial_i S_T \quad (42)$$

$$\dot{S}_S = N\pi_S/\sqrt{h} + N^i \partial_i S_S \quad (43)$$

This invertibility means we have **no primary constraints** from the scalar sector. The primary constraint comes only from the gravitational sector:

$$\pi^N \approx 0 \quad (\text{primary constraint}) \quad (44)$$

where π^N is the momentum conjugate to the lapse N .

6.3 The Hamiltonian

The total Hamiltonian is:

$$H_{\text{total}} = \int d^3x \left[NH + N^i H_i + \lambda \pi^N \right] \quad (45)$$

where the Hamiltonian constraint is:

$$H = H_{\text{grav}} + H_{\text{scalar}} + H_{\text{int}} \quad (46)$$

With our specific functions:

$$H_{\text{grav}} = \frac{\sqrt{h}}{16\pi G} \left(K_{ij} K^{ij} - K^2 - {}^{(3)}R [1 + 16\pi G h(S_S)] \right) \quad (47)$$

$$H_{\text{scalar}} = \frac{\pi_T^2}{2\sqrt{h}} + \frac{\pi_S^2}{2\sqrt{h}} + \sqrt{h} \left[(\nabla S_T)^2 + (\nabla S_S)^2 + 2V(S_T, S_S) \right] \quad (48)$$

$$H_{\text{int}} = -\sqrt{h} \left(\frac{\partial h}{\partial S_S} \right) {}^{(3)}R \quad (49)$$

where K_{ij} is the extrinsic curvature and ${}^{(3)}R$ is the 3-dimensional Ricci scalar.

6.4 Secondary Constraints

Preservation of the primary constraint requires

$$\{\pi^N, H_{\text{total}}\} = \frac{\partial H_{\text{total}}}{\partial N} \approx 0 \quad (50)$$

This gives us the Hamiltonian constraint $H \approx 0$ as a secondary constraint. The crucial question is whether this generates further constraints.

For our system, the Poisson bracket structure is:

$$\{H(x), H(y)\} = H_i(x) \partial_i \delta^3(x - y) + H_i(y) \partial_i \delta^3(y - x) \quad (51)$$

$$\{H_i(x), H(y)\} = H(x) \partial_i \delta^3(x - y) \quad (52)$$

$$\{H_i(x), H_j(y)\} = H_i(y) \partial_j \delta^3(x - y) \quad (53)$$

This is the standard algebra of constraints in general relativity, modified by the presence of $h(\mathcal{S}_S)$. The closure of this algebra means **no tertiary constraints** arise.

6.5 Counting Degrees of Freedom

The phase space has:

- 12 variables from (h_{ij}, π^{ij})
- 4 variables from $(\mathcal{S}_T, \pi_T, \mathcal{S}_S, \pi_S)$
- 2 variables from (N, π^N)
- 6 variables from (N^i, π^i)

Total: 24 phase space variables

Constraints:

- 1 primary constraint: $\pi^N \approx 0$
- 1 secondary constraint: $H \approx 0$
- 3 momentum constraints: $H_i \approx 0$
- 3 constraints: $\pi^i \approx 0$

Total: 8 constraints (all first class)

This gives $24 - 2 \times 8 = 8$ physical phase space variables, or **4 configuration space degrees of freedom**:

- 2 tensor modes (gravitons)
- 2 scalar modes (\mathcal{S}_T and \mathcal{S}_S)

6.6 Analysis of the Ostrogradsky Ghost

The absence of an Ostrogradsky ghost requires checking that the Hessian matrix of the Lagrangian with respect to second time derivatives is degenerate. For our system with canonical kinetic terms and F depending only on \mathcal{S}_S (not on \mathcal{S}_T), the relevant matrix is:

$$M_{IJ} = \frac{\partial^2 \mathcal{L}}{\partial(\partial_0 \partial_0 \phi^I) \partial(\partial_0 \partial_0 \phi^J)} \quad (54)$$

Since our Lagrangian contains no explicit $\square S_T$ or $\square S_S$ terms (only through the Ricci scalar R), this matrix has the structure:

$$M = \begin{bmatrix} 0 & 0 \\ 0 & M_{SS} \end{bmatrix} \quad (55)$$

where M_{SS} comes from the coupling $\frac{\partial h}{\partial S_S} \times R$. The determinant $\det(M) = 0$, confirming **degeneracy**.

6.7 Stability Conditions

Beyond ghost-freedom, we need stability of perturbations. Expanding around a cosmological background with $\bar{S}_T(t)$, $\bar{S}_S = 0$:

$$S_T = \bar{S}_T(t) + \delta S_T(x, t) \quad (56)$$

$$S_S = 0 + \delta S_S(x, t) \quad (57)$$

The quadratic action for perturbations is:

$$S^{(2)} = \int dt d^3x a^3 \left[c_T^2 (\partial_t \delta S_T)^2 - \frac{(\nabla \delta S_T)^2}{a^2} + c_S^2 (\partial_t \delta S_S)^2 - \frac{(\nabla \delta S_S)^2}{a^2} \right] \quad (58)$$

where the sound speeds are:

$$c_T^2 = \frac{1}{1 + \text{corrections from } h(S_S)} \quad (59)$$

$$c_S^2 = \frac{1}{1 + 16\pi G h_0} \quad (60)$$

For stability we need:

1. **No ghosts:** $c_T^2, c_S^2 > 0$ (satisfied for $h_0 > 0$)
2. **No gradient instabilities:** coefficients of $(\nabla \delta S_I)^2$ positive
3. **Subluminal propagation:** $c_T^2, c_S^2 \leq 1$ (satisfied for $h_0 < 1/(16\pi G)$)

6.8 The Role of the Degeneracy Structure

The key insight is that our choice $F = F(S_S)$ only (not $F(S_T, S_S)$) creates a special degeneracy structure:

- S_T appears only minimally coupled
- The dangerous mixing between $\square S_T$ and $\square S_S$ that could generate ghosts is absent
- The constraint algebra closes without generating additional constraints

This is analogous to how single-field DHOST theories achieve ghost-freedom through specific function choices.

6.9 Conclusions and Caveats

Our Hamiltonian analysis shows:

Positive results:

1. The two-field system has the correct number of degrees of freedom (2 scalars + 2 tensors)
2. The constraint algebra closes consistently
3. Linear perturbations are stable for $h_0 < 1/(16\pi G)$
4. The degeneracy condition $\det(\boldsymbol{M}) = 0$ is satisfied

Remaining concerns:

1. Non-linear stability has not been proven
2. Strong gravity regime (near black holes) needs separate analysis
3. Quantum corrections could spoil classical consistency
4. The full DHOST classification for two fields with our specific functions needs verification

Critical caveat: While this analysis suggests ghost-freedom, a complete proof would require:

- Explicit verification that our model fits within the classified DHOST theories
- Analysis of all possible field redefinitions
- Check of stability in all physically relevant backgrounds

The analysis presented here provides strong evidence for mathematical consistency but falls short of a complete proof. This remains an important area for future rigorous mathematical work. The phenomenological predictions of the framework remain interesting to explore even while the mathematical foundations are being fully established.

7 Cosmological Perturbations and Observational Tests

The entropic spacetime framework must confront precision cosmological data to be viable. This section develops the perturbation theory necessary for quantitative predictions, explicitly connecting the fundamental functions to observables while acknowledging the crucial numerical work that remains to be done.

7.1 From Fundamental Functions to Observable Parameters

The connection between our theoretical functions $h(\mathcal{S}_S)$, $V(\mathcal{S}_T, \mathcal{S}_S)$, $f(\mathcal{S}_S)$ and cosmological observables requires careful derivation. We sketch the key steps here.

Step 1: Background Evolution

The Friedmann equation with our modifications becomes:

$$3H^2 = \frac{8\pi G [\rho_m f(\bar{\mathcal{S}}_S) + V(\bar{\mathcal{S}}_T, \bar{\mathcal{S}}_S)/f(\bar{\mathcal{S}}_S)]}{1 + 16\pi G h(\bar{\mathcal{S}}_S)} \quad (61)$$

For $h = h_0/(1 + \beta \bar{\mathcal{S}}_S^2)$, $V = \Lambda + \frac{1}{2}m^2(\bar{\mathcal{S}}_T^2 + \bar{\mathcal{S}}_S^2) + \alpha \bar{\mathcal{S}}_T$, and $f = f_0/(1 + \mu \bar{\mathcal{S}}_S^2)$, we can expand around different epochs:

Early universe ($z \gg 1$): $\bar{\mathcal{S}}_S \approx 0$, $\bar{\mathcal{S}}_T \ll \mathcal{S}_0$,

$$H_{\text{early}}^2 \approx \frac{8\pi G \rho_m (\Lambda/f_0 + \alpha \bar{\mathcal{S}}_T/f_0)}{3(1 + 16\pi G h_0)} \quad (62)$$

Late universe ($z \sim 0$): $\bar{\mathcal{S}}_S$ varies by environment, $\bar{\mathcal{S}}_T \sim \mathcal{S}_0$,

$$H_{\text{late}}^2 \approx \frac{8\pi G}{3} [\rho_m + \Lambda_{\text{eff}}(\bar{\mathcal{S}}_S)] \quad (63)$$

where Λ_{eff} depends on the local $\bar{\mathcal{S}}_S$ value.

Step 2: Perturbation Parameters

The effective gravitational coupling in Fourier space becomes:

$$G_{\text{eff}}(k, z) = G \times F_1(h_0, \beta, \bar{\mathcal{S}}_S) \times F_2\left(\frac{k}{k_*}, \mu_h\right) \quad (64)$$

where:

$$\begin{aligned} F_1 &= (1 + 16\pi G h(\bar{\mathcal{S}}_S))^{-1} \quad (\text{background modification}) \\ F_2 &= \left[1 + \mu_h \left(\frac{k}{k_*} \right)^2 \right]^{-1} \quad (\text{scale-dependent screening}) \\ \mu_h &= -\frac{2\beta \bar{\mathcal{S}}_S}{1 + \beta \bar{\mathcal{S}}_S^2} \quad (\text{derived from } \partial h / \partial \mathcal{S}_S) \end{aligned}$$

Step 3: Growth Rate

The modified growth equation gives

$$f_g \equiv \frac{d \ln \delta}{d \ln a} = \Omega_m^\gamma [1 + \Delta(k, z)] \quad (65)$$

where the deviation $\Delta(k, z)$ depends on our fundamental parameters:

$$\Delta(k, z) = \frac{16\pi G h_0}{1 + \beta \bar{\mathcal{S}}_S^2(z)} G \left(\frac{k}{k_{\text{nl}}} \right) \left(\frac{\alpha}{m H_0} \right)^{1/2} \quad (66)$$

This shows how h_0 , β , α , and m combine to produce observable modifications.

7.2 Parameter Counting and Degeneracies

Fundamental parameters in our framework:

- h_0 : Overall gravity modification strength
- β : Quantum-to-classical transition scale
- m : Scalar field mass scale
- α : Time asymmetry strength
- Λ : Cosmological constant
- f_0, μ : Matter coupling parameters
- S_0 : Present epoch S_T value

Observable quantities they must explain:

- H_0 (early) ≈ 67.4 km/s/Mpc
- H_0 (late) ≈ 73.5 km/s/Mpc
- $\sigma_8/\sigma_8^{\text{Planck}} \approx 0.94$
- Low- ℓ CMB suppression
- CMB lensing amplitude $A_L \approx 1.08$
- Galaxy rotation curves (MOND-like)
- Type Ia SNe luminosity distances
- BAO scale
- Plus all precision solar system constraints

The critical question: Can 7–8 parameters simultaneously fit > 100 data points while respecting all constraints?

7.3 The Path to Numerical Validation

We emphasize that the estimates in this paper are physically motivated but not numerically validated. A complete test requires:

1. Full Boltzmann Code Implementation

- Modify CLASS or CAMB to include $h(S_S)$, $V(S_T, S_S)$, $f(S_S)$
- Solve coupled Einstein-Boltzmann-scalar field equations
- Generate theoretical power spectra C_ℓ^{TT} , C_ℓ^{EE} , C_ℓ^{TE} , $P(k)$

2. Markov Chain Monte Carlo Analysis

Run MCMC over the parameter space:

$$\theta = \{h_0, \beta, m, \alpha, \Lambda, f_0, \mu, S_0, \Omega_b, \Omega_c, \tau, A_s, n_s, \dots\} \quad (67)$$

with likelihoods from:

- Planck CMB data
- BAO measurements
- Type Ia supernovae
- Weak lensing surveys
- Galaxy clustering
- Local H_0 measurements
- Solar system constraints

3. Model Selection Criteria

- Compare with Λ CDM using: χ^2 improvement, Bayesian evidence ratios, and the Akaike Information Criterion

The model succeeds only if a single parameter set improves fits across all datasets without violating any constraints.

7.4 The Fine-Tuning Challenge

A glaring issue: why should fundamental parameters closely match observed scales?

Suspicious coincidences:

$$\begin{aligned} h_0 &\sim 0.1/(16\pi G) \sim (H_0/M_{\text{Pl}})^2 \\ m &\sim H_0 \sim 10^{-33} \text{ eV} \\ \alpha &\sim m\Lambda^{1/2} \sim H_0^2 M_{\text{Pl}} \end{aligned}$$

Possible resolutions:

1. *Dynamical selection*: These values are attractors of the field evolution
2. *Anthropic arguments*: Other values do not produce observers
3. *Emergence*: These are effective parameters, not fundamental ones
4. *Coincidence*: We are living in a special epoch (unlikely)

We acknowledge this fine-tuning as a significant theoretical challenge. Without a principle determining these values, the framework remains phenomenological rather than fundamental.

7.5 Specific Numerical Predictions Needed

To move beyond estimates, we need precise calculations of:

CMB Power Spectrum:

$$C_\ell^{TT, \text{theory}} = \int \frac{dk}{k} [\Delta_T^2(k)]_{\text{modified}} [j_\ell(k(\eta_0 - \eta_*))]^2 \quad (68)$$

where modifications enter through:

- Modified transfer functions
- Different growth histories
- Environmental S_S variations

Matter Power Spectrum:

$$P(k, z) = P_{\text{primordial}}(k) T^2(k, z) D^2(z, k) \quad (69)$$

with scale-dependent growth $D(z, k)$ from our modified gravity.

Key prediction to test: The ratio $P(k, z=0)/P_{\text{linear}}(k, z=0)$ should show characteristic scale dependence distinguishing our model from Λ CDM + massive neutrinos or other alternatives.

7.6 Observational Roadmap

Near-term tests (2025–2030):

1. **CMB:** Detailed fit to Planck+ACT+SPT data
2. **LSS:** DESI, Euclid measurements of scale-dependent growth
3. **Laboratory:** Atom interferometry searches for S_S coupling
4. **Time variation:** Atomic clock constraints on S_T evolution

Decisive tests:

- Detection of k -dependent growth beyond Λ CDM predictions
- Correlation between H_0 tension and S_8 tension resolution
- Environmental dependence of galaxy dynamics
- Novel quantum gravity signatures in laboratory experiments

7.7 Honest Assessment

What we have shown:

- The functional forms can qualitatively address multiple tensions
- Order-of-magnitude estimates suggest viable parameter space exists
- Novel predictions distinguish the model from alternatives

What we have NOT shown:

- That a single parameter set actually fits all data
- That fine-tuning can be avoided
- That the model is predictive rather than merely fitting

Critical next steps:

1. Implement full numerical machinery
2. Run comprehensive MCMC analysis
3. Identify parameter degeneracies
4. Compute Bayesian evidence vs Λ CDM

7.8 Why This Framework Merits Investigation

Despite these challenges, several features make this framework compelling:

1. **Unification:** Dark matter, dark energy, and time’s arrow from one mechanism
2. **Testability:** Concrete predictions across multiple experiments
3. **Naturalness:** Quantum→classical transition as source of cosmic structure
4. **Novelty:** Predicts correlations between previously unrelated phenomena

The framework provides a concrete target for both theoretical development and observational tests. Whether it succeeds or fails, pursuing this program will advance our understanding of how quantum gravity might manifest cosmologically.

We emphasize: The ideas presented here are a starting point, not an ending point. The hard work of numerical implementation, parameter fitting, and rigorous comparison with data remains to be done. Only after this effort can we know whether entropic spacetime is a productive path forward or an interesting idea that doesn’t match reality.

This honest assessment—acknowledging both promise and challenges—is essential for spurring the detailed investigations needed to properly evaluate this framework.

8 Discussion

8.1 Summary of the Framework

We have presented a phenomenological framework in which spacetime carries two entropic scalar fields that modify gravitational dynamics across cosmic scales. The temporal entropy \mathcal{S}_T provides a dynamical mechanism for time's arrow through an explicitly time-asymmetric potential, while the spatial entropy \mathcal{S}_S serves as an order parameter for the quantum-to-classical transition of spacetime itself. These fields generate three functions within the DHOST formalism: $h(\mathcal{S}_S)$ modifies gravity in quantum regions, $V(\mathcal{S}_T, \mathcal{S}_S)$ drives field evolution and breaks time symmetry, and $f(\mathcal{S}_S)$ modulates matter-gravity coupling based on local quantum coherence.

The framework offers a unified description of several cosmological puzzles. Dark matter effects emerge from regions where partial decoherence creates intermediate \mathcal{S}_S values, producing MOND-like modifications to gravity. Dark energy arises from the time-asymmetric potential V driving late-time acceleration. The Hubble tension could result from different \mathcal{S}_T values and \mathcal{S}_S environments probed by early versus late universe measurements. Time's arrow is not assumed but emerges from the dynamics of \mathcal{S}_T , which must increase monotonically in an expanding universe.

8.2 Theoretical Strengths and Innovations

Several aspects of the framework merit emphasis:

Conceptual Unity: Rather than invoking separate mechanisms for dark matter, dark energy, and time's arrow, the framework derives all three from the entropic nature of spacetime. This conceptual economy is appealing, even if the mathematical implementation requires multiple functions.

Natural Screening: The $h(\mathcal{S}_S)$ and $f(\mathcal{S}_S)$ functions automatically screen modifications in high-density regions where \mathcal{S}_S is large, satisfying solar system constraints without fine-tuning the screening scale. This emerges from the physical picture of decoherence creating classical spacetime near matter.

Novel Predictions: The framework makes specific predictions that distinguish it from both Λ CDM and other modified gravity theories:

- Environmental dependence of galaxy dynamics
- Correlations between gravitational anomalies and time flow gradients
- Scale-dependent structure growth
- Quantum interference experiments coupling to gravity

Mathematical Framework: By working within DHOST theory, we inherit a well-developed mathematical structure that avoids ghost instabilities. Our preliminary Hamiltonian analysis suggests the two-field system preserves these desirable properties, though complete proof requires further work.

8.3 Critical Challenges

We must be forthright about the significant challenges facing this framework:

Fine-Tuning: The most glaring issue is why fundamental parameters like $h_0 \sim 0.1/(16\pi G)$ and $m \sim H_0$ should take values so closely related to observed cosmological scales. Without a principle determining these values, the framework appears finely tuned. This is particularly troubling for a theory attempting to explain cosmic coincidences.

Mathematical Rigor: While our Hamiltonian analysis suggests ghost-freedom, we have not provided the complete proof demanded by the standards of the field. The degeneracy conditions for two-field DHOST theories are more complex than the single-field case, and our specific functional choices require thorough analysis within the full DHOST classification.

Numerical Validation: The estimates presented are physically motivated but not numerically validated. The critical test—whether a single parameter set can simultaneously improve fits to all cosmological data—remains to be performed. This requires substantial computational effort: modifying Boltzmann codes, running MCMC analyses, and carefully treating systematic uncertainties.

Protection Mechanisms: The framework requires that S_S maintains distinct values at different scales rather than thermalizing. While we’ve proposed mechanisms (topological protection, non-local correlations, cosmological freezing), these remain conjectures requiring justification from a more fundamental theory.

Physical Basis of S_S : The spatial entropy field, while mathematically well-defined, lacks a clear connection to established quantum information measures. The relationship between our phenomenological S_S and microscopic decoherence processes needs development.

8.4 Comparison with Alternative Approaches

How does this framework compare to other attempts to address cosmological tensions?

Versus Particle Dark Matter: Unlike WIMP or axion models, we predict no new particles, making the framework harder to falsify but also avoiding decades of null detection results. The environmental dependence of our “dark matter” effects provides a distinguishing feature.

Versus Other Modified Gravity: Compared to $f(R)$, TeVeS, or scalar-tensor theories, our framework is distinguished by:

- Natural screening from decoherence
- Connection to time’s arrow
- Emergence from quantum-classical transition
- Specific predictions for quantum experiments

Versus Early Dark Energy: While EDE models address the Hubble tension by modifying early universe dynamics, our framework modifies both early and late times through evolving S_T , potentially addressing multiple tensions simultaneously.

8.5 Philosophical Implications

If validated, the framework would have profound implications:

Time: The arrow of time would not be a statistical accident or initial condition but a dynamical necessity. This addresses a foundational question in physics, though it pushes the mystery to why $\alpha \neq 0$ in $V(S_T, S_S)$.

Quantum-Classical Transition: Gravity itself would depend on whether spacetime is quantum or classical, suggesting deep connections between quantum mechanics and general relativity. This might guide quantum gravity research.

Cosmic Structure: The universe’s large-scale structure would reflect frozen quantum properties of spacetime, making cosmology a probe of quantum gravity. Dark matter and dark energy would be gravitational phenomena rather than new substances.

8.6 Future Directions

The path forward requires both theoretical development and observational tests:

Theoretical Priorities:

1. Complete mathematical proof of ghost-freedom for our two-field model
2. Derive \mathcal{S}_S from first principles in quantum gravity
3. Explain the values of fundamental parameters
4. Develop the connection to quantum information theory

Computational Priorities:

1. Implement the framework in Boltzmann codes
2. Perform comprehensive MCMC analysis with all datasets
3. Run N-body simulations with scale-dependent gravity
4. Calculate CMB and matter power spectra precisely

Observational Priorities:

1. Environmental studies of galaxy dynamics
2. Improved measurements of H_0 and σ_8
3. Quantum experiments testing gravitational coupling
4. Searches for time flow anomalies

8.7 A Measured Perspective

We have presented this framework not as established truth but as a concrete proposal meriting investigation. Its virtues—conceptual unity, natural screening, testable predictions—must be weighed against its challenges—fine-tuning, mathematical incompleteness, unvalidated numerics.

The framework exemplifies a phenomenological approach to quantum gravity: rather than deriving everything from first principles, we parametrize ignorance in a systematic way and explore observational consequences. This approach has proven fruitful in particle physics and may offer insights in cosmology.

Whether the specific proposal succeeds or fails, pursuing this program advances several goals:

- It provides a concrete alternative to Λ CDM for testing
- It suggests new experimental directions
- It highlights connections between disparate phenomena
- It challenges us to think differently about fundamental questions

9 Conclusions

We have developed a phenomenological framework where spacetime carries entropic information through two scalar fields, potentially explaining dark matter, dark energy, and time’s arrow through a unified mechanism. The key insights are:

1. **Spacetime can be quantum or classical:** The spatial entropy \mathcal{S}_S measures spacetime’s quantum coherence, with small values in cosmic voids where quantum properties persist and large values near matter where decoherence creates classical behavior.
2. **Time’s arrow is dynamical:** The temporal entropy \mathcal{S}_T increases monotonically due to an explicitly time-asymmetric potential, providing a dynamical rather than statistical arrow of time.
3. **Gravity depends on quantum state:** The function $h(\mathcal{S}_S)$ modifies gravity in quantum regions while recovering Einstein’s theory in classical regions, naturally explaining both cosmological modifications and local screening.
4. **Structure emerges from decoherence:** The universe’s multi-scale structure reflects the historical pattern of quantum-to-classical transitions, frozen by cosmic expansion.

The framework makes specific predictions: environmental dependence of galaxy dynamics, correlations between gravitational and temporal anomalies, modifications to black hole shadows, and novel effects in quantum experiments. These predictions distinguish it from both Λ CDM and other alternatives.

However, significant challenges remain. The framework requires fine-tuned parameters, lacks complete mathematical proof of consistency, and awaits numerical validation. The protection mechanisms preventing \mathcal{S}_S thermalization are conjectures requiring theoretical justification. Most critically, we have not demonstrated that a single parameter set can simultaneously explain all observations.

Despite these challenges, the framework merits serious investigation for several reasons. It addresses multiple cosmological tensions through one mechanism, makes concrete testable predictions, and suggests deep connections between quantum mechanics, gravity, and time. Even if the specific proposal fails, pursuing this program will advance our understanding of how quantum gravity might manifest cosmologically.

The entropic spacetime framework represents a phenomenological approach to quantum gravity—not deriving everything from first principles but systematically parametrizing our ignorance and exploring consequences. This approach has proven valuable in particle physics and may offer similar insights in cosmology.

We emphasize that this work is a beginning, not an end. The ideas require rigorous mathematical development, detailed numerical implementation, and confrontation with

precision data. Only through this effort can we determine whether entropic spacetime captures deep truths about our universe or remains an intriguing possibility that nature chose not to realize.

The universe presents us with profound puzzles: most of its content is invisible, time flows only forward, and quantum mechanics coexists with classical gravity. The entropic spacetime framework suggests these puzzles are connected, arising from the quantum nature of spacetime itself. Whether this suggestion proves correct, the investigation will deepen our understanding of the cosmos and our place within it.