

## MTH 371: Assignment II

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October 23, 2023

### Instructions

- Use statistical software R/Python for your codes.
- Only basic built-in functions available in R/Python are allowed.
- In each question, show the simulations with relevant graphs.
- Due date is October 31, 2023 (11.59 p.m. IST). No late assignments will be accepted.
- Submit all of your work which include the codes, results, graphs and reports.

1. (10 Marks) Let us assume writing a paper has four stages; read (r), write (w), e-mail (e), surf (s). It is studied as a finite state discrete time Markov chain. The one step transition probability is given by

$$P = \begin{matrix} & \begin{matrix} r & w & e & s \end{matrix} \\ \begin{matrix} r \\ w \\ e \\ s \end{matrix} & \begin{pmatrix} 0.5 & 0.3 & 0 & 0.2 \\ 0.2 & 0.5 & 0.1 & 0.2 \\ 0.1 & 0.3 & 0.3 & 0.3 \\ 0 & 0.2 & 0.3 & 0.5 \end{pmatrix} \end{matrix}$$

Answer the following.

- (a) Suppose the time  $t$  is measured in minutes (discrete). Show how the Markov chain will evolve after 20 minutes. Also, find  $P(X_{20} = s | X_0 = r)$ .
- (b) Similarly, how the Markov chain will evolve after 25 minutes. Also, find  $P(X_{25} = s | X_{20} = s)$ .
- (c) Does the stationary distribution exists, If yes, find it.

(d) Does the limiting distribution exists, If yes, find it.

2. (10 Marks) 1-D Random walk

- (a) Assume a 1-D random walk starts from state  $i$ . Simulate it for  $p = 0.5$  and 500 steps. Plot the results. What is the simulated probability.
- (b) Assume a 1-D random walk starts from state  $i$ . Simulate it for  $p = 0.8$  and 500 steps. Plot the results. What is the simulated probability. Compare the results with part (a).
- (c) Assume a 1-D random walk starts from state  $i$ . Simulate it for  $p = 0.8$  and 500 steps. Repeat the process for 1000 times. Plot the results. Compare the results with part (b).