An Analysis of the Milling Process

By M. E. MARTELLOTTI,[1] CINCINNATI, OHIO

This analysis of the milling process shows that the path of a milling-cutter tooth is an arc of a trochoid, the parametric equation for which can be derived from known variables of the cut. As a result, the milling process is susceptible to mathematical treatment. Practically, the advantage of such analytical methods is that such elements as the radius of curvature of the tooth path, the clearance and rake angles of the length of the tooth path, the radial thickness of the chip, and their effects upon the quality of milled surface may be evaluated. The paper demonstrates the advantages of the up-milling process in achieving machined surfaces of high quality. This form of analysis will also be found useful in comparing different milling methods.

Introduction

Milling is a process of removing the excess material from the workpiece in the form of small individual chips. These chips are formed by the intermittent engagement with the workpiece of a plurality of cutting edges or teeth integral with or inserted in a cylindrical body known as the milling cutter. This intermittent engagement is produced by feeding the workpiece into the field dominated by the rotating cutter.

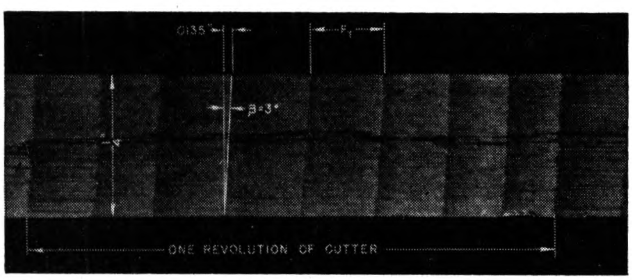

Fig. 1 Tooth Marks on a Milled Surface
(Material, brass; cutter, spiral mill, 8T, 3.89 in. diam, 38 rpm; helix angle, 35 deg; feed rate, 36 in. per min; depth of cut, 1/32 in.; ×3.)

The finished surface, therefore, consists of a series of elemental surfaces generated by the individual cutting edges of the cutter, Fig. 1. The method of milling in which the cutter rotates in a direction opposite to or against the motion of the work is usually called "conventional milling," but is better described as "up milling," since the chips are formed while the teeth of the cutter move in an upward direction, away from the finished surface of the work, Fig. 2.

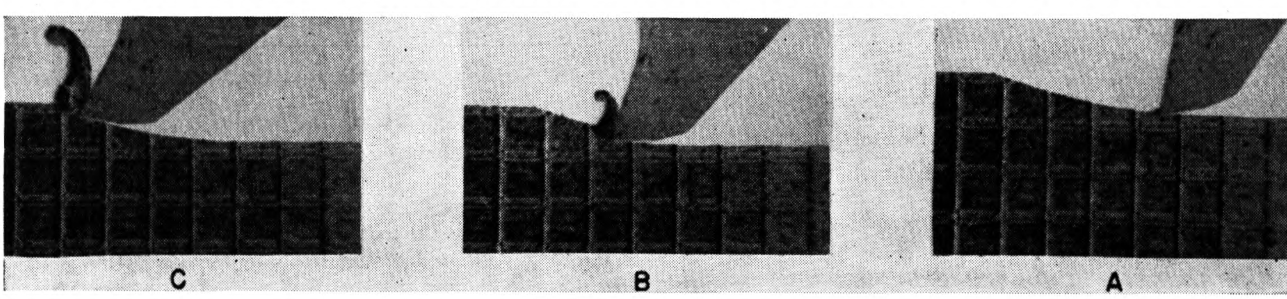

Fig. 2 Milling Chip at Different Stages of Formation
(A, at beginning of tooth path; B, at middle of tooth path; C near end of tooth path.)

Kinematics of Milling

Owing to the limited period of engagement of each tooth, a milling chip is short and of a variable thickness, Fig. 3. This results from the combination of the translatory motion of the work and the rotary motion of the cutter. Hence, the direction of motion of the tool point is continuously changing with respect to the direction of motion of the workpiece, and the path of the tooth resulting therefrom is not circular, but of the type which is properly described as trochoidal.

The name "trochoid" is given to that family of curves which are the locus of a point (taken along a radial line, originating at the center of a circle), generated while the circle rolls on a straight line X. As indicated in Fig. 4, a point P, chosen on this radial line at a distance from the center of the circle which may be either less, equal, or greater than the radius of the circle, will generate the curves A, C, and B, which are respectively called, "prolate trochoid," "cycloid," and "curtate" or "looped trochoid."

In analyzing the milling process, valuable assistance may be realized from a consideration of the kinematic characteristics, defined by the equation of the path of a tooth. In previous investigations of the milling process[2] it has been customary, for the sake of simplicity, to assume that the path generated by the cutter tooth is circular. However, as will appear from the following analysis, it is not necessary to make this assumption, as the various relationships may readily be calculated from the true trochoidal tooth path. This affords a proper understanding of the milling process, and brings to light certain important events, such as the changes in the clearance and rake angles, and radius of curvature of tooth path.

For the purpose of analysis, the system composed of a rotating milling cutter and a translating workpiece may be replaced by an equivalent system in which the work is stationary and the cutter rotating and translating at the same time. In this case, however, the direction of the translatory motion of the cutter or

[1] Research Engineer, The Cincinnati Milling Machine Company. Mem. A.S.M.E.

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[2] "Chip Thickness in Milling," by C. H. Borneman, *American Machinist*, vol. 82, 1938, pp. 189–190.
"How Thick a Chip?" by A. L. DeLeeuw, *American Machinist*, vol. 83, 1939, pp. 991–992.

feed will be opposite to that of the work, which obtains in the ordinary case.

The idealized version of this arrangement, shown in Fig. 5, and applied particularly to the case of slab milling, can be illustrated by means of a mechanical system, of the rack-and-pinion type, in which the pinion Q of the proper dimension is integral with the spindle and cutter and meshes with a rack Z rigidly supported on the stationary base of the machine. Upon rotation of the spindle, the cutter will be fed to the work in a direction which obtains in an ordinary milling machine.

Since the cutter is translated at a rate corresponding to the feed of the work, the pitch radius r of the pinion may be determined by the relation

$$2\pi rn = F \qquad [1]$$

where

F = feed rate, in. per min
n = rpm of cutter and pinion
r = pitch radius of pinion, in.

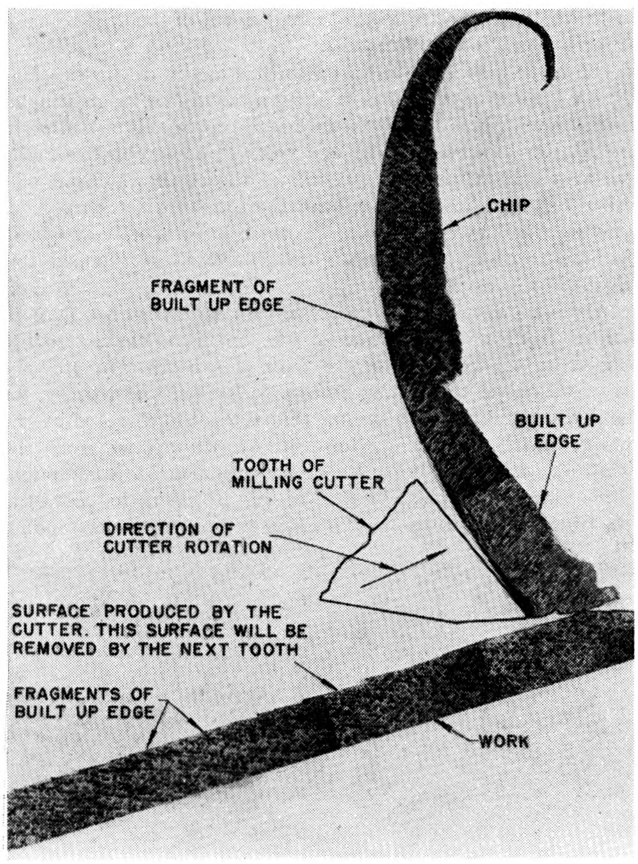

FIG. 3 COMPLETE UP-MILLING CHIP
(Material, S.A.E. 1112; feed rate, $6^1/_4$ in. per min; depth of cut, $^1/_8$ in.; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 40 fpm; $\times$50.)

In practical application, the surface speed of the cutter is always much greater than the feed rate of the work, consequently R, the radius of the cutter, will be much greater than r, the pitch radius of the pinion.

By referring to Figs. 4 and 5, it is evident that, by allowing the pinion Q to roll on the rack Z, the edge of the tooth will generate a looped trochoid. This can be represented by the parametric equations

$$\left.\begin{aligned} X &= r\,\alpha + R\sin\alpha \\ Y &= R\,(1 - \cos\alpha) \end{aligned}\right\} \qquad [2]$$

In Equations [2], α is the angle through which the cutter and pinion have rotated in the direction of the arrow M from the starting point O, which is the origin of the coordinate system XY.

Two consecutive teeth, 2 and 1, Fig. 5, are shown, on an enlarged scale, in their relative position, respectively, at the beginning and near the end of their engagement with the work. Tooth 1 has nearly completed the path A'-N, while tooth 2 is about to contact the work at a point marked A, along the path of the tooth 1.

Tooth and Revolution Marks

As the pinion is rolled on the rack in the direction of the arrow M, tooth 2 moves downward into the work, and not tangentially

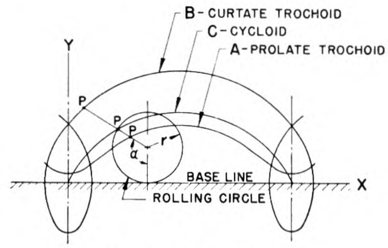

FIG. 4 TROCHOIDS

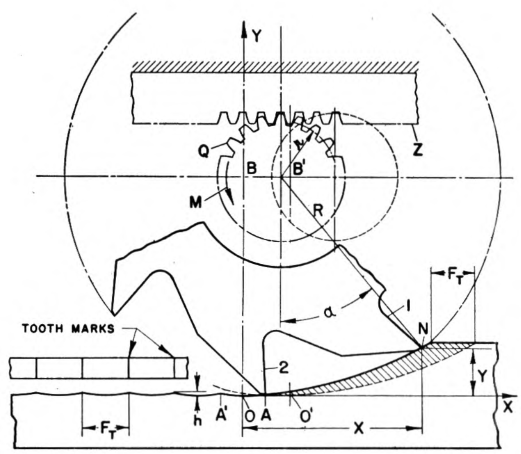

FIG. 5 PATH GENERATED BY A MILLING-CUTTER TOOTH

as is usually believed to be the case, until the point O' is reached. The tooth is then at the greatest perpendicular distance from the path to the center B'. Thereafter, the tooth begins to rise toward the upper surface of the workpiece.

A cusp known as the tooth mark results from the intersection of the path of two consecutive teeth, by the direct action of the edge of the tooth upon the work material. The distance between two adjacent tooth marks is known as the feed per tooth, F_t. This is obviously equal to the distance F_t on the upper surface of the work, Fig. 5.

A milled surface, therefore, is composed of innumerable elements of tooth paths, each delimited by the feed per tooth, as shown in profile and enlarged both in Fig. 5 and in the actual reproduction, Fig. 1.

The uniformity of the tooth-mark spacing depends upon the location of the points $A\ A'$ where the teeth intersect the paths generated by the preceding teeth.

If in the parametric Equations [2] the quantity 2π is added to the angle α, the net result is a translation of a given tooth path, in the direction of the feed, by an amount equal to the feed per revolution; thus, a more general system of parametric equations of the tooth path results

$$\left.\begin{aligned} X &= (2\pi K + \alpha) r + R \sin (2\pi K + \alpha) \\ Y &= R[1 - \cos (2 K\pi + \alpha)] \end{aligned}\right\} \ldots\ldots [3]$$

where K is an integer number.

This indicates that the paths generated by the teeth of a milling cutter are congruous, and that they can be obtained from each other by a displacement equal to the feed per tooth, or a multiple thereof.

By eliminating the parameter α between the parametric Equations [2], the Cartesian form of the looped trochoid of a milling-tooth path will be obtained

$$X = r \cos^{-1} \frac{R - d}{R} \pm [(2R - d) d]^{1/2} \ldots\ldots\ldots [4]$$

where d indicates the instantaneous depth in place of Y.

The equations which have been derived, whether of the parametric or of the more involved Cartesian form, furnish means for determining the shape of the trochoid curve generated by any tooth of a milling cutter, as a function of a few variables of the cut, which are generally known. From the Cartesian equation, for instance, the value of X for any point, taken with respect to the origin O, can be obtained when the radius of the rolling circle r, the radius of the cutter R, and the depth d at different positions of the tooth path are known. Thus the tooth path may be mathematically reconstructed.

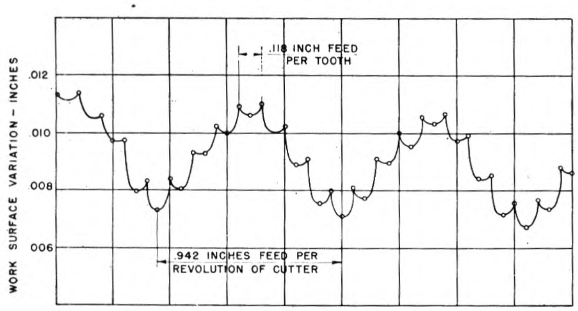

Fig. 6 Variations Produced by Tooth and Revolution Marks on a Milled Surface

(Material, brass; cutter, spiral mill, 8T, 3.89 in. diam, 38 rpm; helix angle, 35 deg; feed rate, 36 in. per min; depth of cut, 1/32 in.)

Since the tooth path, in approaching and leaving the point O, is symmetrical with respect to that point, the points of intersection (such as A and A') will be located at one half the feed per tooth from either side of point O. This is exactly true, however, only under the conditions seldom obtained in practice, viz., that the teeth have the same distance from the center of the cutter, and that the feed per tooth is constant and the teeth are equally spaced.

As can be seen from Fig. 1, any variation in the values of these quantities will also affect the height h, but this effect is usually small, and thus may be disregarded. The magnitude of h can be calculated from the following equation, which has been derived from the parametric Equation [2]

$$h = \frac{F_t^2}{8\left[R + \dfrac{F_t \times T}{\pi}\right]} \ldots\ldots\ldots\ldots\ldots [5]$$

where

h = height of tooth mark above point of lowest level, in.
F_t = feed per tooth, in.
R = radius of cutter, in.
T = number of teeth in cutter

The value of h, calculated with Equation [5], and for 0.118 in. feed per tooth, 8 teeth, 3.894 in. diam of cutter, and using a spiral mill, was 0.000766 in.

The average value of h obtained from the measurement of 52 tooth marks produced in an actual cut on brass, under the foregoing conditions, was 0.000710 in., thus showing a satisfactory agreement between the calculated and observed values. Fig. 1 shows this surface, which was purposely milled at an unusually high feed rate in order to show the tooth marks clearly. The actual variations in the surface are shown in Fig. 6.

In addition to the tooth marks, which correspond to the tooth frequency, a slab-milled surface shows periodic variations having a wavy appearance, the frequency of the waves being equal to the frequency of rotation of the cutter. The amplitude of the wave (or height of the revolution mark) is a function of the eccentricity of the cutter and arbor, the so-called "high tooth" on the cutter, and the periodic variation in the deflection of the arbor, caused by the presence of the keyway and possible uneven conditions on the arbor supports.

The height of a revolution mark, in the case shown in Fig. 6, is 0.004 in. It may be observed also that the tooth marks follow the undulation of the revolution marks. The presence of one or the other or both types of marks alters the physical condition of the surface and consequently the quality of its finish.

While the height of a tooth mark can be reduced by increasing the radius of the cutter, and by decreasing the feed per tooth until the tooth marks become scarcely distinguishable, particularly at the lower feed rates, the revolution marks, on the other hand, cannot be prevented unless special care is taken in grinding the teeth of the cutter within close limits and making sure that the runout of the cutter and arbor is reduced to the lowest possible value.

Radius of Curvature of Tooth Path

The difference in the height of a tooth mark, produced under various operating conditions, is due to the instantaneous radii of curvature of the path of the tooth. This is a function of the radius of the cutter, its speed, the depth of cut, and the feed rate, factors which enter into a milling operation.

In conventional milling (up milling), the radius of curvature at any point in the path of the tooth is expressed as

$$\rho = \frac{\left[R^2 + \left(\dfrac{F_t T}{2\pi}\right)^2 + \dfrac{F_t T}{\pi}(R - d)\right]^{3/2}}{\dfrac{F_t T}{2\pi}(R - d) + R^2} \ldots\ldots\ldots [6]$$

The variables are

d = depth of cut, in. (at point under consideration)
R = radius of cutter, in.
F_t = feed per tooth, in.
T = number of teeth in cutter
ρ = the instantaneous radius of curvature in inches

The radius of curvature obtained from Equation [6] is the instantaneous radius of a small portion of the path of the tooth in the neighborhood of a point determined by a given combination of the variables mentioned.

By assuming certain values for three of these variables, the relationship has been determined between the radius of curvature and the depth of cut, the revolutions per minute of the cutter, and the feed rate, Figs. 7, 8, and 9.

From an inspection of these results, the following conclusions may be drawn:

1 A large radius of curvature is obtained at shallow depth of cut. This condition obtains particularly within the length of a tooth path delimited by the feed per tooth. Beyond this

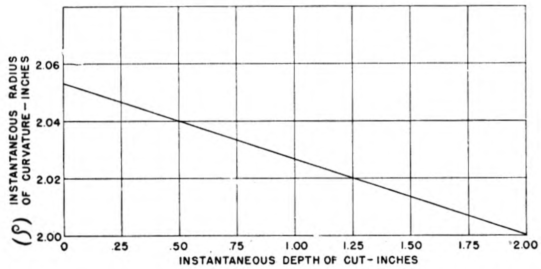

FIG. 7 RADIUS OF CURVATURE OF TOOTH PATH VERSUS DEPTH OF CUT
(Radius of cutter, 2 in.; feed rate, 10 in. per min; cutter speed, 6 fpm.)

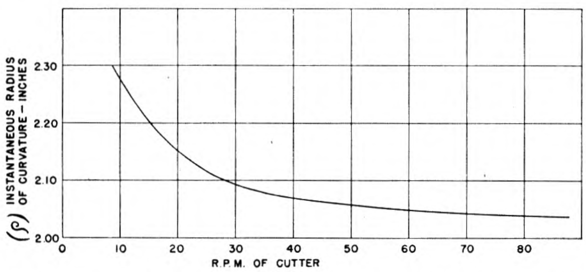

FIG. 8 RADIUS OF CURVATURE OF TOOTH PATH VERSUS REVOLUTIONS PER MINUTE OF CUTTER
(Radius of cutter, 2 in.; depth of cut, $^1/_4$ in.; feed rate, 10 in. per min.)

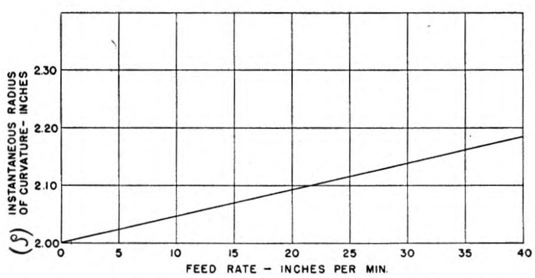

FIG. 9 RADIUS OF CURVATURE OF TOOTH PATH VERSUS FEED RATE
(Radius of cutter, 2 in.; depth of cut, $^1/_4$ in.; cutter speed, 60 fpm.)

elemental portion of the tooth path and, as the tooth travels toward the upper surface of the workpiece, the radius of curvature decreases, reaching a minimum value equal to the radius of the cutter when the depth of cut is likewise equal to the radius of the cutter, Fig. 7.

2 For any given cutting condition, as we increase the cutting speed and, consequently, the revolutions per minute of the cutter, the radius of curvature is correspondingly decreased. For very high values of the revolutions, it tends to approach, as a limiting value, that of the radius of the cutter, Fig. 8.

3 When the feed rate is increased, the radius of curvature increases proportionately with it. The relationship between radius of curvature and feed is shown in Fig. 9.

If we consider the variation of the radius of the curvature, in relation to the variables of the cut, and on the basis of experimental evidence, we can conclude that it is always desirable to have a set of conditions such as will give, at any time, the largest possible radius of curvature. It is well known that more efficient metal removal is usually obtained for relatively large values of the feed per tooth, Figs. 10 and 11.

Since the feed per tooth is a function of the feed rate, the number of teeth and the revolutions per minute of the cutter, as expressed in the equation

$$F_t = \frac{F}{T \times n} \qquad [7]$$

where

F = feed, in. per min
T = number of teeth in cutter
n = rpm of cutter

it is apparent therefore that a large value of the feed per tooth will be obtained for any given number of teeth and feed rate, by choosing a low value for the revolutions per minute of the cutter. This will automatically give us a large radius of curvature.

An illustration of the effect of the cutting speed of a milling cutter on the actual power required to remove a given volume of metal (in this particular case, machinery steel) is given in Fig. 12. From this illustration, it will be seen that, for a constant feed rate, the minimum power required to remove the same amount of metal is always obtained at the lowest cutting speed. The power increases as the cutting speed is increased.

If the same volume of metal is removed under two different conditions, by adjusting the feed rate and the depth so as to give the same metal removal, it is found that the most efficient removal of metal is obtained at a shallow depth and the highest feed rate, as shown in Fig. 13. As an example, if the feed rate in one case is 10 in. per min and the depth of cut $^1/_4$ in., and in the second case the feed rate is 20 in. per min and the depth of cut $^1/_8$ in., the power required at the cutter is 14 and $12^1/_2$ hp, respectively.

It is also evident from the factors entering into the expression for the radius of curvature that, by increasing the radius of the cutter, it is possible to make the arcs of the trochoid yet flatter and thereby reduce the height between the cusps and the trough of each elemental surface, thus improving the quality of the finished surface.

From the foregoing considerations, it is apparent that the radius of curvature can be used to indicate, both qualitatively

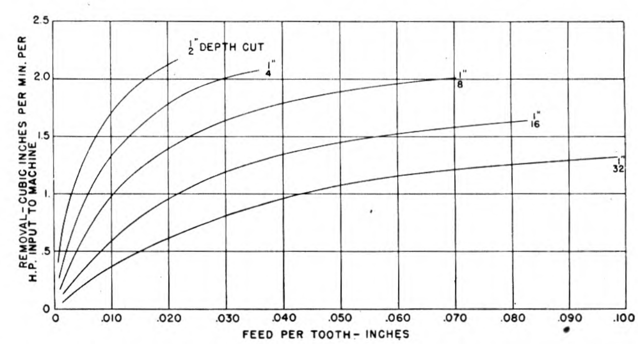

FIG. 10 METAL REMOVAL, CUBIC INCHES PER MINUTE PER HORSEPOWER INPUT TO MACHINE VERSUS FEED PER TOOTH
(Material, cast iron; cutter, spiral mill, 8T, 4 in. diam; rake angle, 10 deg; clearance angle, 3 deg; helix angle, 25 deg; width of cut, 4 in.; cutting speed, 66 fpm.)

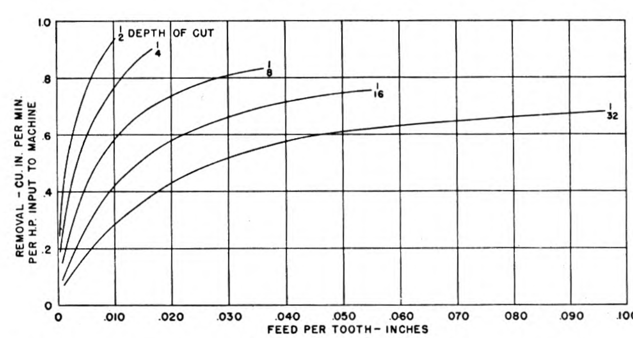

FIG. 11 METAL REMOVAL, CUBIC INCHES PER MINUTE PER HORSEPOWER INPUT TO MACHINE VERSUS FEED PER TOOTH
(Material, S.A.E. 1112 steel; cutter, spiral mill, 8T, 4 in. diam; rake angle, 10 deg; clearance angle, 3 deg; helix angle, 25 deg; width of cut, 4 in.; cutting speed, 66 fpm; cutting fluid, soluble oil and water.)

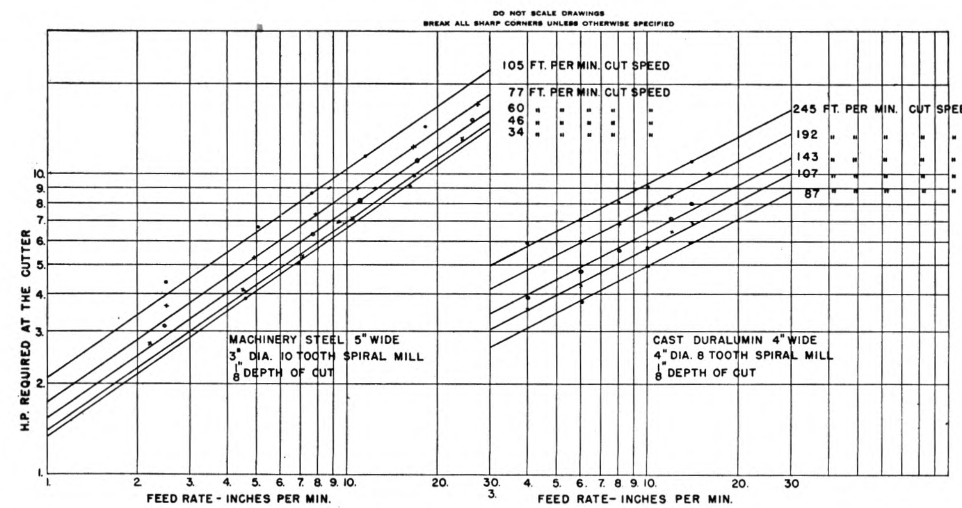

Fig. 12 Horsepower at Cutter When Milling Steel and Duralumin at Different Cutting Speeds Versus Feed Rate

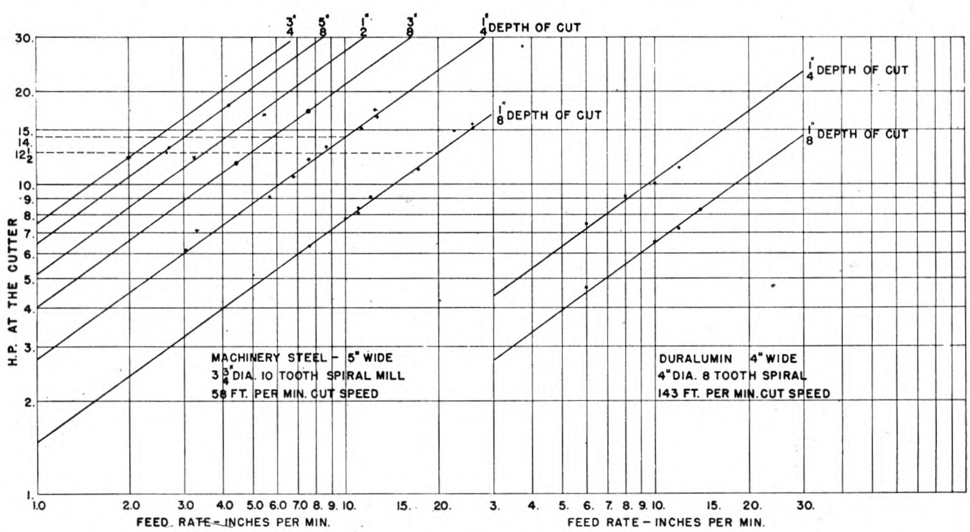

Fig. 13 Horsepower at Cutter When Milling Steel and Duralumin at Different Depths of Cut Versus Feed Rate

and quantitatively, the effects produced on the path of the tooth by the changes in the values of one or more of the variables of the cut. Consequently, these variables can be adjusted to obtain any desired results.

In addition to the effects mentioned, which are inherent in the methods of generating a milled surface, there are other effects which are induced by the flowing metal as the excess material is removed in the form of chips.

Quality of Finish in Milled Surfaces

In all metal-cutting processes, it is found that, at the very beginning of the contact of the cutting edge of the tool with the work material, and for a short distance thereafter, the machined surface is shiny in appearance and of uniformly good finish, while later on, the surface quality changes to that of a dull, coarse finish. This transition is illustrated in Fig. 14.

The length of the shiny surface in the direction of the cut seems to depend upon the kind of material, the depth of cut, the clearance and rake angles, and particularly on the presence, strength, and quality of a film in the nature of an oxide, or an oil film of molecular dimensions, existing on the cutting edge of the tool and its adjacent surfaces at the time of its first contact with the work.

When this film is eventually removed by the combined effect

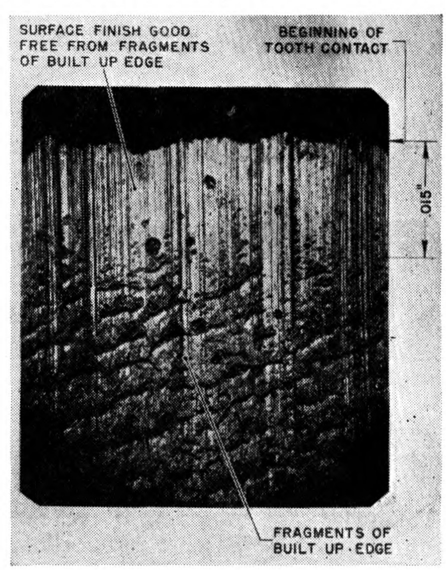

Fig. 14 Photomicrograph of Machined Surface at Beginning of Cut, Showing Gradual Change in Nature of Surface (Material, S.A.E. 1112 steel; ×50.)

of temperature and pressure developed at the root of the chip, and by the wiping action resulting from the motion of the chip, the surface of the tool becomes chemically very active; and therefore the material at the under surface of the chip, which is in a nascent (or freshly formed) condition, will bond readily to the tool.

When this has occurred, the resistance to the motion of the plastic metal of the chip in contact therewith will be increased. This will cause the metal in the vicinity of the cutting edge to be retarded in its motion, and thus the crystals of the chip material become elongated in a direction roughly parallel to the face of the tool.

As the motion of the chip continues, a portion of this material stressed beyond the elastic limit will finally rupture from the main body of the chip and cling to the tool. As the tool continues to advance, the highly stressed material piles up and continues to grow both along the face and the region below the flank of the tool, thus forming the so-called built-up edge. Eventually an unstable condition is reached when a portion of material located on the outer boundary of the built-up edge, no longer sufficiently supported by the tool and under the action of the forces produced in the formation of the chip, will shear off periodically from the underbody and escape as small irregular fragments, both with the chip and the work.

In a metal-cutting process where a tool remains engaged with the chip from the beginning to the end of the cut, and where the path of the tool covers the full length of the final surface, the presence of the "built-up or pseudo edge" is particularly detrimental to the formation of a good surface finish.

The use of a cutting fluid is known to reduce the size of the built-up edge and, in special cases, to prevent its formation.[3] Under normal conditions, however, the built-up edge will never

[3] Symposium, "Machining of Metals," American Society for Metals, 1938.

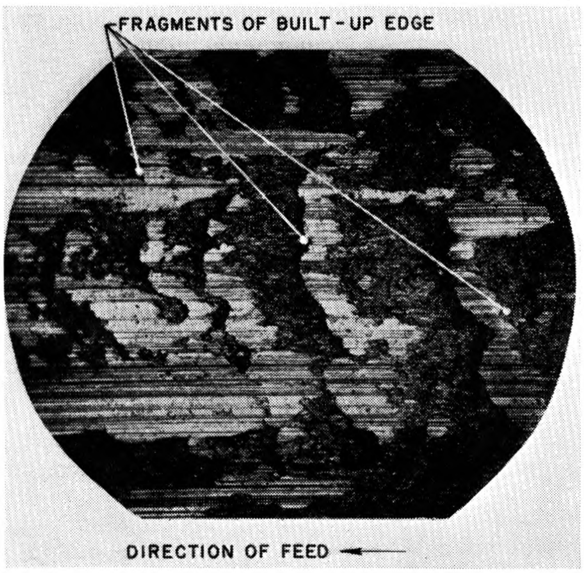

FIG. 15 PHOTOMICROGRAPH OF MACHINED SURFACE SHOWING FRAGMENTS OF BUILT-UP EDGE
(Material, S.A.E. 1112 steel; ×50.)

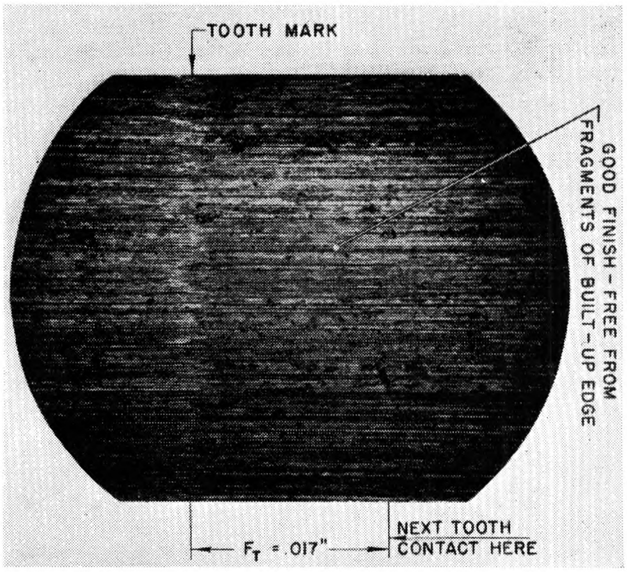

FIG. 17 PHOTOMICROGRAPH OF MILLED SURFACE SHOWING TOOTH MARK AND POSITION OF NEXT TOOTH CONTACT
(Material, S.A.E. 1112; feed rate, 6 1/4 in. per min; depth of cut, 1/8 in.; utter, spiral mill, 10T, 4 in. diam; cutting speed, 40 fpm; cutting fluid, soluble oil and water; ×50.)

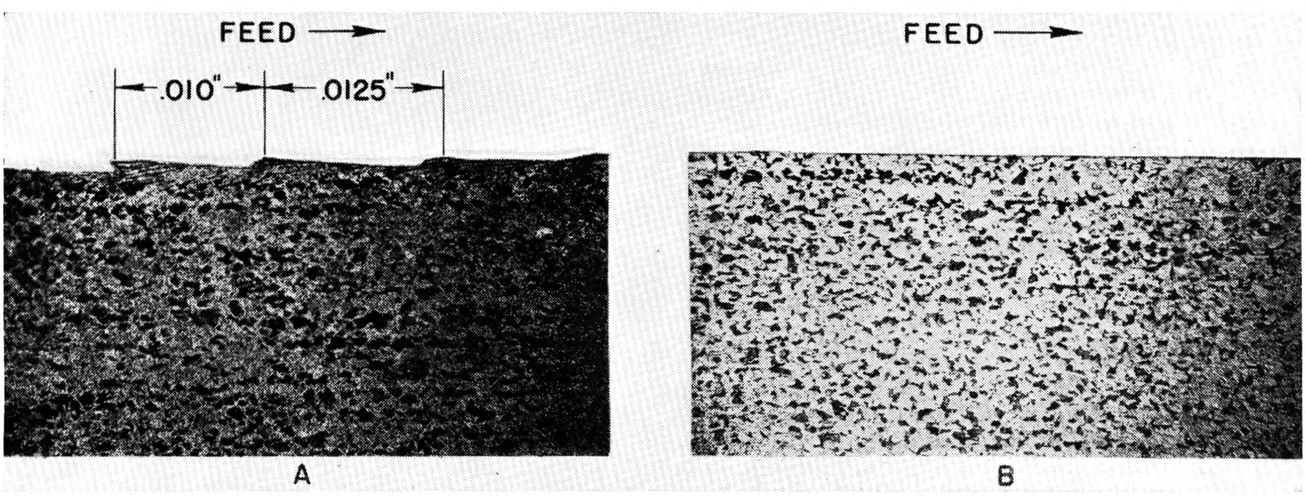

FIG. 16 PHOTOMICROGRAPH OF MACHINED SURFACE CROSS SECTION IN DIRECTION OF FEED
(A, single-point tool; profile of surface shows fragments of built-up edge; B, up milling; profile of surface smooth and free from fragments of built-up edge; ×75.)

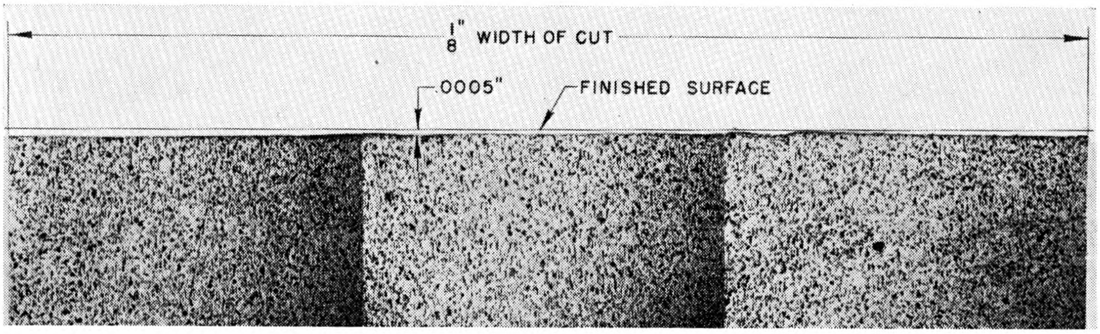

Fig. 18 Photomicrograph of Profile of Milled Surface Normal to Feed
(Produced under same conditions as in Fig. 17; ×50.)

completely disappear, and its fragments will be found on every machined surface in the form of small irregular portions of work material in a highly worked state and aligned in a general direction roughly normal to the direction of the tool path, Fig. 15. A cross section of this surface in the direction of the tool path is shown in Fig. 16 (*A*). Here can be seen the pronounced change in the quality of the finished surface produced by the fragments of the built-up edge.

In the case illustrated in Fig. 14, the element of smooth surface produced at the beginning of the cut extends approximately $^1/_{64}$ in. in the direction of the motion of the tool, but in up milling, this surface extends for a relatively great distance beyond the point where the cut began, Fig. 17.

A milled surface is composed of innumerable elemental surfaces which are generated in the early stages of a tooth path, when the thickness of the chip is extremely small and the cutting edges of the teeth are still covered with the cutting fluid used, and, in addition, with an oxide film formed during the idling period prior to the engagement with the work.

Therefore, as the cutting process continues, successive chips will be formed under nearly identical conditions, and it may be expected to find this quality of surface finish as indicated in Figs. 17 and 16 (*B*), to extend to all elements of the final milled surface. A profile of this surface taken at right angles to the feed within the region of a tooth mark is shown in Fig. 18.

The photomicrographs, Fig. 19, of the actual finish obtained, when milling various kinds of work material with a spiral mill, prove that these deductions are in good agreement with actual practice.

Blemishes on the finished surface are caused, in some cases, by fragments of chips adhering to the teeth and being caught between them and the work in subsequent engagements. This happened in the case of aluminum, Fig. 19 (*A*, *B*), and of stainless steel (Rezistal), Fig. 19 (*C*), all of which were cut dry. When stainless steel was milled with soluble oil as the cutting fluid, a definite improvement in the quality of finish was obtained, as shown in Fig. 19 (*D*). The specimens of copper, brass, and duralumin, Fig. 19 (*E*, *F*, and *G*), show a remarkably good surface finish, particularly copper. The steels including S.A.E. 3115 milled dry, Fig. 19 (*H*), with soluble oil and water, Fig. 19 (*I*), and tool steel, Fig. 19 (*J*), also with soluble oil and water, show indications of small fragments of the built-up edge, due to an early bonding of the material of the chip with that of the tool, thus causing the formation of a small built-up edge almost at the beginning of the cut.

The finish obtained on a cast-iron specimen is shown in Fig. 19 (*K*). The irregularities on this surface are due to successive ruptures caused by the brittleness of the material.

Additional proof that the character of the elemental surfaces is maintained for a certain distance beyond the point *A*, Fig. 5, where the tooth first contacts the work, is shown in the cross section of the chip and the adjacent surface of the work at the beginning of the up-milling cut, performed with a spiral mill on a specimen of S.A.E. 1112 steel, Fig. 20.

The actual point of tooth contact with the work can be determined approximately from a direct measurement of the chip shown in Fig. 20. Here we find that this chip is approximately 0.0008 in. thick and 0.0125 in. long.

Now it has been found that the actual length L_c of a chip bears a definite relation to the length L_s of the resulting surface and the conditions under which it was formed. This may be called the "cutting ratio."

Various values of this ratio obtained on specimens of S.A.E. 1112 steel with a single-point tool, and with various cutting fluids, are listed in Table 1, and the magnified views of the corresponding finished surfaces in Fig. 21.

Comparing the finish of the milled surface, Fig. 17, obtaining at the beginning of the cut, with that of the surfaces shown in Fig. 21, it is found that the finish of the former surface falls between that of the surfaces (2) and (3) of Fig. 21. Hence, the cutting ratio L_c/L_s can be assumed equal to the mean values of the ratios obtaining in these surfaces, or the value of 0.382.

Then, the beginning of the chip, shown in Fig. 20, would be at a distance $L_s = \dfrac{L_c}{0.382} = \dfrac{0.0125}{0.382} = 0.0325$ in. back of its present location, or in the neighborhood of the point marked *A*.

Since the feed per tooth, F_t, used in machining this specimen was 0.0170 in., then the point of intersection of the following tooth will be approximately at *B*, that is, 0.0170 in. from the point where the previous tooth entered. Thus, the element of the finished surface produced by the first tooth is the distance *A*-*B*, Fig. 20, which is well within the region free from fragments of built-up edge, as shown in Fig. 17.

The gradual deterioration of the quality of the milled surface along the path of the milling-cutter tooth is shown in the photomicrograph, Fig. 22 (*A*, *B*, *C*, *D*). At the beginning of the cut, Fig. 22 (*A*), the quality of the finish produced is similar to that obtained in Fig. 14. In Fig. 22 (*B*), taken at a later stage of the tooth passage along its path, the surface shows small fragments of built-up edge; in Fig. 22 (*C*) the surface is rough and shows the extent of nonuniformity produced by large fragments of the built-up edge. This character is maintained in Fig. 22 (*D*), which illustrates the surface at the end of the tooth path.

The cross section of chip and work taken along the ascending portion of the tooth path in up milling is illustrated in Fig. 23. In this case the numerous and rather regularly spaced fragments of the built-up edge are left behind by the advancing tooth at the rate of approximately 2000 per sec. This gives an indication of the rate at which the built-up edge forms and breaks down. This will continue until the chip is finally severed from the work

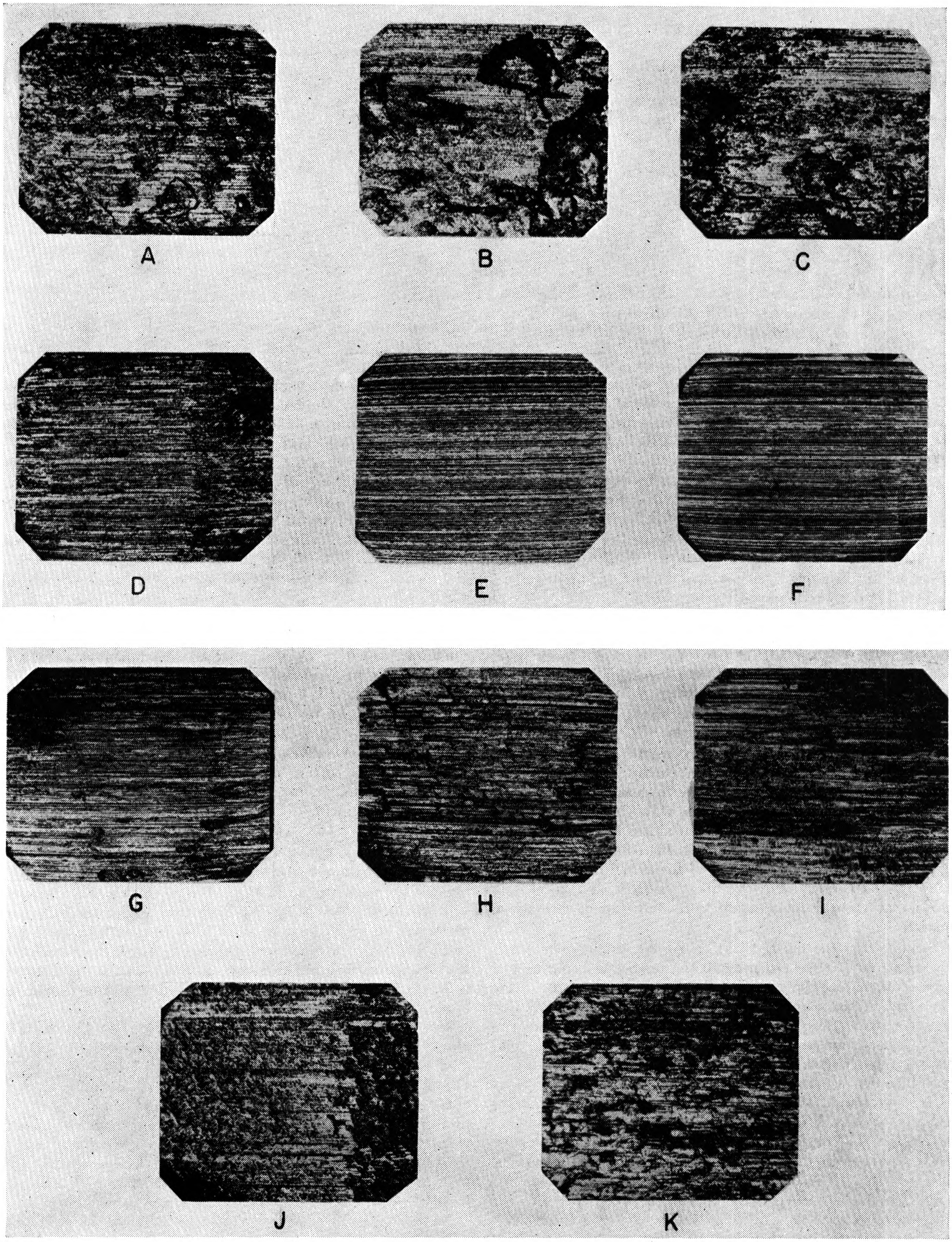

Fig. 19 Quality of Surface Obtained When Milling Various Kinds of Work Material

(*A*, aluminum, cut dry; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 24 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 675 fpm. *B*, aluminum, cut dry; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 6 1/4 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 230 fpm. *C*, stainless steel, Rezistal, cut dry; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 3 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 41 fpm. *D*, stainless steel, Rezistal, soluble oil and water; same as under *C*. *E*, copper, cut dry; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 6 1/4 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 230 fpm. *F*, brass, cut dry; conditions same as in *E*. *G*, duralumin, cut dry; conditions same as in *E*. *H*, S.A.E. 3115 steel, cut dry; depth of cut 1/10 in., width of cut 1/4 in.; feed rate, 3 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 40 fpm. *I*, S.A.E. 3115 steel, soluble oil and water; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 3 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 63 fpm. *J*, tool steel, soluble oil and water; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 3 1/8 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 51 fpm. *K*, cast iron, cut dry; depth of cut 1/16 in., width of cut 1/4 in.; feed rate, 6 1/4 in. per min; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 63 fpm; ×50.)

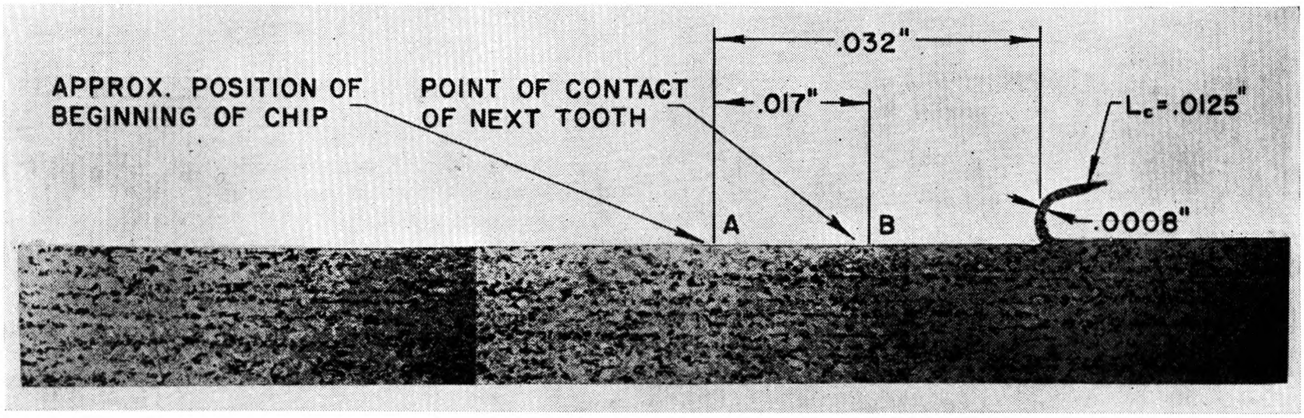

Fig. 20 Photomicrograph of Work and Chip at Beginning of Cut
(S.A.E. 1112 steel; ×75. Other conditions same as in Fig. 17.)

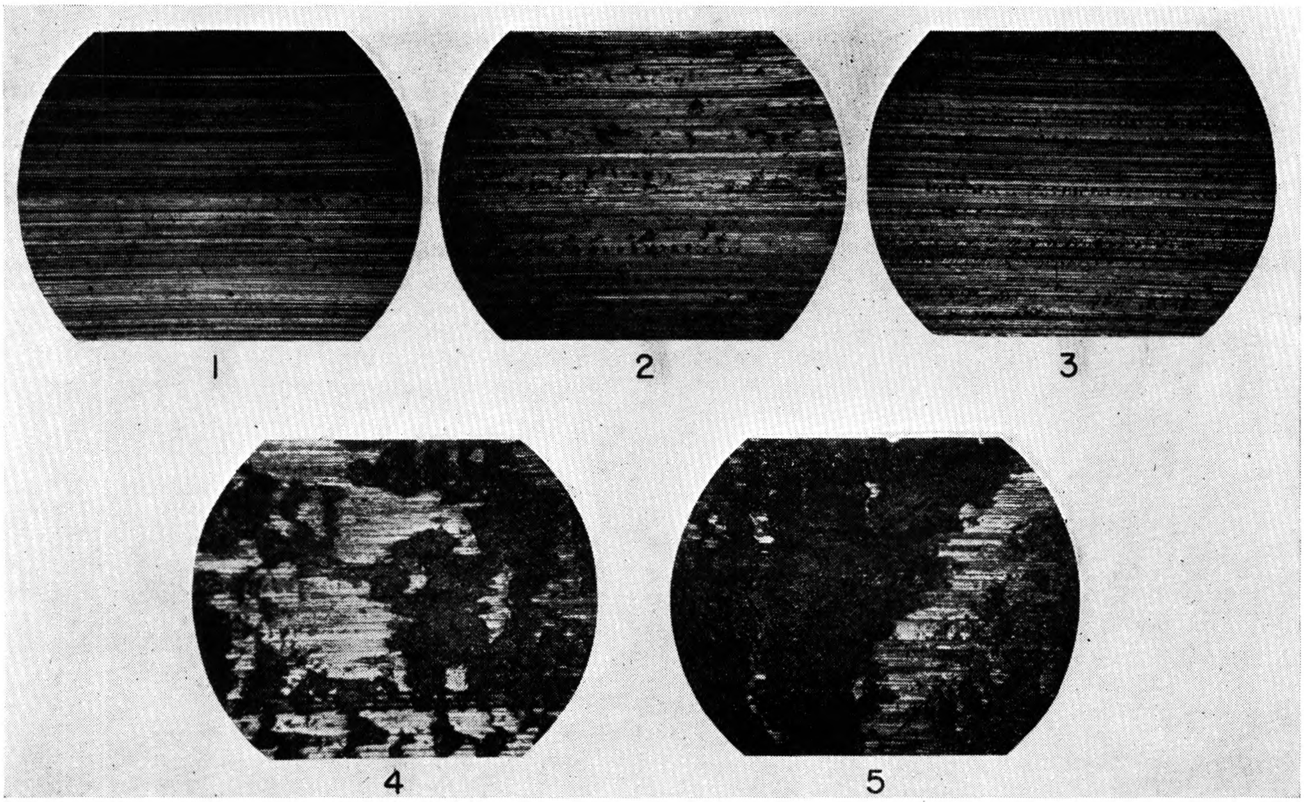

Fig. 21 Quality of Surface Obtained With Single-Point Tool and Various Cutting Fluids
(Material, S.A.E. 1112; depth of cut, 0.005 in.; speed, $5^1/_2$ in. per min; ×75.)

TABLE 1 CUTTING LENGTH RATIO L_c/L_s
(Material, S.A.E. 1112; depth of cut, 0.005 in.; speed, $5^1/_2$ in. per min)

Surface no.	Cutting fluid	L_c/L_s	Magnification
1	Carbon tetrachloride	0.493	×75
2	Acetic acid	0.417	×75
3	Methyl alcohol	0.357	×75
4	Turpentine	0.250	×75
5	Benzene	0.213	×75

when the remnant of the built-up edge will pass off with it, Fig. 3.

In up milling, the fragments of the built-up edge appear always along that portion of the tooth path which will be removed by subsequent teeth. Therefore, it is quite evident that a machined surface, having a desirable quality of finish, can be produced by duplicating indefinitely the conditions obtaining at the very beginning of the cut; and, of all known metal-cutting processes, up milling is perhaps the only one which, by virtue of its characteristics, offers such a possibility.

The statement has often been made that the incipient motion of a milling-cutter tooth into the work is accomplished by a momentary sliding for a certain distance, depending upon the hardness of the metal being cut and on the rigidity of the cutter and work support.

From Fig. 24, however, it will be seen that a tooth contacts the work at a point E on the ascending portion of the path generated by the previous tooth with an angle of approach ϕ.

This angle is included between the tangent to the curve at E and the horizontal tangent to the points such as D' and C'.

The value of this angle can be determined from the equation

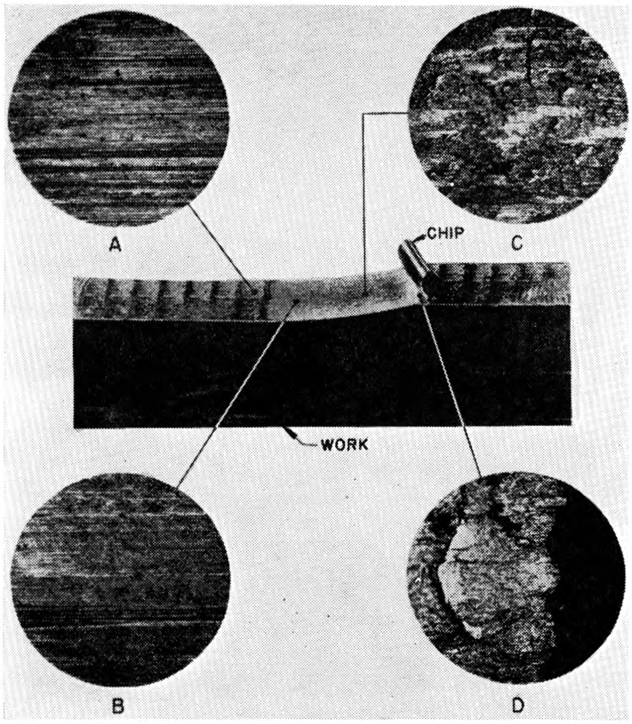

FIG. 22 CHANGE IN SURFACE QUALITY ALONG TOOTH PATH
(Material and conditions same as in Fig. 17. Photomicrographs ×50; workpiece ×4.)

$$\phi = \tan^{-1}\left[\frac{2\,\pi\,(2\,R\,d - d^2)^{1/2}}{2\,\pi\,(R - d) + F_t\,T}\right] \qquad [8]$$

where

ϕ = angle of tooth approach to work, radians
R = radius of cutter, in.
d = depth of cut (instantaneous), in.
F_t = feed per tooth, in.
T = number of teeth in cutter

The depth of cut h, obtaining at this point, is equal to the depth of the tooth mark. It will be seen that this angle at this instant

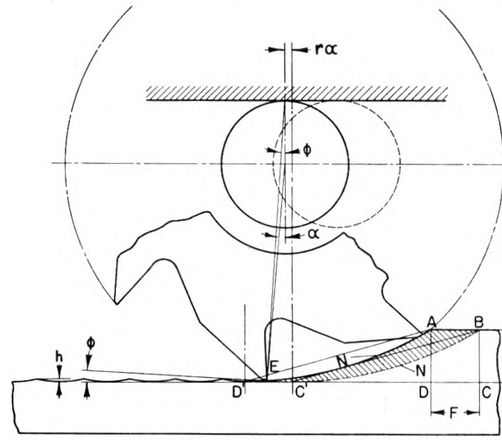

FIG. 24 ANGLE OF APPROACH OF A MILLING-CUTTER TOOTH

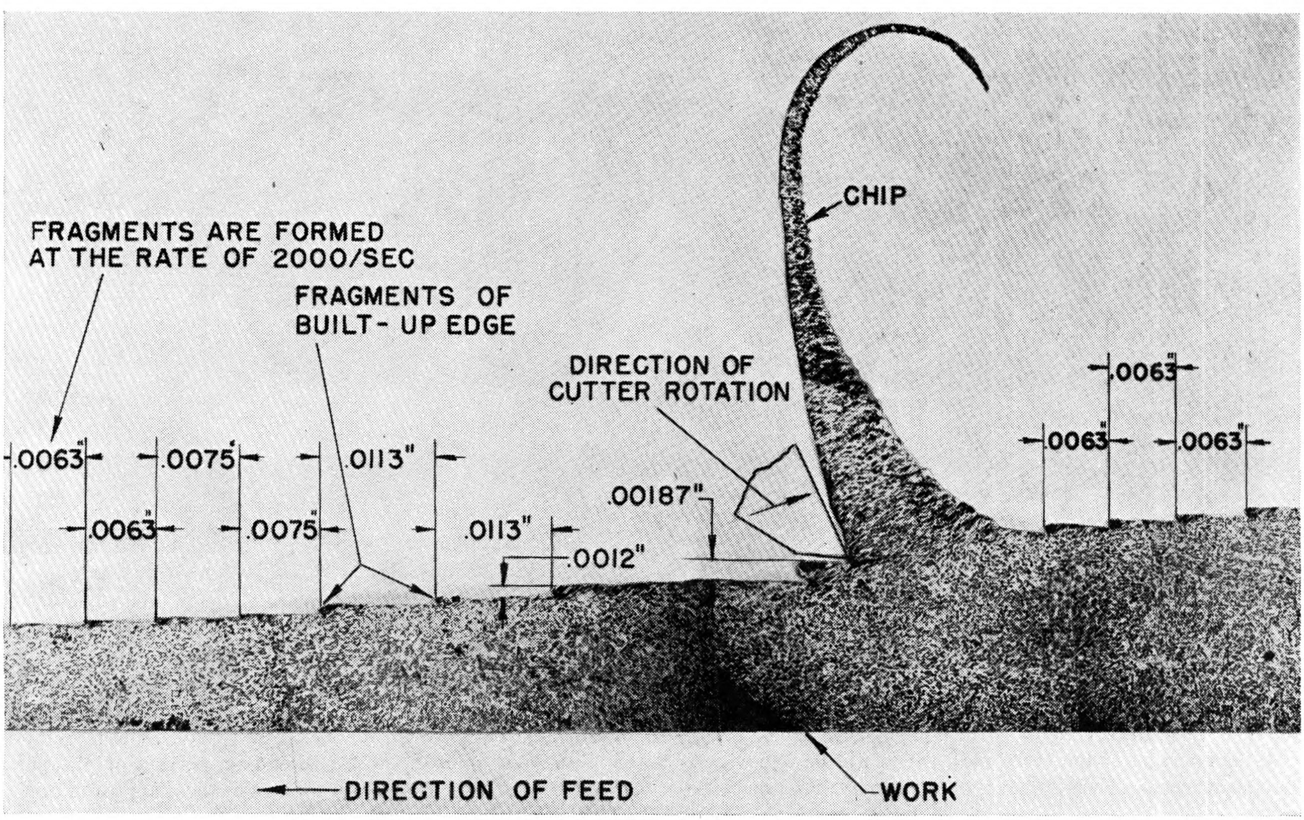

FIG. 23 PHOTOMICROGRAPH OF A MILLING CHIP IN PROCESS OF FORMATION
(Material and conditions same as in Fig. 17; ×50.)

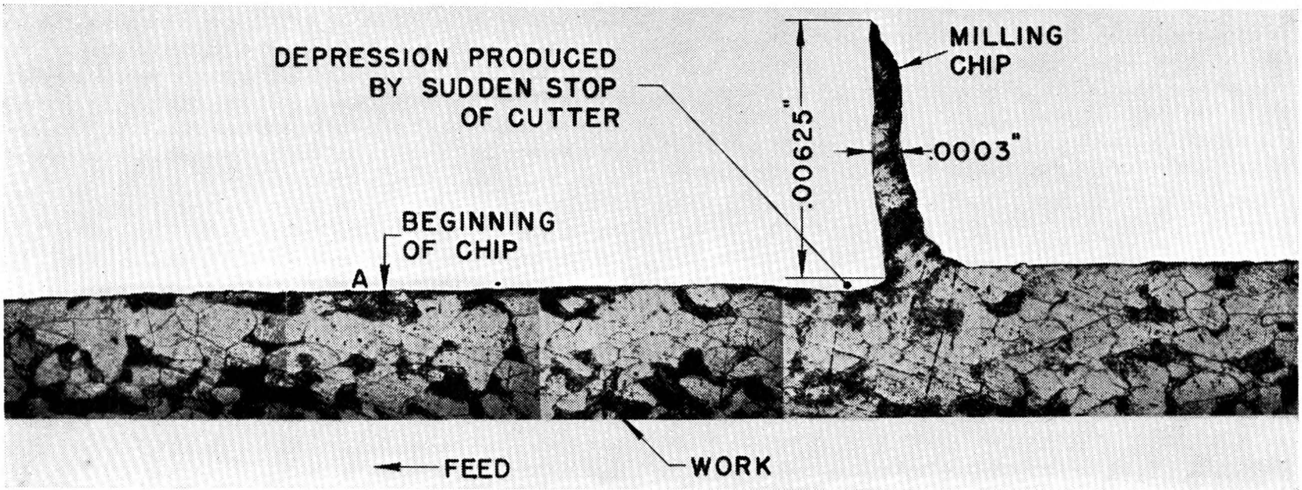

FIG. 25 BEGINNING OF A MILLING CHIP
(Material and conditions same as in Fig. 17; $\times 600$.)

is small but not zero, as is usually believed to be the case. Therefore, a small angular displacement α of the tooth will result with respect to the position C', where the tooth is moving in the direction of the tangent, D'-C'. Incidentally, Equation [8] may be used to determine the angle of approach obtaining at any other point of the tooth path.

For this reason, the tooth approaches and contacts the work while moving in a downward direction, until the point C' is reached, where the upward movement of the tooth begins.

Furthermore, at the beginning of the cut, the thickness of the chip is extremely small, and the length of the cutting edge engaging the work at this instant is usually short for the following reasons:

1 Because the majority of cutters used in milling have either helical teeth with the full length of the cutting edge at a constant distance from the axis of rotation (spiral mills), or straight teeth parallel thereto, but with their cutting edge following a line, the points of which are at variable distances from the center of rotation of the cutter (formed cutters). In either case, there results a gradual engagement of the tooth.

2 In the cutters where the teeth and their respective cutting edges are parallel to the axis of rotation, the length of the cutting edge is inherently limited by their particular use (slotting cutters and saws).

3 Even in those cases (now rare) in which wide straight-tooth milling cutters are used, a full length of contact of the cutting edge at the beginning of the cut can be assured only by grinding the teeth exactly to the same dimensions. Obviously this is a practical impossibility. Therefore, however small the variations in the distance and alignment of each cutting edge relative to the axis of rotation and the fact that each tooth always engages the surface generated by the previous tooth, a limited actual length of engagement with the work at the beginning of the chip will usually result.

4 Since the teeth of a milling cutter are ground to a sharp edge, the area of contact will be exceedingly small. Hence, for materials the hardness of which is within the machinable range, the force applied thereto for producing the intensity of stress needed to cause the material to yield in a plastic manner will also be very small, and penetration of the tooth into the work will readily ensue upon contact.

In view of the foregoing reasons, it is unlikely that sliding of a tooth on the work will ever occur in practice.

It might be argued that, under actual operating conditions, the point of engagement of a tooth is usually very close to the point C', Fig. 24, of maximum distance from the center of the cutter and, therefore, a momentary sliding may be expected.

In the photomicrograph, Fig. 25, in which is shown the cross section of a chip and the adjacent portion of the work at a magnification of 600, there is no evidence whatever of sliding. The minimum thickness of this chip is 0.0003 in. and its length 0.00625 in. This proves, incidentally, that a tooth of a milling cutter can pick up a very thin chip. The beginning of the chip would be placed in the neighborhood of point A for a cutting ratio of 0.5. From an inspection of this illustration, it appears evident that the crystals, of combined carbon and ferrite adjacent to the freshly milled surface, show only very slight indication of local distortion by the action of the cutting edge of the tooth.

In the region A, where the chip began to form, we fail to see a discontinuity in the surface, which may be taken to indicate either sliding or rubbing of the tooth on the work, or sudden digging in, coincident with the picking up of the chip. The chip itself shows, in remarkably clear detail, the crystals composing it. These crystals have maintained their identity intact during the process of being severed from the main body of the work.

The clean-cut outline of the finished surface, and the side of the chip which contacted the face of the tool, suggest that a rather small work of plastic deformation was involved during the formation of this portion of the chip.

Thus, it appears certain that in up milling the cutting edge of a tooth will not slide on the work, but will actually cut as soon as contact between the work and tooth is established.

CLEARANCE AND RAKE ANGLE

In analyzing the trochoidal tooth path generated in milling, it was not necessary to consider the shape of the tooth, since in this case it was sufficient to assume a point on the edge thereof at the maximum distance from the geometric center of the cutter.

In actual milling, however, a tooth has definite dimensions, and its geometric shape must conform with definite requirements, among which are the following:

(*a*) The contact between cutter and work must be established on the cutting edge of the teeth, and at no time should interference develop between the body of the tooth and work.

(*b*) On account of requirement (*a*), the flank of a tooth should be located on a plane deviating from the tangent to the periphery of the cutter, by an angle Δ, known as the "clearance angle" or "relief."

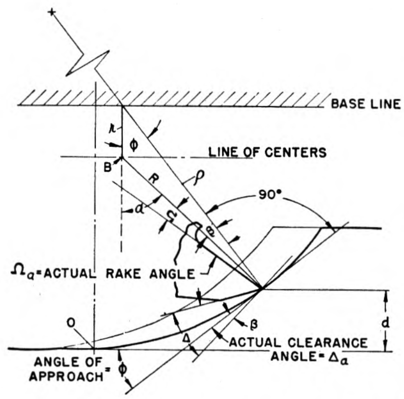

FIG. 26 CLEARANCE AND RAKE ANGLES ALONG PATH OF MILLING-CUTTER TOOTH

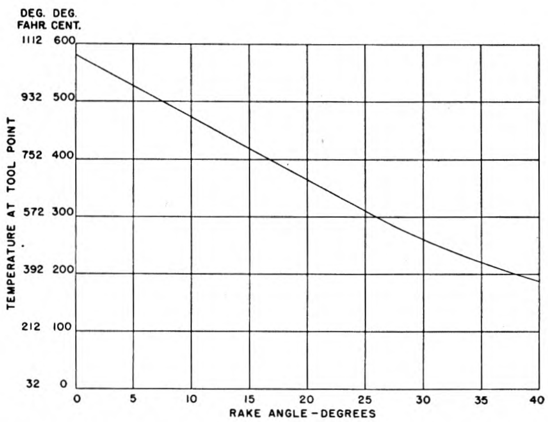

FIG. 27 TEMPERATURE AT TOOL POINT VERSUS RAKE ANGLE
(Data taken from single-point cutting tool, described by Von C. Salomon.[4])

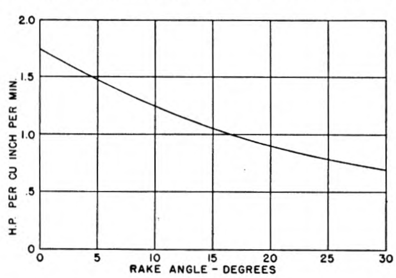

FIG. 28 HORSEPOWER PER CUBIC INCHES PER MINUTE VERSUS RAKE ANGLE[5]

(*c*) To improve cutting action, the face of the tooth is formed by a plane which deviates from a radial plane by a small angle Ω known as the "rake angle" or "undercut."

(*d*) The angle included between these two planes (which intersect each other on the imaginary line constituting the theoretical edge) is the tooth angle. This is always less than 90 deg.

The relative position of these angles is illustrated in Fig. 26.

The clearance angle, therefore, is provided for the specific purpose of preventing an interference between the flank of the tooth and the surface of the work. It also has an important function, i.e., that of limiting the area of the flank exposed to the action of the fragments of the built-up edge escaping with the work. It has no particular influence on the power required to remove metal. This angle must be made sufficiently large to take care of the condition of operation of a milling cutter and thus prevent rubbing on any portion of the tooth path.

The rake angle has a determining influence upon the type of chip produced, the pressure, and temperature at the cutting edge of the advancing tooth, and on the power required in the removal of the excess metal. The relationships between the rake angle temperature and power are shown in Figs. 27 and 28.

A large rake angle is known to ease the flow of metal along the face of a tooth, to reduce the cutting temperature, and to favor the formation of a continuous chip with a small built-up edge, and to lessen the rubbing pressure of the chip on the tool and the abrasion resulting therefrom. As a general rule, with an increase in rake angle, a longer tool life, a better finish, and more efficient metal removal may be expected. In practice constant rake angles greater than 20 deg are seldom used, except in the case of aluminum alloys, since the improvement obtained therefrom may be in some cases offset by an early breakage of the cutting edge of the tooth, resulting from the combination of the wearing away of the nose of the tool, higher local temperature, and decarburization of the material of the tool point.

In a milling cutter, the clearance and the rake angles are by no means constant; they are affected by the ever-changing

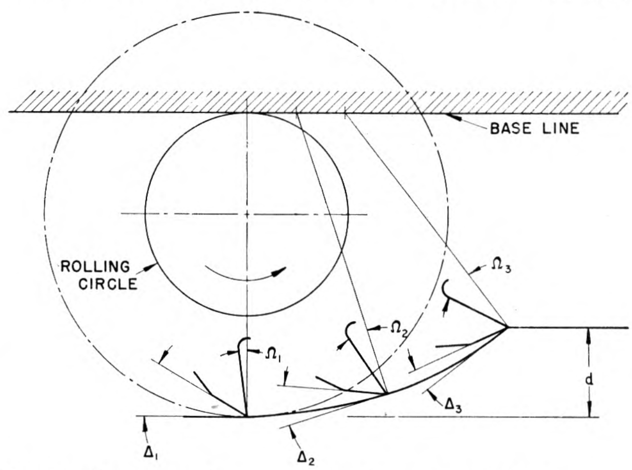

FIG. 29 CHANGES IN CLEARANCE AND RAKE ANGLE WITH POSITION OF TOOTH ON ITS PATH

position of the tooth, and by the curvature of the tooth path. This is illustrated in Fig. 29, showing a tooth in three different positions. It is apparent that the following relation exists among the various angles in the three different positions of the tooth

$$\Omega = \Omega_1 < \Omega_2 < \Omega_3$$
$$\Delta = \Delta_1 > \Delta_2 > \Delta_3$$

where Ω and Δ are the original rake and clearance angles of the cutter.

The actual rake angle or undercut in any point in the tooth path is the angle included between two lines, viz., the parallel to the face of the tooth, and the normal to the trajectory of the tooth at the point under consideration, as is indicated in the case shown in Fig. 26. Since the angle β, included between the radius R of the cutter and the radius of curvature ρ, varies continuously along the path of the tooth in the manner indicated in Equation [9], a variable rake angle will result from the beginning to the end of the engagement of the tooth with the work. Since the teeth of a milling cutter are provided with a rake angle Ω, the actual rake is, therefore, obtained by adding to it the angle β; but

$$\beta = \cos^{-1}\frac{R-d}{R} - \tan^{-1}\left[\frac{2\pi\sqrt{2Rd-d^2}}{2\pi(R-d)+F_t T}\right] \quad \ldots [9]$$

[4] *Die Werkzeugmachine*, December 15, 1929, p. 483.

[5] "Elements of Milling—Part 2," by O. W. Boston and C. E. Kraus, Trans. A.S.M.E., vol. 56, 1938, paper RP-56-1, Fig. 5, p. 358.

hence

$$\Omega_a = \text{actual rake angle} = \Omega + \cos^{-1}\frac{R-d}{R} - \tan^{-1}\left[\frac{2\pi\sqrt{2Rd-d^2}}{2\pi(R-d)+F_t T}\right] \ldots [10]$$

By means of Equation [10], it is possible to compute the actual rake angle obtaining under given cutting conditions. It will be found that, in ordinary cases, the increase in the rake angle is of the order of only a few degrees.

The clearance angle is determined by the amount that a tooth path deviates from a circle, Fig. 26, and is measured by the angle β, made by the radius of the cutter and the radius of curvature ρ.

If the path were circular, the radius of curvature would coincide with the radius R of the cutter, and the angle β would be zero. Hence, the minimum angle required is the angle β. This is obtained from the following equation in which the angle β is expressed as a function of the variables of the cut

$$\text{Minimum clearance angle} = \beta = \cos^{-1}\frac{R-d}{R} - \tan^{-1}\left[\frac{2\pi\sqrt{2Rd-d^2}}{2\pi(R-d)+F_t T}\right] \ldots [11]$$

The actual clearance angle at any point of the tooth path is the result of the difference between the original clearance angle Δ, ground on the flank of a cutter tooth, and the value of the minimum clearance β obtained with Equation [11].

$$\Delta_a = \text{actual clearance angle} = \Delta + \tan^{-1}\left[\frac{2\pi\sqrt{2Rd-d^2}}{2\pi(R-d)+F_t T}\right] - \cos^{-1}\frac{R-d}{R} \ldots [12]$$

At the point O, where the tooth is tangent to the path, the minimum clearance angle is zero, and the actual clearance assumes the value of the original clearance angle. As the depth of cut increases, the actual clearance will be reduced by the increasing minimum clearance angle, and eventual rubbing on the flank of the tooth will develop when the two angles are equal.

Therefore, Equation [11] can be used to compute the minimum theoretical clearance angle required for any particular job. To this must be added a small angle to allow for the slight expansion of the material of the newly formed surface, and of the fragments of the built-up edge escaping with the work. It will be found that, on the basis of this formula, the clearance angle will assume values far below those which are now accepted in shop practice.

Tests which have been conducted to prove the validity of this formula, however, confirm the desirability of using small clearance angles of the order indicated by Equation [11]. This has been corroborated by actual application to difficult cutting jobs. Charts based on this equation, showing the clearance angle required for a given cutter diameter, depth of cut, and feed rate, can readily be prepared for use in the shop.

It is advisable to provide the teeth of a milling cutter with the clearance pertaining to given operating conditions rather than to use a comprehensive angle for all possible applications of the cutter.

The changes, occurring in the actual clearance and rake angles in a tooth of a milling cutter as it travels along its path, are important in so far as they affect the quality of finish, the power consumed, and the life of the cutter.

The derivation of Equations [10] and [11] is given in Appendix 1.

Length of Tooth Path

The total length of engagement of the tool with the work, necessary to complete a machining operation, together with the concomitant abrasive action on the cutting edge by the newly generated surface and the fragments of the built-up edge passing off with the work, has a determining influence upon the life of the cutting tool. In milling, only a small fraction of the actual milled surface, approximately equal to the feed per tooth, is used to form the finished surface; the remainder is removed by the tooth following, and is, therefore, completely lost.

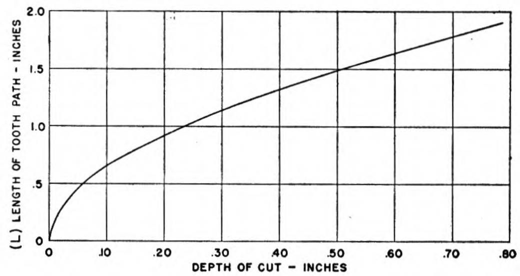

Fig. 30 Relation Between Length of Tooth Path and Depth of Cut

(Diam of cutter, 4 in.; feed rate, 6 in. per min; feed per tooth, 0.021 in.)

Consequently, a minimum actual length of tooth path is a desirable goal. This length is a function of the variables of the cut, as expressed in the equation

$$L = R\cos^{-1}\left(1-\frac{d}{R}\right) + \frac{F_t \times T}{\pi D}(Dd-d^2)^{1/2} \ldots [13]$$

where

L = length of tooth path, in.
d = depth of cut, in.
F_t = feed per tooth, in.
D, R = diameter and radius of cutter, respectively, in.
T = number of teeth in cutter

The derivation of Equation [13] is given in Appendix 2.

For any given condition, in which the depth of cut and the feed rate are known, the only variable on which we can operate to reduce L is the diameter of the cutter and the number of teeth. The feed per tooth, in accordance with the foregoing analysis, should be maintained as large as is consistent with the operating conditions of the job in hand.

A typical example illustrating the results obtained in the application of Equation [13] is shown in Fig. 30. From this chart, we can get some general idea of the advantage obtained in limiting the amount of stock removal to a minimum consistent with the design and the method of manufacturing a given part.

Assuming the cutting conditions, as shown in Fig. 30, and further, that the piece to be machined is 12 in. long, and that the feed per tooth is 0.021 in. with a 10-tooth 4-in-diam spiral mill, and 30-fpm cutting speed, then the total number of engagements made by a single tooth, per piece milled, will be

$$\frac{12}{10 \times 0.021} = 57$$

If the depth of the cut is $^1/_4$ in., then from Fig. 30, the length of tooth path will be 1.04 in. and the total length of surface milled by a tooth will be $57 \times 1.04 = 59.28$ in. If the depth of cut is reduced to $^1/_8$ in., the length of tooth path will be 0.72 in., and then the total length milled per piece will be $57 \times 0.72 = 40.47$ in. This means that the total length of surface milled per tooth in the latter case will be 18.81 in. shorter than the length milled in the former case, under otherwise similar cutting conditions. If the number of pieces milled were 100, then the total saving in milled surface will amount to 1881 in., or approximately 157 ft. Furthermore, by reducing the depth of the cut, the thickness of the chip (and thus the work done by it) is likewise reduced.

Thickness of Undeformed Milling Chip

Knowing the conditions of the cut and the length of the tooth path, it is now possible to determine the average thickness of the undeformed chip. This can be readily accomplished by considering the equivalence between the areas of the rectangle *A-B-C-D* (the sides of which are equal to the feed per tooth F_t and depth of cut d), the parallelogram A-B-C'-D', and the geometric figure A-N-D'-C'-N-B. This area, less the area of the triangle D'-E-C' is equal to the area *E-A-B*, which is the area of the cross section of the undeformed chip, Fig. 24.

However, the area of the triangle D'-E-C' is nearly equal to $\frac{h \times F_t}{2}$ and, due to the very small value of the quantity h, as determined by Equation [5], we can disregard the effects of this area on the total area of the undeformed chip. Consequently, we can write

Area of undeformed chip section =

$$(A\text{-}B\text{-}E) = F_t \times d \text{.................} [14]$$

Average undeformed chip thickness =

$$t_{avg} = \frac{F_t \times d}{L} \text{..................} [15]$$

where L is the length of tooth path, as given in Equation [13].

The variation of the average undeformed chip thickness obtained in a specific case, and as a function of the depth of cut, is shown in Fig. 31.

The average thickness of the undeformed chip can be used to advantage in determining the cutting ratio, namely, the ratio between the length of the actual chip and the length of tooth path, and in estimating the amount of metal removal by a single tooth. Since, in the measurement of either the power or the force involved in a cutting process, average values are generally obtained, it appears more logical to refer these values to the average thickness of the undeformed chip, which is a function of all the variables of the cut, rather than to only one of these variables, as in the case shown in Fig. 10. Here, the efficiency of metal removal is shown plotted against the feed per tooth. These data, replotted on the basis of the undeformed cross section of the chip, are shown in Fig. 32.

By plotting the efficiency of metal removal (cubic inches per minute per horsepower input to the machine) against the average chip thickness, t_{avg}, the difference between the efficiency obtaining in two extreme cases, such as $^1/_{32}$ in. and $^1/_2$ in. depth of cut, is considerably less than on the basis of the feed per tooth F_t. If the efficiency of metal removal is computed on the basis of power consumed at the cutter, then the efficiency obtaining under the various cases represented in Fig. 33 tends to disappear and a single curve could be used to cover the range of depth of cut shown therein. In this case, the average chip thickness could be used in establishing the relationship between the conditions of the cut and the corresponding power required.

This was done for the values shown in Fig. 33, and the following empirical equation was obtained

$$E = 13.2\, t_{avg}^{0.417} \text{..................} [16]$$

where

E = metal removal, cu in. per min per hp at cutter

t_{avg} = average thickness of undeformed cross section of chip, in.

Instantaneous Thickness of Undeformed Milling Chip

The instantaneous radial thickness of the undeformed cross section of the chip, however, is one of the basic elements of the milling process. This is a segment intercepted by two consecutive tooth paths on the normal to the point of the tooth path under consideration. In the case of a circular tooth path, Fig. 34, the instantaneous radial thickness MN is a segment of the radius of the cutter, while in the case of a trochoidal tooth path,

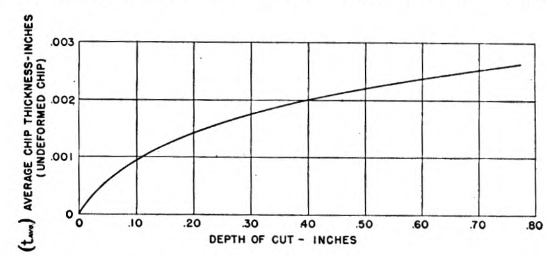

Fig. 31 Average Chip Thickness (t_{avg}) Versus Depth of Cut
(Diameter of cutter, 2 in.; feed rate, 6 in. per min; feed per tooth, 0.0046 in.)

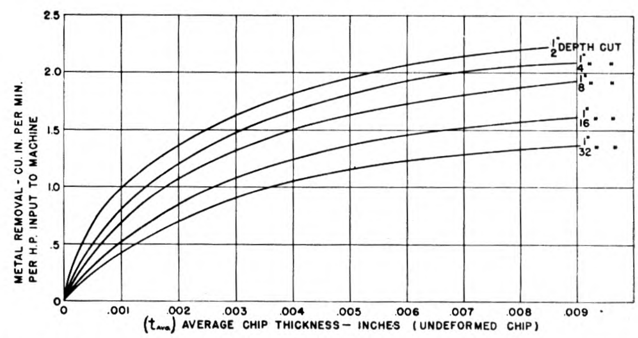

Fig. 32 Metal Removal, Cubic Inches per Minute per Horsepower Input to Machine Versus Average Chip Thickness (t_{avg})
(Material, cast iron; width of cut, 4 in.; cutter, spiral mill, 8T, 4 in. diam; cutting speed, 55 fpm.)

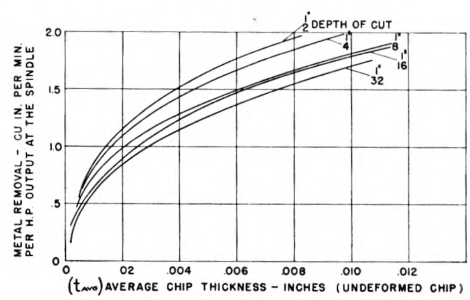

Fig. 33 Metal Removal, Cubic Inches per Minute per Horsepower Output at Spindle Versus Average Chip Thickness (t_{avg})
(Conditions same as in Fig. 32.)

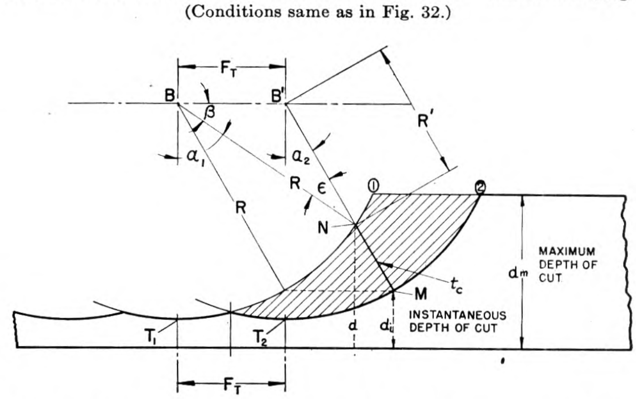

Fig. 34 Instantaneous Thickness (t_c) of Undeformed Cross Section of Chip
(Tooth path approximated to arc of circle.)

Fig. 35, the segment $T_1'' \, T_2'$ is part of the radius vector ρ_2 (on the radius of curvature) normal to the point T_2' and limited by the instantaneous center of rotation C_1 and the point T_2' on the tooth path.

The corresponding analytical expression, for the case in which the path of a tooth is assumed to be circular, is known, but will be derived here in the form which has been found to be more suitable for comparison with that for the trochoidal tooth path.

From the geometry of Fig. 34, and for the position M of tooth 2 therein indicated, the following equation has been derived for the thickness t_c of the cross section of the undeformed chip with the assumed circular path

$$t_c = R + F_t \sin \alpha_2 - (R^2 - F_t^2 \cos \alpha_2^2)^{1/2} \ldots\ldots [17]$$

The derivation of Equation [17] is given in Appendix 3. The angle α_2 appearing in this equation gives the angular displace-

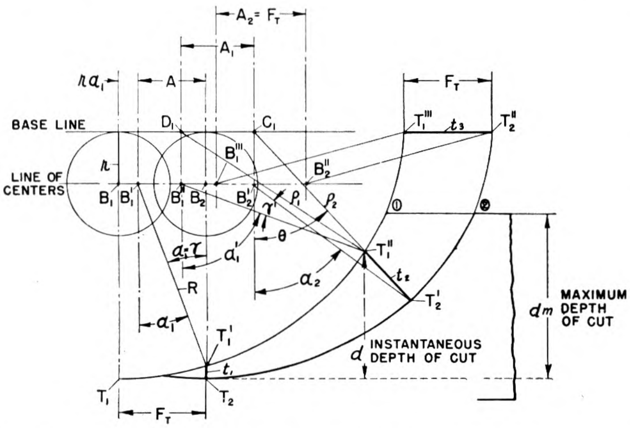

Fig. 35 Instantaneous Thickness (t) of Undeformed Cross Section of Chip
(Actual tooth path.)

ment of the tooth 2 with respect to the vertical drawn from the center B' and corresponding to the position M, of the tooth.

This angle can be obtained from the known data as follows

$$\alpha_2 = \sin^{-1} \frac{(2\,R\,d - d^2)^{1/2} - F_t}{[R^2 - 2\,F_t(2\,R\,d - d^2)^{1/2} + F_t^2]^{1/2}} \ldots [18]$$

where

R = radius of cutter, in.
d = instantaneous depth of cut at point N, in.
F_t = feed per tooth, in.

It has been found, however, that for the values obtaining in practical milling the expression under the radical in Equation [17] is approximately equal to R^2, hence a simplified form of this equation is

$$t_c = F_t \sin \alpha_2 \ldots\ldots\ldots\ldots\ldots\ldots [19]$$

Substituting in this equation the expression for the angle α_2 obtained in Equation [18], the following equation for the thickness t_c is obtained

$$t_c = F_t \frac{(2\,R\,d - d^2)^{1/2} - F_t}{[R^2 - 2\,F_t(2\,R\,d - d^2)^{1/2} + F_t^2]^{1/2}} \ldots\ldots [20]$$

A similar equation can be derived for the thickness t of the undeformed cross section of the chip, obtaining when the true path of the milling-cutter tooth is considered. From the conditions represented in Fig. 35, and for the position T_2' of the tooth 2, the expression for the thickness at this point t_2 is

$$t_2 = \rho_2 + A_1 \sin \theta - (\rho_1^2 - A_1^2 \cos \theta^2)^{1/2} \ldots\ldots [21]$$

This is a more general equation of the thickness of the undeformed cross section of the chip, since it includes the approximate case of the circular tooth path. In fact, Equation [17] can be obtained directly from Equation [21] by eliminating, in this equation, the terms containing r, the pitch radius of the pinion. In Equation [21], the thickness t_2 is expressed as a function of the distance ρ_2 from the center of the instantaneous rotation C_1 to the point T_2' on the path of the tooth (2), the distance ρ_1 from the instantaneous center of rotation D_1 to the point T_1'' of the path of the tooth (1). These segments are, therefore, normal to the tangent to the path at their respective points T_2' and T_1''. The thickness t_2 is also a function of the angle θ made by ρ_2 with the vertical drawn from the center of instantaneous rotation C_1, and of the quantity A_1, which is the distance between the center D_1 and C_1.

When Equation [21] is used within the limits of practical milling operations, it can be simplified without greatly affecting the values obtained therewith. It will be found, therefore, that the term under the radical sign may be assumed equal to ρ_1^2 and that ρ_1 can be taken equal to ρ_2. Hence Equation [21] will assume the form

$$t_2 = A_1 \sin \theta \ldots\ldots\ldots\ldots\ldots\ldots [22]$$

but, since

$$\sin \theta = \frac{R \sin \alpha_2}{\rho_2}$$

it follows that

$$t_2 = \frac{A_1 \, R \sin \alpha_2}{\rho_2} \ldots\ldots\ldots\ldots\ldots [23]$$

The quantity ρ_2 is a function of the variable of the cut, as indicated in the equation

$$\rho_2 = (r^2 + R^2 + 2\,r\,R \cos \alpha_2)^{1/2} \ldots\ldots\ldots [24]$$

In Equation [23] the thickness t_2 is expressed as a function of the variables of the cut in which are included the angles α_1 and α_2. The independent variables in a practical case are the feed rate, depth of cut, and diameter of the cutter. Of these variables the one which determines the value of the angle α_1 and α_2 is the depth of cut (refer to parametric Equation [2]). At any instant, this is given by the maximum vertical distance from the point T_1'' to the finished surface of the work. Eventually this will assume the value dm, the actual depth of cut, which corresponds to the maximum thickness of the undeformed cross section of the chip. When the distance d of point T_1'' is used as the independent variable the angle α_1 can be determined. It is, however, necessary to determine the relationship between the angle α_1 and the angle α_2. An approximate relationship between these two angles can be found by considering the change occurring in both angles as the tooth 2 travels along its path.

For the position T_2 of the tooth 2, the angle α_2 is zero, while the angle α_1 has assumed a value determined by the angular displacement required to bring the radius R from the position T_1 to T_1'. Coincident with the rotation of radius R, there is a translation of the center B_1 of the cutter to B_1' along the line of centers by an amount $r\,\alpha_1$, resulting from the motion of the pinion on the base line. The value of α_1 at this particular instant can be readily determined from the parametric Equations [2] by assuming that $\sin \alpha_1$ is equal to the arc α_1, and this in turn equal to γ. The justification for this assumption is found in the fact that the angle α_1 is, in this case, very small. After making the necessary substitution and solving for γ there results

$$\gamma = \frac{F_t}{r + R} \ldots\ldots\ldots\ldots\ldots\ldots [a]$$

The distance A between centers B_1' and B_2, which was originally equal to F_t, is now

$$A = F_t - r\alpha \qquad [b]$$

As tooth 2 proceeds along its path, the angle γ changes and, for the new position B_1'' and B_2', the centers B_1 and B_2 of the cutter assume a new value γ', which is equal to the difference between α_1' and α_2

$$\gamma' = \alpha_1' - \alpha_2 \qquad [c]$$

When tooth 2 has reached the position T_2'', α_1 and α_2 are equal, hence γ vanishes. Therefore, the following positions are possible

$$\left.\begin{array}{lll} \alpha_2 = 0 & \gamma = \alpha_1 & A = F_t - r\,\alpha_1 \\ \alpha_2 = \alpha_2 & \gamma' = \alpha_1' - \alpha_2 & A_1 = F_t - r(\alpha_1' - \alpha_2) \\ \alpha_2 = \alpha_1 & \gamma'' = 0 & A_2 = F_t \end{array}\right\} \qquad [d]$$

Assuming a linear variation for the angle γ between the point T_2 and T_2'' in which the angle α_2 varies from zero at T_2 to approximately $\pi/2$ at T_2'', any intermediate value γ' of γ, will be obtained as follows

$$\gamma' = \gamma\left(\frac{\pi - 2\,\alpha_2}{\pi}\right) \qquad [e]$$

But from Equation [c], and with the further approximation $\pi = 3$, there results

$$\alpha_2 = \frac{\alpha_1' - \gamma}{1 - {}^2/_3\gamma} \qquad [f]$$

The angle α_1' which appears in this equation is a function of the independent variable d (instantaneous depth of cut) and is obtained by solving the second equation of the system [2]. Hence

$$\alpha_1' = \cos^{-1}\frac{R-d}{R} \qquad [g]$$

Therefore, from Equations [a], [g], and [f], the value of α_2 in radians is obtained

$$\alpha_2 = \frac{(r+R)\cos^{-1}\dfrac{R-d}{R} - F_t}{r + R - \dfrac{2}{3}F_t} \qquad [25]$$

By making the necessary substitution in Equation [d], the quantity A can be expressed as a function of the given variables of the cut

$$A_1 = F_t\left(\frac{R + \dfrac{2}{3}\,r\cos^{-1}\dfrac{R-d}{R} - \dfrac{2}{3}F_t}{R + r - \dfrac{2}{3}F_t}\right) \qquad [26]$$

Returning now to Equation [23], it is possible to substitute for the quantity ρ_2, α_2, and A_1 their corresponding expressions, Equations [24], [25], and [26], which have been obtained in the foregoing analysis. The result is

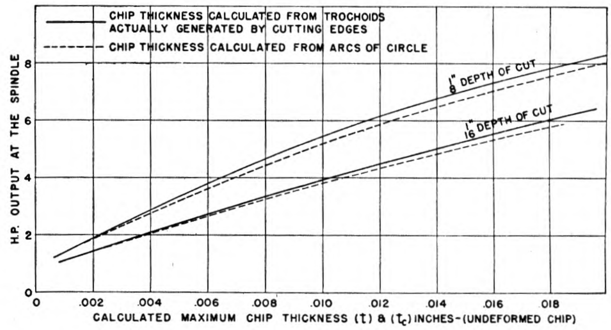

FIG. 36 HORSEPOWER OUTPUT AT SPINDLE VERSUS MAXIMUM THICKNESS (t) AND (t_c) OF UNDEFORMED CROSS SECTION OF CHIP
(Material, cast iron; width of cut, 4 in.; cutter, spiral mill, 8T, 4 in. diam; cutting speed, 63 fpm.)

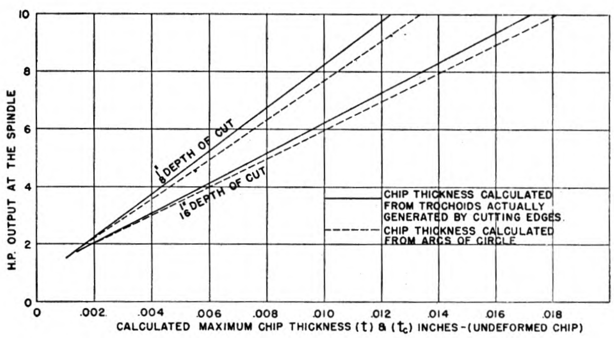

FIG. 37 HORSEPOWER OUTPUT AT SPINDLE VERSUS MAXIMUM THICKNESS (t) AND (t_c) OF UNDEFORMED CROSS SECTION OF CHIP
(Material, S.A.E. 1112; width of cut, 4 in.; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 63 fpm; cutting fluid, soluble oil and water.)

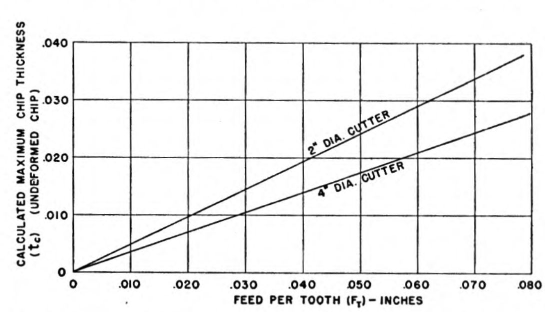

FIG. 38 MAXIMUM THICKNESS (t_c) OF UNDEFORMED CROSS SECTION OF CHIP VERSUS FEED PER TOOTH (F_t)
(Cutter, spiral mill, 8T, 4 in. diam; depth of cut, 1/8 in.; cutting speed, 63 fpm; cutter, spiral mill, 4T, 2 in. diam; cutting speed, 63 fpm.)

The results of the actual power consumed at the cutter with a standard spiral mill on cast iron and S.A.E. 1112 steel have been plotted in Figs. 36 and 37, against the radial thickness t_c and t of the undeformed section of the chip, obtained from Equations [20] and [27], respectively.

On the basis of equal power consumed, it is only when the feed

$$t_2 = \frac{R\,F_t\left[\dfrac{R + \dfrac{2}{3}\left(r\cos^{-1}\dfrac{R-d}{R} - F_t\right)}{R + r - \dfrac{2}{3}F_t}\right] \times \sin\left[\dfrac{180}{3.14}\left(\dfrac{(r+R)\cos^{-1}\dfrac{R-d}{R} - F_t}{r + R - \dfrac{2}{3}F_t}\right)\right]}{\left\{r^2 + R^2 + 2\,r\,R\cos\left[\dfrac{180}{3.14}\left(\dfrac{(r+R)\cos^{-1}\dfrac{R-d}{R} - F_t}{r + R - \dfrac{2}{3}F_t}\right)\right]\right\}^{1/2}} \qquad [27]$$

rate is unusually high that the difference between t and t_c and, consequently, the error in evaluating the thickness, will be increased if Equation [20] is used.

Although the difference between t_c and t becomes increasingly greater as the feed rate is increased, this does not justify the use of the more complicated expression for t, given by Equation [27]. Therefore, it is suggested that the simpler Equation [20] be used in practical application. Particularly appropriate is this choice when we consider that in actual practice the values of t range between 0.001 and 0.01 in. In this region, the error in the evaluation of t is negligible.

It is customary to use the feed per tooth as a criterion in estimating the tooth load on a milling cutter. This, however, in the light of the information contained in Equations [20] and [27], supplemented by the example illustrated in Fig. 38, leads to incorrect appreciation of the actual thickness of the metal being removed by a tooth.

The feed per tooth is determined by the feed rate of the work, the number of teeth, and revolutions per minute of the cutter, Equation [7]. As long as these are maintained constant, a change either in the depth of cut or the diameter of the cutter will not affect the feed per tooth; a change, however, will result in the radial thickness of the undeformed section of the chip.

Moreover, when the feed per tooth is increased, the corresponding radial thickness t_c becomes progressively smaller than the feed per tooth. The foregoing conditions are illustrated in Fig. 38. Since the radial thickness t_c determines the depth of the section of metal which is directly under the action of the advancing tooth, this is a truer indication of the work involved during the process of forming a chip than the feed per tooth.

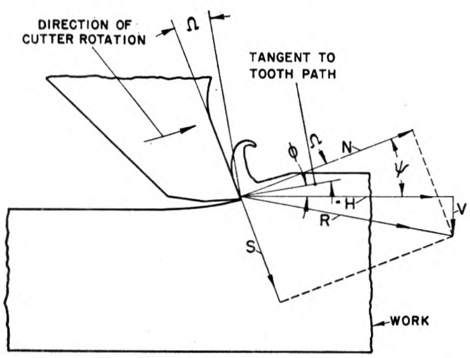

FIG. 39 FORCE DIAGRAM SHOWING RELATIONSHIP OF FORCES AT EDGE OF A MILLING-CUTTER TOOTH

HORIZONTAL AND VERTICAL COMPONENTS OF THE CUTTING FORCE

Of particular interest is the relationship between the radial thickness t_c and the experimentally determined average values of the components of the cutting force R in the direction of the feed and normal thereto.

In the vector diagram, Fig. 39, showing a milling-cutter tooth in the process of forming a chip, these forces are indicated by vectors H and V, respectively, and they are shown as applied to the work.

Their resultant R can be solved in two components, S and N, parallel and normal to the face of the tooth, respectively.

From the known value of R and knowing the angle ψ made by N with the horizontal, it is possible to determine the intensity of N and S.

The angle ψ, however, is equal to the sum of the angle ϕ (Equation [8]) made by the tangent to the tooth path with the horizontal and the actual rake angle Ω, both of which vary with the position of the tooth along its path.

Since the intensities of the vertical and horizontal components of R are average values, it is therefore sufficient in many cases to assume also an average value for ψ.

Hence, the corresponding intensities of N and S could be computed from the known values of H and V. These, for the case of slab milling on cast iron and S.A.E. 1112 steel specimens, are given in Figs. 40 and 41, respectively. The values of the forces were obtained by calibrating with known loads the structure of the milling machine on which these tests were conducted. This method offered the advantage of operating under conditions

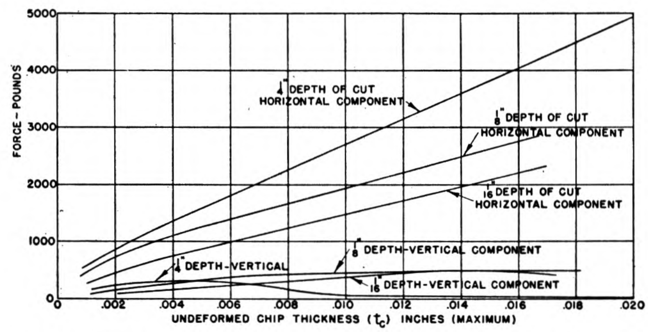

FIG. 40 HORIZONTAL AND VERTICAL COMPONENTS OF CUTTING FORCE VERSUS THICKNESS (t_c) OF UNDEFORMED CROSS SECTION OF CHIP

(Material, cast iron; width of cut, 4 in.; cutter, spiral mill, 10T, 4 in. diam; cutting speed, 63 fpm.)

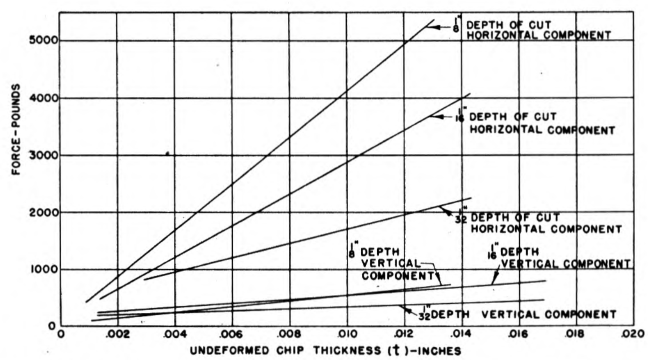

FIG. 41 HORIZONTAL AND VERTICAL COMPONENTS OF CUTTING FORCE VERSUS THICKNESS (t_c) OF UNDEFORMED CROSS SECTION OF CHIP

(Material, S.A.E. 1112; width of cut, 4 in.; cutter, spiral mill, 10T, 4 in diam; cutting speed, 63 fpm; cutting fluid, soluble oil and water.)

identical with those obtaining in practice and, furthermore, of increasing the reliability and accuracy of the results by the simplicity and rigidity of the setup.

The values obtained indicate that:

(*a*) With good approximation the horizontal and vertical components of the cutting force are proportional to the maximum thickness t of the undeformed cross section of the chip.

(*b*) The horizontal component increases rapidly with the depth of cut.

(*c*) The intensity of the vertical component is considerably less than that of the horizontal component. Furthermore, the vertical component shows a marked tendency to remain unchanged at shallow depth of cut and eventually to decrease and possibly become negative with deeper cuts.

In the case of $^1/_4$ in. depth of cut on cast iron, Fig. 40, this decrease of the vertical component begins with a value of t_c = 0.006 in.

For depths of cut above $^1/_8$ in., a reversal of the direction of the vertical component may take place as the tooth approaches the end of its travel through the work.

Conclusions

In the analysis presented, it has been shown that the path of a milling-cutter tooth is an arc of a trochoid, the parametric equations for which can be derived from known variables of the cut.

Thus the milling process may be approached from a mathematical viewpoint. This is particularly helpful in the evaluation of such elements as the radius of curvature of the tooth path, the clearance and rake angles of the length of the tooth path, the radial thickness of the chip, and their effects upon the quality of milled surface, power consumed, and cutter life. This method of analysis, supplemented with actual photomicrographs of surface and chip, has been used to prove that a tooth of a milling cutter will not normally slide on the work but will actually penetrate immediately upon contact.

Furthermore, it has been shown that, in the up-milling process a good quality of machined surface is usually obtained.

When comparing different milling methods, it will be found that a better perspective of the advantages and disadvantages inherent in each method will be obtained by resorting to a more accurate determination of such elements as the length of tooth path, the actual clearance and rake angles, and thickness of the undeformed cross section of the chip.

In metal-cutting problems, it is always necessary to weigh a number of factors in relation to the desired result; this may be either economical operations of the machine, long tool life, or quality of finish on the machined surface. The proper solution of this problem may be greatly aided by a better understanding of the inherent operational functions of the basic elements of the process chosen for carrying out a given machining operation.

Acknowledgment

The author wishes to express his appreciation to Mr. Hans Ernst, research director; to Mr. J. C. Campbell, laboratory engineer, who conducted the tests and assisted in the preparation of the charts; and to Mr. E. Suhr, laboratory assistant, for the preparation of the specimens and relative photomicrographs.

Appendix 1

The derivation of an expression for rake and clearance angles is given as follows:

In the instantaneous position of the tooth, shown in Fig. 26, the actual rake angle is

$$\Omega_a = \Omega + \beta$$

But Ω is the known rake angle of the tooth and

$$\beta = \alpha - \phi$$

From Equation [2]

$$\alpha = \cos^{-1} \frac{R - d}{R}$$

and ϕ is the slope of the tangent to the curve, given by

$$\tan \phi = \frac{dy/d\alpha}{dx/d\alpha}$$

Differentiating Equation [2] with respect to the parameter α, and then replacing y with d

$$\tan \phi = \frac{\sqrt{2\,R\,d - d^2}}{r + (R - d)}$$

Substituting for r, the expression obtained from Equations [1] and [7]

$$\tan \phi = \frac{2\,\pi\sqrt{2\,R\,d - d^2}}{2\,\pi\,(R - d) + F_t\,T}$$

and

$$\phi = \tan^{-1} \frac{2\,\pi\sqrt{2\,R\,d - d^2}}{2\,\pi\,(R - d) + F_t\,T}$$

Therefore

$$\beta = \cos^{-1} \frac{R - d}{R} - \tan^{-1} \frac{2\,\pi\sqrt{2\,R\,d - d^2}}{2\,\pi\,(R - d) + F_t\,T}$$

The actual rake angle is

$$\Omega_a = \Omega + \cos^{-1} \frac{R - d}{R} - \tan^{-1} \frac{2\,\pi\sqrt{2\,R\,d - d^2}}{2\,\pi\,(R - d) + F_t\,T}$$

Since the actual clearance angle is

$$\Delta_a = \Delta - \beta$$

it follows

$$\Delta_a = \Delta + \tan^{-1} \frac{2\,\pi\sqrt{2\,R\,d - d^2}}{2\,\pi\,(R - d) + F_t\,T} - \cos^{-1} \frac{R - d}{R}$$

Appendix 2

The derivation of the length of tooth path is as follows:

The differential of the arc of trochoid from parametric Equations [2] is

$$ds = \sqrt{(r + R\cos\alpha)^2 + R^2\sin^2\alpha} \times d\alpha$$

also

$$ds = \sqrt{r^2 + R^2}\sqrt{1 + \frac{2\,r\,R\cos\alpha}{r^2 + R^2}} \times d\alpha$$

Developing $\sqrt{1 + \dfrac{2\,R\,r\cos\alpha}{r^2 + R^2}}$ in binomial series, and disregarding the terms of higher power than the first

$$ds = \sqrt{r^2 + R^2}\left[1 + \frac{r\,R}{R^2 + r^2}\cos\alpha\right] \times d\alpha$$

Integrating between the limits $\alpha = 0$, $\alpha = \alpha$

$$L = \int_0^\alpha \sqrt{r^2 + R^2}\left[1 + \frac{r\,R}{R^2 + r^2}\cos\alpha\right] \times d\alpha$$

$$= \sqrt{r^2 + R^2} \times \alpha + \frac{r\,R}{\sqrt{r^2 + R^2}}\sin\alpha$$

If r^2 is disregarded, then

$$L = R\,\alpha + r\sin\alpha$$

From Equation [2]

$$L = R\cos^{-1}\frac{R - d}{R} + \frac{r}{R}\sqrt{2\,R\,d - d^2}$$

Appendix 3

The derivation of Equation [17] is as follows:
From Fig. 34

$$t_c = R - R'$$

$$R' = \frac{R \sin (90 - \alpha_1)}{\cos \alpha_2} = \frac{R \cos \alpha_1}{\cos \alpha_2}$$

But

$$\alpha_1 - \alpha_2 = \epsilon$$

and

$$\alpha_1 = \epsilon + \alpha_2$$

therefore

$$R' = \frac{R \cos (\alpha_2 + \epsilon)}{\cos \alpha_2} = R \frac{\cos \alpha_2 \cos \epsilon - \sin \alpha_2 \sin \epsilon}{\cos \alpha_2}$$

$$= R \left(\cos \epsilon - \frac{\sin \alpha_2 \sin \epsilon}{\cos \alpha_2} \right)$$

Since

$$\sin \epsilon = \frac{F_t \cos \alpha_2}{R}$$

$$\cos \epsilon = \sqrt{1 - \frac{F_t^2 \cos^2 \alpha_2}{R^2}} = \frac{1}{R} \sqrt{R^2 - F_t^2 \cos^2 \alpha_2}$$

hence

$$R' = R \left(\frac{1}{R} \sqrt{R^2 - F_t^2 \cos^2 \alpha_2} - \frac{F_t \sin \alpha_2 \cos \alpha_2}{R \cos \alpha_2} \right)$$

$$= \sqrt{R^2 - F_t^2 \cos^2 \alpha_2} - F_t \sin \alpha_2$$

and

$$t_c = R + F_t \sin \alpha_2 - \sqrt{R^2 - F_t^2 \cos^2 \alpha_2}$$

The derivation of Equation [21] follows:
From Fig. 35

$$t_2 = \rho_2 - \rho_2' \quad \text{where} \quad \rho_2' = C_1T_1''$$

but

$$\rho_2' = \frac{\rho_1 \sin \beta}{\sin (90 + \theta)}$$

and

$$\beta = 180 - \{90 + \theta + \epsilon\} = 90 - (\theta + \epsilon)$$

where

$$\epsilon = C_1T_1''D_1 \text{ and } \beta = T_1''D_1C_1$$

therefore

$$\rho_2' = \frac{\rho_1 \cos (\theta + \epsilon)}{\cos \theta}$$

Upon substitution

$$t_2 = \rho_2 - \rho_1 \frac{\cos (\theta + \epsilon)}{\cos \theta} = \rho_2 - \rho_1 \cos \epsilon + \rho_1 \frac{\sin \epsilon \times \sin \theta}{\cos \theta}$$

Since

$$\frac{A}{\sin \epsilon} = \frac{\rho_1}{\cos \theta}$$

$$\sin \epsilon = \frac{A \cos \theta}{\rho_1}$$

and

$$\cos \epsilon = \sqrt{1 - \frac{A^2 \cos^2 \theta}{\rho_1^2}}$$

Results

$$t_2 = \rho_2 + A \sin \theta - \sqrt{\rho_1^2 - A^2 \cos^2 \theta}$$

Discussion

O. W. Boston.[6] This analysis of the actual curvature of travel by the tooth point of a plain milling cutter as it removes a chip, having a definite depth of cut and feed as it deviates from a true circular path, is very interesting. Such analyses have been made previously by Dr. Eng. C. Salomon,[7] who presents a new theory of metal removal by milling, and by N. N. Sawin.[8]

For all cutters which cut on the periphery of the tooth, the length of the chip removed by each tooth depends principally upon its diameter and the depth of cut, as the feed per tooth is relatively very small in comparison. Fig. 42 of this discussion shows an end view of plain- or side-milling cutters removing chips

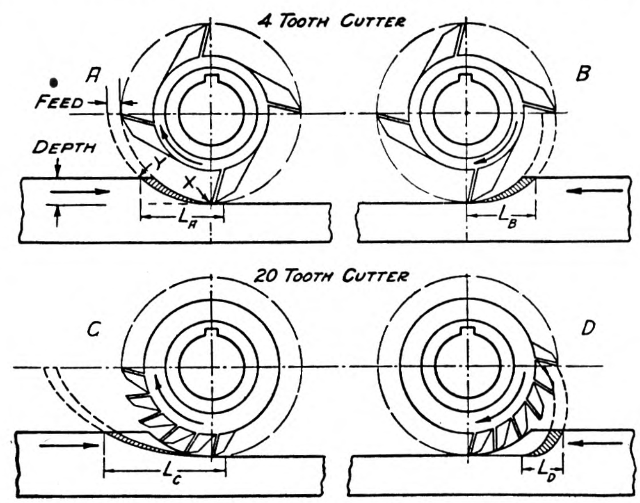

Fig. 42 Graphical Analysis of Chip Formation When Milling Up and Milling Down With Coarse- and Fine-Tooth Cutters

(With thick chips removed more efficiently, from a power standpoint, than thin chips, it is seen that the 4-tooth cutter is more efficient when cutting down at B than when cutting up. It is seen, also, that the fine-tooth cutter is more efficient when cutting down at D than when cutting up. Further, it is seen that the fine-tooth cutter when cutting down is more efficient than the coarse-tooth cutter when cutting down, the feed per tooth in all cases being the same.)

by the peripheral cutting edges of the teeth. The cross-hatch area at A indicates the sectional area of the chip removed by a single tooth. The feed per tooth, exaggerated, and the depth of cut are indicated. The tooth comes in contact with the surface of the work at the point X and leaves the work at the point Y. The curve XY is that discussed by the author. The tooth point rubs over the surface of the work at the point X as the material is fed to the right, as indicated by the arrow, while the cutter rotates clockwise until the normal force between the tooth and the work is sufficient to cause the cutting edge to dig in. If the cutting edge is dull, a considerable force between the work and cutter

[6] Professor of Metal Processing, University of Michigan, Ann Arbor, Mich. Mem. A.S.M.E.

[7] "Die Fräsarheit," by C. Salomon, *Werkstattstechnik*, vol. 20, 1926, pp. 469–474.

[8] "Theory of Milling Cutters," by N. N. Sawin, *Mechanical Engineering*, vol. 48, 1926, pp. 1203–1209.

Note: The articles of ref. (7) and (8) are abstracted in "Bibliography on the Cutting of Metals," A.S.M.E. 1930.

is necessary before a chip starts to be formed. The thickness of the chip at right angles to the path of the cutting edge is practically zero at the point X, and reaches maximum at the point Y. The cross-sectional area of the chip removed is equal to the product of f, the feed per tooth in inches, multiplied by d, the depth of cut in inches. If the diameter of the cutter in inches is D, the average thickness of the chip removed is

$$\text{A.T.C.} = f\sqrt{\frac{d}{D}\left(1-\frac{d}{D}\right)}$$

The horizontal length of the cut as indicated by $L_A = \sqrt{d(D-d)}$. From the A.T.C. formula, it is found that the average thickness of chip having a feed equal to f, and a depth equal to d, is less than the average thickness of chip having a feed equal to $2f$ and a depth equal to $\frac{d}{2}$, even though the cross-sectional areas of the two chips are equal.

The author has used average thickness of chip as a basis for comparing power or energy determinations. This was previously done by Parsons.[9] The writer has proved to his satisfaction that A.T.C. is not a good criterion for such a basis, but rather that a formula for energy in foot-pounds per chip is $E = C\,w f^x d^y$, where C is a constant and w is the width of cut in

[9] "Power Required for Cutting Metal," by Fred Parsons, Trans. A.S.M.E., vol. 45, 1923, pp. 193–227.

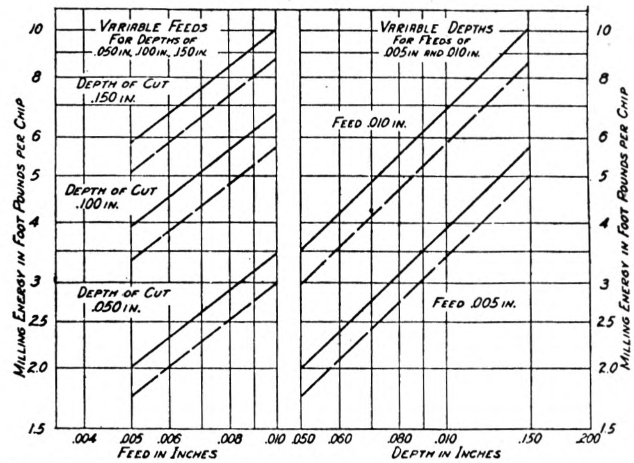

FIG. 43 ENERGY AT TOOL POINT REQUIRED IN FOOT-POUNDS PER CHIP FOR VARIOUS FEEDS AND DEPTHS OF CUT

(Operation of milling brass with lard oil and a side-milling cutter 0.347 in. wide, 3.5 in. diam, and 15-deg rake; when milling up, solid line, and when milling down, dashed line.)

TABLE 2 NET ENERGY AND HORSEPOWER FORMULAS, WITH VALUES OF THE CONSTANT C, FOR MILLING DIFFERENT MATERIALS, BOTH UP AND DOWN, WITH VARIOUS CUTTING FLUIDS

(Formula: $E = Cwf^xd^y$, in which, C = constant for cutter, material, and cutting fluid; E = energy in foot-pounds per chip; w = width of cutter in inches having 15-deg. rake; f = feed in inches per chip; and d = depth of cut in inches)

Material cut	Formulas: Energy, ft-lb. per chip	Formulas: Hp. per cu. in. per min.	Hp./cu. in./min.[a] Up	Hp./cu. in./min.[a] Down	Oil[b] No.	Values of C Up	Values of C Down
S.A.E. 1020 steel, A-1	$E = Cwf^{0.64}d^{0.78}$ both up and down	$= \frac{C}{33{,}000\,f^{0.36}d^{0.22}}$	1.388	1.169	1	5520	4650
			1.084	0.952	4	4320	3790
			1.000	0.796	5	3980	3170
			1.084	1.043	6	4320	4150
			1.043	0.980	8	4150	3900
			0.930	0.854	10	3700	3400
Cold-rolled low-carbon steel	Up: $E = Cwf^{0.73}d^{0.87}$	$= \frac{C}{33{,}000\,f^{0.27}d^{0.13}}$ (Up)	0.936	....	5	6800	..
	Down: $E = Cwf^{0.67}d^{0.87}$	$= \frac{C}{33{,}000\,f^{0.33}d^{0.13}}$ (Down)	....	0.775	5	..	4260
S.A.E. 3150 steel (KF)	$E = Cwf^{0.70}d^{1.00}$	$= \frac{C}{33{,}000\,f^{0.30}}$	1.327	1.278	1	11000	10600
			1.180	1.082	10	9780	8980
S.A.E. 6140 steel (A-13)	$E = Cwf^{0.72}d^{0.90}$	$= \frac{C}{33{,}000\,f^{0.28}d^{0.10}}$	1.44	1.52	1	10630	11220
			1.348	1.427	8	9950	10520
Free-cutting screw-stock steel (No. 414)	$E = Cwf^{0.77}d^{0.86}$	$= \frac{C}{33{,}000\,f^{0.23}d^{0.14}}$	0.954	0.784	6	8180	6720
High-speed steel	$E = Cwf^{0.73}d^{0.84}$	$= \frac{C}{33{,}000\,f^{0.27}d^{0.16}}$	1.437	1.525	1	9700	10400
			1.437	1.525	8	9700	10400
Cast iron (CF)	$E = Cwf^{0.41}d^{0.56}$	$= \frac{C}{33{,}000\,f^{0.59}d^{0.44}}$	0.685	0.740	1	595	643
Leaded screw-stock brass (BF)	$E = Cwf^{0.76}d^{0.96}$	$= \frac{C}{33{,}000\,f^{0.24}d^{0.04}}$	0.381	0.358	5	3830	3600
			0.381	0.358	6	3830	3600
Annealed leaded screw-stock brass (187-A)	$E = Cwf^{0.84}d^{0.87}$	$= \frac{C}{33{,}000\,f^{0.16}d^{0.13}}$	0.307	0.307	5	3700	3700
			0.3095	0.307	6	3730	3700
Annealed unleaded brass (50-A)	$E = Cwf^{0.77}d^{0.96}$	$= \frac{C}{33{,}000\,f^{0.23}d^{0.04}}$	0.573	0.491	5	6040	5170
Pure copper	$E = Cwf^{0.62}d^{0.92}$	$= \frac{C}{33{,}000\,f^{0.38}d^{0.08}}$	1.233	1.049	1	5980	5090
			0.795	0.695	8	3860	3375
Bakelite	Up: $E = Cwf^{0.21}d^{0.75}$	$= \frac{C}{33{,}000\,f^{0.79}d^{0.25}}$ (Up)	0.1432	....	1	74	..
	Down: $E = Cwf^{0.29}d^{0.83}$	$= \frac{C}{33{,}000\,f^{0.71}d^{0.17}}$ (Down)	....	0.1475	1	..	130
Wood fiber	Up: $E = Cwf^{0.49}d^{0.72}$	$= \frac{C}{33{,}000\,f^{0.51}d^{0.28}}$ (Up)	0.0685	....	1	121	..
	Down: $E = Cwf^{0.61}d^{0.91}$	$= \frac{C}{33{,}000\,f^{0.39}d^{0.09}}$ (Down)	....	0.0618	1	..	281

[a] **Feed in inches, 0.010; depth of cut in inches, 0.125.**

[b] Cutting fluid 1 is dry cutting.
Cutting fluid 4 is a soluble oil, 1 part oil to 10 parts water.
Cutting fluid 5 is a No. 2 lard oil.
Cutting fluid 6 is a light mineral oil.
Cutting fluid 8 is a light mineral oil containing 10 per cent No. 2 lard oil.
Cutting fluid 10 is a sulphurized light mineral oil.

inches. Such a formula holds not only for the energy in milling, but for the torque and thrust in drilling, and the cutting force in turning. Values of energy in foot-pounds per chip are shown for various combinations of depth and feed in Fig. 43 of this discussion, in which a log-log scale is used. At the left of Fig. 43, the milling energy in foot-pounds per chip for various values of feed per tooth for three depths of cut is shown, namely, 0.05, 0.10, 0.15 in., when cutting brass, consisting of 65.5 per cent copper, 34.1 per cent zinc, 0.25 per cent lead, and 0.10 per cent iron.[10] The solid lines represent the energy values when cutting up, as shown at A in Fig. 42, and the dashed lines the values when cutting down, as shown at B in Fig. 42. It is interesting to note that all six lines are parallel, the tangent of the angle of the slope being 0.77 which becomes the exponent x of f in the energy equation. It is seen that, if the depth for a given feed is doubled or tripled, the energy increases almost but not quite in the same proportion. The variable-feed curves show that cutting down requires less energy than cutting up, although this relation does not hold true for all metals, as shown in Table 2 of this discussion.

At the right in Fig. 43 are shown the milling-energy values in foot-pounds per chip for variable depths for each of two values of feed per tooth. Again, four straight lines are obtained, all of which are parallel. It is seen that, for a given depth of cut, if the feed is doubled from 0.005 to 0.010 in. per tooth, the energy increase is in the proportion of 2 to 3.5. This shows that thick chips require less energy in proportion than thin chips. The tangent of the angle of slope of these lines is 0.96 which becomes the exponent y of the depth d.

From the results of Fig. 43, an equation giving the relation between the net energy in foot-pounds per chip E, the width of the cutter w, the feed per tooth f, and the depth of cut d, all expressed in inches, may be written

[10] "The Elements of Milling," by O. W. Boston and C. E. Kraus, Trans. A.S.M.E., vol. 54, 1932, paper RP-54-4, pp. 71–92.

$$E = Cwf^{0.77}d^{0.96}$$

C representing a constant which, when cutting up, is 6040 and, when cutting down, is 5171. The energy values have been found to vary directly with the width of the cutter w. Similar equations for other materials when cutting up and down and using various types of cutting fluids are shown in Table 2 of this discussion for relatively light cuts for which the depth of cut is 0.125 in. and the feed per tooth 0.01 in. This equation shows that the energy per chip increases with an increase in feed and depth of cut only as the 0.77 power of the feed and the 0.96 power of the depth of cut. This proves the desirability, from a power standpoint, of taking heavy feeds.

The average thickness of chip (A.T.C.) is a function of the feed and depth of cut as explained. Fig. 44 of this discussion shows the net energy required at the tool point to remove various metals, such as free-cutting brass, cold-rolled steel (low-carbon), S.A.E. 1020 steel, and cast iron for various values of A.T.C., but for three different depths of cut, namely, 0.01, 0.1, and 1 in., respectively. The feed was such as to give the A.T.C. indicated.

The cubic inches of metal removed per horsepower-minute for milling free-cutting brass takes the form of one straight line when plotted over the A.T.C. on log-log paper, when the depth of cut is 0.01 in. Another straight line is obtained, somewhat below the first, for the same feeds when the depth of cut is 0.1, and a third straight line is obtained when the depth of cut is 1 in. This indicates that, for a given depth of cut, a straight power line is obtained, when the feed is varied to obtain different values of A.T.C. for a given depth of cut. For a constant value of A.T.C. such as 0.0006, the cubic inches per horsepower-minute for cutting brass are 2.5 when the depth is 0.01, but only about 2 when the depth of cut is 0.1 in. Again, 2.5 cu in. of the brass are removed per hp per min when the A.T.C. is 0.0006 in., 0.0013 in., and 0.0023 in., respectively, for the three depths of cut. The results for the other metals show also that A.T.C. is not a safe basis for computing energy values. The cold-rolled steel being milled with lard oil is the one exception, as the variation of depth does not destroy the continuity of the three curves.

The results of Fig. 44 indicate the desirability of taking small depths of cut when milling free-cutting brass. The reverse appears to be indicated when milling cast iron and steel. From this it is clear that the A.T.C. alone is not a proper basis for determining power requirements.

Net-energy and horsepower formulas with values of constants for milling different materials both up and down, with a variety of cutting fluids, are given in Table 2 of this discussion. These formulas and values are for the sizes of cut and cutter as indicated in the table heading.

The cutting condition, shown at A, Fig. 42 of this discussion, represents that most commonly used. The cutter rotates clockwise, while the work is fed to the right, both motions being indicated by arrows. In this manner, the cutting tooth cuts against the motion of the work. This is referred to as cutting up, or against the feed. At B the cutter rotates clockwise, while the work is fed to the left. This is called climb or down cutting, in which the cutting action is with the feed. The feed per tooth of the 4-tooth cutter is the same at A as at B. An analysis of the two chips formed

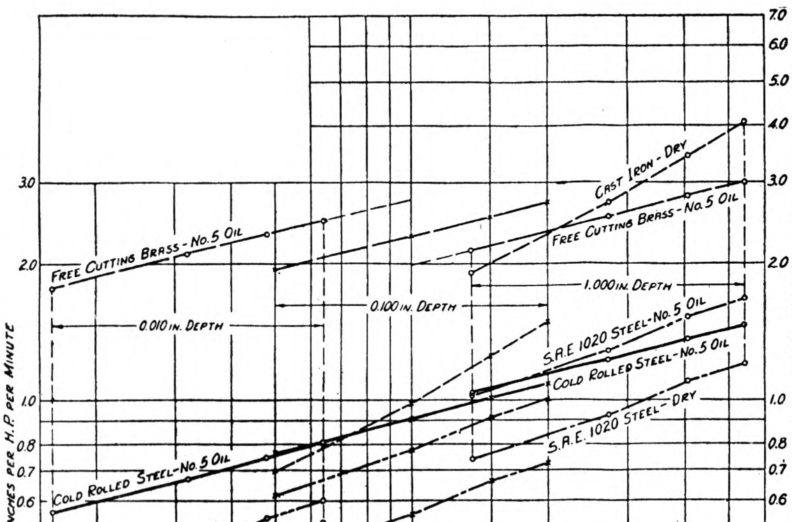

FIG. 44 RELATION BETWEEN A.T.C. AND CUBIC INCHES PER NET HORSEPOWER-MINUTE WHEN CUTTING SEVERAL MATERIALS AT THREE DIFFERENT DEPTHS OF CUT

(The curves are based on the horsepower formula given in Table 2 of this discussion, but plotted over the A.T.C., determined from the formula. All values are for milling up. Oil No. 5 is a No. 1 lard oil.)

shows that the chip at A is thinner and longer than the chip at B. Illustrations at C and D represent, respectively, chips produced by a tooth of a 20-tooth cutter when feeding up and down. Again, it is seen that the chip at C is much longer and much thinner than the chip at D. Therefore, the shape of the chip produced by a cutter of a given number of teeth is influenced by the cutting being done up or down. Also, the difference in shape of the chips is greater, the greater the number of teeth in the cutter.

The author discusses the influence of films of various types on the built-up edge during the formation of a chip. The writer is not in agreement with his statements; the writer's views on the matter were presented in the form of a discussion to a previous paper[11] by Ernst and Merchant.

O. R. Schurig.[12] Thoroughgoing studies of the kind presented in this paper are basic in establishing the fundamentals of everyday shop operations which are in common use and which are so little understood, except for such studies. A more complete understanding of the processes involved will permit operators of machines, or those who control shop practices, to speed up production, or to obtain an improved surface quality, or to increase the life of machine tools, as the case may be.

The author mentions certain advantages resulting from up-milling, including smoother surfaces, because the built-up edge, if any, is not close to the cutting edge when the cut is started, and the initial cut is the one which remains on the final surface. It is also understood that a built-up edge is later formed, as the cutting edge advances and cuts more deeply into the metal.

The writer, therefore, wishes to inquire whether the built-up edge formed on the tool face at the end of the cut remains there as the tool leaves the metal and, if so, why the presence of the built-up edge does not affect the smoothness of cutting at the beginning of the next cut after 1 revolution?

A. E. Richard de Jonge.[13] The author has chosen for his investigation a highly interesting subject which required clearing up beyond what was known heretofore. His analysis of the milling process must, therefore, be received with gratification by the profession. It should be kept in mind, however, that the subject is by no means a new one. The first clear statements about the milling process were made by Prof. F. Reuleaux, in 1900, who published them in the second part of his "Lehrbuch der Kinematik."[14] He was the first to draw attention to the fact that the paths of the teeth of a milling cutter are trochoids (he calls these curves cycloids). He also was the first to discuss the two modes of feeding the material to the revolving cutter, namely, in the direction of rotation of the cutter, and against it. Today, these two processes are called down-milling and up-milling respectively. He had tests made here in the United States, by Messrs. Pratt and Whitney, on whether the first or the second mode of feeding was preferable and produced smoother and better work. The results were indecisive as the marks due to tooth errors were much more pronounced than were the tooth marks. Other tests made at that time for him by Messrs. Ludwig Loewe and Co., of Germany, favored down-milling. Reuleaux also gave a simple formula which showed that in order to obtain tooth marks of small pitch, the circumferential velocity of the cutter must be high, and the speed of the workpiece low. Conversely, if the pitch of the tooth marks is to be great so that the work appears smooth, the speed of the cutter must be low and the speed at which the workpiece is fed to the cutter high.

The author's investigation can be divided into two sections, a practical or experimental one, and a mathematical one. The author has called the latter "Kinematics of Milling." This section forms the greater part of the investigation although the experimental section appears to be the more important one as it gives, for the first time, proof of what actually happens when a chip is removed. Indeed, Figs. 3, 23, and particularly Fig. 25 show what occurs in the formation of a chip, and how little the basic structure of the material is disturbed thereby. The various photomicrographs given by the author amplify and round out the picture of what takes place in milling various metals and what the quality of the surface is that may be obtained. Of value are also the diagrams Figs. 10, 11, 12, 13, 27, 28, 32, 33, 36, 37, 40, and 41, which, as the author has stated privately to the writer, were obtained from actual tests, although the test values are marked only in Figs. 12 and 13. Fig. 6 is of interest in so far as it confirms the statement by Reuleaux that the revolution marks are far more pronounced than are the tooth marks.

Regarding the kinematic analysis, the author, apparently, has gone the long and thorny path of analytical geometry combined with differential calculus, instead of making use of the modern methods of graphical kinematics. Thus, in Appendix 1 for example, there is no need for differentiation to find tan ϕ, because this function can be read off directly from the geometrical figure. The radius of curvature of the tooth path (Equation 6) was also obtained by differential quotients, as stated by the author to the writer in the private communication mentioned. This also is unnecessary as, by the modern kinematic methods, the formula can easily be obtained by similar triangles, making use of the velocities of the tooth generating the trochoid and of the instantaneous center. Most of the other formulas can be read off directly from the respective figures. Yet, the results obtained by the author are correct; only, with respect to Equation 5, it should be mentioned that an approximation has been made and a certain term neglected which, however, is permissible due to the small magnitude of the quantities involved.

As the author has chosen for the title of his paper "An Analysis of the Milling Process," one would have expected that he would have discussed the entire subject. In fact, he has discussed only one half of it, having confined himself to up-milling. Down-milling is just as important, however, and according to Reuleaux and the Loewe experiments cited is even preferable. Yet, this does not actually seem to be the case, and it would be very interesting to see both the mathematical analysis and its experimental verification for the case of down-milling.

In down-milling the rolling centrode (or polode) rolls upon a straight line near the surface of the workpiece instead of, as in up-milling, on a straight line beyond the center of the milling cutter, reckoned from the surface of the workpiece. Consequently, the curvature of the tooth path is far greater, that is, the radii of curvature are very much smaller, than in up-milling. This has an effect on the clearance angle of the teeth, the tooth having to be much sharper. When the cutter has straight instead of helical teeth, it hits the surface of the workpiece at a spot of solid material, hence with an impact, and thus sets up vibrations and chattering; while, in up-milling, the cutter starts its cut at the thinnest part of the chip and not at the thickest part, thus gradually increasing the energy given off to form the chip. If the cutter has helical teeth, there is an impact only at the instant when the front part of the tooth starts cutting, but this impact is greatly reduced if the previous tooth or teeth are

[11] "Chip Formation, Friction, and High Quality Machined Surfaces," by Hans Ernst and M. E. Merchant, Trans. American Society for Metals, 1940, preprint No. 53.

[12] General Engineering Laboratory, General Electric Company, Schenectady, N. Y.

[13] Adjunct Professor, Polytechnic Institute of Brooklyn, Brooklyn, N. Y. Mem. A.S.M.E.

[14] "Die Praktischen Beziehungen der Kinematik zur Geometrie und Mechanik," Lehrbuch der Kinematik, vol. 2, Braunschweig, 1900, pp. 685–689.

still cutting. The distortion of the material forming the chip may, however, be greater than in up-milling, and it would be highly interesting to see the experimental evidence, which the author perhaps is able to supply. As the cutting edges of the teeth have to be much thinner, the life of the cutter seems to be greatly reduced, but here too the author can probably give actual figures which should be of interest.

One would also like to see the actual evidence for the smoothness, or rather roughness, of the surface due to the tooth marks so as to be able to compare the surface finish with that in the case of up-milling. Of interest would also be the formation of the "built-up edge" in down-milling, which should have an important bearing on the actual surface finish. Of importance would, further, be diagrams of the power consumption per cubic inch of metal removed, for various metals, such as the author has given for up-milling. Perhaps, the author might clarify all these points and, thereby, contribute a further valuable piece of information to the very useful facts he has already given in the present paper. It should be stated once more that the profession owes him a debt of gratitude for the very careful investigation he has made of the milling process.

Author's Closure

The author appreciates the comments and the criticisms of Professor de Jonge who has read the paper and checked the derivation of the various equations. He notes the absence of any reference to the down-milling method, although from the title of the paper one would expect a complete analysis of the two methods of milling ("up-milling" and "down-milling"). The paper would have been more appropriately titled: "An Analysis of the Milling Process: Part I—Up-Milling," since the author has actually made a parallel analysis of the two methods and has on hand the information mentioned by Professor de Jonge. It is the author's hope that this material, together with an analysis of the mechanism required to permit down-milling operations, may be presented in the near future in a second paper.

Space limitation prevented the publication of this material in its entirety in the initial paper, and the same reason prevents the author from elaborating on this subject at this time.

The author finds that the analytical method used, supplemented with actual tests, is conducive to a better understanding of the characteristic differences between the two methods of milling, and also makes it possible to visualize, from a practical viewpoint, the advantages and disadvantages inherent in both methods.

Professor Boston takes exception to the author's suggestion that the average thickness of the undeformed cross section of the chip may be used for establishing the relationship between the conditions of the cut and the corresponding power required, on the basis that the results he has obtained with the A.T.C. formula derived by Parsons[9] gave rise to some apparent inconsistencies.

Quoting Professor Boston:[10] "It is obviously inaccurate to use the A.T.C. as the variable in a general milling formula . . . . . This is unfortunate, because the average thickness of the chip seems to be a logical basis for determining cutting qualities."

In his investigation, Professor Boston has used the so-called pendulum-type milling machine, also known as "chip tester." This machine was developed and used by Professor Airey and C. J. Oxford, and was described[15] by them some years ago.

In the chip tester, one fundamental element of milling is eliminated, i.e., the feed of the work. Consequently, in operation, a circular tooth path is obtained instead of the true path which, as shown by the author, is trochoidal. It follows then that with the chip tester there is no geometric difference between up- and down-milling except for the point where the tooth contact with the work begins. For given cutting conditions, the average thickness of the chip is identical in the two cases, and the energy involved in forming the chip should also be the same.

The values of hp per cu in. per min, given by Professor Boston in Table 2 of his discussion, for the two methods of milling, indicate that in some cases there is a difference in favor of down-milling; in other cases, there is no difference; and, in yet other cases, the values for down-milling are higher than those for up-milling. Since these differences are small, one is inclined to believe that, because of the limitations of his experimental apparatus and, furthermore, to the difficulty in setting the work accurately for the required feed per tooth, the variations found by Professor Boston are due to experimental errors rather than to actual differences resulting from the two methods of formation of the chip.

It is to be noted also that, with Professor Boston's chip tester, the rake and clearance angles are constant throughout the tooth path, while in actual milling they vary continuously from the beginning to the end of the chip, as shown by the author.

For the conditions investigated by Professor Boston, the instantaneous radial thickness of the chip can be exactly calculated by means of Equation [17] of the paper. This was derived for the case in which the tooth path is circular. For the purpose of this discussion, the simplified form, Equation [19], of this equation is used

$$t = F_t \sin \alpha \qquad [28]$$

where

t = instantaneous thickness of undeformed section of chip, in.

F_t = feed per tooth, in.

α = angle through which radius vector has rotated from beginning to end of tooth path

The angle α can be evaluated in terms of the elements of the cut from the equation for Y in the system of Equations [2] of the paper. When this is done Equation [28] of this closure, assumes the form given by Parsons for the maximum radial thickness of the chip

$$t = 2f\sqrt{\frac{d}{D}\left(1 - \frac{d}{D}\right)}$$

where

f = feed per tooth, in.

d = depth of cut, in.

D = diameter of cutter, in.

From this formula Parsons derived the following expression for the so-called "average chip thickness" by dividing by 2 his expression of the maximum chip thickness

$$\text{A.T.C.} = f\sqrt{\frac{d}{D}\left(1 - \frac{d}{D}\right)}$$

This is identical with the expression

$$\frac{F_t \sin \alpha}{2}$$

But this obviously corresponds not to the average, but only to one half of the maximum thickness of the chip.

The true average value, however, can be determined from Equation [28] as follows: In general if $f(x)$ is a real function of x, the average value f_{avg} of $f(x)$ with respect to x over a given interval $a \leqq x \leqq b$ is defined to be

$$f_{avg} = \frac{1}{b - a}\int_a^b f(x)dx$$

[15] "On the Art of Milling," by John Airey and C. J. Oxford, Trans. A.S.M.E., vol. 43, 1921, p. 549.

In our case

$$t = f(\alpha) = F_t \sin \alpha$$

and the limits are $0 = \alpha \leqq \alpha_1$; therefore

$$t_{avg} = \frac{1}{\alpha_1} \int_0^{\alpha_1} F_t \sin \alpha \cdot d\alpha \qquad [29]$$

and carrying out the integration

$$t_{avg} = \frac{F_t}{\alpha_1} [1 - \cos \alpha_1] \qquad [30]$$

or

$$t_{avg} = F_t \frac{\sin^2(\alpha_1/2)}{(\alpha_1/2)} \qquad [31]$$

The difference between the values of t_{avg} obtained with the Parsons formula and the more nearly correct Equation [31] of this closure is graphically illustrated in Fig. 45, herewith.

Therefore, it is evident that Professor Boston made a serious mistake in trusting the correctness of the Parsons formula, and in concluding therefrom that the average chip thickness is not a good basis for judging cutting qualities.

It is also interesting to note that, by substituting in Equation [31] for $\cos \alpha$, the expression $(R - d)/R$, this equation becomes identical with Equation [15] of the paper, derived by the author from a direct consideration of the volume of the chip, when L is made equal to the length of the arc of a circle corresponding to the simplified tooth path.

Obviously, for an exact determination of A.T.C., the true trochoidal tooth path must be used as given in Equation [13] of the paper.

Furthermore, it is important to consider that, in milling practice, a variety of combinations of feeds, depths of cut, and speeds are found. Consequently, an approximated formula cannot be used indiscriminately. From Fig. 45 of this closure, it is evident that the Parsons formula will be found satisfactory only for values of α up to 30 deg.

From this it can be concluded that, in using Parsons' formula in the range of depth from 0.01 to 1 in., inconsistencies such as mentioned by Professor Boston will result, and a revision of his conclusions is therefore in order. Professor Boston is still of the opinion that a milling-cutter tooth rubs over the surface of the work at the point of contact until the cutting edge digs into the work. His reference to this action indicates that he has not read the part of the paper in which the author has dwelt at length on the fallacy of this opinion. The author has produced experimental evidence which proves that the cutting edge of a milling-cutter tooth begins to form a chip immediately upon contacting the work.

Professor Boston mentions the works of Salomon[7] and Sawin[8] in connection with the discussion of this paper.

Professor Salomon, without even analyzing the path of a milling-cutter tooth assumes ipso facto that the path is an arc of a circle, and then proceeds by a complicated mathematical procedure to show that the thickness of the undeformed section of the chip, corresponding to a point of the path located at $\alpha/2$ is the true criterion for determining the power required in milling; the angle α being that subtended by the circular arc of the complete tooth path.

His formula for this thickness of the chip is

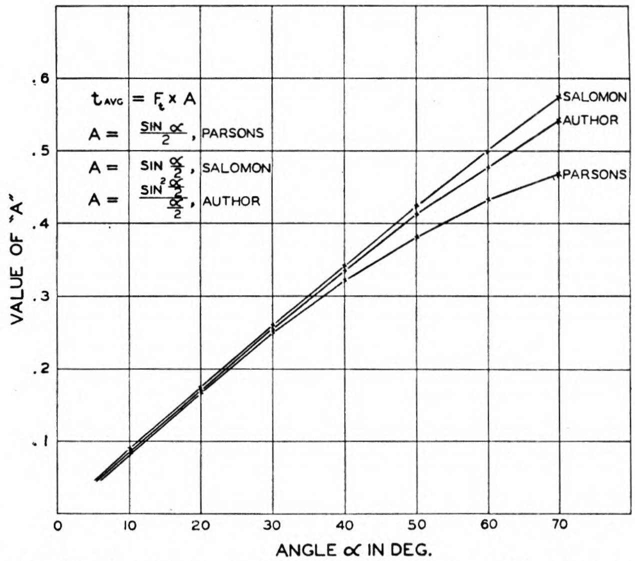

FIG. 45 COMPARISON OF VARIOUS FORMULAS FOR DETERMINING AVERAGE THICKNESS OF UNDEFORMED SECTION OF CHIP

$$t = F_t \sin \frac{\alpha}{2}$$

Fig. 45 of this closure shows the variation between the Salomon formula and that derived by the author for the average value of the thickness of the chip for the case (not true in practice) of a circular tooth path.

Mr. Sawin accepts the approximation often made by various investigators that the path described by a tooth can be simulated by an arc of a circle, and then proceeds to produce a number of formulas for calculating the dimensions of a milling cutter.

The author wishes to state that a better understanding of the physical facts involved in any machining process may be obtained by considering (*a*) the principle on which the process is based, to determine the interdependence of the various elements by analytical methods, regardless of the magnitude of the single elements involved; and (*b*) to make the necessary approximation in certain cases if found admissible within the limits of practical application. Evidently this procedure was not followed by Professor Boston.

The author appreciates Mr. Schurig's interpretation of the studies which formed the object of his paper. In the majority of observations made by the author, covering a number of years, he has found that the built-up edge (formed during the passage of a tooth through the work) remains strongly attached to the chip (it is actually a part of it, Fig. 3 of the paper), and usually passes off with it. The entire built-up edge, therefore, will not be found on the edge of the tooth at the completion of its engagement with the work. There may be small particles of it adhering to the face of the tooth, but these particles will usually be displaced by the oncoming new chip, and thereby a clean edge will result.

In those cases, particularly when dry-cutting, in which a strong bond between the face of the tooth and the material of the chip develops, the completely formed chip with the built-up edge may be carried around by the tooth and fall on the finished surface where it might be caught by the next tooth approaching the work. This will result in blemishes on the finished surface, as shown in Fig. 19 of the paper (*A*, *B*, and *C*).

An effective cutting fluid will invariably improve the conditions mentioned.