ORF435 / ORF535 / FIN535

Homework 5

Instructor: Professor John M. Mulvey

Due date: Friday, 10/27/2017

Question 1: Coherent Risk Measure

Suppose there are two assets with random payoff X and Y, and there are three scenarios A, B and C. Scenario A has a 5% probability, under which X will lose $500 and Y will break even. Scenario B also has a 5% probability, under which X will break even and Y will lose $500. Scenario C has a 90% probability, under which X and Y will both gain $50.

Recall our definition of VaR is $\text{VaR}_h(X) = \min\{v | P(-X > v) \leq h\}$
and our definition of CVaR is $\text{CVaR}_h(X) = -E[X | X \leq -\text{VaR}_h(X)]$.

We also learned that for a coherent risk measure M, we should have the subadditivity:

$$M(X+Y) \leq M(X) + M(Y).$$

1. Show that $\sigma_{X+Y} < \sigma_X + \sigma_Y$.
2. Show that standard deviation, as a risk measure, is coherent (general case).
3. Show that $\text{VaR}_h(X) + \text{VaR}_h(Y) < \text{VaR}_h(X+Y)$ when $h = 5\%$. This inequality demonstrates that VaR is not coherent, and may not always reward diversification effort.
4. Show that CVaR for any h, on the other hand, is coherent.
5. If Z follows standard normal distribution $N(0,1)$, what is VaR and CVaR at $h = 5\%$?

Question 2: More on risk measures

Solve problems 5 and 7 in chapter 10.