ORF435 / ORF535 / FIN535

Homework 7

Instructor: Professor John M. Mulvey

Due date: Thursday, 12/7/2017

Question 1: Growth Optimal Analysis
Solve exercises 2, 7, 10, and 12 in chapter 18 of "Investment Science", 2nd edition.

Question 2: Multi-regime Scenario Simulation with Mean-CVaR Optimization
This time, we will build some portfolios under a multi-regime framework and watch their performance during the financial crisis period and a few years after that.

(1) We will use the data from 1994 to 2007 as sample and save 2008 to 2013 for testing. In addition to the seven assets data, we also consider an indicator for stock market growth. Read the column "S&P 500 Growth" separately into your computing software.

(2) Simulate 1000 annualized return for the assets based on their annualized historical mean and covariance.

(3) For the SP500 return series $x = (x_1, \ldots, x_n)'$, the first-order trend-filtering algorithm is given by

$$\underset{\beta \in \mathbb{R}^n}{\operatorname{argmin}} \ (||x - \beta||_2^2 + \lambda \sum_{i=2}^{n} |\beta_{i-1} - \beta_i|).$$

Solve this optimization problem for $\lambda = 0.1$ and set $y_i = \operatorname{sign}(\beta_i)$. Report the number of 1's and -1's of y.

(4) We call the quarters with $y = 1$ "growth periods" and the quarters with $y = -1$ "crash periods". Compute, respectively, the annualized geometric mean and covariance for growth and contraction periods.

(5) Simulate 10000 annualized data (single period) with the two sets of mean and covariance. The proportion of the "growth" and "crash" scenarios should be the same as historical data. (For example, if your answer to (3) is 42 positive and 14 negative, then you should generate 7500 "growth" data and 2500 "crash" data.)

(6) Compare the VaR and CVaR (at 5% level) of each asset simulated in (2) and (5). Report your findings.

(7) The mean-CVaR optimization shares the same constraints (budget, non-negativity, and expected return) as mean-variance optimization. The only difference is that we will minimize CVaR at 5% level, instead of volatility. For the simulated scenarios in (5), solve for optimal mean-variance and mean-CVaR portfolio at expected return level $\bar{r} = 12\%$ annually (no shorting or borrowing allowed).

(8) Then solve for the mean-variance portfolio with simulated data in (2), also at expected return level $\bar{r} = 12\%$ annually (no shorting or borrowing allowed either).

(9) Now we have three "optimal" portfolios in total. Use the following steps, compare their performance during the testing periods (from 2008 to the end of 2013).

- Assuming quarterly rebalance and fixed mix, plot the wealth path for each portfolio.
- Compute the total return of the portfolio in each quarter, then report the mean and volatility.
- What do you think causes the difference between the performance? Use your computations from (6) to illustrate.