DRAFT

Freelancing and the Value of Flexible Work

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Abstract

The last decade has seen a rise in non-traditional work arrangements, with freelancing, contract work, gig apps, and remote work becoming more prevalent across a range of industries. This paper provides new information on how these types of workers search for, select, and complete jobs, and provides insights into the value that these workers gain from being able to choose their hours flexibility. The paper is motivated by an empirical observation that freelancers have large week-to-week variation in the hours that they work, even when working on the same project for many months. To explain this high-frequency variation in hours-worked, I develop a structural model of freelancing, using a particular type of work arrangement unique to freelancers: a job is a fixed quantity of work, and the freelancer chooses an optimal time-horizon and hours-schedule over which to complete the work. The model is estimated using using tax data for a panel of New Zealand freelancers. I then use an estimated version of the model to quantify the trade-offs that freelancers face, including which jobs to accept and which to reject, and how to optimally complete tasks. I show that freelancers value the flexibility of shorter-term jobs versus the stability of longer-term jobs. I also conduct welfare analysis comparing the 'flexible work schedule' freelancing model to a 'fixed hours' baseline, to show that freelancers are made on average 27 percent better off by being able to flexibly choose when to work. Finally, I document a subset of freelancers termed 'flex-or-quit' workers, whose value placed on flexibility is so high that they would be unwilling to take on any traditional job with a fixed-hours work schedule.

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1 Introduction

The recent rise in online job platforms and remote work has led to an increase in non-traditional employment – labor market participation in work arrangements other than familiar firm-worker pairings (Cohany, 1996; Katz & Krueger, 2019). While gig-work has dominated the public discourse in this space (particularly for ride-sharing and food-delivery), other types of non-traditional labor are also becoming common. Platforms to quickly match workers with clients have led to a rise in longer-term task-based employment, such as freelancing and contract work. This is evident in a range of industries, including legal services, medical practice, education, and professional services.[1] This paper explores the job search, job selection, and task completion behavior of freelancers, and quantifies the benefits to workers from freelancing.[2]

One commonly-cited benefit of freelancing is the flexibility it offers in terms of when and how much to work (Dawson, Henley, & Latreille, 2009). Given a job or a project, it is often the case that a freelancer can pick which hours in they day they work and scale their hours up or down, subject to the constraint that the work is completed within the client's time-frame. This flexibility can be particularly beneficial for certain groups in the labor force whose value of non-work time is both variable and unable to be predicted perfectly at the time of contracting. For example, freelancers who are new parents can work around their child's sleep schedule; freelancers who are students can work around their class requirements; and freelancers who like to surf can work less when the waves are good and work more when the waves are bad. Indeed it may be the case that a worker is *only* able to work if they have this type of flexibility. If an individual's non-work commitments are strong enough and random enough, then any fixed roster of hours may prevent them from being able to take on work.

In this paper, I provide a structural link between flexible work and random non-work commitments, by building a model of freelancing in which workers are able to choose their hours at a high frequency in response to changes in their time-value of non-work (or, equivalently, their time-value of work). In the model, a freelancer takes on a contract to work a fixed total number of hours on a job. For example, a contract for a freelance web developer might be to produce a website that will take 200 hours in total.

[1] The associated platforms include HireAnEsqiure.com (legal services), LocumTenens.com (medical practice), Preply.com (education), and Upwork.com (professional services).

[2] In order to be concise, I'll refer to this type of non-traditional worker as a *freelancer*, and this type of non-traditional work as *freelancing* throughout the paper. However, a larger group of worker-types under the umbrella of non-traditional work could also fit my definition, including independent contractors. Moreover, given that there are some similarities in the search process across different types of non-traditional workers (gig workers, day laborers, on-call workers), the findings of the paper may be more-generally applicable across worker types.

The freelancer can then choose each week how many hours to work on this job, until the job is finished. The optimal choice of hours to work each week will reflect unpredictable changes in the disutility of work – sometimes the freelancer has a low disutility of work, and so will work more hours, while at other times the freelancer has a high disutility of work, and so will work fewer hours. I can then quantify the value of flexible work by comparing the freelancer's utility when they are able to pick and choose hours, to a world in which they face the same disutility shocks but are required to work a constant hours schedule.

At a fundamental level, freelancers engage with clients to complete tasks in exchange for a fee. While this arrangement appears simple, there are a number of complex trade-offs and decisions that must be made by a freelancer while searching for jobs and completing tasks. For example, accepting a job adds to a freelancer's 'stack' of work. If this stack of work keeps the freelancer sufficiently busy, they may have to forgo search for other jobs, and may even need to reject lucrative offers in the future, in order to complete their current work. Thus, freelancers may face a trade-off between the stability of accepting a given job and maintaining capacity to take on future offers. Another trade-off concerns the speed of work. Given a quantity of work to complete, the freelancer must decide how many hours to work each week. Working more hours reduces time for leisure or non-work commitments, but means the job will be completed more quickly, freeing them to search for more opportunities. This paper studies and quantifies these trade-offs and decisions.

For my empirical setting I use a novel dataset of freelancer earnings, as recorded by Hnry, a New Zealand firm providing financial services (tax and social security calculation) for freelance workers.[3] The dataset covers a wide range of industries and worker demographics, offering a comprehensive view of freelancer decision-making. This stands in contrast to many other papers in the broader literature on non-traditional work arrangements, which have tended to focus on a particular type of worker or a particular job-finding platform.

A number of differences exist between freelancing and traditional firm employment, meaning that existing labor-market models may be insufficient to fully capture and explain the nuances of this type of work. The model I develop is built with these differences in mind, allowing for an exploration of freelancer-specific mechanisms, and hence a targeted analysis of the trade-offs and decisions inherent to

[3] Reflecting the broadness of this category of workers, the company uses the term *independent earner* to refer to their clients. I refer to the workers as *freelancers*, despite the fact that the platform covers a broader range of work arrangements than just freelancing.

a freelancing work-arrangement.

Firstly, most traditional models of job search assume that a job is a possibly-indefinite match, perhaps with dissolution due to an exogenous 'layoff' shock, to a match-productivity breakdown, or to the worker finding a better job through on-the-job-search (OTJS). Freelancing on task-based work differs in that the client-worker match is usually contracted for a particular project or piece of work, with the freelancer needing to work on the job until it is complete. It is unlikely, for example, that a freelance web developer will refuse to complete a website because they are offered a more lucrative contract, or that an Uber driver will cancel a ride mid-way due to finding a more attractive trip with another rider. In the model I develop, a job has a pre-specified size (in terms of the number of hours to complete) and the freelancer must continue to work until the job is finished, at which time the match breaks indefinitely.

Secondly, traditional labor market arrangements generally have fixed remuneration per unit of work associated with a job, whether in terms of a wage (as in McCall (1970) for example), or a piece-rate (as in Bagger, Fontaine, Postel-Vinay, and Robin (2014) for example). In combination with near-constant weekly hours, this typically leads to reasonably stable earnings for workers in traditional labor arrangements. For freelancers, remuneration may be negotiated over an entire job or task, meaning that the quantity of work and compensation may vary considerably over short horizons. This can lead to very large fluctuations and uncertainty in earnings. The model I develop allows for a distribution of compensation for tasks, and hence higher income uncertainty for workers.

Thirdly, in a traditional labor-market setting, working more on a given job is typically associated with higher total pay. Given a fixed wage and variability in hours, an increase (decrease) in hours-worked will raise (lower) total compensation. For freelancers, this *intensive margin* is less clear – often workers will receive a known fee for completing an entire project, and hence have fixed total compensation. Thus, the decision to work more hours can instead be interpreted as reducing the total time to complete a given task. A freelance web developer who works twice as fast, will finish the client's website in half the time. In the model, the job-completion mechanism makes this explicit – the worker chooses an optimal time frame and an optimal work schedule in order to complete the task, with higher hours-worked reducing the time to completion.

This job structure and task-completion mechanism is inherent to freelancing, and is one of the main differences between my model and previous studies on the flexibility of work. The prior literature quantifying the value of flexibility has tended to look at workers where a change in hours-worked necessarily comes with a change in income, and on jobs where the worker-client relationship is indefinite (Angrist, Caldwell, & Hall, 2021; Mas & Pallais, 2017, 2020). In these types of work arrangements, flexibility has a different trade-off – choosing a more-flexible hours schedule means that income will be more volatile too. The indefinite job-match means that the intensive margin (changing hours) affects only compensation, rather than time-to-completion. For a freelancer, compensation for a project is often fixed, and hence varying hours-worked doesn't mean that income varies too, but instead changes the time until the freelancer completes the job. This means that the trade-off between working more and and having more time for leisure is very different for freelancers.

A key mechanism in this paper is *work un-smoothing* – I observe in the data that freelancers have large variations in the amount that they work from week to week, even when working on the same job for months at a time. This fact runs contrary to the predictions of a classical labor market model, in which workers would prefer to smooth their labor supply over time.[4] I rationalize this behavior through the use of stochastic shocks to the dis-utility of work. This gives a clear mapping between the data and the model – any deviations in hours-worked from a *smoothed* labor provision path are attributed to disutility shocks. For example, a new parent whose child unexpectedly needs more attention one week (hence higher disutility of work) is able to work less, and a surfer who finds that their local beach has unexpectedly bad waves (hence lower disutility of work) is able to work more.

By building a model around freelancer-specific characteristics and work un-smoothing, I am able to quantify the decisions and trade-offs that freelancers make. I find that freelance workers value the flexibility of shorter jobs, rather than the stability of longer-term jobs. For example, a doubling of job length from ten weeks to twenty weeks requires a weekly-pay premium of 15 percent. I also use the estimated model to provide some quantification of the welfare impacts of flexible freelancing arrangements. Traditional jobs typically require workers to work a near-constant roster of hours each week, irrespective of any shocks to the disutility of work. By comparing actual paths of hours-worked to counterfactual outcomes with constant hours-worked, I can construct *compensating differentials* – the increase in pay required for a freelancer to complete a job under a fixed-hours schedule. I show that

[4] The terminology *work un-smoothing* is chosen deliberately to parallel the ubiquitous economic concept of *consumption smoothing*. In classical economic models (whether of consumption or labor provision), convex preferences give rise to optimal behavior in which the choice variable doesn't vary much over the choice horizon. For example, optimizing agents are predicted to consume out of lifetime income, and to provide a relatively constant share of time endowment to labor.

freelancers are made on average 25 percent better off by having flexible work, and that a non-negligible fraction of workers are made so much better off by flexible work that they would likely be unwilling to work in a fixed-hours job.

Related literature

Building on insights from prior work, this paper aims to make contributions to three broad strands of the literature.

The first literature I contribute is focused on quantifying the value of flexible work. Earlier research in this literature focused on motivations for taking on flexible work, often using reduced-form modeling. This literature includes Dawson et al. (2009), Mas and Pallais (2017), and Garin, Jackson, Koustas, and McPherson (2020). Papers in this strand of the literature have identified benefits to flexible work including ability to focus on parenting (Lim, 2017), smoothing income shocks (Koustas, 2018), and allowing time for home production Gurley-Calvez, Biehl, and Harper (2009). More recent papers with structural models include Frazier (2018), Dolado, Lalé, and Turon (2021), and Kass (2022). My contribution to this literature is to focus directly on freelance workers, who may be more likely to value flexible work. I build a model of flexible work explicitly based on freelancer-specific characteristics, where other papers have tended to use models of traditional firm-worker pairings.

The second literature I contribute to is the nascent but growing literature using data on workers in non-traditional employment arrangements (freelancing, gig-work, contracting, etc.) to study high-frequency labor market dynamics. This literature has tended to leverage gig-economy platforms as an experimentation setting or data source. Uber is a commonly-used platform with Castillo (2020) and Castillo, Knoepfle, and Weyl (2017) using spatial equilibrium frameworks of gig worker decision making to study the welfare gains associated with surge pricing and search, Cook, Diamond, Hall, List, and Oyer (2021) using data on driver earnings to decompose the gender wage gap, Hall, Palsson, and Price (2018) discussing the complementarity of ride-sharing and public transport in urban planning, and Chen, Rossi, Chevalier, and Oehlsen (2019) studying the value of flexibility to drivers. Other researchers have used platforms such as Angi's HomeAdvisor to study the impact of occupational licensing (Blair & Fisher, 2022) and Upwork to study high-skilled migration flows (Horton, Kerr, & Stanton, 2017).

Given that my empirical setting is freelancers in New Zealand, I note that there is little work on the flexible in the New Zealand context, perhaps due to the recency of the rise in the gig economy. Riggs, Sin, and Hyslop (2019) use aggregate survey measures produced by Statistics New Zealand to characterize the gig economy, and Sarina and Riley (2018) discuss the New Zealand gig-economy in the employment relations context.

The third literature I contribute to looks at optimal time allocation on a job, and the value to spending time on non-work commitments. Early theoretical contributions such as Becker (1965), Gronau (1973), and Gronau (1977) discussed how workers and entrepreneurs should optimally allocate time across work, leisure, family, and non-work commitments. More recently, studies such as Aguiar and Hurst (2007), Ramey (2009), Ramey and Francis (2009), and Pabilonia and Vernon (2020) have used time-use surveys to quantify the amount of time individuals allocate to work and other activities. A particularly salient branch of this literature looks at time allocation during the Covid-19 pandemic. The uprooting of traditional work arrangements, and increased flexibility of working-from-home (WFH) caused many workers and households to change their time-allocation habits. Papers in this strand of the literature include Andrew et al. (2020), Meekes, Hassink, and Kalb (2020), Hupkau and Petrongolo (2020), and Alon, Doepke, Olmstead-Rumsey, and Tertilt (2020).

Layout

The remainder of the paper is arranged as follows. Section 2 outlines a behavioral model of freelancer job search, and documents some theoretical predictions from the model. Section 3 discusses the novel administrative dataset of disaggregated workers in New Zealand used throughout the paper, and section 4 outlines some reduced-form empirical facts on freelancer job-search and task-completion arising from the data. Section 5 the empirical estimation approach and section 6 discusses identification of the parameters in the model. Section 7 discusses the results of this model, and quantifies the decisions and trade-offs in the freelancer search process. Finally, section 8 concludes and discusses avenues for further research.

2 Model

Time is continuous, and the future is discounted at rate ρ. The economy is made up of infinitely-lived freelancers who search for jobs. A job in this model takes on a specific form, based on the types of work contract that freelancers engage in. Namely, a job is a commitment to perform T hours of work, in exchange for some up-front payment P.[5] For example, a job for a freelance web developer might be to produce a website that will take 100 hours in total to build (so $T = 100$), for which they will be paid some fee P. The worker gets utility $u(P)$ from a payment P, where u is increasing, concave, and $u'(0) = \infty$.

Reflecting the flexible nature of freelancing, the worker can choose exactly how they want to complete the T hours of the job. In particular, worker who accepts a job of size T chooses: (1) an optimal time horizon over which to complete this work, W; and (2) an optimal schedule of hours $(h_t)_0^W$ to work over this horizon.

The hours schedule $(h_t)_0^W$ need not be constant. However, in order for a schedule to be *consistent*, it must satisfy the following condition.

Definition 1. *Consistency Given a job size T and a time horizon W, an hours schedule $(h_t)_0^W$ is consistent if*

$$\int_0^W h_t \, \mathrm{d}t = T \tag{1}$$

That is, the total amount of work performed over the time horizon W is equal to the required amount T.

Note that this definition implies that for any consistent hours schedule, a lower W requires a higher average h_t and *vice versa*. This choice of how quickly to complete work captures the idea of the *intensive margin* for freelancers – working more reduces the time to completion. After the time W has passed, the job is complete and the worker returns to search. In the model, the value of search is positive to the freelancer, since they can find jobs (and get paid). Given the discount rate ρ, the discount factor of

[5] I assume that the size of the job and the payment are both known with certainty. An alternative approach may be to model a job as a a specific task whose hours to completion are known only noisily. In such a model, the job size T could be treated as the mean of a distribution, with the worker learning about the true job size over time using Bayesian updating.

returning to search is $e^{-\rho W}$ and so higher W means that the benefit of returning to search is discounted more.

The condition in equation 1 also requires that the worker must complete all the work, and that the job cannot be terminated early (i.e. before T amount of work is completed). This is a specific characteristic of freelancing – client-worker matches break down only after the job is finished, and not due to random layoff shocks or to match-productivity breakdown. There are a number of 'real-world' explanations for this inability to terminate jobs early. For instance, it may reflect legal contracts which stipulate the amount of work to be done, the withholding of (partial) payment until the job is satisfactorily completed, or the fact that reputation effects matter significantly when engaging in the type of freelancing work where clients are searched for directly by the worker. I take the restriction as binding, without explicitly modeling the reason.

The freelancer does not like to work, and the amount that they don't like to work is stochastic. I assume that working h_t hours incurs a stochastic dis-utility $a_t v(h_t)$, where v is a known convex function, and $\log(a_t)$ follows a Gaussian noise process,[6] with $a_t \sim G$ and $\mathbb{E}_G[a] = 1$ and the process variance σ_G^2 is known.

Given a job of size T, a time horizon W, a consistent hours schedule $(h_t)_0^W$, and a path of shocks a_t, the discounted disutility from completing the job is

$$\int_0^W e^{-\rho t} \times -a_t v(h_t)\,\mathrm{d}t$$

This functional form for the disutility of work, in combination with the consistency definition in equation 1, captures one of the key trade-offs that the freelancer faces. Namely, working lower average hours decreases the disutility of task completion, but also requires a longer completion horizon W and hence delays the return to search.

One interpretation of the disutility shocks a is in terms of the random value of the freelancer not working. For a new parent who freelances, the model ascribes a high disutility of working to periods when the child needs maximum attention. For a freelancer who likes to surf, the model ascribes a high

[6]The use of a Gaussian noise shock (as opposed to a shock with more structure such as Brownian motion) simplifies the modeling significantly and keeps the model tractable enough to be empirically estimated with maximum likelihood methods.

dis-utility of working to days when surf conditions are ideal, and a low disutility to days when the waves are bad. I don't model the outside force generating the diutility shocks (e.g. surf conditions), but ascribe any differences between optimal and actual hours worked to these shocks.

An unemployed worker receives no flow benefit.[7]

Job search

An unemployed worker can search for a job, and jobs opportunities arrive at a Poisson rate α. A job opportunity is modeled as a draw from a known distribution $F(T, P)$ over the total time commitment of a job and the upfront payment. The worker may choose to accept the job, in which case she gets paid P immediately[8] and becomes employed with a stock of work T, or may reject the job, in which case she continues to search. There is no on-the-job search, in order to maintain tractability and ensure that the model can be estimated empirically. However, in Appendix A I discuss potential extensions to the model which would allow for multiple job holding.

In either the one-job case or the multi-job case, the trade-off between accepting a job and maintaining flexibility for future offers is clear. In the one-job case, accepting *any* job precludes the worker from searching for other jobs. In the multi-jobs case this trade-off can either be made exogenous or endogenous. An exogenous trade-off could work by imposing a maximum number of jobs J, in which case each job $j \leq J$ limits the ability of the freelancer to accept more jobs in the future. An endogenous trade-off could also be generated since there is an effective upper limit on the total amount of work outstanding $\sum_{j \in J} S_j$ at which the disutility associated with finishing the jobs, no matter how slowly, outweighs the utility of returning to search.

The non-degenerate distribution of P and optimal choice of W mean that there is variation in payment amounts and 'per-week' earnings. This allows me to explicitly model a key feature of freelancing – namely that the average freelancer faces considerably more week-to-week variability in pay than the average worker in traditional firm employment.

[7] In New Zealand, 'self-employed' workers such as freelancers may be eligible for a Jobseeker's Benefit if they are out of work for sufficiently long. See https://www.workandincome.govt.nz/products/a-z-benefits/jobseeker-support.html for details. I abstract from this, since most of the spells without work in the data are short enough to not qualify for a benefit.

[8] I assume that the pay is upfront, rather than paid out week-to-week based on hours worked, in order to simplify the decision problem.

Recursive problem

The problem of an unemployed searcher concerns whether to accept or reject any jobs she meets. The problem of the employed worker concerns the optimal time horizon and work schedule. These two problems can be combined to build a recursive representation of the overall worker problem.

For an employed worker who has just accepted a job (T, P), we can consider her problem in two steps – firstly she chooses an optimal W (taking into account the optimal future choice of hours), and secondly given W and realizations of a_t she chooses $h_t(S_t, a_t)$ where S_t is the amount of work still to be completed at time t. The accept / choice of the unemployed worker, and the W choice of the employed worker can be expressed as policy solutions to infinite-horizon (stationary) Bellman equations, while the choice of $h_t(S_t, a_t)$ can be expressed as the policy function to a finite-horizon Bellman equation equivalent to a cake-eating problem.

Formally, the Bellman Equations characterizing the workers problem are

$$\rho U = \alpha \int \max_{A,R}\{V_0(T) + u(P) - U, 0\}\,\mathrm{d}F(T, P) \tag{2}$$

$$V_0(T) = \max_{W,\,(h_t)_0^W} \mathbb{E}_0 \left[\int_0^W e^{-\rho t} \times -a_t v(h_t(S_t, a_t))\,\mathrm{d}t\right] + e^{-\rho W} U \tag{3}$$

where U denotes the value of unemployed job search and $V_0(T)$ is the value of having a job with time commitment T, at the start of that job.

Equation 2 says that the capitalized value of employment is equal to the Poisson rate α times the expected maximum value of accepting a job (the value of becoming employed with a job of size T plus the utility of payment, less the value of being unemployed: $V_0(T) + u(P) - U$), where the expectation is taken over the distribution F.

Equation 3 gives the value of being employed with a stock of work T – the agent chooses the time horizon W and optimal work schedule $\{h_t\}_0^W$ to maximize utility. The utility has two components – the term in the square braces is the disutility associated with completing T amount of work optimally (and subject to stochastic shocks a_t), and $e^{-\rho W}U$ is the discounted value of returning to unemployed search after finishing the job.

The dynamic process of the stock of work is governed by a time derivative and two boundary conditions, namely

$$\dot{S}_t = -h_t(S, a_t) \quad S_W = 0 \quad S_0 = T$$

The hours choice of a worker who has already chosen their optimal time-frame W can be characterized in terms of the Hamilton-Jacobi-Bellman partial differential equation

$$\rho \mathcal{V}(S, a, t) = \max_h -av(h) + \mathbb{E}\, \mathcal{V}'(S, a, t)\dot{S}(h) \tag{4}$$

For a generic dis-utility function v, equation 4 does not have a closed-form solution. As such, I solve for a numerical approximation to a discretized form of the problem. The numerical solution approach is outlined in section 5.1.

The effect of dis-utility shocks on optimal work schedules

A key consideration in the model is how dis-utility shocks affect the optimal work schedule. Intuitively, a higher dis-utility shock at a particular point in time will result in the agent working less, while a lower dis-utility shock will result in her working more. To provide an illustration of this mechanism, figure 1 shows the optimal work schedule according to the model for an agent who was $T = 400$ hours to complete in a horizon of $W = 10$ weeks, first in the absence of shocks, and then with random dis-utility shocks.

In the absence of shocks, the freelancer works (essentially) constant hours each week, but with dis-utility shocks she works less when dis-utility shock is greater than unity, and more when it is less than unity.

[Figure 1 about here.]

This mapping of dis-utility shocks to hours worked, and the inverse from hours to shocks, is a key explanatory mechanism in the model. The estimated model is able to generate a series of dis-utility shocks corresponding to a given work schedule, which I use to quantify the value of flexible work.

3 Data description

The data used in this analysis are payment-level data on freelancers, collected from users of the Hnry platform. Hnry is a New Zealand company that carries out financial services for independent earners, including invoicing clients, calculation of Pay-As-You-Earn taxes and social security contributions, and budgeting.[9] The data-set covers all gross earnings made by a subset of Hnry users between January 31, 2018 and March 2, 2021, or equivalently, all gross payments made from clients to Hrny users.[10][11] A single worker-client relationship can have multiple associated payments, for example if the client is invoiced weekly.

The initial sample covers 2,218 of Hnry's users. I restrict the sample to cover only working-age individuals (aged between 20 and 65, inclusive), and who report having at least 70 percent of their annual income from independent earnings.[12] I also restrict the sample to exclude any worker who held more than one job concurrently. Full information on creation of the analysis sample can be found in Appendix B, but the final sample contains information on 9,863 payments for 1,131 jobs, performed by 569 freelancers, and constituting 23,917 worker-weeks.

Calculating hours worked

A key variable of interest in this paper is the amount that a freelancer works in a given week. This is not directly measured in the Hnry dataset – I observe the date and amount of each payment, but not the amount of work that the payment is for. In order to estimate the amount that the freelancer worked per week, I consider an *Implied Wage*: the amount that a worker typically earns for a week of their time. I can then estimate Time Worked worked using the following identity and considering different horizons.

[9] Workers in most 'traditional' jobs in New Zealand have taxes withheld by their employer, and are able to file an individual tax return at year-end using an IR3 form. This is similar to the use of a W-2 form in US tax jurisdictions. For freelancers who work directly with clients, this type of withholding is not done during payment or invoicing stages, and hence freelancers must do tax calculation manually. Hnry simplifies the tax-calculation process for freelancers, by providing a service that calculates and withholds the correct amount of tax from gross earnings.

[10] Hnry's user base has grown considerably since January 2018. As such, the payments are heavily backloaded – the median payment date is August 19, 2020, and 15 percent of payments were made since the start of 2021.

[11] The sample period includes the Covid-19 Pandemic. I don't explicitly control for the pandemic in my estimation, since New Zealand's response to the pandemic meant that there was relatively little disruption in day-to-day life (less than four weeks of 'lock-down') for many New Zealanders before August 2021.

[12] Some of Hnry's users use independent earnings to supplement traditional employer-employee income sources. In order to calculate accurate tax withholding, they report estimated off-platform income to Hnry. I calculate independent income share using reported 2021 estimates.

$$\text{Total Pay} = \text{Implied Wage} \times \text{Time Worked} \tag{5}$$

In particular, I observe *Total Pay* and *Time Worked* over the full sample in which the worker is on the platform, and so am able to estimate the worker's Implied Wage by taking the total amount earned by the worker on the platform over their entire time on the horizon, and dividing by the total number of weeks the worker has been employed for on the platform.[13]

In order for this to be accurate, I need the following assumption.

Assumption 1. *The implied wage is constant for a given worker and job over time*

The Time Worked per week is then the payment received in a given week, divided by Implied Wage of the worker. This is simply the identity in equation 5, expressed in weekly terms (since I observe *Total Pay* for the week).

In order to map these payments to when the work is performed, I also need the following assumption.

Assumption 2. *The delay between performing work and receiving payment is constant for a given worker.*

This assumption means that I can treat the payment date and the work date as identical for the purposes of the model. A final adjustment I make is to *smooth* time worked across less-than-weekly payments for a job. For example, if a worker receives payment bi-weekly for a job (every period $t = 2n$ for $n = 1, \ldots, T/2$), I assume that the payment was made equally for work in periods t and $t-1$ (i.e. I smooth the estimated time worked across the immediately-preceding period without payment). This assumption also means that I can consistently allocate the first payment received by a worker work in a given period (or set of periods). In particular, if the first payment is received in period S, and the worker gets paid on average every n periods,[14] then I count the first payment as counting for periods $S-(n-1), \ldots, S$.

Note that this procedure produces an estimate of 'relative hours' worked by the freelancer each week – that is, what fraction of the freelancer's typical hours did they work in a given week. A value of one

[13] This is a *weekly wage*, i.e. the amount that the worker earns in a typical week. I focus on this in order to avoid any heterogeneity in terms of full-time versus part-time employment – I instead consider everything in terms of the typical week for the worker.

[14] I calculate n as the `floor` of the mean difference in weeks between payment dates over the duration of the job.

indicates that the freelancer worked their typical hours in a given week, while a value below (above) one indicates that the freelancer worked more (less). In some cases, I will scale this value by 40, to give results that are interpretable in terms of a standard work week.

An advantage of this approach to creating a work index is that any fundamental differences in how much a freelancer wants to work on average are stripped out, allowing for a more in-depth look into variation in work. This is a useful feature since freelancers are likely to work different average amounts of time, for example full-time vs part-time. One drawback, however, is that only within-worker comparisons are valid – since I don't know the typical hours worked of any freelancer in the dataset, I cannot compare the hours worked across freelancers.

Example work paths

Figure 2 shows some example work paths across jobs for a characteristic worker in the Hnry sample. While some of the jobs have relatively stable work amounts, some jobs see considerable variation in the amount worked from week to week.

[Figure 2 about here.]

4 Model-free results

Before estimating the model, it is useful to consider some reduced-form correlations and results from the data. In all cases, I discretize the hours and wages to a weekly level. This provides an approximation of the continuous-time optimization problem considered in the model.

A key part of understanding the task-completion behavior of freelancers is to assess how workers choose their hours over the duration of the job (i.e. what the optimal hours schedule looks like). I am interested in both the level and variability of work over time.

The first empirical result I find is that freelancers tend to 'front-load' work, completing more hours at the start of a job than at the end of the job. Formally, I run a regression of the number of hours worked each week on the 'time left' on a job (how many weeks they have until the job is finished).[15]

[15] My assumption on job deadlines is that the worker chooses a work horizon W first, and then selects an optimal hours

The regression is of the form

$$\log h_{ij} = \beta_0 + \beta_1 \log(TL_{ij}) + \delta_i + \delta_j + \varepsilon_{ij}$$

where h_{ij}) is the weekly hours worked by worker i in job j, TL_{ij} is the number of calendar weeks until the job is finished, and δ coefficients are worker and job fixed effects.

The results of the regression are shown in table 1. I find that the elasticity of weekly hours-worked with respect to outstanding work is around 0.16.

[Table 1 about here.]

As well as considering how the level of optimal hours changes over the course of a job, I'm interested in how the variability in hours changes over the course of a job. This cannot be done at a worker-job level, since we only have a single observation. As such, I take all jobs of a similar length, and consider the cross-sectional variance (across jobs) in the number of hours worked in each week of that job. Figure 3 shows the variance of hours across week for eight-week long jobs. I find that there is considerably higher variance in the number of hours worked in the early period of the job than at the end of the job. This is consistent with a framework of procrastination, in which workers get more focused as the end of a job nears, and hence the distribution of hours-worked becomes tighter.

[Figure 3 about here.]

Next, I consider the reduced-form relationship between the flexibility of work and the pay required to perform that work. This will provide a reduced-form estimate of the value of flexibility – lower wages for jobs with higher variance in hours could indicate that workers are willing to accept less pay in return for more-flexible work.

I estimate a regression of the average pay for a job on the variability in the hours worked per week on that job, of the form

schedule h_t given W so as to complete the job of size T. Under this assumption, W reflects a hard deadline, and the number of calendar weeks left on the job is a well-defined and exogenous object. However, in the data, this 'time-left' variable is potentially a source of endogeneity. Namely, a freelancer who works more over the duration of a job (higher h_t) will end up completing the job faster (lower W), which influences the deadline and hence the time left. Thus, while the reduced-form regression results are potentially useful for understanding mechanisms, the parameters should be be treated with caution, and should be compared to the structural parameters from the estimated model.

$$\log P_{ij}/T_{ij} = \beta_0 + \beta_1 \log \sigma^2(h_{ij}) + \delta_i + \delta_j + \varepsilon_{ij}$$

where P_{ij}/T_{ij} is the pay per week for worker i in job j, $\sigma^2(h_{ij})$ is the week-to-week variance in hours worked in that job, and δ coefficients are worker and job fixed effects.

Table 2 presents the results of this regression. The negative coefficients suggest that jobs with higher variability are associated with lower pay. While this regression is only correlative, one interpretation is that jobs which allow for more week-to-week variance in hours (i.e. are more flexible) are more attractive to freelancers, and hence have a lower reservation wage.

[Table 2 about here.]

5 Estimation

5.1 Numerical approximation

For the chosen functional form of v equation 4 does not admit a closed form solution.[16] Thus, I solve a discretized version of the model and obtain an approximate solution.[17]

My approximate solution approach proceeds by imposing a functional form on the optimal number of hours h_t. For a given work horizon W, equation 3 is solved by a sequence $\{h_t\}_0^W$.[18] I impose a flexible functional form for this sequence, which allows me to parameterize the solution rather than maximizing over an infinitely-dimensional object. In particular, given that the worker problem has continuous-time exponential discounting, I use the functional form[19]

$$h_t(S, \cdot) = \phi \frac{S}{W} e^{\gamma t}$$

[16] To my knowledge, this is true for any non-trivial function v.

[17] Estimation code is available at https://www.jedmsarmstrong.com/

[18] This is the advantage of having a relatively structure-free process for the stochastic shock a – in expectation a is 1, and so the worker's optimization is not affected.

[19] This functional form was chosen as it is approximately equal to the disutility-minimizing choice of hours calculated using polynomial approximation or a neural net. Appendix C provides more details.

where $\gamma \in \mathbb{R}$ is a parameter to be estimated which controls how much the worker likes to vary work over time, and ϕ is a scalar chosen to ensure that the consistency condition in equation 1 holds.[20] Note that for $\gamma < 0$ the freelancer will work more at the start of the job and less at the end, and for $\gamma > 0$ the freelancer will work more at the end of the job and less at the end.

Given a functional form for h_t, the optimal value of W can be found using numerical integration and an optimization toolbox.[21]

I also discretize the distribution $F(T, P)$ over a grid of values for T and P to allow for numerical integration of the expected value in equation 2. I set the minimum and maximum grid values using the minimum and maximum values from the data. I oversample for lower values (reflecting the approximate log-normal marginal distributions of F seen in the data), by taking the exponent of a linear grid between the log minimum value and the log maximum value.

The overall solution algorithm for a given parameter vector is as follows.

1. Initialize V_e as a vector of zeros and U as a scalar 0
2. For each T in the grid of T values
   (a) Choose the optimal $W^*(T)$, given the functional form for h_t and the value T
   (b) Update the V_e vector to the optimal value, using $W^*(T)$
3. Calculate the Accept / Reject policy function by comparing V_e to U
4. Calculate the value of the expected maximization in equation 2 and update U
5. Go to 2. and repeat until convergence in V_e and in U

Convergence of the algorithm is guaranteed by the following Theorem, which is proved in Appendix F.

Theorem 1. *The discretized set of Bellman equations obtained from equations 3 and 2 represent a contraction mapping, and hence convergence is the solution algorithm is guaranteed.*

[20] The use of the scalar ϕ is simply notational convenience, since the consistency condition implies that

$$\int_0^W \phi \frac{T}{W} e^{\gamma t}\, \mathrm{d}t = T \implies \phi = \frac{1}{\int_0^W \frac{1}{W} e^{\gamma t}\, \mathrm{d}t}$$

[21] I use MATLAB's `quadgk` to integrate over the function h_t, and then find the optimal W using `fminbnd`.

5.2 Functional forms and parameterization

I assume that the work disutility function takes the form

$$v(h) = \delta e^{bh}$$

where $\delta > 0$ and $b \in [0, 1]$. Note that v is differentiable and convex.

I assume that $F(T, P)$ takes the form of a multivariate log-normal distribution with mean and covariance given by

$$\mu_F = [\mu_T \ \mu_P]'$$

$$\Sigma_F = \begin{bmatrix} \sigma_T^2 & \rho\sigma_T\sigma_P \\ \rho\sigma_T\sigma_P & \sigma_P^2 \end{bmatrix}$$

I calibrate the discount rate $\rho = 0.001$, which corresponds to a yearly discount rate of 5 percent. I also calibrate the utility function over payment to be a CRRA utility function

$$u(P) = \frac{P^{1-\eta} - 1}{1 - \eta}$$

with $\eta = 0.7$ which is in line with the literature. The upfront payment P is the net present value (discounted using the discount rate ρ) of all payments received by the freelancer over the duration of the job.

The parameters of interest to be estimated are: α, the arrival rate of job offers; a and b, the disutility function parameters; γ the curvature parameter in the optimal hours function; the parameters of the F distribution, namely μ_F and Σ_F; and the variance parameters for the shock distributions: σ_W, σ_h, and σ_a.

5.3 Maximum Likelihood Estimation strategy

For notation purposes, I combine the parameters to be estimated into a vector

$$\theta \coloneqq (\alpha, \delta, b, \gamma; \mu_F, \Sigma_F; \sigma_W, \sigma_h, \sigma_a)$$

I also define the following vector of observations

$$y \coloneqq ((h_{ijt})_{ijt}; (W_{ij})_{ij}); (d_{is})_{is}; (T_{ij})_{ij}; (P_{ij})_{ij})$$

That is, the observable data are the hours worked by freelancer i in job j in period t, the durations of unemployment for freelancer i in unemployment spell s; and the observed time commitments T and payments P for each freelancer-job pair ij.

The MLE procedure is summarized by the following optimization problem

$$\hat{\theta} = \arg\max_{\theta} \mathcal{L}(\theta; y) \tag{6}$$

The likelihood function $\mathcal{L}$ is made up of three components: 1) the likelihood of the observed unemployment-spell durations; 2) the likelihood of the observed job-completion horizons; and 3) the likelihood of the disutility shocks implied by the observed hours-worked paths.

Likelihood of unemployment-spell durations

The likelihood of an unemployment spell of length $d-1$ followed by finding a job T, P is

$$((1-\alpha) + \alpha F(T^*, P^*))^{d-1} \times \alpha \tilde{F}(T, P)$$

where the first term is the probability of either not receiving a job offer $(1-\alpha)$ or meeting a job that is unacceptable $(\alpha F(T^*, P^*))$ for $d-1$ periods, and the second term is the probability of finding an acceptable job offer with commitment T and pay P.

Likelihood of W and h observations

The model produces prediction for the optimal job horizon $\hat{W}$. I then assume that the true W is observed with Gaussian error ε^W, which gives a log-likelihood function

$$\sum_{ij} \log(P(\varepsilon_{ij}^W | \sigma_W))$$

Likelihood of imputed disutiility shocks a

Finally, I use the model to back out the un-observed disutility shocks A_{ijt}. These shocks measure the difference between the actual hours-worked and the model prediction of the optimal shock-free hours-worked.

Given the Gaussian noise process outlined in section 2, this implies a log-likelihood function

$$\sum_{ijt} \log(P(A_{ijt} | G))$$

The overall likelihood function is then

$$\mathcal{L}(\theta; y) = \mathcal{L}_d(\theta; y) + \mathcal{L}_W(\theta; y) + \mathcal{L}_h(\theta; y) + \mathcal{L}_a(\theta; y)$$

Numerical procedure for obtaining the MLE

The maximization in equation 6 is performed numerically using MATLAB's `fminsearch` function, which uses the Nelder-Mead simplex algorithm (Lagarias, Reeds, Wright, & Wright, 1998). An advantage of this algorithm is that it doesn't require the use of gradients, and hence is computationally less expensive. However, for highly non-convex problems such as the maximization in equation 6, the algorithm is not guaranteed to converge to a global minimum. In order to check the robustness of the minimization procedure, I test a range of starting parameter values and optimization algorithms. These robustness checks are outlined in Appendix E. I find that the MLE estimator $\hat{\theta}$ is relatively robust to alternative specifications.

6 Identification

A key element to the identification model is the manifold of minimally-acceptable (T, P) values. This manifold can be written as a correspondence outlining the lowest possible payment P requeried to perform a task of size T: $P^*(T)$.[22] This is the two-dimensional analogue to the reservation wage w^* in a search model with a one-dimensional job contract in which a job is determined only by a wage. Using similar arguments to Flinn and Heckman (1982), it is possible to show that $P^*(T)$ is identified as long as we impose some structural form on the underlying offer distribution $F(T, P)$. The use of a multivariate log-normal distribution thus means that $P^*(T)$ is identified, and hence we can identify the parameters of the underlying distribution $F(T, P)$ using the observed (T_{ij}, P_{ij}) pairs in the data.

The identification of the offer-arrival rate α using unemployment durations is described in Eckstein and Wolpin (1990). In my particular setting, it's useful to note that the distributional assumption for job arrival times (an Exponential distribution, arising from a Poisson process) allows for robust identification even when some spells are right-censored (if freelancers are unemployed at the end of the sample period).

7 Model Results

Model fit

The model fits key moments of the data well, and the functional form assumptions seem to be well justified. Table 3 shows how the model matches the data along some important moments, and Appendix D shows the fit graphically.

[Table 3 about here.]

[22] Under the relatively innocuous assumption that workers strictly like money and strictly don't like to work (i.e. that $u'(P) > 0$ and $v'(h) < 0$, this correspondence will be a function.

Compensating differentials

In order to quantify the value of flexible work to freelancers, I use the estimated model to compare the dis-utility of completing jobs with a counterfactual work schedule requiring constant hours (as would be the case in many traditional jobs). In particular, I compare the flexible-work dis-utility of job j worked by worker i

$$V_{ij}^F = \int_0^{W_{ij}} e^{-\rho t} \times -a_{ijt} v(h_{ijt}) dt$$

to the constant-work dis-utility

$$V_{ij}^C = \int_0^{W_{ij}} e^{-\rho t} \times -a_{ijt} v(T/W) dt$$

where a_{ijt} are the estimated dis-utility shocks from the model. Note that the constant-work disutility has $h_t = T/W$ for all t. I then calculate the minimum acceptable payment required for the worker to accept the job – the payment that delivers exactly enough utility to offset the dis-utility from completing tho job. Namely

$$P_{ij}^F = u^{-1}(-V_{ij}^F) \qquad P_{ij}^C = u^{-1}(-V_{ij}^C)$$

The compensating differential of job j worked by worker i is then defined as the (percentage) difference between the flex-work minimum acceptable payment and the constant-work minimum acceptable payment

$$CD_{ij} = \frac{P_{ij}^F - P_{ij}^C}{P_{ij}^F} \times 100$$

The distribution of compensating differentials is shown in figure 4. The mean CD is .[23] This is in line

[23]There are a small number of negative estimated compensating differentials – situations in which the worker would have been *better off* engaging in constant-hours work rather than flexible work. This is typically situations in which very large high (or low) dis-utility shocks at the end of the job required them to work more (or less) than the expected, and their overall dis-utility could have been reduced by working less (or more) at the start of the job.

with the literature – for example Mas and Pallais (2017) finds that workers are willing to give up 20 percent of wages to avoid a rigid schedule set by their employer, with higher willingness to pay identified for women, particularly new mothers, and Lim (2017) finds that the value of flexibility to new mothers is valued at around $7,000 annually, or 20 percent of average wage earnings.

[Figure 4 about here.]

The extreme right tail: Flex-or-Quit workers

An interesting subset of the sample is the group of workers in the far right tail of the distribution of compensating differentials. These are workers whose compensating differentials are so large that they would be unlikely to find a fixed-schedule job that offered their required pay. Thus, these workers may be in the position of *only* supplying labor if they are able to pick and choose their hours flexibly. Around 10 percent of jobs have compensating differentials of more than 100 percent, indicating that workers would need to have their compensation doubled in order to move away from a flexible work arrangement.

From a fundamental perspective, this type of worker makes sense – if non-work commitments are random enough and the associated disutility of work is great enough, then *any* fixed-hours schedule with a reasonable salary may be too costly for the worker to take on. For example, some individuals with unpredictable health conditions or some new parents may find that the cost to forgoing non-work commitments is so large that they would prefer to not work at all than to work in a non-flexible arrangement.

8 Conclusion

In this paper, I study the value to freelancers of flexible work. Formally, I quantify the value of an increasingly-common feature of modern labor market arrangements: being able to choose exactly when and how much to work. My starting point is a comprehensive dataset of earnings for New Zealand freelancers. I document large variations in hours-worked from week-to-week, even when a freelancer is working on the same project with the same client for months at a time. I rationalize this by building a model in which freelancers working on some task can vary their labor supply up or down in response to stochastic variation in the disutility of work. My model suggests that the value of high-frequency variation in labor provision is considerable. Workers in the model would need to have their compensation increased by 25 percent on average to complete their tasks under a rigid fixed-hours schedule.

While I focus on freelancing and 'non-traditional' work arrangements in this paper, insights into the value of flexibility work are applicable to many traditional worker-firm pairings as well, particularly in the service industry. The Covid-19 pandemic led to a rise in working-from-home for many, pushing workers away from rigid 9-to-5 schedules, and allowing for flexibility to take short breaks for exercise, child-care, or to run errands (Bloom, Han, & Liang, 2022). Although my empirical setting is a self-selected group who have decided to become freelancers (and hence who may place higher valuation on flexibility than the average worker), the results of this paper can be used to shed light on why many workers are reticent to return to the office and to binding schedules.

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Appendix A Extension to multiple job holding

The core model of this paper assumes that a freelancer can only work one job at a time, and hence precludes on-the-job search (OTJS). This ensures that the model remains tractable and is able to be estimated empirically, but it may not be a strictly realistic assumption. Indeed, Armstrong (2022) (a companion paper in this dissertation) shows that over half of workers in the full Hnry sample work with two or more clients concurrently over the sample period.

Extending the model to allow for multiple job holding is relatively straightforward from a theory perspective, but makes estimation considerably more complex. Indeed, I am unable to estimate even the two-job version of the model in a reasonable amount of time with access to current computational resources. As such, in this Appendix I outline the theoretical extensions to the model to allow for two (or more) concurrent jobs, and briefly discuss some ways in which computational estimation of the model could proceed.

A key difficulty of the multi-job version of the model is that the state-space of the model and dimensionality of the worker optimization problem both increase with each additional job. Keeping track of the number of hours outstanding on a single job (denoted S in the exposition in section 2) is computationally inexpensive. Similarly, determining the optimal number of hour worked as a function of outstanding work and the labor disutility shock (denoted $h(S, a)$) is fairly computationally easy. When moving to multiple jobs (for instance, the case when the worker can hold a maximum of J jobs concurrently), it is necessary to keep track of the amount of work outstanding for each job: $S_1, \dots, S_J$, and to calculate the optimal hours worked in each job as a function of all of the hours outstanding in all jobs: $h_1(S_1, \dots, S_J; a), \dots, h_J(S_1, \dots, S_J; a)$. For even fairly reasonable numbers of J, the state-space becomes unmanageable: (Armstrong, 2022) shows that 80 percent of freelancers in the data work with no more than 5 clients concurrently in a month, but even this is a large enough state space to make computation prohibitively intensive.

Appendix B Creation of the analysis sample

The original dataset is at a worker-client-payment level. I first aggregate to a daily level, to capture instances in which a client paid a worker multiple times on the same day. I trim the sample to those aged between 20 and 65, and then remove any workers whose off-Hnry (i.e. non-SE) share of annual income is greater than 30 percent. This drops less than 20 percent of the sample (see figure 5 for a distribution). I then restrict to only workers who have no more than one job concurrently at any time in the sample.

[Figure 5 about here.]

A judgment call required in the sample creation process is how to treat extended periods of non-payment between a client and a worker. It is common in the data to have long-lasting client-worker relationships with periods of no pay. I assume that a period of 60 days without payment constitutes the end of a job, and so if a client-worker relationship has a payments separated by at least 60 days I assume that to be multiple jobs. Figure 6 shows a plot of payments for a typical client in the Hnry data-set. The size of the points is proportional to the payment amount, while the color indicates the job number between the worker and the client. For three of the clients this worker worked for, there were 60-day gaps between payments, meaning I classified them as new jobs.

[Figure 6 about here.]

Appendix C Approxmiting the functional form for hours-worked

As noted in section 5.2, I impose a functional form restriction on the optimal 'shock-free' hours schedule, namely

$$h_t(S, \cdot) = \phi \frac{S}{W} e^{\gamma t}$$

This functional form assumption makes the solution method and estimation procedure considerably more simple.

I argue that this particular functional form is a good approximation to the 'actual' optimal hours schedule for a given T and W. To test this, I find the actual optimal hours schedule using the following utility maximization (or equivalently, disutility minimization) procedure.

$$\max_{\{h_s\}_{t\in\mathcal{T}}} \int_0^W e^{-\rho t} - v(\mathcal{F}(t;\{h_s\}_{t\in\mathcal{T}}))\,\mathrm{d}t$$

Where $\mathcal{T}$ is a fixed set of points along a grid from 0 to W, and $\mathcal{F}(\cdot;\{h_s\}_{t\in\mathcal{T}})$ is a functional approximation of the optimal hours schedule generated using either a polynomial approximation or a simple neural net.[24]

Essentially this maximization procedure involves finding the true optimal hours for a fixed number of times s along the completion path, and constructing a flexible function approximation to the rest of the path. I test for the validity of the functional form assumption in section 5.2 by comparing the function $\mathcal{F}(\cdot;\{h_t\}_{t\in\mathcal{T}})$ to my functional form assumption. I find that the functions differ only a very small amount, indicating that the functional form assumption is acceptable. Some example differences for various T and W values is shown in table 4.

[Table 4 about here.]

Appendix D Model fit

[Figure 7 about here.]

[Figure 8 about here.]

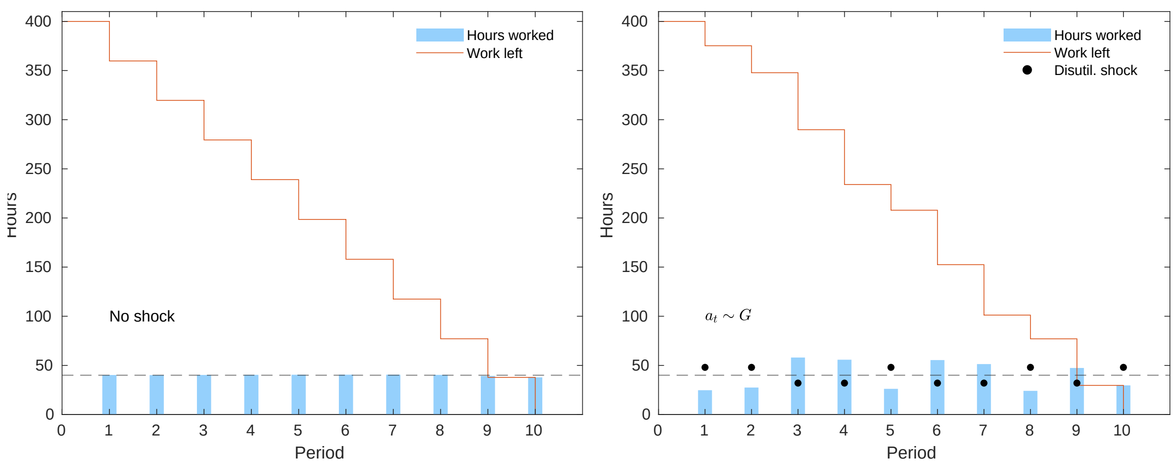
[24] I used a polynomial with order $p = 3$, and a neural net with 1 hidden layer, in order to avoid over-fitting. I used MATLAB's Deep Learning Toolbox to train the neural net, and performed a numerical integration using the `trapz` function.

Appendix E Robustness of MLE to alternative optimization specifications

Appendix F Proof of Theorem 1

The discretized set of Bellman equations obtained from equations 3 and 2 represent a contraction mapping, and hence convergence is the solution algorithm is guaranteed.

Proof. □



(a) No dis-utility shocks

(b) Random dis-utility shocks

Figure 1: Effect of dis-utility shocks on optimal work schedule

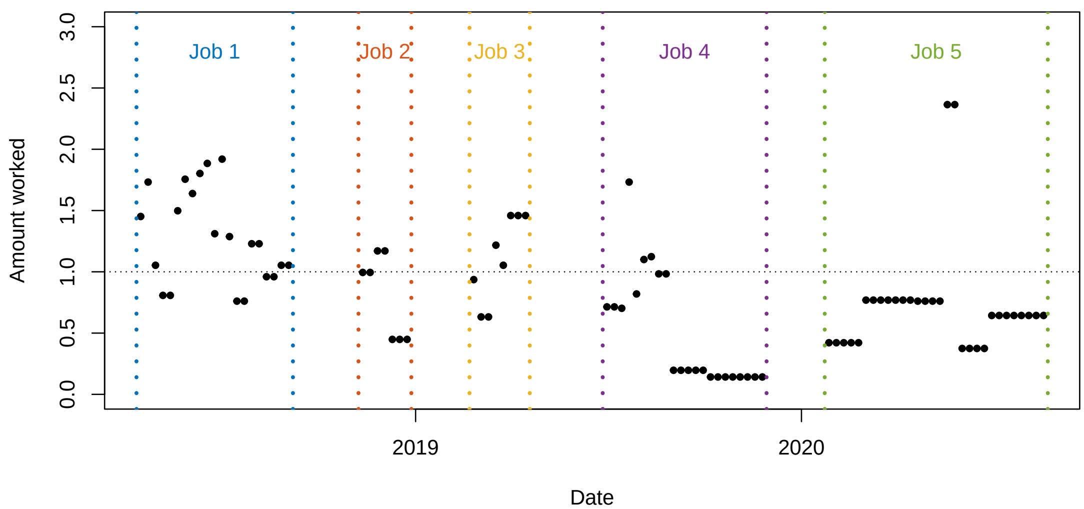


Figure 2: Weekly amount worked for a representative freelancer

Note: Each point is a weekly hours observation, showing what fraction of 'average' hours the freelancer worked that week. The dashed line at 1 corresponds to the 'average' amount that the freelancer worked over the sample period (across all jobs), and so points above the line are when the freelancer worked more that their average, and points below the line are when the freelancer worked less than their average. The freelancer worked five separate jobs over the sample period, the start and end dates of each job are indicated by the dashed lines.

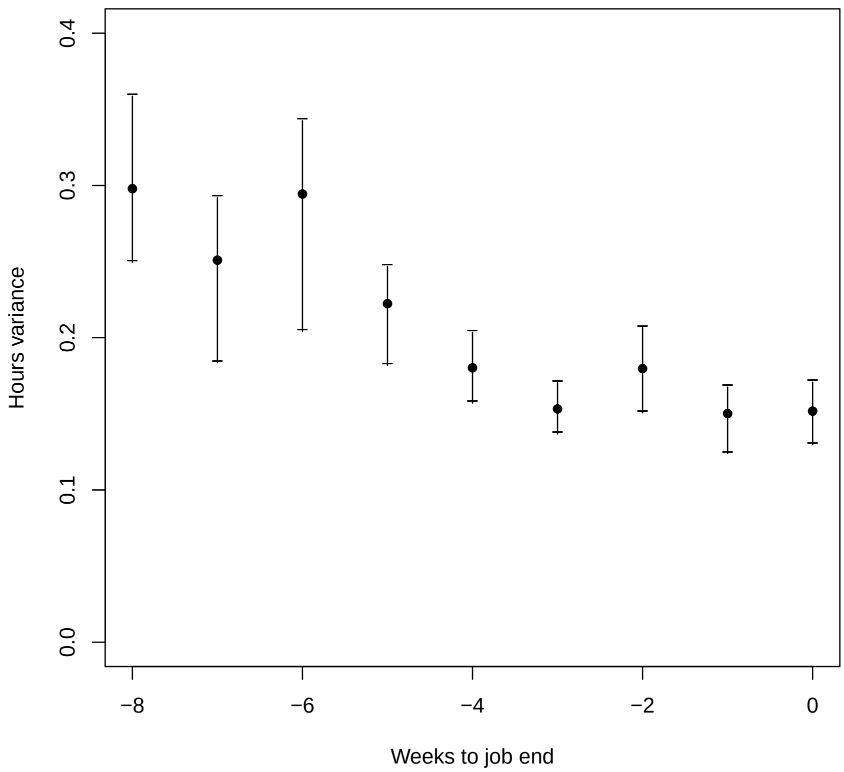


Figure 3: Cross-sectional variance in hours worked by job week for all jobs of length 8 weeks

Note: The line segments are 95 percent confidence intervals generated by running $N = 100$ 80 percent bootstraps. A regression line through the points has slope -0.0198, and is statistically significantly different from 0 at a 1 percent level. Due to the potential for jobs to be missed due to discretization error, I include all jobs whose 'calendar length' in the data is between six and ten weeks.

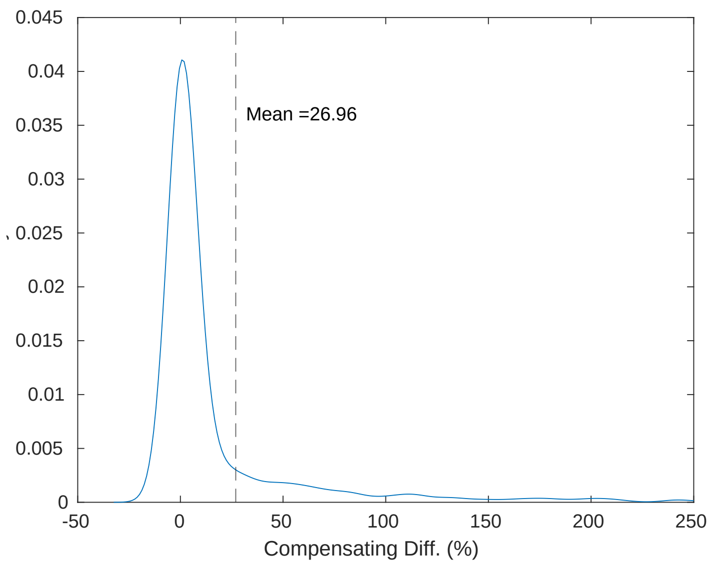


Figure 4: Compensating differentials (in percentage terms) for shifting from flexible work to constant-hours work

Note: This is the distribution of compensating differentials for *jobs* – how much more would the worker have to be paid in order to perform a given job under a fixed-hours schedule.

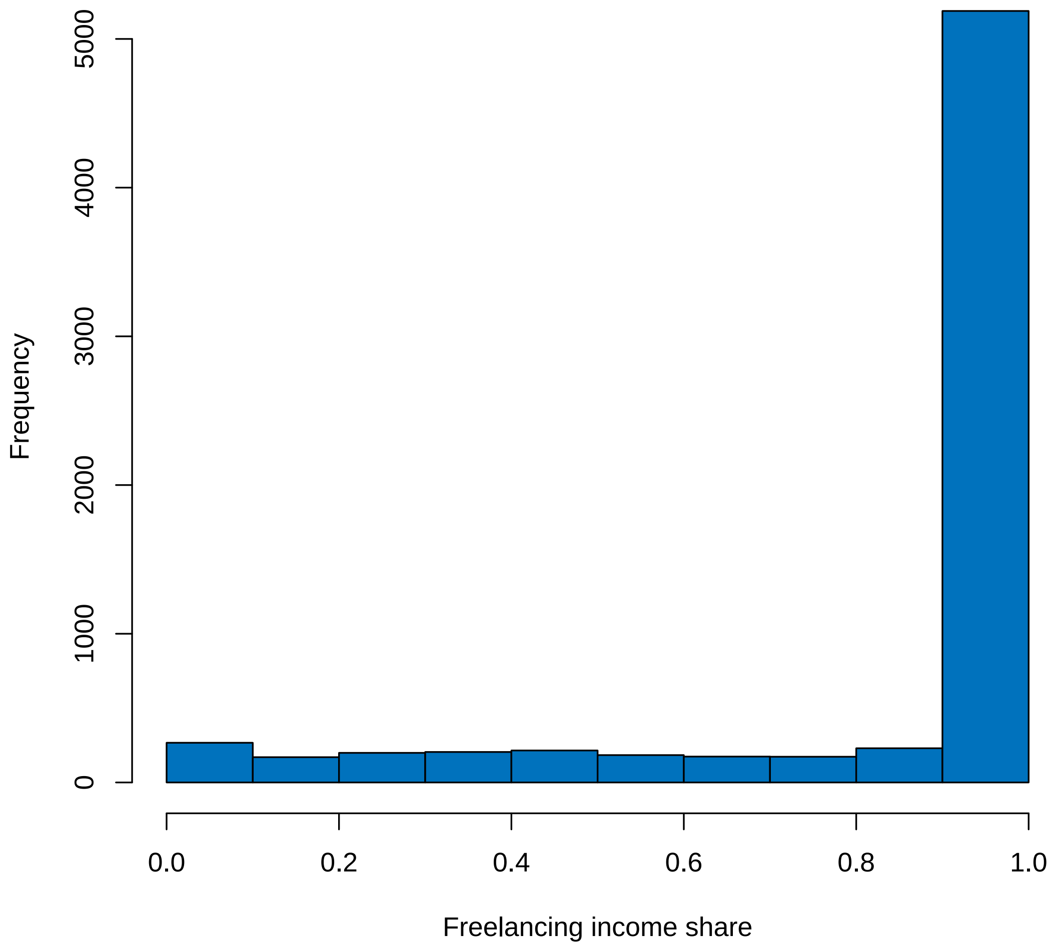


Figure 5: Share of freelancer income

Note:

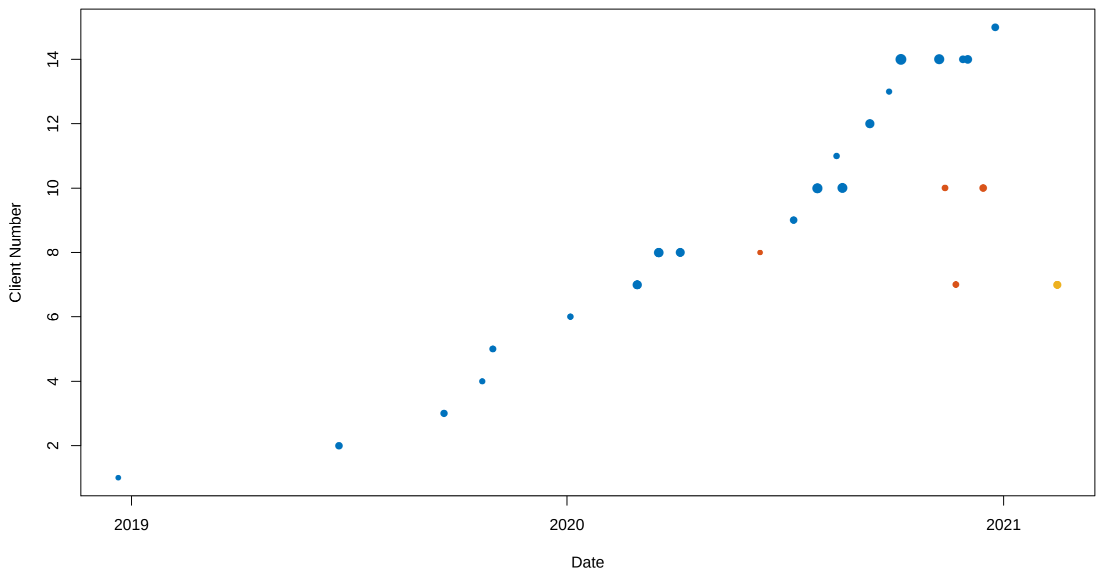


Figure 6: Jobs worked for a representative freelancer

Note: Each point is a payment made between the client and the worker. The freelancer worked five separate jobs over the sample period, the start and end dates of each job are indicated by the dashed lines. Different colored dots represent different 'jobs' worked by the freelancer for the same client. I have added randomness to dates, some client numbers, and payment amounts in order to preserve anonymity.

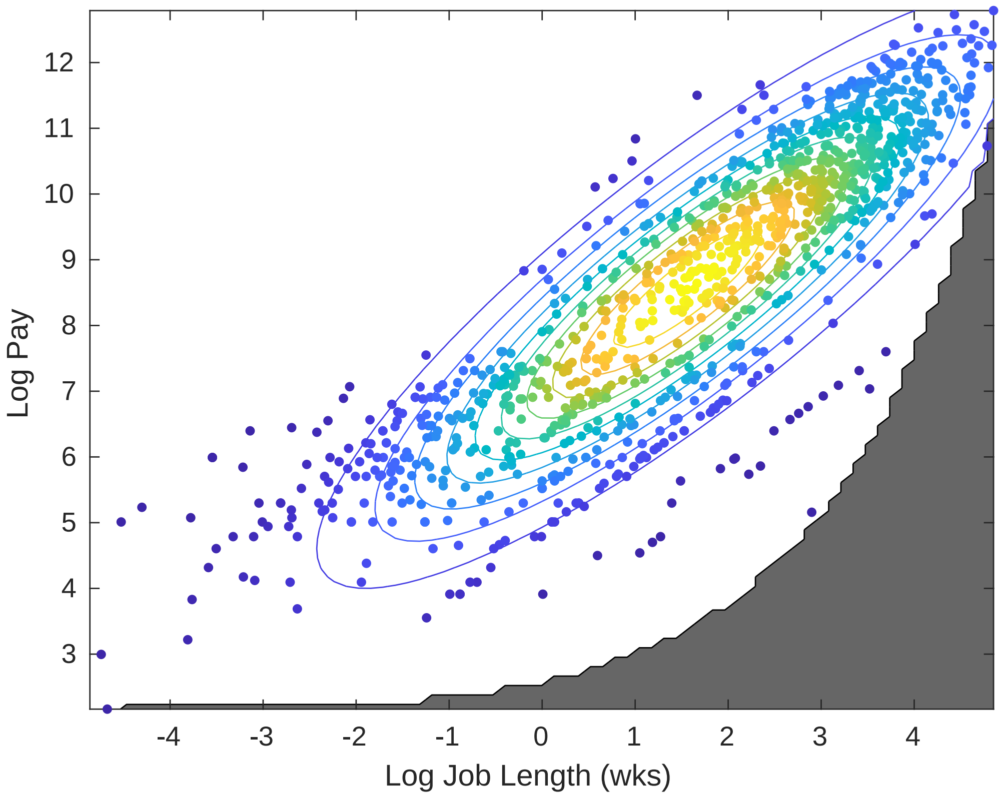


Figure 7: Model fit for the distribution $F(T, P)$

Note: Each point is a worker-job, showing the time commitment T and the (equivalent) upfront payment P. The contours show the distribution likelihood from the model, and the color of each point is relative to the likelihood of the observed T-P pair in the model. The grey area is the set of T-P pairs which are rejected in the model – anything above the grey line will be acceptable to agents in the model.

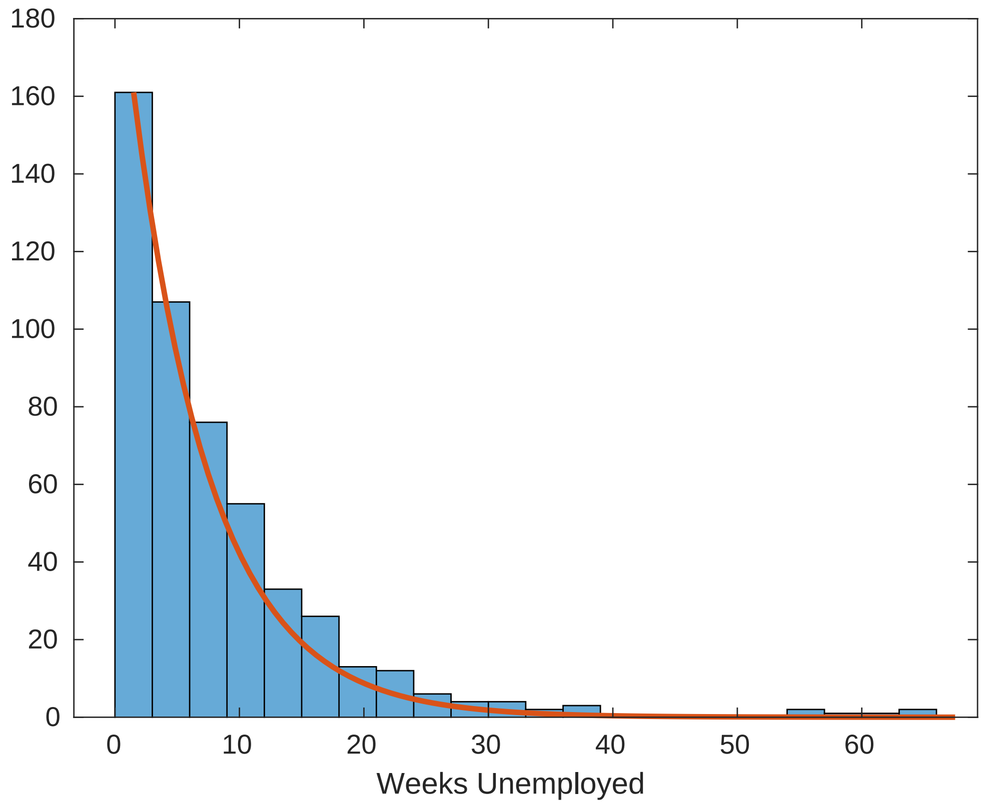


Figure 8: Model fit for the distribution of unemployment spells

Note: The blue bars show a histogram of the actual unemployment spells in the data, while the red line shows a fitted exponential distribution with parameter α as estimated in the model.

Table 1

Log Weeks Left on Job	0.1270	0.1654	0.1629
	(0.0044)	(0.0037)	(0.0038)
Controls	-	Work Type	Worker
N	19,666	19,666	19,666
Adj. R^2	0.09329	0.1263	0.2632

Table 2

Hours Variability	-0.161	-0.124
	(0.034)	(0.032)
Controls	-	Work Type
N	704	704
Adj. R^2	0.030	0.275

Table 3

Moment	Model	Data
Average unemployment duration		
Average job length		
Average job payment		
corr(job length, job payment)		

Table 4

Param. Vals		$\mathcal{F}$ form	
T	W	Poly.	Neural Net
	3		
120	5		
	10		
	10		
400	16		
	25		

Note: The job length T and the time horizon W can be interpreted in terms of hours and weeks respectively. The differences in columns (3) and (4) are the $L-2$ norm between the optimal functional approximation $\mathcal{F}$ and the functional form outlined in section 5.2.