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PHILOS 12A / DIS 102

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Problem Set #5

Exercise 5.2

This is not valid. In order to make $P \lor Q$ True, either P or Q has to be True. P can also be True in this case. Because of the given inference ($\neg P$), it is possible for P to be True and (not only) False. In a Truth Table, it is seen that P is and can be True. This possibility and others can be seen in the Table. Nevertheless, it is not valid.

To further clarify my point, As an example, P can be “I went to the library to study.” For Q, it can be “I went to the kitchen to cook.” Not P would be “I didn’t go to the library to study.” Joining the sentences together with the disjunction would be “I didn’t go to the library to study, or I went to the kitchen to cook.” As a result, Not P does not make this valid.

P	Q		$P \lor Q$	Q	$\neg P$
T	T		T	T	F
T	F		T	F	F
F	T		T	T	T
F	F		F	F	T

Exercise 5.4

This is valid. The Truth Table can show why it is valid. The conclusion is $\neg(P \land Q)$ having to infer $\neg Q$. When inputting the necessary components, the Table shows a row of True values with the given conditions. As a result, it is valid.

P	Q		¬(P ∧ Q)	P	¬Q
T	T		F	T	F
T	F		T	T	T
F	T		T	F	F
F	F		T	F	T

Exercise 5.8

The proof presented is a valid argument. The premises do lead to a conclusion.

Starting off by using proof by cases, it can be seen in Premise 1 that A is to the left of B OR A is to the right of B.

There is a disjunction for Premise 2 which states that A is to the back of B OR A is NOT to the left of B. In order to make this True, the first component (BackOf(a, b)) depends on the second component (¬LeftOf(a, b)). In this case, the A being to the right of B is True.

For the third premise, there is another disjunction. This time, it is that B is to the front of A OR A is NOT to the right of B. Because of the previous premise, it is known that A is to the right of B, so the second part of the conjunction is False. This also means that B being in front of A is True.

The last premise before the conclusion is a conjunction: C and A are in the same column AND C and B are in the same row. This deems to be irrelevant information as there is no mention of C in the conclusion.

Despite everything, it can be seen that based on the premises, the final conclusion is A is to the back of B. The proof is complete.

Exercise 5.14

Because of the nature of this question/proof, it should be assumed that there is a joint Truth Table with the corresponding True/False values.

If S is a tautological consequence of P, then the True row values should be the same for both P and S. As a result, S is a tautological consequence of P because S and P will have every True value reflected.

This same ideology can be directed towards S and Q. If we look at S being a tautological consequence of Q, then the two would have the same True value rows. S and Q would both reflect the same True rows. As a result, S is a tautological consequence of Q because they share the same results (True values).

In order for S to be a tautological consequence of P ∨ Q, then all True values for S must also be True for P ∨ Q. Because of the previous two paragraphs, it can be seen that P, Q, and S all reflect True values. P and S show the same True values. Q and S show the same True values. Therefore, since P and Q are true, S is also True. S is a tautological consequence of P and Q.

After everything, it can be concluded that when P ∨ Q is True, S will also be True. The final conclusion is S is a tautological consequence of P ∨ Q.

Exercise 5.23

Proof by contradiction is necessary for proving that n is odd when assuming that n^2 is odd. This will be similar to what was talked about in lecture.

Suppose that n^2 is odd but n is even. Because of this, there is an integer k such that n = 2k (by the definition of an even number). n^2 = $(2k)^2$ = $4k^2$ = 2($2k^2$) is then valuable information. This leads to n^2 having an integer m where m = $2k^2$. It can then be given that $n^2 = 2m$.

It can be concluded by the definition of an even number that n^2 will be even. This thus contradicts the given assumption/original statement. n^2 is not odd. Because of proof by contradiction, it can be concluded that if it's assumed n^2 is odd, n is also odd.