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PHILOS 12A / DIS 102

GSI: Mathias Boehm

Problem Set #9

Exercise 9.22

	Malcev's	Bolzano's	Boole's	Wittgenstein's
1.	False	False	True	<i>False</i>
2.	True	False	False	False
3.	True	False	<i>False</i>	False
4.	True	False	False	False
5.	<i>True</i>	False	True	False
6.	False	False	False	False
7.	False	False	True	False
8.	False	<i>True</i>	True	False
9.	True	True	True	True
10.	False	False	True	True

Exercise 10.1

	Annotated Sentences	Truth-Functional Form	a/b/c
1.	$\frac{A_x \ X = X}{A}$	A	b
2.	$\frac{\exists_x \text{Cube}(x) \rightarrow \text{Cube}(a)}{A \quad B}$	$A \rightarrow B$	c
3.	$\frac{\text{Cube}(a) \rightarrow \exists_x \text{Cube}(x)}{A \quad B}$	$A \rightarrow B$	b
4.	$\frac{\forall_x (\text{Cube}(x) \wedge \text{Small}(x)) \rightarrow \forall_x (\text{Small}(x) \wedge \text{Cube}(x))}{A}$	$A \rightarrow B$	b
5.	$\frac{\forall_v (\text{Cube}(v) \leftrightarrow \text{Small}(v)) \leftrightarrow \neg \forall_v (\text{Cube}(v) \leftrightarrow \text{Small}(v))}{A}$	$A \leftrightarrow \neg \neg A$	a
6.	$\frac{\forall_x \text{Cube}(x) \rightarrow \neg \exists_x \neg \text{Cube}(x)}{A \quad B}$	$A \rightarrow \neg B$	c
7.	$\frac{\forall_z (\text{Cube}(z) \rightarrow \text{Large}(z)) \wedge \text{Cube}(v) \rightarrow \text{Large}(b)}{A \quad B} (A \wedge B) \rightarrow C$	$(A \wedge B) \rightarrow C$	b
8.	$\frac{\exists_x \text{Cube}(x) \rightarrow (\exists_x \text{Cube}(x) \vee \exists_y \text{Dodec}(y))}{A \quad B} A \rightarrow A \vee B$	$A \rightarrow A \vee B$	a
9.	$\frac{(\exists_x \text{Cube}(x) \vee \exists_y \text{Dodec}(y)) \rightarrow \exists_x \text{Cube}(x)}{A \quad B} A \vee B \rightarrow A$	$A \vee B \rightarrow A$	c
10.	$\frac{[(\forall_u \text{Cube}(u) \rightarrow \forall_u \text{Small}(u)) \wedge \neg \forall_u \text{Small}(u)] \rightarrow \neg \forall_u \text{Cube}(u)}{B \quad A} ((A \rightarrow B) \wedge \neg B) \rightarrow \neg A$	$((A \rightarrow B) \wedge \neg B) \rightarrow \neg A$	a

Exercise 10.3 & Exercise 10.4

$$\frac{\frac{\frac{\forall_x \text{Cube}(x) \rightarrow \exists_y \text{Small}(y)}{A \quad B} \quad \neg \exists_y \text{Small}(y)}{B}}{\exists_x \neg \text{Cube}(x)}_C$$

A \rightarrow B
B
C
· B: logically
but not tautologically
valid

$$\frac{\frac{\frac{\forall_x \text{Cube}(x) \rightarrow \exists_y \text{Small}(y)}{A \quad B} \quad \neg \exists_y \text{Small}(y)}{B}}{\neg \forall_x \text{Cube}(x)}_A$$

A \rightarrow B
B
A
· A: tautologically
valid

Exercise 10.9

	Paraphrase	Logical Truth	FO Validities
1.	If all the blocks are to the left of B, then B is to the right of the block	Yes	No
2.	If all the blocks are small and the blocks are in the back of C, then the block is a dodecahedron	No	No
3	If all the blocks are cubes and not b, then the blocks are larger than B or the blocks are smaller than B	No	No
4.	If D is a dodecahedron, then all the blocks' named D is a dodecahedron	Yes	No
5.	There are some blocks that if A is smaller than the block and the block is smaller than B, then A is smaller than B	Yes	No
6.	If all the blocks are larger than C, then the block is not C	Yes	No
7.	All the blocks are between A and D or the blocks are not between A and D	Yes	Yes
8.	All the blocks are between A and D or the blocks are not between D and A	Yes	Yes
9.	If all the blocks are dodecahedrons, then the block is D or the block is small	No	No
10.	If all the blocks are to the left of E, then there does not exist a block that is a cube and the block is not to the left of E	Yes	Yes

Part 4

1. $\forall x (\text{LeftOf}(x, b) \rightarrow \text{RightOf}(b, x))$

Counterexample: Replace the red text with SameRow and the blue text with

SameColumn

4. $\text{Dodec}(d) \rightarrow \forall x (x = d \rightarrow \text{Dodec}(x))$

Counterexample: Replace the red text with Dodec

5. $\exists y (\text{Smaller}(a, y) \wedge \text{Smaller}(y, b)) \rightarrow \text{Smaller}(a, b)$

Counterexample: Replace the red text with DifferentShape

6. $\forall x (\text{Larger}(x, c) \rightarrow x \neq c)$

Counterexample: Replace the blue text with SameShape

Exercise 10.12

$\frac{\frac{\frac{\forall x \text{Cube}(x) \rightarrow \exists y \text{Small}(y)}{A} \quad \frac{}{B}}{\neg \exists y \text{Small}(y)}_B}{\exists x \neg \text{Cube}(x)}_C$	$\frac{A \rightarrow B \quad \frac{\frac{\forall x \text{Go}(x) \rightarrow \exists y \text{Bears}(y)}{A} \quad \frac{}{B}}{\neg \exists y \text{Bears}(y)}_B}{\exists x \neg \text{Go}(x)}_C$	$\cdot B: \text{FO consequences, not tautological consequences}$
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This is not tautologically valid. From the TFF, there cannot be a conclusion based on the given premises, so the conclusion is not always True with the premises in the TNF. The argument is logically valid since the conclusion can be proven based on the premises.

Exercise 10.13

$\frac{\frac{\frac{\forall x \text{Cube}(x) \rightarrow \exists y \text{Small}(y)}{A} \quad \frac{}{B}}{\neg \exists y \text{Small}(y)}_B}{\neg \forall x \text{Cube}(x)}_A$	$\frac{A \rightarrow B \quad \frac{\frac{\forall x \text{Go}(x) \rightarrow \exists y \text{Bears}(y)}{A} \quad \frac{}{B}}{\neg \exists y \text{Bears}(y)}_B}{\neg A}_C$	$\cdot A: \text{tautological consequences}$
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This is tautologically valid. From the TTF, the conclusion can be conned based on the premises. The conclusion is always True with the premises in the TNF. The argument is logically valid since we can prove the conclusion based on the premises.