Name: ____

SID: ____

Name of your GSI: ____

Philosophy 12A. Introduction to Logic. Spring 2023
Prof. Mancosu
MIDTERM EXAM – Monday, March 6

Instructions:

- Please write your name, SID, and GSI's name on this sheet. Write your SID on the top of each of the following sheets of paper as well.
- You will have **50 minutes** to complete the exam. Do not forget to hand in your exam.
- If you finish before 9:50, come to the front of the room and hand your exam in.
- If you finish during the last 10 minutes, please remain in your seat quietly. When time is called, we will collect the exams from each row. Please do not leave until we announce that all exams have been collected.
- Please answer all questions on the exam itself (space is provided as needed for each of the questions). Cross out any work that you don't want us to grade. For problems with multiple parts, be sure to answer each part.
- Note: you may not appeal to any of the Con rules when doing proofs on the exam. Good luck!

Question #:	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**Total**
Points possible:	15	10	10	11	8	7	7	5	5	**78**
Points earned:										

SID: ______________________________

Question 1: (15 points) Using the given names, predicates, and function symbols (provided in the table below), plus the identity symbol "=", translate the following five sentences into FOL. (3 points for each answer)

Names	**Predicates**	**Function symbols**
m: Mike	$S(x)$: x is a student	$b(x)$: the best friend of x
e: Eleven	$O(x,y)$: x is older than y	
n: Nancy	$R(x,y)$: x is related to y	
j: Joyce	$L(x,y)$: x lives in y	
h: Hawkins		

Provide your final answer on the line below the respective sentence.

(1) Nancy lives in Hawkins, and either Eleven or Joyce lives there too.

$L(n,h) \wedge \big(L(e,h) \vee J(j,h)\big)$

(2) Mike is a student who lives in Hawkins, while Eleven lives in Hawkins but is not a student.

$\big(S(m) \wedge L(m,h)\big) \wedge \big(L(e,h) \wedge \neg S(e)\big)$

(3) Either Nancy and Joyce are both students or at least one of them is older than Mike.

$\big(S(n) \wedge S(j)\big) \vee \big(O(n,m) \vee O(j,m)\big)$

(4) Nancy is not Mike's best friend, but she is related to him.

$\neg\big(b(m) = n\big) \wedge R(n,m)$

Alternative answer: $\neg\big(b(m) = n\big) \wedge R\big(n, b(m)\big)$

(5) Though Joyce's best friend is older than Nancy's best friend, neither Mike nor Eleven are older than Nancy's best friend.

$O\big(b(j), b(n)\big) \wedge \neg\big(O(m, b(n)) \vee O(e, b(n))\big)$

SID: ______________________________

Question 2: (10 points) For each of the following, fill in the blank with "in all cases","in some cases" or "never" so that the resulting sentence is TRUE. Note that "in some cases" should be taken to mean "in some but not all cases", and "valid" and "sound" should be taken to have the precise logical definitions we've been using in this course. (2 points for each blank)

(1) Arguments with all false premises and a false conclusion are in some cases valid.

(2) Arguments with all true premises and a false conclusion are never valid.

(3) Arguments with all true premises and a true conclusion are in some cases valid.

(4) Arguments with the negation of a tautology as the conclusion are

in some cases valid and never sound.

SID: ______________________________

Question 3: (10 points) Consider the following sentence of FOL:

$$\neg P \vee (Q \wedge P)$$

(a) Construct a complete truth table for this sentence. Notice that there will be two columns for the sentence letters on the left, and six columns for the main sentence on the right. (5 points)

(b) Is the sentence a tautology? (5 points) **Y** **N**

(Circle **Y**, if if your answer is 'yes' and circle **N**, otherwise.)

P	Q	$\neg$	P	$\vee$	$(Q$	$\wedge$	$P)$
T	T	F	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	T	F	T	T	F	F
F	F	T	F	T	F	F	F

SID: ______________________________

Question 4: (11 points) Consider the following two sentences of FOL:

$P \vee (Q \wedge \neg P)$ $\qquad$ $\neg(P \wedge Q) \wedge Q$

(a) Construct a complete joint truth table for these sentences (as in question (3)). (5 points) (Provide the truth table in the blank space below.)

(b) On the basis of your joint truth table, determine the answer to the following questions. (3 points each)
(Circle **Y**, if your answer is 'yes' and circle **N**, otherwise.)

Is $\neg(P \wedge Q) \wedge Q$ a tautological consequence of $P \vee (Q \wedge \neg P)$? **Y** **N**

Is $P \vee (Q \wedge \neg P)$ a tautological consequence of $\neg(P \wedge Q) \wedge Q$? **Y** **N**

P	Q	P	$\vee$	$(Q$	$\wedge$	$\neg$	$P)$	$\neg$	$(P$	$\wedge$	$Q)$	$\wedge$	Q
T	T	T	T	T	F	F	T	F	T	T	T	F	T
T	F	T	T	F	F	F	T	T	T	F	F	F	F
F	T	F	T	T	T	T	F	T	F	F	T	T	T
F	F	F	F	F	F	T	F	T	F	F	F	F	F

Explanation for (b): Notice that in every row of the TT in which $\neg(P \wedge Q) \wedge Q$ is true, $\neg(P \wedge Q) \wedge Q$ is true as well but not vice versa. Hence, by the definition of 'tautological consequence', the latter is a tautological consequence of the former but not vice versa.

SID: ______________________________

Question 5: (8 points) Consider the language with a binary relation R, two individual constants (names) a and b and two binary function symbols f and g. Check the box next to the correct answer. (2 points each)

1)	$f(a,a) = g(b,a)$ is	a) a term	☐
		b) a well-formed formula	■
		c) none of the above	☐
2)	$R\big(f(a,a)\big)$ is	a) a term	☐
		b) a well-formed formula	☐
		c) none of the above	■
3)	$f\big(R(a,b), R(a,a)\big)$ is	a) a term	☐
		b) a well-formed formula	☐
		c) none of the above	■
4)	$\neg\big(f(a,b) = g(b,a)\big)$ is	a) a literal	■
		b) a connective	☐
		c) none of the above	☐

The correct answers are marked by a black box.

SID: ______________________________

Question 6: (7 points) Given the premises $\neg(A \wedge B)$ and A, prove in Fitch[1] that $\neg B$.

Make sure to justify all the steps that require justification by appealing to the appropriate rules and lines of the derivation. (You may use Reiteration as well as all introduction and elimination rules for $\bot$, $\wedge$, $\vee$, and $\neg$. You are **not** allowed to use any other rules or to substitute logically equivalent sentences.)

1	$\neg(A \wedge B)$	
2	A	
3	B	
4	$A \wedge B$	$\wedge$Intro, 2, 3
5	$\bot$	$\bot$Intro, 1, 4
6	$\neg B$	$\neg$Intro, 3–5

[1] What the book calls system $\mathcal{F}_{\mathcal{T}}$.

SID: ______________________________

Question 7: (7 points) Given the premises $A \lor B$ and $\neg B$, prove in Fitch $A \lor (B \land \neg B)$.

Make sure to justify all the steps that require justification by appealing to the appropriate rules and lines of the derivation. (You may use Reiteration as well as all introduction and elimination rules for $\bot$, $\land$, $\lor$, and $\neg$. You are **not** allowed to use any other rules or to substitute logically equivalent sentences.)

Here are two possible solutions:

1	$A \lor B$	
2	$\neg B$	
3	$\quad A$	
4	$\quad A \lor (B \land \neg B)$	$\lor$Intro, 3
5	$\quad B$	
6	$\quad B \land \neg B$	$\land$Intro, 5, 2
7	$\quad A \lor (B \land \neg B)$	$\lor$Intro, 6
8	$A \lor (B \land \neg B)$	$\lor$Elim, 1, 3–4, 5–7

1	$A \lor B$	
2	$\neg B$	
3	$\quad A$	
4	$\quad A \lor (B \land \neg B)$	$\lor$Intro, 3
5	$\quad B$	
6	$\quad \bot$	$\bot$Intro, 2, 5
7	$\quad A \lor (B \land \neg B)$	$\bot$Elim, 6
8	$A \lor (B \land \neg B)$	$\lor$Elim, 1, 3–4, 5–7

SID: ____________________

Question 8: (5 points) Show that the following argument is invalid by assigning the appropriate truth values to A, B, and C (i.e. circle the corresponding truth values, **T** or **F** below)

1	$(A \vee \neg B) \wedge \neg C$
2	$A \vee C$
3	$\neg B \vee C$

$A:$ **T** **F**

$B:$ **T** **F**

$C:$ **T** **F**

SID: ______________________________

Question 9: (5 points) For the sentence below find an equivalent sentence in negation normal form. Be sure to show all your work by giving a series of equivalences and providing explicit justifications (by citing the de Morgan's laws or double negation).

$$\neg((A \wedge B) \vee \neg(A \vee \neg D))$$

$$\begin{aligned}
\neg\big((A \wedge B) \vee \neg(A \vee \neg D)\big) &\Leftrightarrow \neg(A \wedge B) \wedge \neg\neg(A \vee \neg D) && \text{by De Morgan} \\
&\Leftrightarrow \neg(A \wedge B) \wedge (A \vee \neg D) && \text{by double negation} \\
&\Leftrightarrow (\neg A \vee \neg B) \wedge (A \vee \neg D) && \text{by De Morgan}
\end{aligned}$$