Problem Set - LHP and Allocation to PSP versus LHP

Lionel MARTELLINI

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Problem 1 *Consider the following market prices for 4 default risk-free Treasury bonds of maturity 1 to 4 years:*

	Annual Coupon	*Maturity*	*Price*
Bond 1	5	*1 year*	$P_0^1 = 101$
Bond 2	5.5	*2 years*	$P_0^2 = 101.5$
Bond 3	5	*3 years*	$P_0^3 = 99$
Bond 4	6	*4 years*	$P_0^4 = 100$

1. *Derive the pure discount rates for maturities 1Y, 2Y, 3Y and 4Y.*

2. *Assume that are facing consumption needs of \$20,000 per year for the next 4 years (\$20,000 in 1Y, \$20,000 in 2Y, \$20,000 in 3Y and \$20,000 in 4Y). Derive the minimum amount of money that is needed to finance these consumption cash-flows.*

3. *Find the corresponding replicating portfolio (LHP).*

Problem 2 *Today is 1/1/2023. On 6/30/2024 we will have to make a payment of \$100. We can only invest in a riskfree pure discount bond (nominal \$100) that matures on 12/31/2023 and in a riskfree coupon bond, nominal \$100, which pays an annual interest (on 12/31) of 8% and matures on 12/31/2025. Assume a flat term structure of 7%. How much should be invested in each of the bonds so as to match the duration of ther liabilities?*

Problem 3 *The Sensitivity/Convexity Liability Portfolio Hedge*

At date J, the liability portfolio to be hedged has the following characteristic features:

Price	*IRR*	*Sensitivity*	*Convexity*
328.635	5.143%	-6.76	85.329

Make the following assumptions or requirements (some of which are obviously an oversimplification of the real situation):

- *hedging assets are Treasury bonds (T-bills and T-bonds)*
- *at date J, the value of the portfolio to be hedged and the value of the hedging portfolio should be the same*
- *the dynamic strategy corresponding to the global portfolio (portfolio to be hedged and hedging portfolio) is self-financing*
- *the portfolio is rebalanced every month*
- *financing costs are negligible*
- *unlimited short-selling is allowed*
- *any number of assets may be traded (including non integer numbers)*

Find the hedging portfolio that matches sensitivity and convexity, and also satisfies a self-financing constraint, based on the following three bonds.

	Price	*Coupon rate*	*Maturity date*
Asset 1	108.039	7%	3 *years*
Asset 2	118.786	8%	7 *years*
Asset 3	97.962	5%	12 *years*

	IRR	*Sensitivity*	*Convexity*
Asset 1	4.097%	-2.704	10.168
Asset 2	4.779%	-5.486	38.962
Asset 3	5.232%	-8.813	99.081

At date $J+1$, market conditions have changed and the new features of the portfolio to be hedged and the hedging assets are given as follows.

	Price	*IRR*	*Sensitivity*	*Conve*
Portfolio to be hedged	341.583	4.635%	-6.886	88.57
Asset 1	109.947	3.452%	-2.723	10.30
Asset 2	122.742	4.184%	-5.538	39.61
Asset 3	103.035	4.664%	-8.935	101.2

Estimate the change of value of the liability portfolio between dates J and $J+1$ with and without the hedging strategy in place. What would be the new hedging portfolio at date $J+1$?

Problem 4 *Provide a Python or Excel simulation of the performance of a CPPI strategy. For this problem you may use the S&P500 index price data (which needs first to be converted into index return data) - see attached excel file that contains daily prices on the period ranging from early November 2014 to November 2024. Please analyze how the results vary as a function* *of the rebalacing frequency, the floor level (you may want to try 80%, 85%, 90% and 95%), the multiplier value (you may want to use m=2, 3, 4 or 5) the starting date and investment horizon. Note: you may want to impose a no-leverage constraint, meaning that the allocation to the risky asset (here the S&P500 index) remains between 0% and 100% (no short position and no borrowing).*

Problem 5 *(Optional) Propose an extended version of the CPPI strategy that can be used to protect a maximum drawdown level (the maximum drawdown (MDD) is a measure of an asset largest price drop from a peak to a trough). Provide a Python or Excel simulation of the performance of the dynamic max drawdown strategy on the same historical sample as in the previous problem, and comment of the pros and cons of this strategy compared to the standard CPPI strategy.*