Decarbonized S&P500

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1 Introduction

The **Decarbonized S&P500** products are an innovative low-carbon investment solution derived from the S&P500 index. Developed by *Alves-Trochon*, this product line offers tailored strategies designed to meet various investment objectives while maintaining a strong commitment to sustainability. Our solutions provide institutional investors with an opportunity to align their portfolios with climate goals maintaining reasonable financial performance.

Our approach employs advanced optimization techniques to reduce the carbon intensity of the S&P500 index while carefully balancing risk and return. The portfolio is designed to preserve market exposure while adapting to the specific needs of our clients.

2 Investment Strategy

The proposed strategies are structured around three key pillars, which are dynamically adjusted based on client preferences.

- **Carbon Intensity Reduction**: The portfolios can achieve a **20% to 80% reduction in carbon intensity** compared to the standard S&P500.
- **Risk-Adjusted Performance**: The strategies aim to at least maintain the average **Sharpe ratio** of the S&P500, estimated at 0.7 ± 1.7, while reducing variance over backtested periods through optimized asset selection and weight allocation.
- **Tracking Error Control**: The strategies are designed to achieve a maximum tracking error of **5%** relative to the benchmark.

To ensure different risk appetites and sustainability ambitions, we offer four distinct investment solutions:

- **Low Deviation Strategy**: Focused on minimizing deviations from the benchmark while incorporating a moderate reduction in carbon intensity.
- **High Expected Return Strategy**: Designed for investors seeking an aggressive approach with higher risks and lower carbon reduction while accepting higher tracking error for higher overall returns.
- **Near Zero Carbon Strategy**: Aiming for the highest carbon intensity reduction while maintaining diversification, risk control and a lower expected return.
- **Balanced Strategy**: A well-rounded approach optimizing all three axes while maintaining a measured trade-off between risk, return, and sustainability.

All strategies leverage the full spectrum of S&P500 stocks, ensuring a holistic and diversified approach. The optimization process carefully calibrates the weight of each pillar to align with the specific investment goals of our clients and is dinamically adjusted each 3 months with respect to the previous 2 years of optimization.

In this document, all strategies will be presented and explained comparetivelly, along with their backtesting results on U.S. stock market data from 1989 to 2019. The data is sampled weekly, and the carbon footprint and intensity are calculated only once in 2019 for all companies, using their Scope 1 and Scope 2 emissions relative to their market capitalization and revenue, respectively. The technical assumptions, as well as the reasoning behind certain specific choices are discussed in the last section, though they are not essential for understanding the results.

Additionally, the sector compositions of the portfolios and their contributions to carbon intensity in the portfolio are analyzed, as no prior assumptions were made regarding sector selection or constraints in portfolio construction.

3 Strategies

The results of the optimized portfolios over 121 steps during the back testing are presented below. The section *Tracking Errors* (3.0.1) compares the tracking errors for different levels of carbon intensity constraints, with respect to the S&P500 for out-of-sample data. Similarly, section *Annualized Expected Returns and Volatility* (3.0.2) evaluates both metrics with respect to out-of-sample data.

Section *Cumulative Growth* (3.0.3) also presents out-of-sample results for each portfolio if a single 1000 $ were invested in the beginning of the period in 1989. Meanwhile, section *Sharpe Ratios* (3.0.4) is calculated using in-sample data, as the Sharpe ratio is the primary measure for assessing return-to-risk trade-offs in portfolio evaluation and decisions.

Finally, the section *Sector Dissection over Periods* (3.0.5) provides a sectorial analysis of each portfolio across different back testing windows, considering both sector weight distribution in each portfolio and sectorial carbon intensity percentage by sector.

One must notice that all portfolios were optimized to reduce carbon intensity to a certain $1 - \xi$ percentage of the S&P500 index. The metric is calculated as a Weighted Average Carbon Intensity (WACI) of the Carbon Intensity CI_i of each stock i, described below and the constraint of the portfolios is that $WACI_{Portfolio} \leq \xi \cdot WACI_{S\&P500}$.

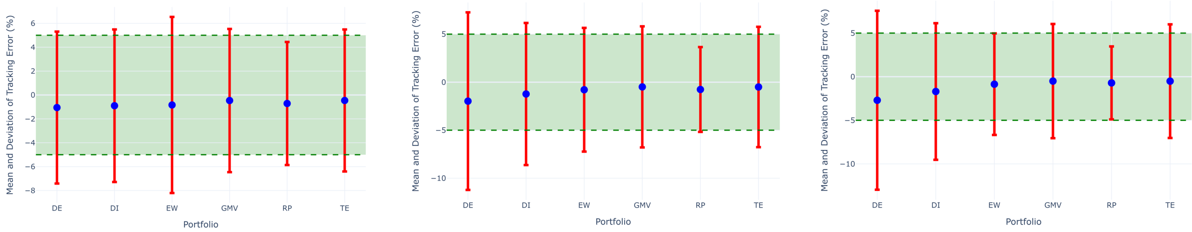
$$CI_i = \frac{Scope\ 1_i + Scope\ 2_i}{Revenue_i}$$

$$WACI = \sum_{i=1}^{N} \omega_i \cdot CI_i$$

Over the whole document, the portfolios are the following: Max De-correlation (**DE**); Max Diversification (**DI**); Equally Weighted (**EW**); Global Minimum Variance (**GMV**); Risk Parity (**RP**); Tracking Error (**TE**).

3.0.1 Tracking Errors

Most portfolios exceed the tracking error of 5% when calculated with respect to out-of-sample data, especially when the carbon intensity constraint enforces a low-carbon portfolio (e.g., $\xi = 0.2$), as shown in Figure 1. The portfolio with the lowest overall tracking error is the Risk Parity portfolio (RP). Notably, with looser carbon constraints ($\xi = 0.8$), its tracking error remains within the desired 5% margin. Although specifically optimized for this purpose, the Tracking Error (TE) portfolio achieves a relatively low tracking error overall but still exceeds the expected threshold and it is really similar to the Global Minimum Variance (GMV).



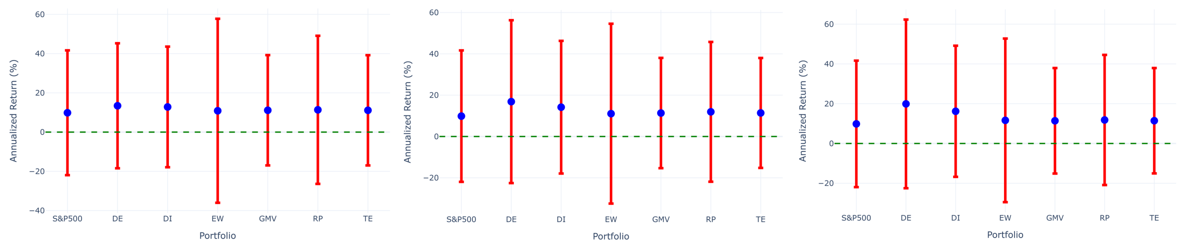
(a) $\xi = 0.2$ (b) $\xi = 0.5$ (c) $\xi = 0.8$

Figure 1: Tracking Errors for Portfolios with different ξ values

3.0.2 Annualized Expected Returns and Volatility

For the annual expected returns and volatility calculated with out-of-sample data, the portfolios with the least standard deviation over the entire testing period are the Global Minimum Variance (GMV) and the Tracking Error (TE) portfolios, as they minimize portfolio variance. These portfolios exhibit the lowest minimum annual return overall but also the lowest maximum return, reinforcing their low-risk characteristics as investment strategies.

The Equally Weighted (EW) portfolio, however, exhibits the highest variation among all strategies, regardless of the carbon constraint applied. For $\xi = 0.2$, it has the lowest possible return when considering the standard annual deviation, reaching an annual return close to -40 %, as in Figure 2. Other strategies are also strongly impacted by the carbon constraint when analyzed in terms of their annual volatility and expected annual return. In general, as the carbon constraint becomes more restrictive (ξ decreases), volatility increases while returns decrease.



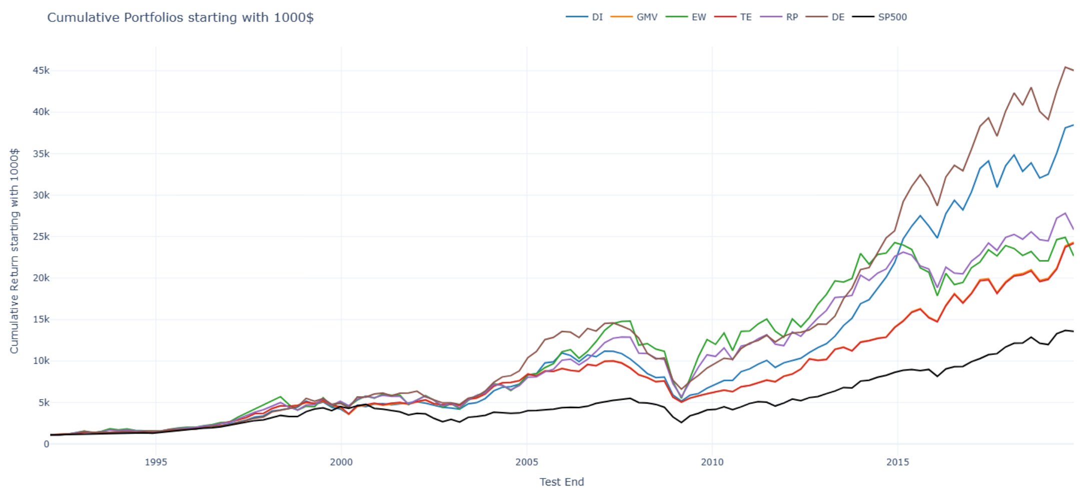
(a) $\xi = 0.2$ (b) $\xi = 0.5$ (c) $\xi = 0.8$

Figure 2: Returns and Volatility of Portfolios with Different ξ Values

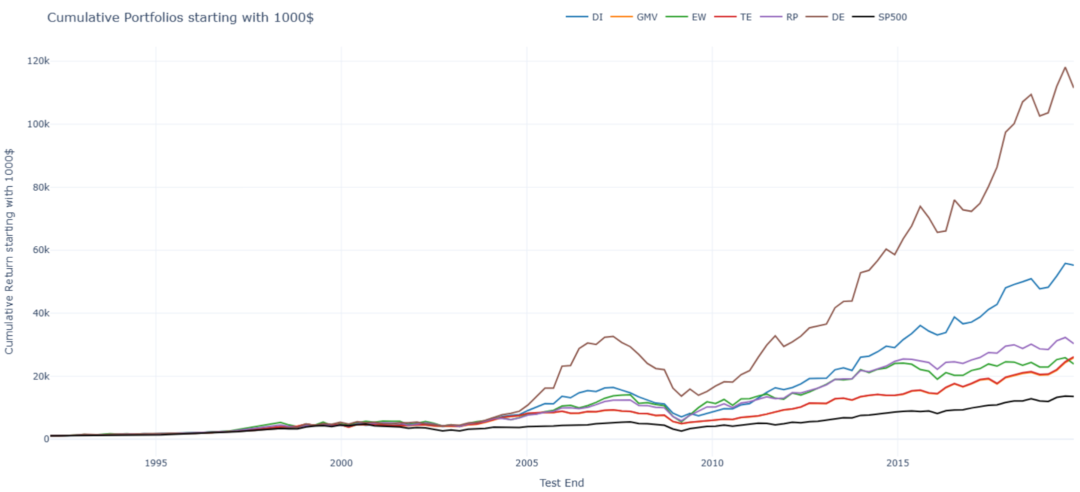
3.0.3 Cumulative Growth

This section presents the final portfolio values for a single initial investment of 1000 \$ also with out-of-sample data for the whole period from 1989 to 2019. As shown in Figure 3, the more restrictive the carbon constraint, $\xi = 0.2$, the lower the overall returns for all portfolios. A looser carbon constraint, $\xi = 0.8$, can lead to significantly higher returns. For instance, the Max Decorrelation (DE) portfolio achieves a final return of 24168.4 %, whereas the S&P500 yields a final return of 1257.3 % over the entire period.

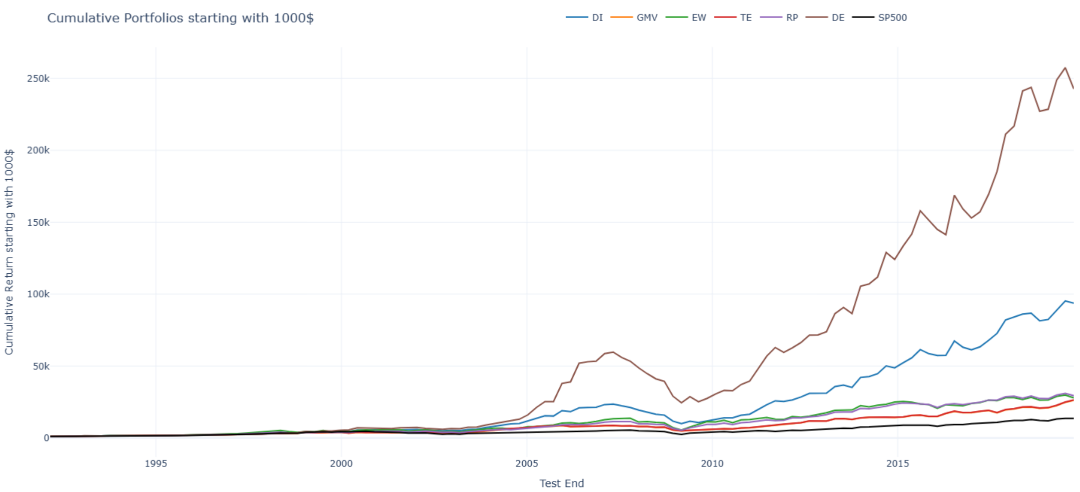
It is important to notice that all portfolios are more effective than the S&P500 independently of the carbon constraint, since in the long run their final values are always above the S&P500 index.



(a) $\xi = 0.2$



(b) $\xi = 0.5$



(c) $\xi = 0.8$

Figure 3: Returns and Volatility of Portfolios with Different ξ Values

3.0.4 Sharpe Ratios

Unlike the previous sections, the Sharpe ratios are calculated in-sample, as they serve as the decision metric for selecting one portfolio over another during the optimization process within the two-year window. The Sharpe ratios are computed using the equation below, where the risk-free rate is set to $\mu_{risk\ free} = 3\%$:

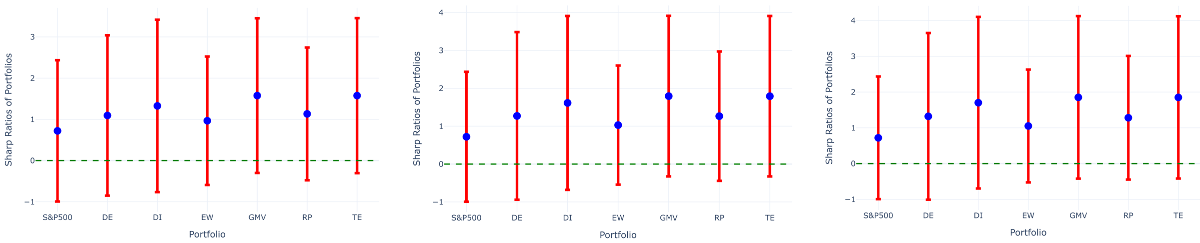
$$SR = \frac{(\mu_{annual} - \mu_{risk\ free})}{\sigma_{annual}}$$

In Figure 4, the Sharpe ratio of the S&P500 over the 121 backtesting windows is, on average, lower than that of all other portfolios. The Global Minimum Variance (GMV) portfolio exhibits the lowest negative Sharpe ratio when considering the standard deviation over the period.

All other portfolios display greater Sharpe ratio variation, except for the Equally Weighted (EW) and Risk Parity (RP) portfolios. These two maintain relatively stable Sharpe ratios close to 1, with RP having a slightly lower negative margin, around -0.5. These results reinforce the risk-adjusted performance of the portfolios over long periods.

Another important observation is the positive increase in Sharpe ratios as the carbon constraint becomes less restrictive, while maintaining a relatively stable negative Sharpe ratio margin. This indicates that for higher values of ξ, all portfolios tend to perform better from a return-risk perspective compared to when they are subject to stricter carbon constraints.

For example, consider the Max Decorrelation (DE) portfolio with $\xi = 0.2$ and $\xi = 0.8$. When $\xi = 0.2$, the Sharpe ratio is $SR_{DE} = 1.1 \pm 1.9$, whereas for $\xi = 0.8$, it increases to $SR_{DE} = 1.3 \pm 2.3$, achieving higher Sharpe ratios during the period (up to 3.6) while keeping the lower Sharpe ratios close to -1, similar to the previous value of -0.8.



(a) $\xi = 0.2$ (b) $\xi = 0.5$ (c) $\xi = 0.8$

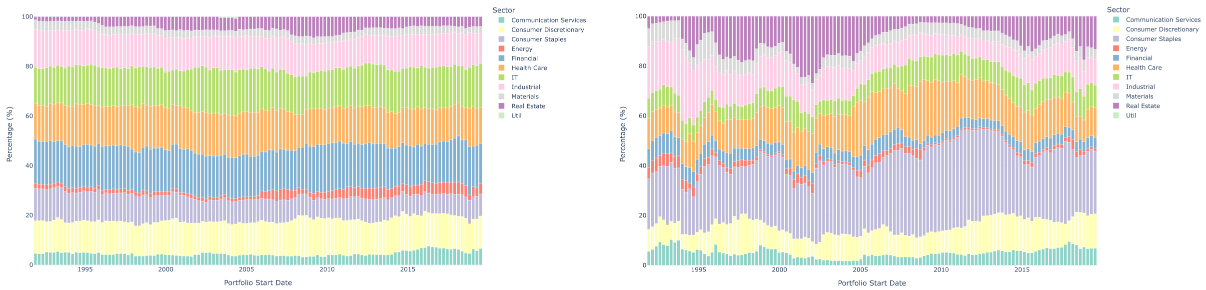
Figure 4: Sharp Ratios for Portfolios with different ξ values

3.0.5 Sector Dissecation over Periods

Finally, this section highlights the sectors that were predominantly used to construct each portfolio strategy over the back-testing windows, along with their contribution to the overall carbon intensity of the portfolios. Since each strategy and carbon reduction level has two corresponding graphs, to streamline the analysis, only two portfolios will be selected for comparison under the highest and lowest carbon intensity constraints. The following figures illustrate the sector compositions of the Risk Parity (RP) and Max Decorrelation (DE) portfolios for carbon reductions of 20% and 80% relative to the S&P500 index.

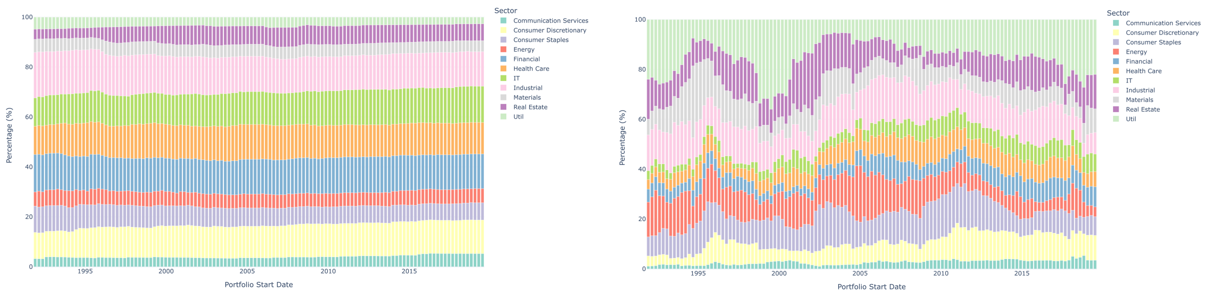
In all graphs, it is easy to observe that some sectors can have higher carbon intensity even when they are less represented in the portfolio proportion. For instance, in the Risk Parity (RP) portfolio under a more restrictive carbon constraint ($\xi = 0.2$), the Utilities and Energy sectors are barely present in the portfolio compositions over the periods. However, if the carbon intensity constraint is less strict, these two sectors appear more and dominate the carbon intensity proportions in the portfolios, as illustrated in Figures 5 and 6. The same reasoning applies to other portfolios, such as the Maximum Decorrelation (DE) portfolio, as shown in Figures 7 and 8.

Another important remark is that there were no sector equalization constraints for any strategy. Nevertheless, the Risk Parity (RP) portfolio seems to achieve this naturally, attempting to balance the sectors even under strict carbon constraints, as shown in Figure 5. In contrast, other portfolios, such as Maximum Decorrelation (DE), may completely exclude certain sectors from their portfolio weights during some periods, even without a strict carbon constraint, as observed in Figure 8.



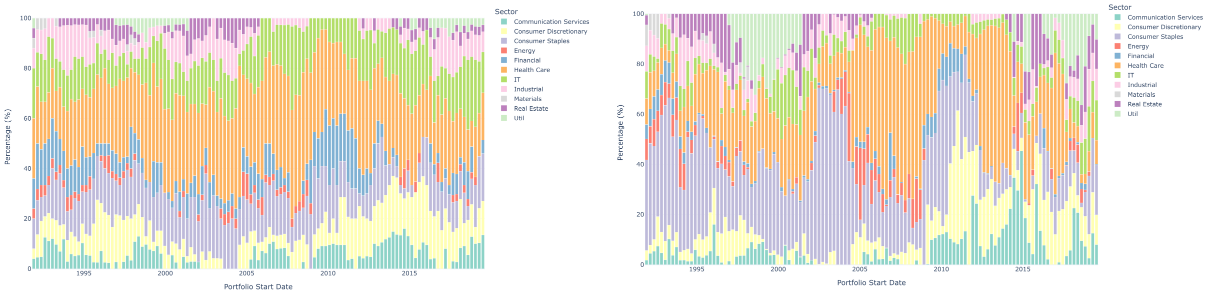
(a) Portfolio weights distribution by sector (b) Portfolio carbon intensity by sector

Figure 5: Risk Parity (RP) for carbon intensity constraint $\xi = 0.2$



(a) Portfolio weights distribution by sector (b) Portfolio carbon intensity by sector

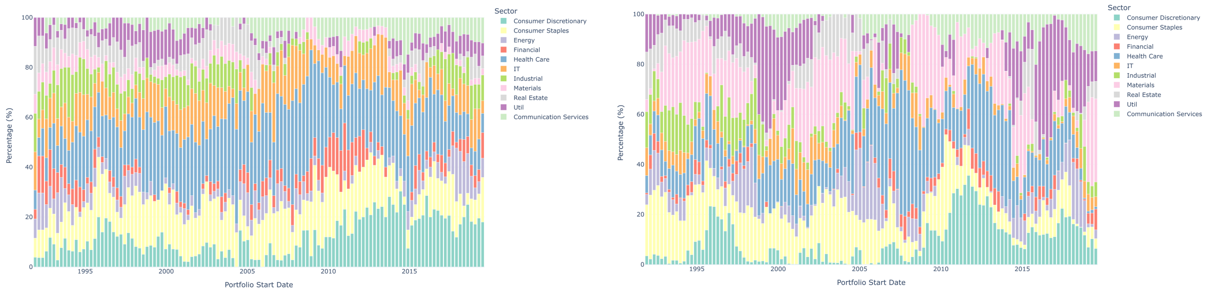
Figure 6: Risk Parity (RP) for carbon intensity constraint $\xi = 0.8$



(a) Portfolio weights distribution by sector

(b) Portfolio carbon intensity by sector

Figure 7: Max Decorrelation (DE) for carbon intensity constraint $\xi = 0.2$



(a) Portfolio weights distribution by sector

(b) Portfolio carbon intensity by sector

Figure 8: Max Decorrelation (DE) for carbon intensity constraint $\xi = 0.8$

4 Which Parameters to Choose?

After analyzing multiple performance metrics, Table 1 summarizes the selected strategies tailored to different client profiles, as previously introduced. Each strategy reflects a specific balance between risk, return, and carbon intensity reduction, aiming distinct investment objectives.

The Low Deviation Strategy prioritizes stability with minimal volatility relative to the benchmark, making it suitable for investors who have greater confidence in the S&P 500 index while seeking to reduce their portfolio's carbon intensity.

In contrast, the High Expected Return Strategy focuses on maximizing potential gains, appealing to those willing to accept higher risk-reward strategies.

The Near Zero Carbon Strategy is designed for environmentally conscious investors, emphasizing a substantial reduction in carbon intensity, 80 %, while maintaining competitive performance and a relatively low tracking error.

Lastly, the Balanced Strategy offers a middle ground, optimizing risk-adjusted returns while controlling tracking error and carbon exposure.

	Low Deviation Strategy	High Expected Return Strategy	Near Zero Carbon Strategy	Balanced Strategy
Portfolio Type	RP	DE	TE	RP
Carbon Intensity Reduction	20%	20%	80%	50%
Risk-Adjusted Performance (Sharpe Ratio)	1.3 ± 1.7	1.3 ± 2.3	1.6 ± 1.9	1.3 ± 1.7
Tracking Error Control	$(-0.7 \pm 4.2)\%$	$(-2.7 \pm 10.3)\%$	$(-0.5 \pm 6)\%$	$(-0.8 \pm 4.4)\%$
Expected Annual Return	$(11.8 \pm 32.7)\%$	$(19.9 \pm 42.4)\%$	$(11.1 \pm 28.1)\%$	$(11.9 \pm 34.0)\%$

Table 1: Performance Metrics for Different Portfolio Strategies

5 Methodological Considerations

For those seeking a more mathematical foundation of each strategy and the underlying hypotheses, this section further dissects the equations behind the strategies, as well as the assumptions for missing data and other technical implementation challenges.

As explained in the introduction, back-testing is conducted over a 30-year period from 1989 to 2019. At each step, starting in 1991, the past two years of data are used to calculate the optimal portfolio weights. These weights are then applied to buy stocks, which are held for three months before being sold.

The return after this three-month period is stored in a vector for further use in calculating the annual expected return per period using the geometric mean:

$$GM = 100 \cdot \left[\left(\prod_{i=1}^{N} (1 + r_i) \right)^{\frac{1}{N}} - 1 \right]$$

where the calculation accounts for the fact that there are only 52 weeks in a year, as weekly data is used. The annual volatility is derived by first computing the weekly volatility from the samples and then scaling it by a factor of $\sqrt{52}$.

The portfolio return for a given test period is defined as:

$$r_p = \frac{100 \cdot \Delta P}{P_{start\ test}} \%$$

where

$$P_{start\ test} = \omega_*^t \cdot p_{start\ test}$$

represents the portfolio price at the beginning of the test period, and

$$\Delta P = P_{end\ test} - P_{start\ test}$$

is the difference between the portfolio prices at the start and end of the test window.

As a major hypothesis, it is assumed that revenue, market capitalization, Scope 1 and Scope 2 emissions data remain constant across all periods for all stocks, as these values are based on 2019 data. For missing carbon intensity values, they are replaced by the WACI of the S&P500, effectively assigning the carbon intensity of the S&P500 index itself. Additionally, the weights of the S&P500 are assumed to remain constant over the entire period. No transaction fees are considered for buying or selling stocks.

For missing stock price data, the following procedure is applied: within each two-year training window, if a stock has more than 5% of missing data, the entire stock price series is excluded from the optimization. If the missing percentage is below 5%, the data is interpolated using a second-degree spline.

The optimization equations used for each strategy are as follows:

- Global Minimum Variance (GMV)

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top \Sigma \mathbf{w}$$

  subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \in \{1, 2, \ldots, n\}$$

$$\mathbf{w}^\top CI \leq \xi \, \frac{e^\top CI}{e^\top e}$$

  Where $e^\top = (1, 1, 1, \cdots, 1)$, Σ is the covariance matrix, $\mathbf{w}$ the weights of the portfolio, and CI is the vector of carbon intensities for all stocks

- Max Diversification (DI)

$$\max_{\mathbf{w}} \quad \frac{\mathbf{w}^\top \sigma}{\sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}}$$

  subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \in \{1, 2, \ldots, n\}$$

$$\mathbf{w}^\top CI \leq \xi \, \frac{e^\top CI}{e^\top e}$$

  Where $e^\top = (1, 1, 1, \cdots, 1)$, Σ is the covariance matrix, $\mathbf{w}$ the weights of the portfolio, and CI is the vector of carbon intensities for all stocks

- Max Decorrelation (DE)

$$\min_{\mathbf{w}} \quad \mathbf{w}^\top C \mathbf{w}$$

  subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \in \{1, 2, \dots, n\}$$

$$\mathbf{w}^\top CI \leq \xi \; \frac{e^\top CI}{e^\top e}$$

Where $e^\top = (1, 1, 1, \cdots, 1)$, C is the correlation matrix, $\mathbf{w}$ the weights of the portfolio, and CI is the vector of carbon intensities for all stocks

- Max Effective Number of Constituents applied to dollar (EW portfolio)

$$\max_{\mathbf{w}} \quad \frac{1}{\sum_{i=1}^{n} w_i^2}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \in \{1, 2, \dots, n\}$$

$$\mathbf{w}^\top CI \leq \xi \; \frac{e^\top CI}{e^\top e}$$

Where $e^\top = (1, 1, 1, \cdots, 1)$, $\mathbf{w}$ the weights of the portfolio, and CI is the vector of carbon intensities for all stocks

- Max Effective Number of Constituents applied to risk contributions (RP portfolio)

$$\min_{\mathbf{w}} \left(\frac{\omega^\top \Sigma e}{\sigma_p} \cdot \omega - \frac{1}{n} e\right)^\top \cdot \left(\frac{\omega^\top \Sigma e}{\sigma_p} \cdot \omega - \frac{1}{n} e\right)$$

subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \in \{1, 2, \dots, n\}$$

$$\mathbf{w}^\top CI \leq \xi \; \frac{e^\top CI}{e^\top e}$$

Where $e^\top = (1, 1, 1, \cdots, 1)$, Σ is the covariance matrix, $\mathbf{w}$ the weights of the portfolio, and CI is the vector of carbon intensities for all stocks

- Min Tracking Error with respect to the S&P500 index

$$\min_{\mathbf{w}} \; \omega^\top \Sigma \omega - 2\omega^\top \Sigma_{SP500}$$

subject to:

$$\sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0 \quad \forall i \in \{1, 2, \dots, n\}$$

$$\mathbf{w}^\top CI \leq \xi \; \frac{e^\top CI}{e^\top e}$$

Where $e^{\top} = (1, 1, 1, \cdots, 1)$, Σ is the covariance matrix between stocks, Σ_{SP500} is the covariance vector between stocks and the index, $\mathbf{w}$ the weights of the portfolio, and CI is the vector of carbon intensities for all stocks

For more information please contact mathias.trochon@etu.minesparis.psl.eu or mathias.trochon@etu.minesparis.psl.eu. For all plots and the applied code in python follow the link: Google Drive Folder.