AN APPROACH TO THE DESIGN OF DIGITAL ALGORITHMS FOR MEASURING POWER CONSUMPTION CHARACTERISTICS

A.V. Voloshko O.I. Kotsar
Kiev Polytechnic Institute
Ukraine

O. P. Malik
University of Calgary
Canada

Abstract

An approach to the design of digital algorithms based on multi-function correlators applied to current and voltage samples to measure power consumption characteristics is described in this paper. Based on this approach, a new technique for algorithm design is developed. The main advantage of this approach is the determination of all power consumption characteristics by the same algorithm. The algorithm has a high accuracy and regular structure. The regular structure is a very important property for the development of specialized chips.

Keywords: Power measurement; power consumption characteristics; Active and reactive power; Correlator

Introduction

The use of electric energy is expanding, and it is getting complicated to maintain balance between generation and consumption. An effective solution to this problem is possible with the help of automatic control of energy consumption, also known as demand side management.

An increasing number of electrical and electronic loads that produce non-sinusoidal waveforms are being connected to the system. They have started to cause 'power quality' problems in the systems and affect the accuracy of the electric power measurements. It is thus necessary to develop and apply new techniques for the control of electric energy to support the optimal performance of the electric supply network. This development requires that proper information be provided about the quality and quantity of the electric supply.

An analysis of the existing digital algorithms for the determination of the electrical power measurements given in Ref. [1], shows that all of them were originally developed for an analog system model and represent digital approximations to the analog processing schemes. An analysis of the processing methods shows that these methods are ineffective in providing information for the problem of electrical energy supply control from the point of view of accuracy and speed.

The objective of this paper is to investigate and describe how power consumption can be determined digitally under sinusoidal and non-sinusoidal conditions. A new algorithm for measuring the power consumption using a multi-function correlator is described. The proposed technique is an extension of the correlation technique proposed in Ref. [2] and is applicable when signals contain harmonics and frequency deviates from the nominal.

94 SM 491-1 PWRD A paper recommended and approved by the IEEE Power System Instrumentation & Measurements Committee of the IEEE Power Engineering Society for presentation at the IEEE/PES 1994 Summer Meeting, San Francisco, CA, July 24 - 28, 1994. Manuscript submitted January 31, 1992; made available for printing May 3, 1994.

Basic Principle

The power system voltage, $u(t)$, and current, $i(t)$, signals can be represented as a sum of their Fourier Components as follows:

$$u(t) = \sum_{k=0}^{M} U_k \sin\left(\frac{2\pi kt}{T} + \phi_k\right) \quad (1)$$

$$i(t) = \sum_{k=0}^{N} I_k \sin\left(\frac{2\pi kt}{T} + \psi_k\right) \quad (2)$$

where: U_k - is the peak voltage of kth harmonic, I_k - is the peak current of kth harmonic, and ϕ_k, ψ_k are the phase angles of the kth harmonic of voltage and current, respectively. As a general case, it is assumed that the voltage and current signals have different number of components. For simplicity, an algorithm for a single - phase system only is considered.

Consider the periodic single phase voltage $u(t)$ and current $i(t)$ signals. The cross correlation function $B_{i,u}(\tau)$ of these signals is given by:

$$\begin{aligned} B_{i,u}(\tau) &= \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} i(t)\, u(t+\tau)\, dt \\ &= \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} u(t)\, i(\tau - t)\, dt \end{aligned} \quad (3)$$

In digital form this function is:

$$S(kT) = \lim_{N \to \infty} \frac{1}{N} \sum_{l=1}^{N} i(lT)\, u((l+k)T), \quad (4)$$

for k=0,1,2, . . .

T is the sampling interval and N is the number of samples in the integration interval.

Assume that the signals $u(t)$ and $i(t)$ are sampled at equidistant time instants, t_l:

$$t_l = \frac{l\, T_o}{2n+1}, \quad (5)$$

where

l is an integer number, 0, . . . , 2n,

n is the highest order of harmonic and 2n + 1 = N, the number of samples in one period, T_0, of the fundamental.

The cross correlation function has the form:

$$S(k) = \frac{1}{2m+1} \sum_{l=0}^{2n} u_{l-k}\, i_l, \; k = 0, \ldots, 2n \quad (6)$$

According to the above description, these are two periodic sequences

$$u = \{u_l,\ l=0, \ldots, N-1\}$$
$$i = \{i_l,\ l=0, \ldots, N-1\}$$

and the linear convolution is a new sequence

$$S = \{S_n , n=0 , \ldots , N-1\}$$

given by the equation

$$S(n) = \sum_{l=0}^{N-1} i_l \, u_{n-l} \qquad (7)$$

The convolution is written with the understanding that $u_{n-l} = 0$ if $n-l$ is less than zero.

Effort involved in the solution of eqn. (7) depends upon the number of samples, (2n+1 =) N. The presence of powerful nonlinear polluting loads (harmonics, non-sinusoidal and so on) causes degradation of electric power quality [3], and to maintain the necessary accuracy, the number of samples, N, may have to be increased somewhat. As the sampling rate is related to the highest harmonic, the solution of eqn. (7) may become slow and require large computational time for high order harmonics.

Discrete Fourier Transform (DFT) of eqn. (7) is

$$S(k) = \sum_{n=0}^{N-1} S(n)\, W_N^{kn} = \sum_{n=0}^{N-1} [\sum_{l=0}^{N-1} i_l \, u_{n-l}] \, . \exp (j\frac{2\pi kn}{N}) \qquad (8)$$

where: k=0, 1, . . . , 2n - the number of Fourier components.

Changing the order of the sum in eqn. (8) and after conducting several mathematical transformations, one obtains

$$S(k) = \sum_{l=0}^{2n} i_l \, [\sum_{n=0}^{N-1} u_{n-l} \, e^{-j\,(\frac{2\pi}{N}(n-l)k)}] \, e^{-j\,\frac{2\pi}{N}\, ln} \qquad (9)$$

The expression

$$\sum_{n=0}^{N-1} u_{n-l} \; e^{-j\,(\frac{2\pi}{n}(n-l)\,k\,)} = U(k)$$

is the DFT of the sequence $u(l)$.

Then,

$$S(k) = U(k) \sum_{l=0}^{2n} i_l \; e^{-j\,(\frac{2\pi}{N})(l)(n)} \qquad (10)$$

where

$$\sum_{l=0}^{2n} i_l \; e^{-j(\frac{2\pi}{N})(l)(n)} = I(k)$$

is the DFT of the sequence i_l.

Thus,

$$S(k) = I(k) \cdot U(k) \qquad (11)$$

That is, the calculation of the convolution S(n) is obtained as a product of the DFTs of the functions u_l and i_l.

The cross correlation function may also be written in the form:

$$S(k) = \frac{1}{2n+1} \sum_{l=0}^{2n} u_{l-k} \, i_l \; , k = 0, \ldots , 2n \qquad (12)$$

This equation shows that the cross correlation function can be computed as the convolution (7) simply by reading one of the two sequences backwards.

The Proposed Algorithm

As mentioned above, all of the algorithms for digital power measurement [1, 2, 4,-6] were originally developed for an analog systems model. They perform the same computations but on discretized values of voltage and current signals, and thus represent digital approximations of the analog processing schemes. For a digital system, direct application of digital signal processing seems more appropriate. This means that all the algorithms have the same digital form just as suggested in Ref. [1]. This form and some results described in Refs. [7-9], are the basis of the approach to the design of the algorithm proposed in this paper.

A DFT of eqn. (12) has the form:

$$S(k) = S_0 + \sum_{m=1}^{n} (S_{1m} \sin \frac{2\pi ml}{2n+1} + S_{2m} \cos \frac{2\pi mk}{2n+1}) , \qquad (13)$$

where

$$S_0 = \frac{2}{2n+1} \sum_{k=0}^{2n} S_k \; , \qquad (14)$$

$$S_{1m} = \frac{2}{2n+1} \sum_{k=1}^{2n} S_k \sin \frac{2\pi mk}{2n+1} \; , \; m = 1, \ldots , n \qquad (15)$$

$$S_{2m} = \frac{2}{2n+1} \sum_{k=0}^{2n} S_k \cos \frac{2\pi mk}{2n+1} \; , \; m = 1, \ldots , n \qquad (16)$$

Now consider the algorithm which was proposed in Ref. [2]. The authors assumed that the current and voltage signals are purely sinusoidal, and proposed the following procedure for active and reactive power measurements.

First, the input signals can be resolved along the orthogonal reference axes as follows:

$$V_d = \frac{1}{T_w} \int_0^{T_w} u(t)\, F_d\,(t)\, dt = \frac{1}{2}\, U_m \cos \phi \qquad (17)$$

$$V_q = \frac{1}{T_w} \int_0^{T_w} u(t)\, F_q\,(t)\, dt = \frac{1}{2}\, U_m \sin \phi \qquad (18)$$

$$I_d = \frac{1}{T_w} \int_0^{T_w} i(t)\, F_d\,(t)\, dt = \frac{1}{2}\, I_m \cos (\phi - \theta) \qquad (19)$$

$$I_q = \frac{1}{T_w} \int_0^{T_w} i(t)\, F_q\,(t)\, dt = \frac{1}{2}\, I_m \sin (\phi - \theta) \qquad (20)$$

where $T_w = KT$,

T is the period of the waveform,

K is an integer number,

F_d and F_q, a pair of orthogonal reference functions, are $F_d = \sin \omega t$, $F_q = \cos \omega t$, and subscripts d and q represent the components of corresponding quantities along orthogonal axes.

Summing the products of in-phase components and simplifying gives

$$V_d\, I_d + V_q\, I_q = \frac{1}{2} (\frac{1}{2}\, u_m\, I_m \cos \theta) = \frac{1}{2} P \qquad (21)$$

The difference of the products of out-of-phase components gives

$$V_d\, I_q - V_q\, I_d = \frac{1}{2} (\frac{1}{2}\, U_m\, I_m \sin \theta) = \frac{1}{2} Q \qquad (22)$$

Now return to the sequences (14) - (16) and write the following expressions

$$S_0 = U_0 I_0 = P_0 \; , \qquad (23)$$

$$S_{1m} = \frac{1}{2}\,(U_{2m}\,I_{1m} - U_{1m}\,I_{2m}) = Q_m\;,\; m = 1,\ldots,n \quad (24)$$

$$S_{2m} = \frac{1}{2}\,(U_{1m}\,I_{1m} + U_{2m}\,I_{2m}) = P_m\;,\; m = 1,\ldots,n \quad (25)$$

where: S_0, S_{1m}, S_{2m} are Fourier components (the expressions S_k are defined by analogy with eqns. (14-16)), P_0 is the power corresponding to the unidirectional component; and Q_m, P_m are the reactive and active powers respectively of the mth harmonic.

Now write eqn. (12) in the matrix form

$$\begin{bmatrix} S_0 \\ S_1 \\ \cdot \\ \cdot \\ \cdot \\ S_{2n} \end{bmatrix} = \frac{1}{2n+1} \begin{bmatrix} u_0\, u_1 & \cdots & u_{2n} \\ u_{2n}\, u_0 & \cdots & u_{2n-1} \\ \cdot & & \\ \cdot & & \\ \cdot & & \\ u_1\, u_2\, u_3 & \cdots & u_0 \end{bmatrix} \begin{bmatrix} i_0 \\ i_1 \\ \cdot \\ \cdot \\ \cdot \\ i_{2n} \end{bmatrix} \quad (26)$$

In the case, when 2n is small, eqn. (26) can be solved directly using the procedure given in Ref. [10]. Solving the cross correlation function from eqn. (26), and substituting into eqn. (13), the active and reactive power can be obtained using eqns. (15) and (16). The computing speed can be increased by using tables with the data functions $\sin\frac{2\pi mk}{2n+1}$ and $\cos\frac{2\pi mk}{2n+1}$, [11], and the regular structure of eqn. (26) can be realized with the help of small blocks, such as described in Ref. [10].

For illustration, consider as an example the design of an algorithm to determine the reactive power (the fundamental and the highest harmonic).

To determine the reactive power, substitute eqn. (24) in $Q = \sum_{m=1}^{n} Q_m$ and taking into consideration eqn. (15) gives

$$Q = \frac{2}{2n+1}\sum_{m=1}^{n}\sum_{k=1}^{2n} S_k \sin\frac{2\pi mk}{2n+1}\,. \quad (27)$$

After some mathematical manipulation one can obtain (Appendix)

$$Q = \frac{2}{2n+1}\sum_{k=1}^{n}\,(S_k - S_{2n+1-k})\sum_{m=1}^{n}\sin\frac{2\pi mk}{2n+1}\,. \quad (28)$$

Considering the cross-correlation function, eqn. (12), eqn. (28) can be written as

$$Q = \frac{2}{2n+1}\sum_{l=0}^{2n} i_l \sum_{k=1}^{n} W_{nk}\,(u_{l-k} - u_{l-(2n+1-k)})\,, \quad (29)$$

where

$$W_{nk} = \frac{\sin\frac{\pi nk}{2n+1}\,\sin\frac{\pi\,(n+1)k}{2n+1}}{(2n+1)\,\sin\frac{\pi k}{2n+1}}\;,\; k = 1,\ldots,n \quad (30)$$

Substituting m=1 in eqn. (29), the expression for the determination of the fundamental harmonic reactive power becomes

$$Q_1 = \frac{2}{2n+1}\sum_{l=0}^{2n} i_l \sum_{k=1}^{n} W'_{nk}\,(u_{l-k} - u_{l-(2n+1-k)})\,, \quad (31)$$

where

$$W'_{nk} = \frac{1}{2n+1}\sin\frac{2\pi k}{2n+1}\;,\; k = 1,\ldots,n \quad (32)$$

Taking m = 2, . . . , n and substituting eqn. (12) in eqn. (28), the expression for the determination of the reactive power of the higher harmonics becomes

$$Q_h = \frac{2}{2n+1}\sum_{l=0}^{2n} i_l \sum_{k=1}^{n} W''_{nk}\,(u_{l-k} - u_{l-(2n+1-k)})\,, \quad (33)$$

where

$$W''_{nk} = \frac{\sin\frac{\pi(n-1)}{2n-1}\,\sin\frac{\pi(n+2)k}{2n+1}}{(2n+1)\,\sin\frac{\pi k}{2n+1}}\;,\; k = 1,\ldots,n \quad (34)$$

The analysis of eqns. (31) - (34) shows that the reactive power is determined by the same algorithm. The difference lies only in the weight W.

Based on this algorithm, other power consumption characteristics can be determined. Substituting eqn. (12) in eqn. (16) and after some mathematical transformations, the expression to determine the fundamental harmonic active power can be determined as

$$P_1 = \frac{2}{2n+1}\sum_{l=0}^{2n} i_l\, u_l + \sum_{k=1}^{n} W'''_{nk}\,(u_{l-(2n+1-k)})\,. \quad (35)$$

where

$$W'''_{nk} = \frac{1}{2n+1}\cos\frac{2k}{2n+1}\;,\; k = 1,\ldots,n \quad (36)$$

Taking m = 2, . . . , n and substituting eqn. (12) in eqns. (16) and (25), the expression to determine the active power of the higher harmonics becomes

$$P_h = \frac{2}{2n+1}\sum_{l=0}^{2n} i_l\,[\,(n-1)\,u_l + \sum_{k=1}^{n} W''''_{nk}\,(u_{l-k} + u_{l-(2n+1-k)})\,]\,, \quad (37)$$

where

$$W''''_{nk} = \frac{\sin\frac{\pi(n-1)k}{2n+1}\,\sin\frac{\pi(n+2)k}{2n+1}}{(2n+1)\,\sin\frac{\pi k}{2n+1}} \quad (38)$$

Substituting u_l for i_l in eqns. (35) and (37), the expressions for the following characteristics can be determined:

effective value of the fundamental and harmonic voltage

$$U = \sqrt{\frac{2}{2n+1}\sum_{l=0}^{2n} u_l\,[\,u_l \sum_{k=1}^{n} W'''_{nk}\,(u_{l-k} + u_{l-(2n+1-k)})\,]} \quad (39)$$

effective value of the fundamental

$$U_1 = \sqrt{\frac{1}{2n+1}\sum_{l=0}^{2n} u_l^2}\,, \quad (40)$$

effective value of the higher harmonic voltage

$$U_2 = \sqrt{\frac{2}{2n+1}\sum_{l=0}^{2n} u_l\,[\,(n-1)\,u_l + \sum_{k=1}^{n} W''''_{nk}\,(u_{l-k} + u_{l-(2n+1-k)})\,]}$$

and so on.

A schematic structure for the proposed algorithm for the determination of the power consumption characteristics is shown in Fig. 1. In this figure, the output Q is given as:

$$Q = Q_1^o + Q_2^1 + \ldots$$

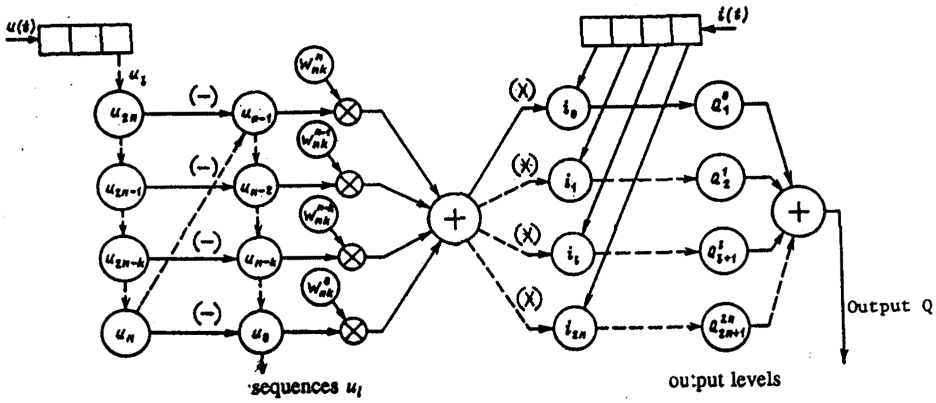

Fig. 1. Schematic structure for the design of the proposed algorithm.

where

$$Q_1^0 = i_0 \sum_{k=1}^{n} W_{nk} \left[u_{0-k} - u_{0-(2n+1-k)} \right]$$

$$Q_2^1 = i_l \sum_{k=1}^{n} W_{nk} \left[u_{1-k} - u_{1-(2n+1-k)} \right]$$

$$\vdots$$

$$Q_{2n+1}^{2n} = i_{2n} \sum_{k=1}^{n} W_{nk} \left[u_{2n-k} - u_{2n-(2n+1-k)} \right]$$

As shown, this scheme can be implemented in the form given in Fig. 2.

Detailed analysis of this scheme shows that the proposed device can be constructed on a chip using standard blocks i.e. multiplication, division, addition and subtraction.

Simulation Studies

The algorithm has been tested on the following examples:

(i) voltage $u(t)$ and current $i(t)$ are sinusoidal with 45° phase angle between them;

(ii) voltage $u(t)$ is sinusoidal and current $i(t)$ is a 60° phase-controlled waveform [9];

(iii) voltage $u(t)$ and current $i(t)$ both contain harmonics. The signals used were:

$$u(t) = 310.9 \sin \omega t + 11.51 \sin 3\omega t + 2.487 \sin 5\omega t$$

$$i(t) = 35.11 \sin(\omega t + 30^o) - 3.912 \sin(3\omega t - 90^o) + 1.416 \sin(5\omega t + 150^o) - 0.729 \sin(7\omega t + 30^o) + 0.446 \sin(9\omega t - 90^o) - 0.303 \sin(11\omega t + 150^o)$$

(iv) voltage $u(t)$ and current $i(t)$ both contain harmonics and the frequency also varies from 60Hz by a small amount.

The active P and reactive Q powers for each of the above cases were computed to an accuracy of 16 digits by the proposed algorithm and the algorithms proposed in Refs. [1, 2, 4, 9]. A comparison of the relative errors in P and Q with this algorithm and the minimum relative error obtained by any of the other three techniques, i.e. the most accurate out of the three techniques [1, 2, 4], is given in Tables 1 through 4 for the above four cases. The integration interval used was one period.

Table 1. Relative Errors for Sinusoidal Waveforms

	Relative error in P	Relative error in Q
Other methods	7.4E-8	7.06E-8
Proposed algorithm	7.012E-8	6.92E-8

Table II. Relative Errors for $u(t)$ sinusoidal and $i(t)$ a 60° phase-controlled waveform

	Relative error in P	Relative error in Q
Other methods	6.12E-7	7.11E-7
Proposed algorithm	574E-8	8.02E-8

Results in Table 1 and 2 for cases 1 and 2 respectively show that the accuracy of the proposed algorithm under normal conditions of power supply is good. The highest relative error compared with the theoretical value is 8.02E-8 in the determination of Q. Also, the error with this algorithm is lower

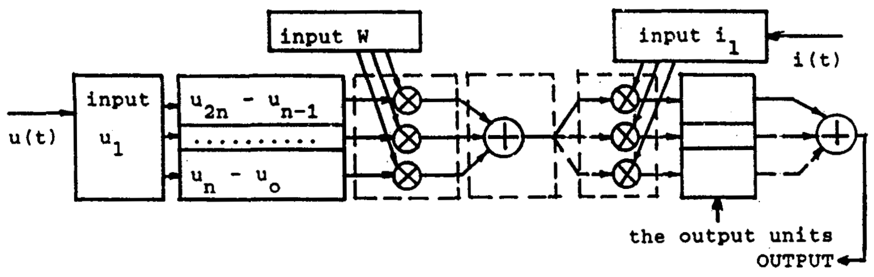

Fig. 2 Implementation of the proposed algorithm.

Table III. Relative Errors for $u(t)$ and $i(t)$ containing harmonics

	Relative errors in					
	$P_{sum.}$	$P_{fund.}$	$P_{harm.}$	$Q_{sum.}$	$Q_{fund.}$	$Q_{harm.}$
Other methods	4.1E-7	1.62E-6	7.4E-6	1.2E-7	1.14E-6	6.2E-6
Proposed algorithm	6.2E-7	2.13E-6	1.12E-6	2.01E-7	9.61E-7	8.15E-7

Table IV. Relative errors for $u(t)$ and $i(t)$ containing harmonics and with frequency deviation from 60 Hz.

Methods	Relative errors in					
	$P_{sum.}$ 1	$P_{fund.}$ 2	$P_{harm.}$ 3	$Q_{sum.}$ 4	$Q_{fund.}$ 5	$Q_{harm.}$ 6
Frequency deviation	-0.15%					
Other methods	5.16E-5	5.09E-5	4.14E-4	7.08E-5	9.1E-5	4.5E-4
Proposed algorithm	6.01E-5	5.94E-5	5.99E-5	6.41E-5	6.42E-5	6.8E-5
Frequency deviation	+0.15%					
Other methods	4.11E-5	4.97E-5	3.96E-4	6.12E-5	5.87E-5	3.8E-4
Proposed algorithm	5.16E-5	5.17E-5	6.14E-5	7.16E-5	6.92E-5	7.4E-5
Frequency deviation	-0.45%					
Other methods	2.99E-4	3.16E-4	6.12E-3	9.11E-4	8.64E-4	7.1E-3
Proposed algorithm	4.82E-4	5.13E-4	8.19E-4	7E-4	6.06E-4	8.8E-4
Frequency deviation	+0.45%					
Other methods	3.14E-4	3.26E-4	9.65E-3	5.88E-4	9.12E-4	4.6E-3
Proposed algorithm	6.11E-4	8.87E-4	7.13E-4	8.26E-4	6.71E-4	9.1E-4

than with other algorithms in every case. The non-zero errors for all algorithms are due to the finite precision in the computer number representation.

It is seen from Table III that the accuracy of the proposed algorithm is good even in the presence of harmonics. The highest relative error is 2.13E-6 for Q, and the error in each case is lower than in the other methods.

Results for the case of signals containing harmonics and their frequency not being the same as the nominal frequency are given in Table IV.

It is noticed that the relative errors in this case are higher than that of the case of nominal frequency. Also, the magnitude of error increases as the frequency deviation from the nominal frequency increases.

For the cases studied the biggest relative error with the proposed algorithm is 9.1E-4 for Q as compared to 9.65E-3 in P for the other methods. The largest relative error occurred in the determination of the harmonics. This is due to the deviation in the frequency being more significant for higher harmonics.

The results show that the accuracy of the proposed algorithm is acceptable under all conditions. It is somewhat independent of the harmonic content of the voltage and current signals and has good accuracy even if the frequency deviates from the nominal value.

Conclusions

An approach to the design of digital algorithms for measuring power consumption characteristics is described in this paper. The main advantage of the proposed technique is the calculation of all power consumption characteristics by the same algorithm.

The proposed approach is applicable to both sinusoidal and non-sinusoidal signal waveforms. Results show that this method has a high accuracy even with signals containing harmonics and for frequency that deviates from the nominal frequency. Thus the sphere of application of this method is wider than other techniques.

The algorithm proposed can be implemented on IC Chips using standard computation blocks which will keep the implementation cost relatively low. Using hardware implementation, the computational time can be kept very low compared to purely software based algorithms.

References

1. M. Kezunovic, E. Soljanin, B. Perunicic, S. Levi, "*New approach to the design of digital algorithms for Electric Power Measurements*", IEEE Trans. on Power Delivery, Vol. 6, No. 2, pp. 516-523, April 1991.
2. G.S. Hope, O.P. Malik, J. Chang, "*Microprocessor-based active and reactive power measurement*", Journal of Electric Power and Energy Systems, Vol. 3, No. 2, pp. 75-83, April 1981.
3. D.M. Woskobojnikow, "*A technique for the analysis of electric consumption in industry*", in 'Optimization and Forecast of Electric Consumption in Industrial Plants', Swerdlowsk, DNTP, 1980 (In Russian).
4. M. Savino, "*Measurements of voltage current power and energy in nonsinusoidal electric systems*", Energia Elektrica, Vol. IX, No. 10, 1983.
5. R.S. Turgel, "*Digital wattmeter using a sampling method*", IEEE Trans. on I & M, Vol. IM-23, pp. 337-341, December 1974.
6. A.J. Baggot, "*The effect of wave shape distortion on the measurements of energy tariff meters*", *IEE Metering, Apparatus and Tariffs for Electricity Supply, Conference Publication No. 156, pp. 280-284, London, 1977.*
7. H. Reck, "*Basis and development of correlation techniques in measurement*", Die Technik, Bd. 33, No. 1, 1978 (In German).
8. Z. Nowomiejski, "*Generalized theory for electric power*", Archiv fur Elektrotech., Vol. 63, No. 3, pp. 177-182, 1981.
9. I.H.R. Enslin, I.D. Van Wyk, "*Measurement and compensation of fictitious power under nonsinusoidal voltage and current conditions*", IEEE Trans. on I & M, Vol. 37, No. 3, pp. 403-408, September 1988.
10. R.E. Blahut, "*Fast algorithms for digital signal processing*", N. Y., 1985.

11. S.M. Mahmud, "*High precision phase measurement using reduced sine and cosine tables*", IEEE Trans. on I & M, Vol. 39, No. 1, pp. 56-60, 1990.

Appendix

Rewriting eqn. (27)

$$
\begin{aligned}
Q &= \frac{2}{2n+1} \sum_{m=1}^{n} \sum_{k=1}^{2n} S_k \sin \frac{2\pi mk}{2n+1} \\
&= \frac{2}{2n+1} \sum_{k=1}^{2n} S_k \sum_{m=1}^{2n} \sin \frac{2\pi mk}{2n+1}
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
&= \frac{2}{2n+1} \left[\sum_{k=1}^{n} S_k \sum_{m=1}^{n} \sin \frac{2\pi mk}{2n+1} + \right. \\
&\left. \sum_{k=n+1}^{2n} S_k \sum_{m=1}^{n} \sin \frac{2\pi mk}{2n+1} \right]
\end{aligned} \tag{42}
$$

Taking $k' = 2n+1-k$ or $k = 2n+1-k'$, the second expression in eqn. (42) can be written as

$$
\sum_{k'=n}^{1} S_{2n+1-k'} \sum_{m=1}^{n} \sin \frac{2\pi m(2n+1-k')}{2n+1} \tag{43}
$$

Now substituting

$$
\sin \left(2\pi m - \frac{2\pi mk'}{2n+1} \right) = -\sin \frac{2\pi mk'}{2n+1}
$$

in eqn. (43), the second expression in eqn. (42),

$$
\sum_{k=n+1}^{2n} S_k \sum_{m=1}^{n} \sin \frac{2\pi mk}{2n+1} = -\sum_{k=1}^{n} S_{2n+1-k} \sum_{m=1}^{n} \sin \frac{2\pi mk}{2n+1} \tag{44}
$$

Substituting eqn. (44) in eqn. (42) leads to eqn. (28).

O. P. Malik, (M'66-SM'69-F'87) graduated in 1952 and obtained M.E. in 1962. In 1965 he received Ph.D. from the University of London. From 1952 to 1961, he worked with electric utilities in India. He is at present a Professor at the University of Calgary.

Anatoly Voloshko received his Dipl. Ing. degree in Electrical Engineering and Ph.D. degrees in the Information Measurement Systems in 1983 and 1986 respectively from Kiev Politechnik Institute, Ukraine. Since 1978 he has been working in the Kiev Politechnik Institute, and at present is an Associate Professor there. His current research interest is in the areas of digital algorithms for measurement and control, and development systems for the measurement of energy supply.

Oleg Kotsar received his Dipl. Ing. degree in Electrical Engineering from Kiev Politechnik Institute in 1986. Since 1982 he is working in the Kiev Politechnik Institute, Ukraine as a scientist. His current research interest is in the development of measurement systems.