technische universität
dortmund

Case Studies

Summer 2022

Project I

One-Quarter-Ahead Forecasts of US GDP Growth
A Vector Autoregression Approach

Jorge Eduardo Durán Vásquez
May 5, 2022

in collaboration with
Christopher Gerlach, Felix Fikowski, Sven Pappert

Lecturers: Prof. Dr. Matei Demetrescu, M.Sc. Karsten Reichold
M.Sc. Alexander Gerharz

Contents

1 Introduction

Governments, business planers and investors take the real Gross Domestic Product (GDP) growth as an important macroeconomic indicator to assess the performance of a country's economy. The GDP is one of the most important indicators that reflects societies' material living standards (OECD, 2009) and is often used for defining strategies and monetary policies. For example, policymakers use historical macroeconomic indicators, among others GDP, to forecast and evaluate possible expansions or recessions in the economy (Kitchen J. Monaco R., 2003). These predictions allow governments and companies to develop strategies that respond to the corresponding economic situation.

The purpose of this report is to perform various Vector Autoregressive (VAR) models to forecast the US real GDP growth. The models use time series that starts from the Q1 of 1959 and are provided by the Federal Reserve of Economic Data. We use time series of GDP, the unemployment rate, manufacturing industry capacity, inflation, Federal Funds rate, as well as stock market indexes and measures of liquidity in the economy like M1 Real. We start fitting a simple autoregressive (AR) model with only the information of the GDP itself. Then the six additional macroeconomic indicators are added to create a VAR model of order one to predict GDP growth. With this result Granger Causality is used to test possible causalities between the mentioned indicators and GDP. In addition, with the Akaike information criteria (AIC) the most suitable lag-order for VAR Techniques is selected. After applying the methods outlined above, we found that even though the AR(1) model underestimates variations in GDP growth it achieves a better value of root mean square error than VAR(1). The VAR(1) model gives reasonable and consistent predictions of expansions and recessions in the economy, however it is sensitive to extreme observations, which diminish its performance. The lag-one GDP growth and Inflation are the variables that are not Granger causal for GDP growth. Finally, according to the AIC, the three-lag-order provides the most information in the VAR model.

This report is structured as follows. In section 2, the data set as well as the process of data collection and data quality is described. Section 3 explains the used software tools and statistical methods, which include definition, parameter estimation and forecasting in VAR models, Granger Causality and AIC for lag-order selection. In section 4, the results of the analysis are presented. Finally, section 5 concludes the report and provides recommendations for future research.

2 Data

Since January 1959 the research department at the Federal Reserve Bank of St. Louis in the US provides monthly and quarterly frequency data on US macroeconomic variables. The databases are called "FRED" for Federal Reserve Economic Data. The data set for quarterly data is called FRED-QD and consists of 247 quarterly time series of macroeconomic indicators. It goes from the first quarter (Q1) of the year 1959 until last quarter (Q4) of the year 2021 (McCracken, M.W. & Ng S., 2020).

To perform the following macroeconomic analysis 7 cardinal time series are extracted from FRED-QD. The first variable is `GDPC1`, real (inflation adjusted) GDP in billions of chained 2012 US dollars. It belongs to the group National Income and Product Accounts (NIPA). The term "Gross" in GDP means that no deduction has been applied for the depreciation of capital products (machinery, buildings) that were used in the production. "Domestic" means that the production belongs to resident companies of the respective country. And "Product" refers to final goods and services that are purchased during a period (OECD, 2009). `GDPC1` is transformed to `GDP growth` because that provides a more stationary measure for the periodic change in the volume of production in the US economy than the absolute GDP.

The second variable, the manufacturing industry capacity utilization `CUMFNS`, is given in percentage of capacity. It refers to the percentage ratio of actual production over the potential production of a country. This variable relates to the future performance of the economy in one nation and therefore can be related with the total production of that country in the next period according to the US Federal Reserve Board.

The third variable `UNRATESTx`, which represents the unemployment rate, is given in percentage. This variable measures the rate of the number of unemployed people for less than 27 weeks compared to the total labor force, where the labor force is the sum of the employed and unemployed people during the respective time reference (U.S. Bureau of Labor Statistics, 2021).

The consumer price index for all urban consumers `CPIAUCSL` is the fourth variable. It measures the price changes over time for goods and services that a reference population, in this case urban consumers, buys for consumption (OECD, 2002). For the analysis in this report the quarterly growth for `CPIAUCSL` is used which is generally known as `Inflation`.

The real M1 money stock `M1REAL` is given in billions of 1982-84 US dollars which are deflated by the consumer price index. M1 money stock is a measure of the liquidity in the economy and consists of mainly three components. First, the money outside of the US Treasury, Federal Reserve Banks, and the vaults of depository institutions. Second, the demand deposits at commercial banks and finally, other liquid deposits (Federal Reserve, 2021). The quarterly growth of `M1REAL` is calculated and incorporated in the analysis as `M1REAL growth` in percentage.

Finally, the last variable is `S&P 500`, which is the Standard & Poor's 500 stock price index. It is composed of the 500 biggest companies in the US. This indicator refers to a high proportion of the total US equity market value and is affected by the health of the US economy and investor expectations. We transform the variable to a quarterly growth percentage and call it `S&P 500 growth`.

The time series given in FRED-QD are not transformed in any way. For the subset of seven variables selected for this study no missing values are presented. The quality is ensured by the Federal Reserve Bank of St. Louis. Many of the macroeconomic indicators have outliers that in many cases are related with economic crises. In the case of `M1REAL` a change in the definition of the indicator at the beginning of May 2020 leads to a high variation of its value, which generates an extreme outlier in the data.

3 Methods

A series of random variables that are sorted and equally spaced by time is called time series. For this analysis, the variable y_1 is the value that the random process takes at time $t = 1$, the variable y_2 is the value of the random variable at time $t = 2$ and so on for y_3, y_4, $\ldots$, y_T, where T represents the time of the last observation of the random variable Y (Brockwell, J.P. & Davis, R.A., 1996).

To work with time series, it is important to analyze the trend, seasonal patterns, the long-term cycles and the random error. A time series may or may not have one or all of these components. The trend refers to the overall pattern of the time series, which can be either an increasing or decreasing polynomial trend. The seasonal patterns refer to a fluctuation in the variable that is stable over time and happens with relative similar timing, direction, and magnitude. The cycles are those systematic fluctuations in the data that appear periodically because of a regular behavior (Sharpe et al., 2012). Models also contain the component of noise or random error which is the remaining values after subtracting all other components. For the purpose of this work the error will be assumed independent and identically distributed (iid) with mean 0 and variance σ_w^2 (Brockwell, J.P. & Davis, R.A., 1996)

3.1 Stationary Time Series

Stationarity is a special assumption about the behaviour of the time series and looks for regularity of the variable over time. A stationary time series, Y_T, is a finite variance sequence of values such that the mean value function, μ_t, is constant and does not depend on time t (Brockwell, J.P. & Davis, R.A., 1996). The mean for a time series is defined as:

$$\mu_{yt} = \int_{-\infty}^{\infty} y f(y) dy$$

In addition, a stationary process needs to fulfill that the autocovariance function does not depend on time (Brockwell, J.P. & Davis, R.A., 1996). The autocovariance function a time h is given by

$$\gamma(h) = E[(y_{t+h} - \mu)(y_t - \mu)].$$

To achieve any meaningful statistical analysis in the modeling of time series, it is important that the mean and the autocovariance functions of the variables satisfy the conditions of stationarity, which means that the time series shows no trend over time and has a relatively constant variance (Sharpe et al., 2012). To achieve a stationary time series in most of the cases the data can be transformed. Some of the most common transformations are first difference ΔY_t, first log difference $\Delta log(Y_t)$ and growth rates (Brockwell, J.P. & Davis, R.A., 1996).

3.2 Vector Autoregressive Model

The next section is mainly based on (Kilian, L., Lütkepohl, H. 2017). VAR Models are used for multivariate stationary time series analysis, which comprise a system of regression equations. Each time series has a regression model which is formed by lags of its own and lags of the other time series. The maximum number of lags is given by the order "P" of the model. In general, the structural VAR(P) model which approximates the data generating process best, is of the following form:

$$\boldsymbol{y_t} \; = \; A_0 \; + \; A_1\boldsymbol{y_{t-1}} \; + \; ... \; + \; A_p\boldsymbol{y_{t-p}} \; + \; e_t \tag{3.1}$$

where $\boldsymbol{y_t}$ is the vector of the observations in the time series and has the dimensions Kx1. $t = 1, \ldots, T$ where T is the total time units of the time series and K is the number of different time series. A_0 is the intercept and A_i are the KxK parameter matrices with $i = 1, \ldots, p$ and $\boldsymbol{e_t}$ is the k-dimensional white noise. The matrix representation for a system of three variables is of the form:

$$\boldsymbol{y_t} = \begin{pmatrix} y_{1,t} \\ y_{2,t} \\ y_{3,t} \end{pmatrix}, \; A_i = \begin{bmatrix} a_{10,i} & a_{11,i} & a_{12,i} & a_{13,i} \\ a_{20,i} & a_{21,i} & a_{22,i} & a_{23,i} \\ a_{30,i} & a_{31,i} & a_{32,i} & a_{33,i} \end{bmatrix}, and \; e_t = \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix} \tag{3.2}$$

3.3 Parameter estimation in VAR Models

For estimation of VAR(P) models we use the Least-Squares estimation method. For this, we consider the following compact form:

$$\boldsymbol{y_t} \; = \; [A_0, A_1, \; ... \; A_p]\boldsymbol{Z_{t-1}} \; + \; \boldsymbol{e_t} \tag{3.3}$$

Where the $\boldsymbol{Z}$ matrix is given by $\boldsymbol{Z_{t-1}} \equiv (1, \boldsymbol{y'_{t-1}}, \; ... \; \boldsymbol{y'_{t-p}})'$, $\boldsymbol{A_0}$ is the intercept vector and the error term is assumed to be iid white noise. According to the order P, the model requires the pre-sample vectors $\boldsymbol{y_{-p+1}}, \ldots, \boldsymbol{y_0}$. The LS estimator is given by

$$\boldsymbol{\hat{A}} \; = \; [\boldsymbol{\hat{A}_0}, \boldsymbol{\hat{A}_1}, \; ... \; \boldsymbol{\hat{A}_p}] = (\sum_{t=1}^{T} \boldsymbol{y_t Z'_{t-1}})(\sum_{t=1}^{T} \boldsymbol{Z_{t-1} Z'_{t-1}})^{-1} = \boldsymbol{Y Z'(Z Z')^{-1}} \tag{3.4}$$

Where $\boldsymbol{Y} \equiv [\boldsymbol{y_1, \; ... \; y_T}]$ is KxT and $\boldsymbol{Z} \equiv [\boldsymbol{Z_0, \; ... \; , Z_{T-1}}]$ is $(Kp+1)$xT. Staking the columns of the estimated parameter matrices we obtain the vector $\alpha = vec(\boldsymbol{A})$ with dimensions $(pK2 + K) \times 1$. Which under the assumptions of iid errors $\boldsymbol{e_{it}}$ and the asymptotic normal distribution of the LS estimator follows:

$$vec(\boldsymbol{\hat{A}}) \sim \mathcal{N}(\boldsymbol{vec(A), (ZZ')^{-1}} \otimes \widehat{\Sigma}_u)$$

where $\otimes$ denotes the Kronecker product and $\widehat{\Sigma}_u$ is the estimator of the white noise covariance matrix given by:

$$\widehat{\Sigma}_u = \frac{\hat{U}\hat{U}'}{T - Kp - 1} \tag{3.5}$$

The numerator is the product of the LS residuals matrix, where $\boldsymbol{\widehat{U} = Y - \widehat{A}Z}$.

3.4 Prediction VAR Models

Following the structural VAR(P) equation 3.1. The corresponding forecast vector of values $\hat{\boldsymbol{y}}_{\boldsymbol{T+h|T}}$ at time $T+h$ given the information up to time T, is calculated as follows:

$$\hat{\boldsymbol{y}}_{\boldsymbol{T+h|T}} \;=\; \hat{\boldsymbol{A}}_{\boldsymbol{0}} + \;\hat{\boldsymbol{A}}_{\boldsymbol{1}}\boldsymbol{y}_{\boldsymbol{T+h-1|T}} \;+\; ... \;+\; \hat{\boldsymbol{A}}_{\boldsymbol{p}}\boldsymbol{y}_{\boldsymbol{T+h-p|T}} \tag{3.6}$$

3.5 Autoregressive process AR(P)

When a time series has patterns or regular fluctuations with a consistent trend, autoregressive models are used. Weight based modeling facilitates reproducing such structures by multiple regression models (Sharpe et al., 2012). In the case of an autoregressive model, the time series is weighted with respect to a lagged, of order P, version of itself. An autoregressive model of order P (AR(P)) has the form:

$$y_t \;=\; a_0 \;+\; a_1 y_{t-1} \;+\; ... \;+\; a_p y_{t-p} \;+\; e_t$$

where y_t represents a unique stationary time series, a_0 the intercept, and a_1, a_2 ,... ,a_P are constants different from zero. The element e_t is a Gaussian white noise with mean zero and variance σ_w^2. The first order autoregressive model AR(1) is given by $y_t \;=\; a_0 + a_1 y_{t-1} + e_t$. AR(P) processes are a particular case of VAR(P) models with $K = 1$. For that reason for estimation and forecasting the same approach that was presented in VAR(P) is applicable.

3.6 Mean square forecasting error

The measure of the error in the prediction is given by the root mean square error (RMSE) (Fahrmeir et al, 2013):

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T-1}(y_{t+1} - \hat{y}_{t-1|t})^2}$$

3.7 Granger Causality Analysis

In 1969, Granger proposed a method to evaluate the dynamic relationship between economic variables based on the VAR model. Granger proposed that a variable y_{2t} is causal for a variable y_{1t} if the information in past, lagged observations, and in present values of y_{2t} helps to reduce the expected square prediction error for y_{1t}.

For the general VAR(P) model described in equations 3.1, 3.2 and 3.6 not rejecting the hypothesis $\mathbb{H}_0 : \hat{a}_{12,1} = \ldots = \hat{a}_{12,p} = 0$ means that y_{1t} does not Granger cause y_{2t}. To find Granger Causality a multi-parameter testing is required, Kilian, L. and Lütkepohl, H. (2017) suggest the Walt test. However, in the case of VAR(1) a t-test with significance level of 0.05 can be applied. The VAR(1) regression equation is reduced to a set of one parameter per explanatory variable in the vector $\boldsymbol{y_t}$. The hypothesis for one parameter is given by:

$$\mathbb{H}_0 : \hat{a}_{12} \ = \ 0 \, against \, \mathbb{H}_1 : \hat{a}_{12} \ \neq \ 0$$

And the t-statistic is calculated with:

$$t_j = \frac{\hat{a}_{1j}}{\widehat{var}(\hat{a}_{1j})^{1/2}} \sim t_{T-(k+1),(1-\frac{\alpha}{2})}$$

where $(k+1)$ is the number of parameters plus the intercept. $\widehat{var}(\hat{a}_{1j})$ is the estimated variance of the least square estimators $\hat{a}_{1j}$, which is possible to obtain from the diagonal elements of the covariance matrix defined in equation 3.5. In addition, we can apply the big sample approximation of the t-distribution to a standard normal distribution when the value of T is big.

3.8 Akaike Information Criterion

VAR(P) models in practice do not have an explicit value for P. Information criteria are used for choosing a suitable lag-order. It is required to choose the best compromise between a good model fit and the complexity of the model to avoid overfitting (Fahrmeir et al, 2013). The AIC is one method used to find the best model based on computing one equation for each possible model and then directly comparing this value. Models with small values of AIC are considered better models (Fahrmeir et al, 2013). The AIC is defined by:

$$AIC(m) = log(det(\tilde{\Sigma}_u(m))) + \frac{2}{T}(mK^2 + K)$$

where the value of $det(\tilde{\Sigma}_u(m)))$ is the determinant of the residual covariance matrix calculated with the method of maximum likelihood estimation. m indicates the number of lags. The value $(mK^2 + K)$ is related with the number of parameters in the model. To obtain the value of the maximum likelihood residual covariance matrix the following transformation can be used:

$$\hat{\Sigma}_u(m) = \frac{T}{T - Km - k}\tilde{\Sigma}_u(m)$$

where k is the number of deterministic regressors in each equation. For evaluating AIC models Kilian, L. and Lütkepohl, H. (2017) suggest compare 4 lags for quarterly data. For calculating the AIC it is important to be aware that each time that a model with different number of lags is calculated the evaluation period (number of observations) of AIC also changes. Therefore, it is essential that the computation of $\tilde{\Sigma}_u(m)$ for AIC comparison takes the same starting time in all models. The evaluation period is given by $t = P_{max} + 1, ..., T$ for all m models. P_{max} is the number of pre-sample vectors needed in the model with the highest lag.

3.9 Software tools

The Software used for this report is R in the version R 4.0.5 GUI 1.74 for MAC iOS (R Core Team, 2021).

4 Results

4.1 Analysis of the Data Set

Each of the seven time series variables that were subtracted from the FRED-QD database consists of 252 observations. These entries correspond to quarterly measures of each variable between 1959 and 2021. In order to achieve stationary time series, some of the variables were transformed using growth rates. After transforming the variables, the first observation of the variables that were not transformed is dropped in order to have a consistent dimension. The seven variables over the period 1959Q2–2021Q4 are presented in Figure 1.

For many of the seven macroeconomic indicators there are extreme values that correspond to different economic crises that occurred in the US between 1959 and 2021. For example, the Oil crisis in 1973, "Black Monday" in 1987, the financial crisis in 2008 and the Covid-19 pandemic in 2020 have had a negative effect on the normal behavior of the economy and are reflected in a decay of many of the growth rates in Figure 1. In addition, as can be seen in the Appendix, almost all of the variables have a symmetric variation around the median. However, the variable `FEDFUNDS` shows a slightly left skewed distribution since most of the values of Federal Funds rate are in between 0 and 10%.

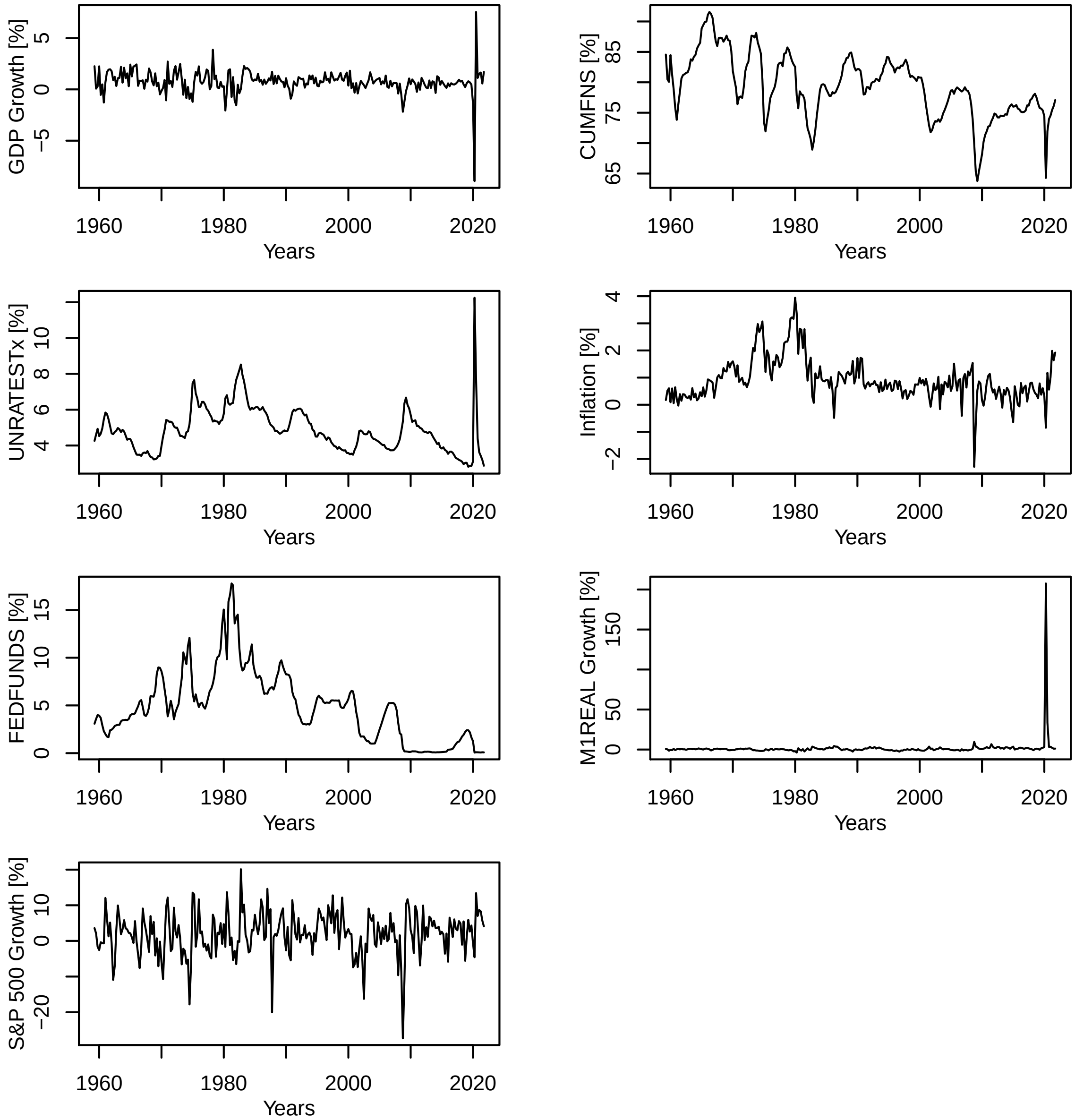

Figure 1: Macroeconomic indices

After applying the growth transformation of the variables, the trend component of the time series `GDP`, `CPIAUCSL`, `M1REAL` and `S&P 500` was removed. This can be seen by the stable fluctuation around the mean of the transformed time series. Furthermore, many of the variables show a stable fluctuation of the variance and the mean over time and do not show clear seasonality effects in the observed time window. Stationarity is a fundamental property of time series analysis and for this work we assume that all variables follow a stationary process. This assumption allows us to apply the AR(P) and VAR(P) models in order to forecast the GDP growth.

4.2 One-quarter-ahead forecasts of GDP growth AR(1)

We forecast the `GDP growth` by applying the AR(1) model in the data between the third quartile of the year 1959 and the last quartile of the year 2021. We start forecasting the values from the first quartile of 1960 because the first observation has been used for data transformation purposes, and the next three are used for parameter estimation in the autoregressive model for the first forecast. The forecast is implemented starting from the second measure and going until the last observed period. Each time all information of `GDP growth` before the forecasted value is used in the model. The forecasting result of the AR(1) model can be seen in Figure 2.

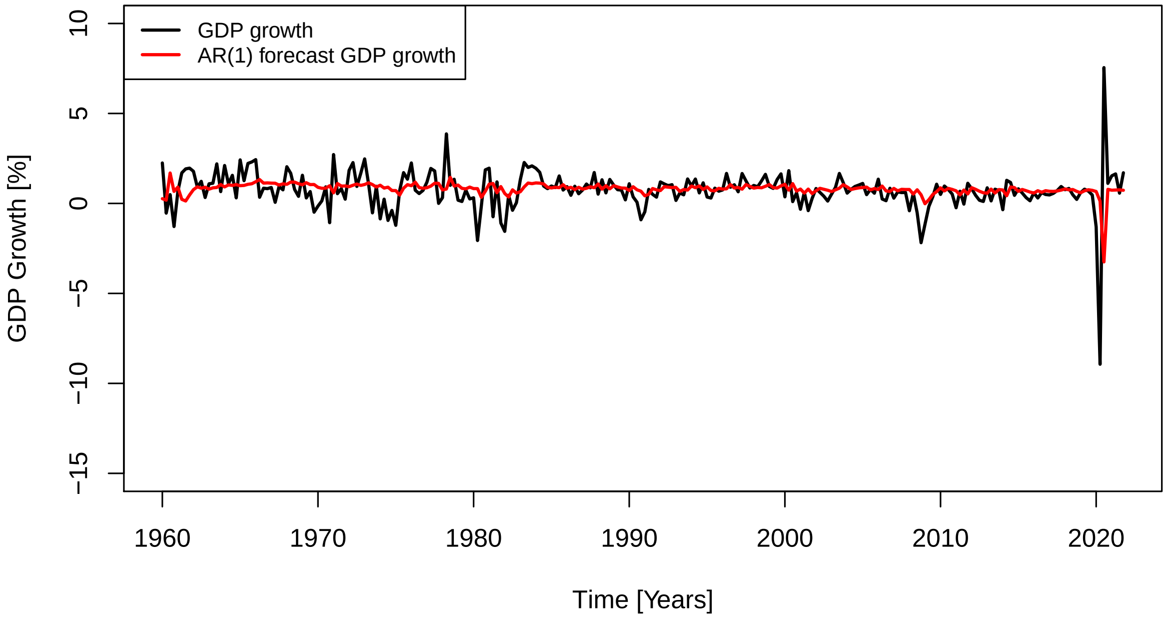

Figure 2: AR(1) one-quarter-ahead forecast of `GDP growth`

The AR(1) forecast shows instability between 1960 and 1962 because of the limited number of observations that are available for making predictions. From 1963 on the model shows a stable and constant behavior, and it looks like it is fitting the mean value over time. However, some extreme changes are represented in the forecasting as well, for instance in 2008 and 2020. The root mean square forecasting error over the period between 1960 and 2021 corresponds to 1.20.

4.3 One-quarter-ahead forecast of GDP growth VAR(1)

The `GDP growth` is now forecasted using the model VAR(1) and the time series `GDP growth`, `CUMFNS`, `UNRATESTx`, `Inflation`, `FEDFUNDS`, `M1REAL` and `S&P 500`. As was explained in the method section, The VAR(1) model requires one pre-sample vector and at least $k+1$ observations for parameter estimation. In this case our number of dimensions k is seven, which means that the model requires the first nine observations for forecasting. The forecasting is given between the Q3 of the year 1961 and Q4 of 2021.

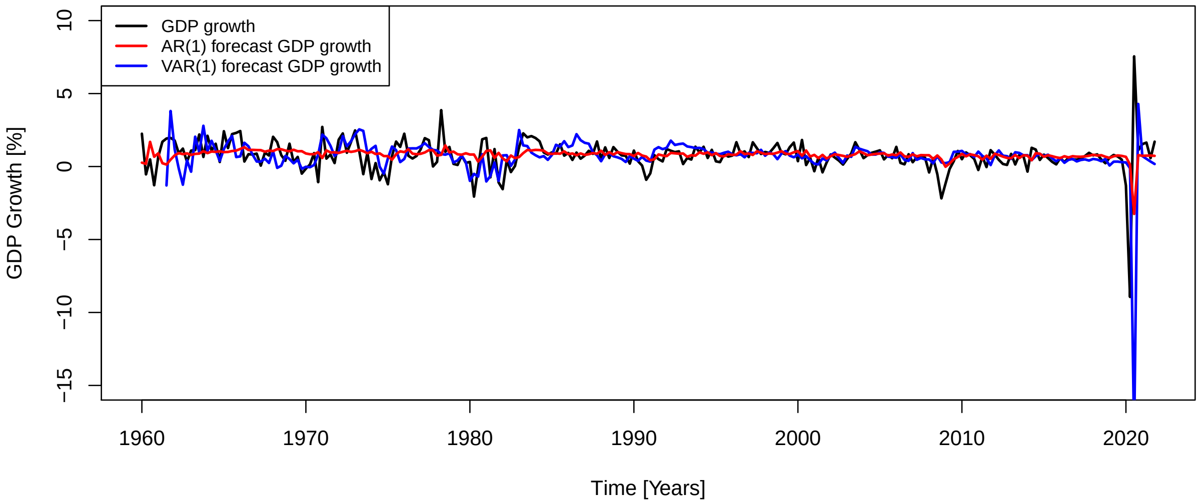

Figure 3: VAR(1) one-quarter-ahead forecast of `GDP growth`

Figure 3 illustrates the forecast of VAR(1) together with the official `GDP growth` rates and the forecast of the AR(1) model. In the VAR(1) model the first 10 observations are missing a prediction because they do not have enough past data to base their prediction on. Also, the graph illustrates that the VAR(1) predictions replicate some of the behaviours of the real GDP growth. However, the model is also more sensitive to a low number of historic data as well as to extreme values. This generates a higher root mean square forecasting error, namely 1.98.

4.4 Granger Causality

To test the Granger causality of the time series that were used in the model VAR(1), we use the t-test because in this model we only have one coefficient estimation per regression variable. Also, we can use the large sample approximation of the t-distribution to a standard normal distribution. Given a level of significance alpha of 0.05 and the 2-sided test, the critical value for the t- test is $(+/-)1.96$. This means that when the t-statistic is higheror lower than (+/−)1.96 we reject the null hypothesis, which indicates that the estimated parameter of the respective variable is not zero and then Granger causal.

The hypothesis test was performed for the estimated coefficient of each lagged macroeconomic indicator in the model. We find that the t-statistics for `GDP growth` (1.44) and `Inflation` (-1.60) are in the non-rejection region and therefore are not Granger causal for forecasting GDP.

4.5 Lag-Order Selection Akaike Information Criterion (AIC)

Determining the number of lags that is optimal for modeling time series data is important to find the best number of past values that helps forecasting one quarter ahead behavior. With the objective of finding the lag-order that provides most information in the prediction of VAR(p) models for GDP growth the AIC is used. We run the model for the values of "P" equal to 1, 2, 3 and 4 and for each case the AIC value was calculated. The results are 2.05, 1.40, 1.33 and 1.44, respectively. According to the AIC criterion, the model with the lowest AIC value, in this case the VAR(3) model with an AIC value of 1.33, is the best lag-order. Figure 4 shows the comparison of the VAR(3) model with the previous fitted models.

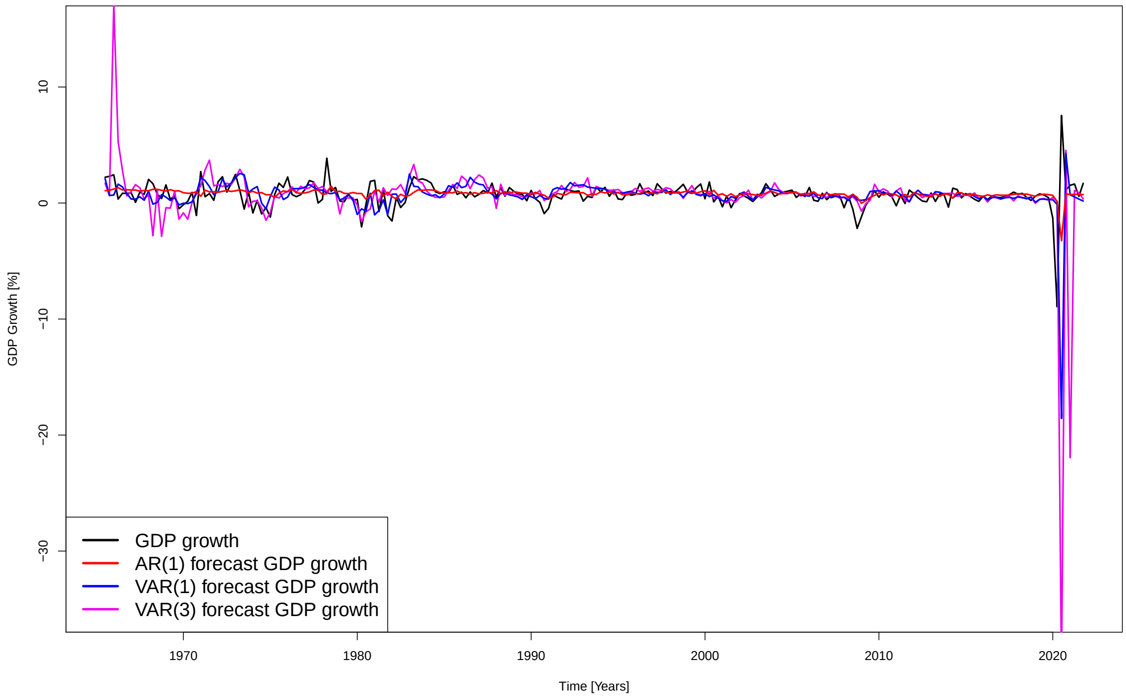

Figure 4: Comparison One-quarter-ahead forecast of `GDP growth`

For fitting the VAR(3) model we require the first 25 observation, which is the reason why the forecasting illustrated in Figure 4 starts from the Q3 of 1965 and ends in 2021 Q4. The plot shows a big variation in the forecasting values due to low historic data at the beginning of the 60s and extreme values in 2020. These extreme variations impact the value of the RMSE for VAR(3) models, which is 3.87, considerably.

4.6 Model Comparison

If the performance of the AR(1), VAR(1) and VAR(3) models is only based on the error value given by the root mean square error (RMSE), the AR(1) model is the model with the best performance. The RMSE value for AR(1) is 1.20, while it is 1.98, respectively 3.87, for VAR(1) and VAR(3). AR(1) only requires three pre-sample observations for parameter estimation, which means that the prediction starts from the fourth time point. For the VAR(1) and VAR(3) models the number of required pre-sample observations grows proportionally to the number of variables K and the lag-order P. The VAR(1) model requires at least eight observations and VAR(3) 25, and the first predictions are respectively the ninth and 26.

Table E.1 in the appendix shows a comparison of the three models in different time widows. In most of the cases the RMSE of AR(1) is lower than the others. There are two periods, from 1970 to 1979 and 2000 to 2009 where, respectively, the model VAR(1) and VAR(3) have a better RMSE than the others. If we only compare the period in which all models has prediction, which is 1965Q3 to 2021Q4, the RMSE for AR(1) is still better than the others.

The AR(1) model shows an overall good RMSE value and requires few pre-sample observations, but still the model has problems to fit expansions or recessions in the economy given that the predicted values follow a nearly flat distribution. On the other hand, the VAR(1) and VAR(3) models fit the variations in the economy better, and hence they are advantageous for predicting expansions or recessions. However, they have problems in prediction when outliers are present in any of the explanatory variables. The VAR(1) model is the model that predicts expansions and recessions the best. In general, for all models the predictions that are based on few data are problematic as well as those that are made with extreme values (economic crises).

5 Conclusion

This report presents the results of forecasting the US real `GDP growth` by performing various vector autoregressive (VAR) models. The macroeconomic data is provided by the Federal Reserve Economic Data and for the models we used seven variables with 252 observations. The variables that are used in the models are unemployment rate, manufacturing industry capacity, inflation, Federal Funds rate, S&P 500 growth and measures of the variation of liquidity in the economy like the growth in M1 Real.

The prediction of `GDP growth` obtained with an autoregressive model AR(1) underestimates high variations in the behavior of the economy and is close to the mean of the historical data. Using all additional indicators, the VAR(1) model gives reasonable and consistent predictions of expansions and recessions in the economy, however it is sensitive to extreme observations in any of the explanatory variables used for predictions. This sensitivity for extreme observations increases the root mean square error (RMSE) in comparison to the AR(1) model. Using the coefficients that were estimated for the VAR(1) model we test the Granger Causality between the seven time series and `GDP growth`. In this regard, the lag-one version of `GDP growth` and `Inflation` are the variables that are not Granger causal for `GDP growth`. From the noncausality of the lag-one observation of `GDP growth` we can conclude that this variable loses importance in the presence of the other variables, which have more economical relation with the value of GDP. In the case of Inflation, we can conclude that the inflation of the previous quartile does not affect the predictions of the `GDP growth`, however we suggest being careful with this result and to perform further studies with higher lag-order, ensuring stationarity and make previous outlier treatments. Finally, with the Akaike information criteria, we were looking for the most suitable lag-order (P) for VAR techniques which, for the given data, is order 3. This means that the combination of the first three lagged observations add the most information in our model and can enhance the predictions.

In addition, further research on exploring different lag-orders in AR(P) and VAR(P) models as well as implementing the variable selection criteria is recommended. Also, ensuring the stationarity condition in all explanatory variable can give more reliable results. Further works could also implement methods of outlier treatment for the variables `M1REAL` and `Unemployment Rate`. In addition, data inconsistency of the `M1REAL` variable, due to the change of its definition in 2020, could be addressed.

References

Bernanke, Ben S. and Alan S. Blinder (1992), "The Federal Funds Rate and the Channels of Monetary Transmission" The American Economic Review, pp.901-921.

Brockwell, J.P. & Davis, R.A. (1996). Introduction to Time Series and Forecasting. Springer, New York: excellent introduction.

Fahrmeir, L., Kneib, T., Lang, S. & Marx, B. (2013). Regression Models, Methods and Applications. Berlin: Springer-Verlag.

FEDERAL RESERVE statistical release (2021) MONEY STOCK MEASURES USA. Retrieved May 2 2022, www.federalreserve.gov/releases/h6/current/default.htm

Kilian, L., Lütkepohl, H. (2017). Structural Vector Autoregressive Analysis. Cambridge University Press, Cambridge.

Kitchen J. Monaco R. (2003), REAL-TIME FORECASTING IN PRACTICE. Department of the Treasury. United States of America.// URL https://mpra.ub.uni-muenchen.de/21068/

McCracken, M.W., Ng S. (2020). FRED-QD: A Quarterly Database for Macroeconomic Research. Federal Reserve Bank of St. Louis Working Paper 2020-005. URL https://doi.org/10.20955/wp.2020.005

OECD (2009). Gross Domestic Product (GDP), Organisation for Economic Co-operation and Development Publishing,, URL: https://www.oecd.org/berlin/44681640.pdf

OECD (2002). MAIN ECONOMIC INDICA TORS. Organisation for Economic Co-operation and Development Publishing, URL: https://www.oecd.org/sdd/prices-ppp/1947731.pdf

R Core Team (2021). R: A language and environment for statistical computing. R Foundation for Statistical Computing, Vienna, Austria. URL: https://www.R-project.org/.

Sharpe, N. R., De Veaux, R. D. & Velleman P. F. (2012). Business Statistics, 2nd Edition. United States of America Pearson Education, inc.

U.S. Bureau of Labor Statistics. (2021). Concepts and definitions. Retrieved May 2, 2022, from https://www.bls.gov/cps/definitions.htm

Appendix

A Additional figures

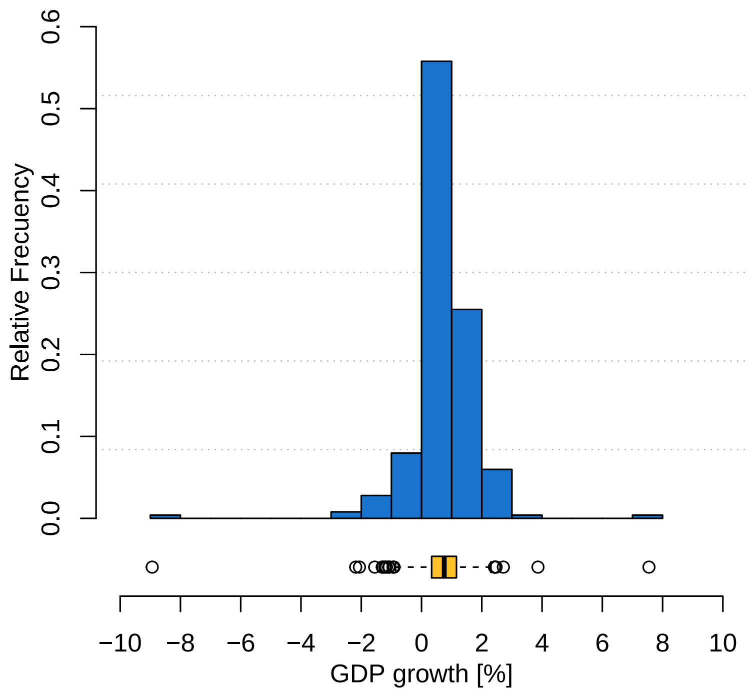

Figure E.1: Histogram of `GDP growth`

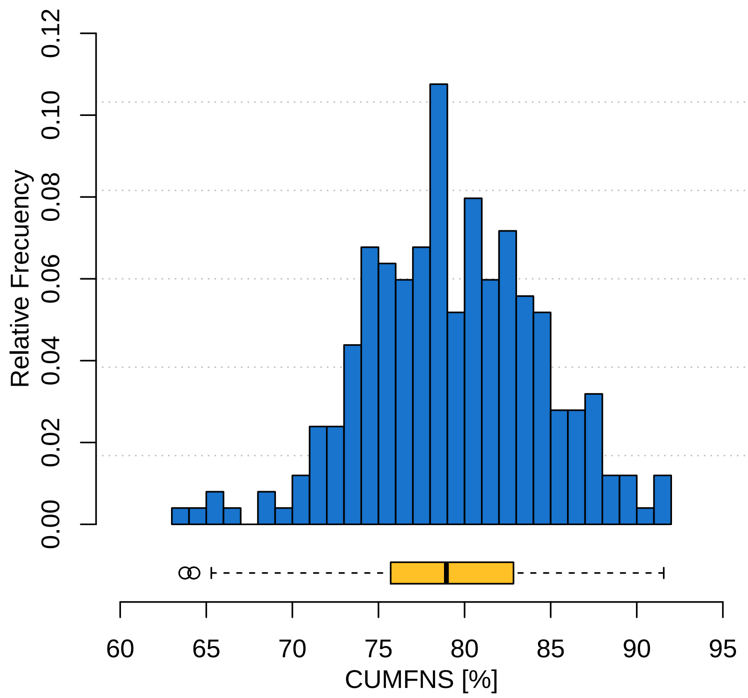

Figure E.2: Histogram of `CUMFNS`

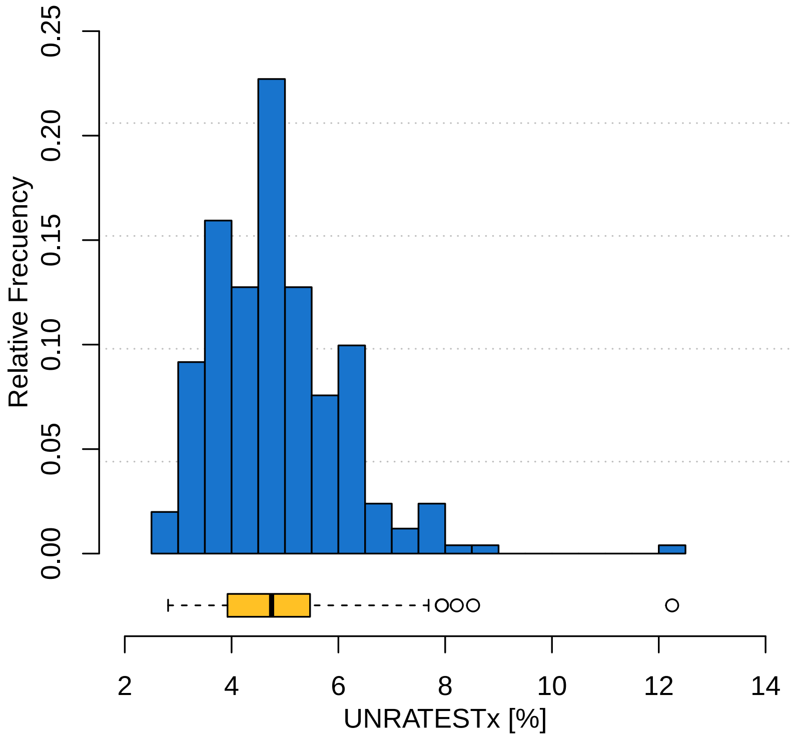

Figure E.3: Histogram of `UNRATESTx`

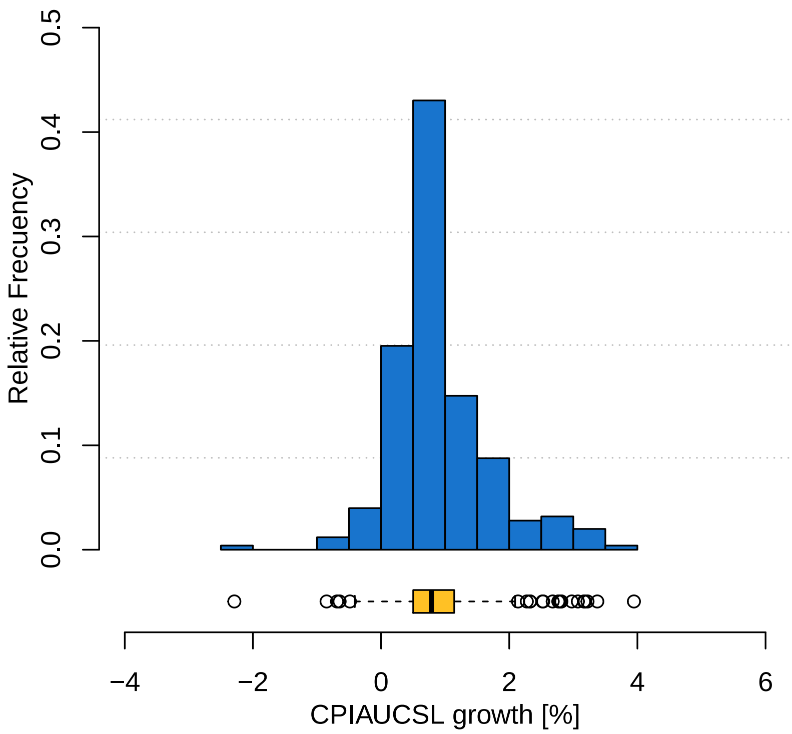

Figure E.4: Histogram of `CPIAUCSL`

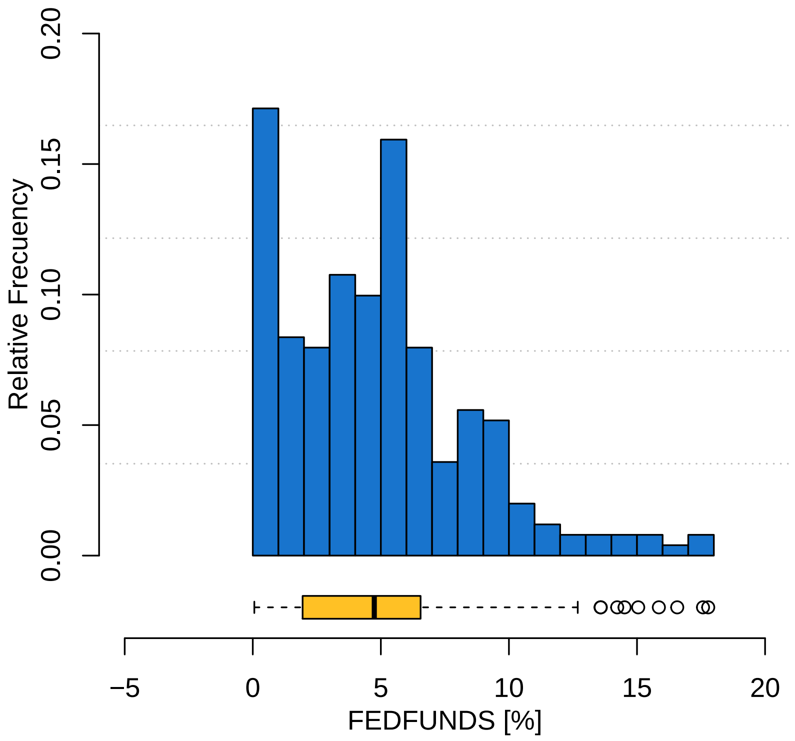

Figure E.5: Histogram of `FEDFUNDS`

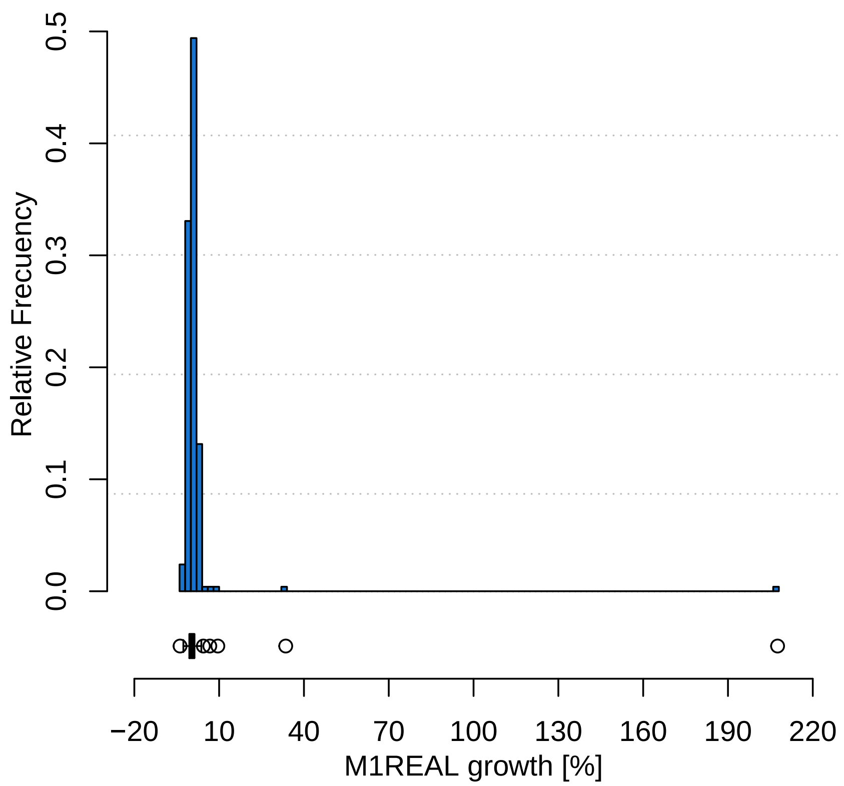

Figure E.6: Histogram of `M1REAL Growth`

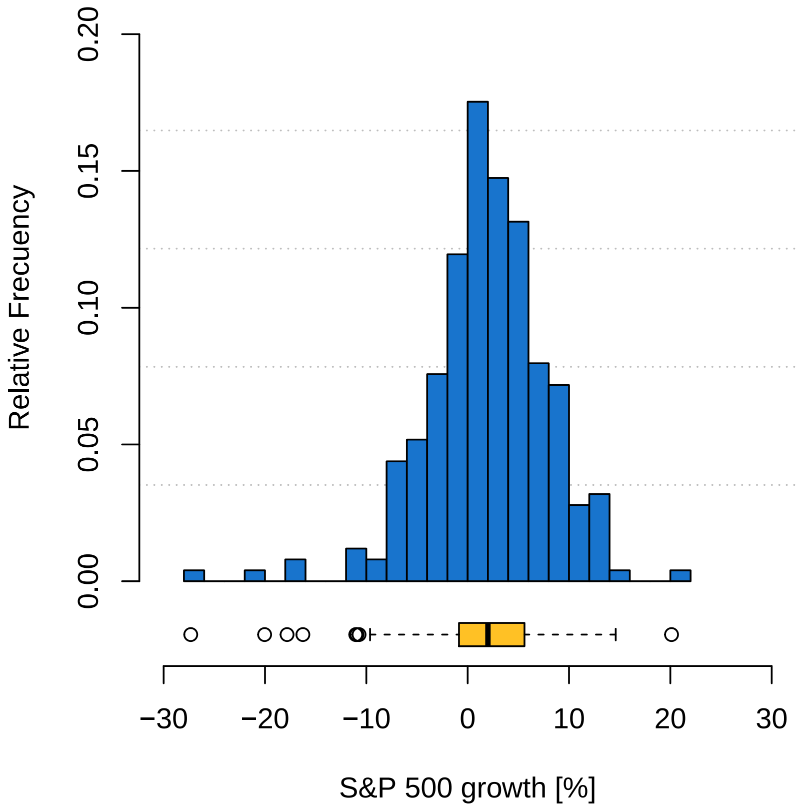

Figure E.7: Histogram of `S&P 500 Growth`

	65Q3 -69Q4	70Q1 -79Q4	80Q1 -89Q4	90Q1 -99Q4	00Q1 -09Q4	10Q1 -19Q4	20Q1 -21Q4	Total 65Q3-21Q4
AR(1)	0.8214	1.0958	0.9099	0.4979	0.7163	0.4079	5.0643	1.2185
VAR(1)	0.8991	1.0758	0.9581	0.6032	0.6607	0.4796	9.8531	2.0126
VAR(3)	3.9980	1.2208	1.0136	0.6340	0.6002	0.4602	19.2739	3.8783

Table E.1: Comparison RMSE AR(1), VAR(1) and VAR(3) for different time periods