CHRIST (DEEMED TO BE UNIVERSITY),BENGALURU - 560029

End Semester Examination March/April - 2019

Bachelor of Science (CME/CMS/EMS/PCM/PME) VI SEMESTER

Code: MAT631   Max.Marks: 100

Course: COMPLEX ANALYSIS   Duration: 3Hrs

SECTION A

Answer any EIGHT of the following.   **8X3=24**

1   Sketch the region $|z + 3i| \geq 2$ of the complex plane.

2   Find a complex number z such that $|z| - z = 2 - i$.

3   Examine whether the function $f(z) = z^3 - \dfrac{1}{z}$ is continuous at the point $3i$.

4   Verify whether $f(z) = cos z$ is analytic.

5   Evaluate $\displaystyle\int_{|z-1|=2} z^2 dz$.

6   Evaluate $\displaystyle\oint_{|z|=1} Â\dfrac{e^z}{z^3} dz$.

7   Prove that if $f(z)$ is analytic in a region D and if a and b are two points in D, then $\int_a^b f(z)dz$ is independent of the path joining a and b.

8   Find the fixed points of the bilinear transformation $w = \dfrac{i - z}{i + z}$.

9   Find the radius of convergence of the power series $\sum_{k=1}^{\infty} \left(\dfrac{4k + 3}{2k + 5}\right)^k (z + i)^k$.

10   Find the zeros of the function $f(z) = \dfrac{z - 2}{z^2} sin\left(\dfrac{1}{z - 1}\right)$.

11   Define essential singularity. Give an example.

12   Obtain the Maclaurin's series expansion for the function $f(z) = \dfrac{1}{(z - 1)^2}$.

SECTION B

Answer any SEVEN of the following   **7X8=56**

13   Prove that if a function is differentiable at a point, then it is continuous at that point but not conversely.

14   Find the real part of the analytic function whose imaginary part is $e^{-y}(x sin x + y cos x)$.

15   If $f(z)$ is analytic, show that $\left(\dfrac{\partial}{\partial x}|f(z)|\right)^2 + \left(\dfrac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$.

16   Evaluate $\displaystyle\int_C (y - x - 3ix^2)dz$ where C consists of two line segments; one from $z = 0$ to $z = i$ and the other from $z = i$ to $z = 1 + i$.

17   State and prove Cauchy's inequality. Using this prove Liouville's theorem and fundamental theorem of algebra.

18   Evaluate $\displaystyle\int_{|z+1+i|=2} \dfrac{z + 4}{z^2 + 2z + 5} dz$.

19   Discuss the nature of the transformation $w = \dfrac{1}{2}\left(z + \dfrac{1}{z}\right)$.

20   Expand $f(z) = \dfrac{1 + z}{1 - z}$ as a Taylor series about $z = i$.

21 Find the Laurent series expansion of $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ valid in the annular region $2 < |z| < 3$.

<div align="center">

SECTION C

</div>

Answer any TWO of the following.   **2X10=20**

22 State and prove the sufficient conditions for a function to be analytic.

23 Find the bilinear transformation which maps the points $1, -i, -1$ of z-plane to the points $0, i, \infty$ respectively of the w-plane. What is the image of the upper-half of the z-plane under this transformation?

24 State and prove Laurent's Theorem.