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The Role Of Complex Numbers In Interdisciplinary Integration In Mathematics Teaching

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Abstract

Complex numbers are an obligatory content of mathematics education. However, almost without exception, in all degrees of education their application is restricted to geometrical interpretation of the complex number and solving algebraic equations. It is well known that opportunities for mathematics teaching integration, offered by specific content topics, are not fully utilized, especially in higher degrees of education. In this paper, we will present program content that can significantly improve integration of teaching, and improve the training of future mathematics teachers. Needles to say, we can achieve the last with the introduction of a new teaching course *Geometry of complex numbers*, as well as with the study of the mentioned content within the existing courses in elementary mathematics found at the majority of faculties that prepare mathematics teachers.

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1. Introduction

The idea for curriculum integration stems from the young generation's ambition to be presented with a complete and unique idea for the nature, society and their place in them. The traditional classification of teaching content into distinct independent subjects is initiated by the ambition to provide young generations with profound knowledge from a specific area of interest, knowledge that the individual can independently connect to a whole. Therefore, the cross and inter disciplinary correlation of teaching content is of significant importance.

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Practice shows that interdisciplinary correlation in mathematics teaching is accomplished on a relatively high level. However, it seems that the opportunities for mathematics teaching integration, offered by specific content topics, are not fully utilized, especially in higher degrees of education. For instance, this is the case with the study of complex numbers, the study of which in teacher training programmes in university courses the following aims are achieved:

- realize the need to expand the set of real numbers,
- adopt the notions for complex numbers, conjugate complex number, modulus of a complex number and operations with complex numbers,
- adopt the geometric interpretation of complex numbers,
- adopt the notions complex plain and Riemann sphere,
- solve equations in the set of complex numbers, and
- adopt the metric and topology properties of set of complex numbers, which enables students to prepare for the study of complex analysis.

At that, the current teaching content of university courses only correlates complex numbers with the study of specific content from algebra, and of course, the content found in the complex analysis subject that studies conformal translation, topic that needs to be an integrative factor, especially in teacher training program. At that, the topics complex numbers, algebraic structures, and plane movements and similarities are not sufficiently integrated, although there is an excellent basis to do exactly that. Namely, while preparing future mathematics teachers it is inevitable to study algebra, therefore the following aims need to be achieved:

- to study the binary relations, especially the order and equivalence relations,
- to study binary operations, groupoids, semigroups, groups, and
- to give examples of groupoids, semigroups, and groups, that is to perceive the possible application of these algebraic structures.,

and Euclid's plane geometry studies motion and similarities in a plane, where the following aims need to be achieved:

- adopt the notions of movement and similarity and their classification,
- study the group properties of movements, and
- adopt the application of movements and similarities when solving tasks.

We can notice that different teaching disciplines study contents that are already partially integrated, but there is no real integration because while teaching separate content no attention is paid to the possibility that these can be presented to students using a different mathematical apparatus that is available to us. This refers to the study of movements, where the group properties are proved in a constructive way, which creates fixed representations for both movements and complex numbers. Surely, this type of positioning of teaching has its advantages; however, one of the major disadvantages is the disintegration of mathematics teaching and its partialization of seemingly non-relatable scientific discipline. Partial step forward can be achieved through a more thorough extensive study of geometry of complex numbers, where it is necessary to use the transformation $S:\mathbf{C}\to\mathbf{C}$, $S(z)=az+b, a\neq 0$ in order to learn the algebraic interpretation of similarities, their classification and of course, prove group properties of movements.

The changes in the training of future teachers, in the before-mentioned direction, will facilitate:

- future teachers to obtain yet a different view of this part of mathematics, which in the future will enable them to fulfill their responsibilities as teachers in a better way,

- realizing the significance of complex numbers will be an impetus for future teachers to pay more attention when teaching certain contents in secondary education,
- adoption of the application of complex numbers in Euclid's plane geometry provides future teachers with a strong analytical apparatus which can be used in secondary education while working with talented students, and
- this subtle example of interdisciplinary integration positively influences future teachers to continually take care of interdisciplinary integration of teaching.

2. Proposal for better integration of complex numbers and Euclid's plane geometry

In the introductory part of this paper, we addressed complex numbers, algebraic structures, and Euclid's plane geometry studied in many separate teaching disciplines at many of the faculties that prepare future teachers of mathematics. At that, as we can see from the aims that need to be achieved while studying the mentioned content we have minimal integration of content studied with complex numbers and Euclid's plane geometry. Further in this paper we will present program content that can significantly improve integration of teaching, and also improve the training of future mathematics teachers. Also, we can achieve the last with the introduction of a new teaching course *Geometry of complex numbers*, as well as with the study of the mentioned content within the existing courses in elementary mathematics found at the majority of faculties that prepare mathematics teachers. Faculties that train future mathematics teachers, without any exception, have the subject geometry, most frequently, with the following syllabus:

- Introduction to geometry: basic elements and basic assertions, axioms of incidence, order, congruence, continuity and their consequences, and axiom of parallelism,
- Congruence: isometric transformations, congruence of figures, congruence of line segments, congruence of angles, congruence of triangles, angles of a transversal, (sum of angles in a triangle), inequality of triangles, rectangle, parallelogram, midsegments of a triangle, important points of a triangle,
- Application of congruence: application of circle congruence, central and inscribed angle, tangential rectangle, cyclic quadrilateral, normal lines and planes, dihedral, orthogonality of planes, angle between a line and a plane, angle between skew lines,
- Isometric plane transformations: direct and indirect transformations, reflection symmetry, central rotation, central symmetry, translation, slide symmetry, classification of plane isometries,
- Similarities: proportion of segments (Thales' theorem), homothety, similarity transformations (similar figures), similarity of triangles, Appolonius circle, degree of a point in relation to a circle, selected geometry theorems, and
- Inversion: definition and basic properties of inversion, Appolonius problems.

On other hand, within the subject of Complex analysis, the following content are frequently moved:

- Complex numbers (basic properties) algebraic form of complex numbers and complex conjugate, trigonometric form of a complex number, roots of complex numbers and exponential form of complex numbers, and
- Geometrical interpretation of a complex number, extended complex plane and Riemann's interpretation of complex number.

Later follows the study of the topological properties of the set of complex numbers, then moves on to differentiability and integrability, and then at a certain point there is the study of conformal translation. Clearly, this positioning of study of complex numbers does not allow integration of teaching with Euclid's geometry, which ca be significantly improved if the following contents are studied together with complex numbers:

- Equation of a line, self-conjugate equation of a line and distance from a point to a line,
- Equation of a circle, self-conjugate equation of a circle,
- Direct similarities, movements, homothety, indirect similarities and inversion,
- Möbius transformation, geometric properties of Möbius' transformation,

- Central and inscribed angle of a circle, radical axis and radical center,
- Important points of a triangle, right triangle, area of a triangle, circumcircle and incircle of a triangle,
- Euler line and Euler circle, theorems of: Manelaus, Desargues, Pascal, Ceva, Stewart and Ptolemy, Simson line, etc.

However, this approach enables not only integration of the teaching content, but also classification of the similarities, proving group properties of similarities and movements, as well as analyzing their compositions with the usage of the following transformation: $S:\mathbf{C}\to\mathbf{C}$, $S(z)=az+b, a\neq 0$ and $S_1:\mathbf{C}\to\mathbf{C}$, $S_1(z)=a\overline{z}+b, a\neq 0$. Furthermore, the adoption of the mentioned content helps a great number of non-standard geometry tasks to be solved using simple basic calculations, as shown in the following example.

Task 1. Let the diagonals of a convex rectangle $ABCD$ intersect in point O and let T_1 and T_2 be the medians of triangles AOD and BOC, and H_1 and H_2 be the orthocenters of triangles AOD and BOC, respectively. Prove that $T_1T_2 \perp H_1H_2$.

Solution. Let point O be the coordinate origin. Then for the affixes of points H_1 and H_2 and medians T_1 and T_2 we have

$$h_1=\frac{(a-b)(\overline{a}b+a\overline{b})}{a\overline{b}-\overline{a}b},\ h_2=\frac{(c-d)(\overline{c}d+c\overline{d})}{c\overline{d}-\overline{c}d},\ t_1=\frac{a+d}{3} \text{ and } t_2=\frac{b+c}{3}.$$

Points A,C and O are collinear and points B,D and O are collinear, therefore $\overline{c}=c\overline{a}/a$ and $\overline{d}=d\overline{b}/b$, that is

$$h_2=\frac{(c-d)(\overline{b}a+b\overline{a})}{a\overline{b}-\overline{a}b}.$$

Further on,

$$h_1-h_2=\frac{(a+d-b-c)(\overline{a}b+a\overline{b})}{a\overline{b}-\overline{a}b},\ t_1-t_2=\frac{a+d-b-c}{3}$$

and with an immediate test we get that

$$\frac{t_1-t_2}{\overline{t_1}-\overline{t_2}}=-\frac{h_1-h_2}{\overline{h_1}-\overline{h_2}}.$$

Thus follows that $T_1T_2 \perp H_1H_2$. ■

The last is of great importance especially when we work with talented students form secondary education because mastering the analytical apparatus of complex numbers and its application in Euclid's plane geometry enables teachers with effective work with students in preparation for mathematical competitions. Hence, for instance, even the following tasks assigned on prestigious mathematical competitions can be solved using complex numbers

Task 2 (IMO '07). Consider five points A,B,C,D and E such that $ABCD$ is a parallelogram and $BCED$ is a cyclic quadrilateral. Let l be a line passing through A and intersect segment DC at its inner point F, and line BC at point G. Suppose that $|EF|=|EG|=|EC|$, prove that l is the bisector of $\measuredangle DAB$.

Task 3 (BMO '09). In triangle ABC, M and N are points on sides AB and AC, respectively, so that $MN \parallel BC$. Let $P=BN\cap CM$. The circumcircles of triangles BMP and CNP intersect in two different points P and Q. Prove that $\measuredangle BAQ=\measuredangle CAP$.

Task 4 (IMO '10). Point P lies inside triangle ABC. Lines AP, BP and CP meet the circumcircle Γ of triangle ABC again at points K, L and M, respectively. The tangent to the circumcircle Γ at point C meets line AB at S. Suppose that $|SP|=|SC|$, prove that $|MK|=ML|$.

Task 5 (IMO '11). Let ABC be an acute triangle and Γ be its circumcircle. Let line l be a tangent line to circumcircle Γ and l_a, l_b and l_c be the lines obtained by reflecting line l in the lines BC, CA and AB, respectively. Prove that the circumcircle of the triangle determined by the lines l_a, l_b and l_c is tangent to the circumcircle Γ.

3. Conclusion

Integrating mathematics teaching with other teaching disciplines is of great importance and interest, however, it cannot be successfully achieved without firstly providing strong interdisciplinary integration of mathematics itself. Previously we discussed how we can achieve this in secondary education using complex numbers. However, this is not possible if we don not make the necessary changes in the education of future mathematics teachers. It is very important to guide the education of teachers towards their future profession, which means that the same should not be burdened with content that will not be used while they teach students, and at the same time be deficient in advanced knowledge that will become their daily routine (number theory, trigonometry, complex numbers and similar). The previous arguments are frequently used, advanced mathematics courses in faculties for mathematics teachers are replaced by subjects from didactics, psychology and pedagogy, and the issues discussed here are trivialized. Surely, this has a positive side, as well as negative consequences because we need to consider the following: *A person can have all the skill to use cutlery, but if the plate is empty, he will for sure stay hungry*.

References

Gogovska, V., Malcheski, R. (2012). "*Improvement intra-disciplinary integration of mathematics instruction*", Elsevier, Procedia - Social and Behavioral Sciences,Vol.46 , pp. 5420-5424

Grozdev, S., (2007). For High Achievements in Mathematics. The Bulgarian Experience (Theory and Practice). ADE, Sofia, (ISBN 978-954-92139-1-1), 295 pages

Krtinic, Dj. (2012). Hight school Maтhematical competitions 2007-2011, DMS, Belgrade

Malcheski, R. (2012). Didactic of mathematics teaching *(basic part),* FON University, Skopje

Malcheski, R. Anveska, K., Malcheski, S. (2014). Complex number – their application in , geometry , ALFA '94, Skopje