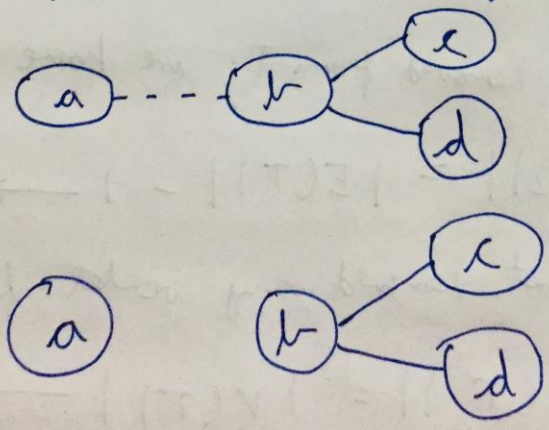

28/2/20

Qn. 1 ÷ A graph G is minimally connected iff it is a tree.

Proof ÷ Let the Graph G be minimally connected, i.e removal of 1 edge makes it disconnected. Therefore, there is no circuit. Hence, graph G is a tree.

Conversely; let the graph G be a tree i.e there exists 1 & only 1 path between every pair of vertices & we know that removal of 1 edge from the path makes the graph disconnected.

Hence; graph G is minimally connected.



(i)

Write a note on Chinese Postman Problem (CPP)

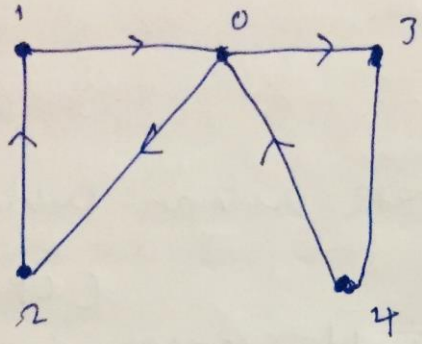
(ii) Travelling Salesman Problem (TSP)

A network is a weighted directed graph.

Chinese Postman Problem (Route Inspection):-

- A variation of Eulerian circuit problem for undirected graphs. An Euler circuit is a closed walk that covers every edge once starting & ending position are the same.
- It is defined for connected & undirected graphs the problem is to find the shortest path or circuit that visits every edge of the graph atleast once.
- If input graph contains Euler circuit, then a solution of the problem is Euler circuit.

- A directed & connected graph has Eulerian cycle if all vertices have even degree."



Chinese Postman tour is same as Eulerian cycle

$\Longrightarrow$ The graph has Eulerian cycle.

Eg :- "2103402". (all vertices have even degree)

Chinese Postman Route; if exists - is always the same as Eulerian Problem for both weighted as unweighted.

Weighted graph : Minimum possible weight of Postman tour : Sum of all edges we obtain through Eulerian circuit

$\longrightarrow$ Travelling Salesman Problem (TSP) :-

- Given a set of cities & distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once & returns to the starting point.

• Difference between Hamiltonian cycle & TSP :-

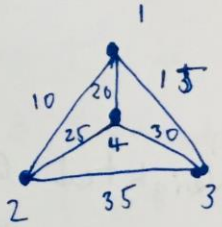
Hamiltonian cycle problem is to find if there exists a tour that visits every city exactly once.

Here, we know that Hamiltonian tour exists (∵ graph is complete) & in fact many such tour exists; the problem is to find a minimum weight Hamiltonian cycle.

• For eg :-

- A TSP tour in the graph is : 1 - 2 - 4 - 3 - 1.
- the cost of the tour is :-

$$10 + 25 + 30 + 15$$
$$= 80$$



* Naive solution :-

① Consider city 1 as starting & ending point
② Generate all $(n-1)!$ Permutations of cities
③ Calculate cost of every permutation & keep track of minimum cost permutation.
④ Return the permutation with minimum cost.

Time complexity :- $\Theta(n!)$