
10/2/20 CIA - 3

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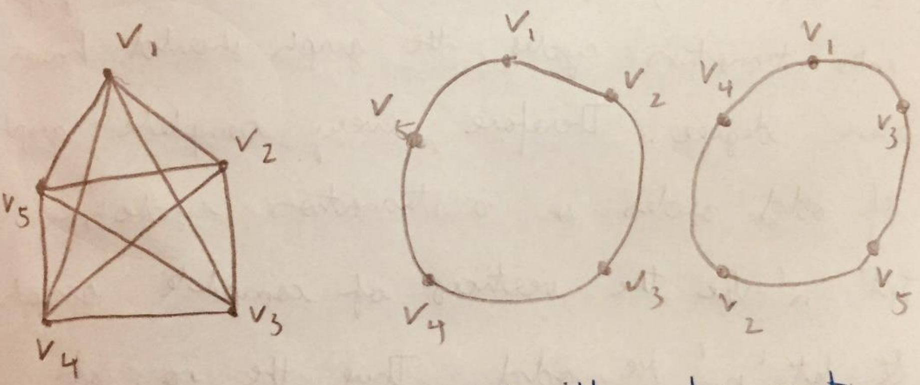
GRAPH THEORY

1. Explain a Hamilton cycle decomposition (i) with an illustration.

Ans A Hamilton cycle decomposition of a graph is a decomposition of graph into Hamilton cycles.

Eg:- Consider the graph K_5 which is a complete graph with 5 vertices.



Hamilton decomposition

In the above case, C_5 has been decomposed into 2 Hamilton cycles.

$C: V_1 - V_2 - V_3 - V_4 - V_5$

$C': V_1 - V_3 - V_5 - V_2 - V_4$

② Prove or disprove every complete graph (ii) of odd order is Hamilton cycle decomposible. Also determine the number of Hamilton cycles in that decomposition.

Ans Consider a complete graph K_n; where n is odd. Thus the degree of each vertex in the complete graph is even.

Hence, for a cycle - every vertex has degree 2 and thus; in order to decompose a graph into transition cycles, the graph should have even degree. Therefore, every complete graph of odd order is a transition cycle.

Let 'n' be the vertices of complete graph & let 'n' be odd. Thus, the no. of transition cycles is $\frac{n(n-1)}{2}$.

② Is a complete graph of even order hamilton cycle decomposible? Explain.

Ans It is not decomposible.

Consider a ~~g~~ complete graph K_n; where 'n' is even.

The degree of each vertex in the graph is $(n-1)$ which is odd. Now for a cycle to every vertex should have degree 2, which is even. But in this case, degree of each vertex is odd. Hence, we cannot decompose a graph of even order into Hamilton cycles.

③ Prove or disprove the following:-

(i) An Eulerion graph does not cut edges.

Ans Let us assume that Eulerian graph has cut edges on removal of a particular edge 'e' from the Euler graph G; the graph becomes disconnected. But the end vertices of the graph 'G - e' have odd degree. Hence it cannot be decomposed into cycles.

Therefore, it violates the iff condition that is it is not a Eulerian graph. Hence an assumption is wrong. Thus the Eulerian graph does not have cut edges.

(ii) A hamilton graph does not ensure or have cut vertices.

Ans Let us assume that a transition graph has cut vertices.

The removal of vertices from the graph, the graph becomes disconnected. Hence it does not cover all vertices of the graph.

Hence, it is not a Hamiltonian graph.

There is a contradiction & our assumption is wrong.

∴ Hamiltonian graph does not have cut vertices.