Some Additional Questions in Graph Theory

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1. If the size of a graph is equal to its order, then show that it contains exactly one cycle.
2. Prove or disprove: Any non-trivial subgraph of a bipartite graph is also bipartite.
3. Prove or disprove: A graph G is regular if and only if its complement is regular.
4. Prove or disprove: If a graph and its complement are r-regular, then G has odd number of vertices.
5. Prove or disprove: For $k \geq 2$, a k-regular bipartite graph has no cut-edges.
6. Is the join of two bipartite graphs bipartite? Justify your answer.
7. Define a semi-Eulerian graph. Show that a graph is semi-Eulerian if and only if it has exactly two odd degree vertices.
8. If a disconnected graph G with two components has two odd degree vertices, show that both of them will be in the same component.
9. Prove or disprove: Every graph of order n is a spanning subgraph of the complete graph K_n.
10. A graph has the property that every edge of it joins an odd degree vertex with an even vertex. Then, show that
    (a) G has even number of edges.
    (b) G is bipartite.
11. Using graph theoretical notions, show that it is impossible to have a group of nine people at a party such that each one of them knows exactly five of the others in the group.
12. Show that $rad(G) \leq diam(G) \leq 2 * rad(G)$.
13. If the size of a graph is greater than its order, then show that contains at least one cycle.

14. Prove or disprove: If all vertices of a connected graph G have even degree, then show that G has no cut-edge.

15. Show that a graph of order n and size $\frac{n^2}{4}$ is not bipartite.

16. Define an $r-$ partite graph and a complete r-partite graph. Also, draw the complete 6-partite graph $K_{3,3,2,2,2,1}$.

17. Prove or disprove: Two graphs G and H are isomorphic if and only if their complements are isomorphic.

18. For any simple connected graph G of order n and size m, show that $n-1 \leq m \leq \frac{n(n-1)}{2}$.

19. Prove or disprove: If a graph G on n vertices is r -regular, then either n or r is even.

20. Construct a simple connected r–regular graph on 6 vertices and a simple connected s-regular graph on 7 vertices for all possible values of r and s.

21. Let G be a graph of order $2n+1$ such that the degree of every vertex of G is either $n+1$ or $n+2$. Then, show that G has at least $n+1$ vertices of degree $n+2$and at least $n+2$ vertices of degree $n+1$.

22. Two graphs G and H have the same order, size and degree sequence. Are they necessarily isomorphic? Justify your comment with suitable illustrations.

23. The degree of each vertex of a certain graph of order 12 and size 31 is either 4 or 6. Find the number of vertices falling in each category.

24. Show that an Eulerian graph cannot have a cut-set of an odd number of edges.

25. If the degree of every vertex of a graph is at least 2, , then show that G has at least one cycle.

26. Let G be a connected r-regular graph of even order n such that its complement is also connected. Then, show that
    (a) either G or $\bar{G}$ is Eulerian.
    (b) either G or its complement is Hamiltonian.

27. Show that the rank of the incidence matrix of a connected graph G of order n is $n-1$.

28. Explain Hamilton cycle decomposition of a graph. Show that all complete graphs of odd order are Hamilton cycle decomposable.

29. Prove or disprove: Every subgraph of a planar graph is planar.

30. Find all integers n for which the complete graph K_n is planar.
31. Prove or disprove: A bipartite graph with $\delta(G) \geq 4$ is non-planar.
32. Prove or disprove: Every Hamilton path of a graph G is a spanning tree of G.
33. Distinguish between a tournament and a complete digraph. Comment on the number of directed edges in a tournament and a complete digraph on the same number of vertices.
34. Prove or disprove: The size of an acyclic graph of order n with k components is $n - k$.
35. Show that every graph of order n and size m has at least $m - n + 1$ cycles.
36. Is a graph with degree sequence $(4, 3, 3, 3, 2, 1)$ planar? If so, find the number of faces in G.
37. Prove or disprove: An edge e of a connected graph G is a cut-edge of G if and only if it belongs to every spanning tree of G.
38. Prove or disprove: If G^* is the dual of a planar graph G, then $(G^*)^*$ is isomorphic to G.
39. If it exists, draw a tree whose complement is also a tree. Show that every tournament has a Hamilton path.
40. Prove or disprove: If $r \geq 6$, there exists no r–regular planar graphs.
41. An r-regular graph of order 12 is embedded on a plane, resulting in 8 faces. Then find the value of r.
42. Show that a simple connected planar bipartite graph G has each face with even degree.
43. Is it necessary that a graph with n vertices and $n - 1$ edges is a tree? Justify your answer.
44. Show that a pendant edge of a graph G will be a part of every spanning tree of G.
45. Every graph with fewer edges than its vertices will contain a tree as its component.
46. Are the geometric duals of isomorphic graphs isomorphic? Justify your comment.