CHRIST (DEEMED TO BE UNIVERSITY),BENGALURU - 560029

End Semester Examination October/November - 2018

Bachelor of Science V SEMESTER

Code: MAT531 | **Max.Marks: 100**

Course: LINEAR ALGEBRA | **Duration: 3Hrs**

SECTION A

Answer EIGHT questions: **8X3=24**

1 Find K so that $(1, K, 5)$ is a linear combination of $(1, -3, 2)$ and $(2, -1, 1)$.

2 Show that $\alpha = (1,3,2), \beta = (1,-7,-8), \gamma = (2,1,-1)$ of $V_3(R)$ form a set of linearly dependent vectors.

3 If α, β, γ are linearly independent vectors of $V(F)$, then show that $\{\alpha+\beta, \beta+\gamma, \gamma+\alpha\}$ are also linearly independent.

4 Prove that a set consisting of a single non zero vector is linearly independent.

5 Let $T : V \to V$ be a function defined by $T(x) = x, \forall\, x \in V$, then prove that T is linear.

6 If $T : V_2 \to V_1$ defined by $T(x,y) = x^2 + 2y^2$ then show that T is not a linear transformation.

7 Find the matrix of the linear transformation $T : V_3(R) \to V_2(R)$ defined by $T(x,y,z) = (x+y, y+z)$ w.r.t standard bases.

8 Prove that there is no non singular linear transformation from R^4 to R^2.

9 If $u = (x_1, x_2, \ldots, x_n)$ and $v = (y_1, y_2, \ldots, y_n)$ are two vectors in V_n^C. Define $<u, v> = x_1\bar{y}_1 + x_2\bar{y}_2 + \ldots + x_n\bar{y}_n$. Prove that <u, v> is an inner product .

10 In V define $< u, v > = \int_{-\infty}^{0} uv\,dx$. Show that this defines an inner product on V.

11 If W is a subspace of an inner product space V . Then prove that orthogonal complement of W is a subspace of V.

12 Show that if u and v have the same norm and u & v are orthogonal then $u+v$ is orthogonal to $u-v$

SECTION B

Answer any SEVEN questions: **7X8=56**

13 If α, β, γ are linearly independent vectors in a vector space V(F), where F is any sub-field of complex numbers, then show that the vectors $\alpha+\beta, \alpha-\beta, \alpha-2\beta+\gamma$ are also linearly independent.

14 Prove that in an n - dimensional vector space V(F) any set of n linearly independent vectors is a basis.

15 Find the dimension and basis of the subspace spanned by the vectors $(2,4,2), (1,-1,0), (1,2,1)$ and $(0,3,1)$ in $V_3(R)$.

16 Let $T = U \to V$ be a linear map, then prove the following, (i) R(T) is a subspace of V, (ii) N(T) is a subspace of U.

17 Find the matrix of the linear transformation. $T : V_3(R) \to V_2(R)$ by $T(x,y,z) = (x+y, y+z)$ relative to the bases $B_1 = \{(1,1,1)(1,0,0)(1,1,0)\}$ of $V_3(R), B_2 = \{(1,1)(-1,1)\}$ of $V_2(R)$.

18 Let $T : V_4 \to V_3$ be a linear map by $T(e_1) = (1,1,1),\ T(e_2) = (1,-1,1),$ $T(e_3) = (1,0,0),\ T(e_4) = (1,0,1)$ then verify Rank – nullity theorem.

19 Prove that any orthogonal set of non zero vectors in an inner product space is linearly independent.

20 Find an orthonormal basis of $P_3[-1,1]$ starting from the basis $\{1, x, x^2, x^3\}$ using the inner product $f.g = \int_{-1}^{1} f(t)g(t)dt$.

21 Prove that every inner product space is a metric space.

SECTION C

Answer any TWO questions: **2X10=20**

22 Let W_1 , W_2 and W_3 be subspaces of a finite dimensional vector space V. Show that $\dim(W_1 + W_2 + W_3) \leq \dim(W_1) + \dim(W_2) + \dim(W_3) - \dim(W_1 \cap W_2) - \dim(W_2 \cap W_3) - \dim(W_1 \cap W_3) + \dim(W_1 \cap W_2 \cap W_3)$

23 Prove that every vector space V over the real field R of dimension n is isomorphic to $V_n(R)$.

24 Explain geometrical interpretation of Schwarz inequality.