Partial Differential Equations and its Applications (MAT231)

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Practice Problems: Partial Differential Equations

I. Linear Partial Differential Equations of Order One: $Pp + Qq = R$

Solve the following:

1. $(y+z)p + (z+x)q = (x+y)$	2. $\left(\frac{y-z}{yz}\right)p + \left(\frac{z-x}{zx}\right)q = \left(\frac{x-y}{xy}\right)$
3. $(bz-cy)p + (cx-az)q = (ay-bx)$	4. $(y^2+z^2-x^2)p - 2xyq + 2xz = 0$
5. $x(y-z)p + y(z-x)q = z(x-y)$	6. $(mz-ny)p + (nx-lz)q = (ly-mx)$
7. $xzp + yzq = xy$	8. $x(y^2-z^2)p + y(z^2-x^2)q = z(x^2-y^2)$
9. $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$	10. $y^2p - xyq = x(z-2y)$
11. $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$	12. $(x^2+y^2+yz)p + (x^2+y^2-zx)q = z(x+y)$
13. $(1+y)p + (1+x)q = z$	14. $(x^2-y^2-yz)p + (x^2-y^2-zx)q = z(x-y)$
15. $\frac{y^2z}{x}p + zxq = y^2$	16. $(z^2-2yz-y^2)p + x(y+z)q = x(y-z)$
17. $x^2p + y^2q = z^2$	18. $(x+2z)p + (4xz-y)q = 2x^2+y$
19. $px + qy = \sqrt{x^2+y^2}$	20. $(x^2-yz)p + (y^2-zx)q = (z^2-xy)$

II. Non-linear Partial Differential Equations of Order One: $f(x,y,z,p,q) = 0$

Solve the following.

1. $p^2 - q^2 = x - y$	2. $\sqrt{p} + \sqrt{q} = x + y$
3. $p + q = sinx + siny$	4. $yp + xq + pq = 0$
5. $p^2 - y^3q = x^2 - y^2$	6. $pe^y = qe^x$
7. $q = 2yp^2$	8. $q = xyp^2$
9. $pq = xy$	10. $yp = 2xy + log\ q$
11. $x^2p^2 = yq^2$	12. $q = px + p^2$
13. $p^2y(1+x^2) = qx^2$	14. $p^2 + q^2 = x + y$
15. $p - x^2 = q + y^2$	16. $\sqrt{p} + \sqrt{q} = 2x$
17. $q(p - cosx) = cosy$	18. $q - p + x - y = 0$
19. $z(p^2 - q^2) = x - y$	20. $z(p + q) = tanx + tany$
21. $z(p^2 + q^2) = x^2 + y^2$	22. $q^2y^2 = z(z - px)$

23.	$z = pq$	24.	$z^2(p^2 + q^2 + 1) = 1$
25.	$p(1 + q) = zq$	26.	$p(1 + q^2) = q(z - a)$
27.	$p^2 = z^2(a - q^2)$	28.	$4(1 + z^3) = 9z^4 pq$
29.	$p(1 - q^2) = q(1 - z)$	30.	$p^3 + q^3 = 3pqz$
31.	$p^3 + q^3 = 27z$	32.	$p^2 = z^2(1 - pq)$
33.	$pz = 1 + q^2$	34.	$9(p^2 z + q^2) = 4$
35.	$z^2(p^2 + q^2 + 1) = c^2$	36.	$p + q = \dfrac{z}{a}$
37.	$q^2 = z^2 p^2(1 - p^2)$	38.	$p^2 + q^2 = 1$
39.	$p^2 q^3 = 1$	40.	$p = e^q$
41.	$pq = 1$	42.	$p^2 - q^2 = 1$
43.	$q = e^{-\frac{p}{a}}$	44.	$pq + p + q = 0$
45.	$p + q = pq$	46.	$\sqrt{p} + \sqrt{q} = 1$
47.	$p^2 + q^2 = npq$	48.	$2p^2 + 6p + 2q + 4 = 0$
49.	$p = e^{-\frac{q}{a}}$	50.	$2p + 3q = 1$
51.	$z = px + qy + p^2 + q^2$	52.	$z = px + qy + \log pq$
53.	$z = px + qy + pq$	54.	$z - px - qy = c\sqrt{1 + p^2 + q^2}$
55.	$\sin(z - px - qy) = pq$	56.	$pq = e^{z - px - qy}$
57.	$z = px + qy + \sqrt{\dfrac{pq}{p+q}}$	58.	$(px + qy - z)^2 = 1 + p^2 + q^2$
59.	$x^2 p^2 + y^2 q^2 = z^2$	60.	$z^2(p^2 x^2 + q^2) = 1$

Solve the following using Charpit's method

1. $q = px + q^2$	2. $px + qy = pq$
3. $p(1 + q^2) + (b - z)q = 0$	4. $z = p^2 x + q^2 y$
5. $(p^2 + q^2)y = qz$	6. $(p + q)(px + qy) = 1$
7. $2xz - px^2 - 2qxy + pq = 0$	8. $p^2 + q^2 - 2px - 2qy + 1 = 0$
9. $2z + p^2 + qy + 2y^2 = 0$	10. $p^2 + q^2 - 2px - 2qy + 2xy = 0$
11. $p = (qy + z)^2$	12. $z^2(p^2 z^2 + q^2) = 1$
13. $16p^2 z^2 + 9q^2 z^2 + 4z^2 - 4 = 0$	14. $(p^2 + q^2)x = pz$
15. $2(z + px + qy) = yp^2$	16. $z^2 = pqxy$