

ORDINARY DIFFERENTIAL EQUATIONS

Equations of Order One and Degree One

TYPE-V: Exact Equations

*This is one of the easiest set of problems that I am sure, all of you can do yourself.
All the best!*

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Note: You need to always check whether the given equation $Mdx + Ndy = 0$ by

testing whether $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Solve the following.

1. $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$.
2. $\left[y\left(1 + \frac{1}{x}\right) + \cos y\right]dx + [x + \log x - x \sin y]dy = 0$.
3. $(y^2 e^{xy^2} + 4x^3)dx + (2xy e^{xy^2} - 3y^2)dy = 0$.
4. $(x^2 + y^2 + x)dx - (2x^2 + 2y^2 - y)dy = 0$.
5. $(12x + 5y - 9)dx + (5x + 2y - 4)dy = 0$.
6. $(x^2 - ay)dx + (y^2 - ax)dy = 0$.
7. $(x^2 + y)dx + (y^3 + x)dy = 0$.
8. $(a^2 - 2xy - y^2)dx - (x + y)^2 = 0$.
9. $x(y^2 + x^2 - a^2x)dx + y(y^2 + x^2 - b^2y)dy = 0$.
10. $\left(1 + e^{\frac{x}{y}}\right)dx + \left(1 - \frac{x}{y}\right)e^{\frac{x}{y}}dy = 0$.

$$11. \frac{dy}{dx} + \frac{y \cos x + \sin y + y}{\sin x + x \cos y + x} = 0.$$

$$12. (\sin x \cos y + e^{2x}) dx + (\sin y \cos x + \tan y) dy = 0.$$

$$13. \left(x - \frac{y}{x^2 + y^2} \right) dx + \left(y + \frac{x}{x^2 + y^2} \right) dy = 0.$$

$$14. [\cos x \tan y + \cos(x + y)] dx + [\sec^2 y + \cos(x + y)] dy = 0.$$

$$15. [2 \cos 2x + \sin(x + 2y)] dx + [\cos y + 2 \sin(x + 2y)] dy = 0.$$

$$16. x(x^2 + 3y^2) dx + y(y^2 + 3x^2) dy = 0.$$

$$17. \frac{1}{x} [x(y - \sin y) + y] dx + [x(1 - \cos y) + \log x] dy = 0.$$

$$18. [\cos 2y - 3x^2 y^2] dx + [\cos 2y - 2x \sin 2y - 2x^3 y] dy = 0.$$

$$19. [6x^2 y^2 - 8xy^3 + 10xy + 3y] dx + [4x^3 y - 12x^2 y^2 + 5x^2 + 3x] dy = 0.$$

$$20. \left[\left(\frac{y}{x} \right) \cos \left(\frac{y}{x} \right) + \sin \left(\frac{y}{x} \right) + \cos y \right] dy - \frac{y^2}{x^2} \cos \left(\frac{y}{x} \right) dx = 0.$$

$$21. \left[1 + \left(\frac{x}{y} \right) \cos \left(\frac{x}{y} \right) + \sin \left(\frac{y}{x} \right) \right] dx - \frac{x^2}{y^2} \cos \left(\frac{x}{y} \right) dy = 0.$$

$$22. \left(5x\sqrt{x} + \frac{7y^2}{\sqrt{x}} \right) dx + 28y\sqrt{xdy} = 0.$$

ANSWERS

$$1. 2x^2 + y^2 + 3xy + x + y = c.$$

$$2. y(x + \log x) + x \cos y = c.$$

$$3. x^4 - y^3 + e^{xy^2} = c.$$

$$4. 2x - 4y + \log(x^2 + y^2) = c.$$

$$5. 6x^2 + y^2 + 5xy - 9x - 4y = c.$$

$$6. x^3 + y^3 - 3axy = c.$$

$$7. 4x^3 + 3y^4 + 12xy = c.$$

$$8. a^2x - x^2y - y^2x - \frac{y^3}{3} = c.$$

$$9. 3x^4 + 3y^4 + 6x^2y^2 - 4a^2x^3 - 4b^2y^3 = c.$$

$$10. x + ye^{\frac{x}{y}} = c.$$

$$11. y \sin x + x \sin y + xy = c.$$

$$12. -\cos x \cos y + \frac{1}{2}e^{2x} + \log(\sec y) = c.$$

$$13. x^2 + y^2 - 2 \tan^{-1} \left(\frac{x}{y} \right) = c.$$

$$14. \sin x \tan y + \sin(x + y) = c.$$

$$15. \sin 2x + \sin y - \cos(x + 2y) = c.$$

$$16. \frac{x^4}{4} + \frac{3}{2}x^2y^2 + \frac{y^4}{4} = c.$$

$$17. x(y - \sin y) + y \log x = c.$$

$$18. 2x \cos 2y - 2x^3y^2 + \sin 2y = c.$$

$$19. 2x^3y^2 - 4x^2y^3 + 5x^2y + 3xy = c.$$

$$20. y \sin \left(\frac{y}{x} \right) + \sin y = c.$$

$$21. x \sin \left(\frac{x}{y} \right) + x = c.$$

$$22. (x^2 + 7y^2)\sqrt{x} = c.$$