# ORDINARY DIFFERENTIAL EQUATIONS

## Equations of Order One and Degree One

## TYPE-III: *Linear Equations of Order One and*

## TYPE-IV: Bernoulli's Equation

*Try solving all these problems independently. All the best!*

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**Level I**

| Q. No. | Problem | Q. No. | Problem |
|---|---|---|---|
| 1. | $\frac{dy}{dx} + xy = xy^3.$ | 2. | $\frac{dy}{dx} + ycotx = 2cosx.$ |
| 3. | $x\frac{dy}{dx} + y = x^3y^6.$ | 4. | $\frac{dy}{dx} + ytanx = secx.$ |
| 5. | $2x\frac{dy}{dx} - y = 10x^3y^5.$ | 6. | $\frac{dy}{dx} + ysecx = tanx.$ |
| 7. | $\frac{dy}{dx} + ytanx = y^3secx.$ | 8. | $(1+x^2)\frac{dy}{dx} + 2xy = 4x^2.$ |
| 9. | $x\frac{dy}{dx} + y = x^3y^2cosx.$ | 10. | $\cos^2 x\frac{dy}{dx} + y = tanx.$ |
| 11. | $\frac{dy}{dx} + 2ytanx = y^2.$ | 12. | $\frac{dy}{dx} + \frac{2x}{1+x^2} = \frac{1}{(1+x^2)^2}.$ |

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|---|---|---|---|
| 13. | $x\frac{dy}{dx} - ycosx + y^2 = 0.$ | 14. | $\frac{dy}{dx} + 2xy = 4x.$ |

**Level II**

| No. | Problem | No. | Problem |
|---|---|---|---|
| 1. | $\frac{dy}{dx} + 3x^2y = x^5e^{x^3}$ | 2. | $(1-x^2)\frac{dy}{dx} - xy = 1\,.$ |
| 3. | $\frac{dy}{dx} + ycotx = sinx.$ | 4. | $(x+1)\frac{dy}{dx} - y = e^x(x+1)^2$ |
| 5. | $\frac{dy}{dx} - \frac{2}{x}y = x + x^2$ | 6. | $\frac{dy}{dx} + ycosx = sinx\,cosx$ |
| 7. | $xlogx\frac{dy}{dx} + y = 2logx$ | 8. * | $\frac{dy}{dx} + 3y = e^{-x}cosx$ |
| 9. | $xcosx\frac{dy}{dx} + y(xsinx + cosx) = 1$ | 10. | $(1-x^2)\frac{dy}{dx} + 2xy = x\sqrt{(1-x^2)}$ |
| 11. | $(1+x^2)\frac{dy}{dx} + 2xy - 4x^2 = 0\,.$ | 12.* | $sinx\frac{dy}{dx} + 3y = cosx\,.$ |
| 13. | $\frac{dy}{dx} + ycotx = 4xcosecx$ | 14. | $(1+x^2)dy + (y - \tan^{-1}x)dx = 0$ |
| 15. | $(1+x^2)\frac{dy}{dx} + y = e^{\tan^{-1}x}$ | 16. | $sinx\,cosx\,\frac{dy}{dx} = y + sinx$ |
| 17. | $\frac{dy}{dx} + ytanx = sin2x.$ | 18. | $\sin^2 x\,\frac{dy}{dx} - y = cotx$ |

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|---|---|---|---|
| 19. | $\frac{dy}{dx} + ycotx = 4xcosecx$ | 20.* | $\frac{dy}{dx} + 3x^2y = x^5e^{x^3}$ |
| 21. | $sinx\ \frac{dy}{dx} + ycosx = xsinx$ | 22.* | $x^2(x^2-1)\frac{dy}{dx} + x(x^2+1)y = (x^2-1)$ |
| No. | Problem | No. | Problem |
| 23. | $\frac{dy}{dx} = ycosx + cosxsin^2x$ | 24. | $x\ \frac{dy}{dx} = y + x^3 + 3x^2 - 2x$ |
| 25. | $(x-2)\frac{dy}{dx} = y + 2(x-2)^3$ | 26. | $\frac{dy}{dx} + ycotx = 5e^{cosx}$ |
| 27. | $x^3\ \frac{dy}{dx} + (2-3x^2)y = x^3$ | 28. | $\frac{dy}{dx} - 2ycot2x = 1 - 2xcot2x - 2cosec2x$ |
| 29. | $dx + 2xydy = ye^{-y^2}dy$ | 30. | $ydx + (3x + 2 - xy)dy = 0$ |
| 31. | $(1+y^2)dx = (\tan^{-1}y - x)dy$ | 32. | $sin2ydx = (x + tany)dy$ |
| 33. | $(1+y^2) + \left(x - e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0$ | 34. | $ylogydx + (x - logy)dy = 0$ |
| 35. | $\frac{dy}{dx} + \frac{1}{x}sin2y = x^3\cos^2 y$ | 36. | $x\frac{dy}{dx} + ylogy = xye^x$ |
| 37. | $\frac{dy}{dx} + \frac{y}{x}logy = \frac{y}{x^2}\ (logy)^2$ | 38. | $\frac{dy}{dx} - \frac{tany}{1+x} = (1+x)e^xsecy$ |
| 39. | $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$ | 40. | $tany\frac{dy}{dx} + tanx = cosy\cos^2 x.$ |
| 41. | $x\frac{dy}{dx} + y = y^2logx$ | 42. | $x\frac{dy}{dx} + (1-x)y = x^2y^2$ |

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| 43. | $\frac{dy}{dx} + xy = xy^3$ | 44. | $x^3\frac{dy}{dx} - x^2y + y^4cosx = 0.$ |

***Level II: Answers.***

1. $ye^{x^3} = \frac{1}{3}\left(\frac{x^3}{2}e^{2x^3} - \frac{1}{4}e^{2x^3}\right) + c$

2. $y\sqrt{(1-x^2)} = sin^{-1}x + c.$

3. $ysinx = \frac{x}{2} - \frac{1}{4}sin2x.$

4. $y = (x+1)(e^x + c).$

5. $\frac{y}{x^2} = logx + x + c.$

6. $y\,e^{sinx} = e^{sinx}(sinx - 1) + c.$

7. $ylogx = (logx)^2 + c.$

8. *Search!*

9. $xysecx = tanx + c.$

10. $\frac{y}{(1-x^2)} = \frac{1}{\sqrt{(1-x^2)}} + c.$

11. $y(1+x^2) = \frac{4}{3}\,x^3 + c.$

12. $\left(y + \frac{1}{3}\right)\tan^3\left(\frac{x}{2}\right) = 2\tan\left(\frac{x}{2}\right) - x + c.$

*13.* $y\ sinx = 2\ x^2 + c.$

*14.* $y = \tan^{-1} x - 1 + c\ e^{-\tan^{-1} x}$

*15.* $y\ e^{-\tan^{-1} x} = \frac{1}{2} e^{2\tan^{-1} x} + c.$

*16.* $y\ cotx = \log(cosec\ x - cotx) + c.$

*17.* $y \sec x + 2\ cosx = c.$

*18.* $y = 1 - cotx + c\ e^{-cotx}.$

*19.* $y\ sinx = 2\ x^2 + c.$

*20.* $y\ e^{x^3} = \frac{1}{12}\ e^{x^3}\ (2\ x^3 - 1)c.$

*21.* $(y + 1)sinx + x\ cosx = c.$

*22.* $\frac{y}{x}\ (x^2 - 1) = logx + \frac{1}{2x^2} + c.$

*23.* $e^{-sinx}\ (y + \sin^2 x + 2\ sinx + 2\ ) = c.$

*24.* $2y = x^3 + 6x^2 - 4x\ logx + cx.$

*25.* $y = (x - 2)^3 + c(x - 2).$

*26.* $y\ sinx + 5\ e^{cosx} = c.$

*27.* $2y = x^3 + c\ x^3 e^{\frac{1}{x^2}}$

*28.* $y = x + cos2x + c\ sin2x.$

*29.* $x\ e^{y^2} = \frac{y^2}{2} + c.$

*30.* $xy^3 = (2y^2 + 2y + 2) + ce^y.$

*31.* $e^{\tan^{-1} y}(x + \tan^{-1} y - 1) = c.$

32. $x\sqrt{coty} = \sqrt{tany} + c.$

33. $xe^{\tan^{-1} y} = \frac{1}{2}e^{2\tan^{-1} y} + c.$

34. $x \log y = \frac{1}{2}\ (\log y)^2 + c.$

35. $x^2 \tan^{-1} y = \frac{x^6}{6} + c.$

36. $x \log y = (x - 1)e^x + c.$

37. $\frac{1}{x\ logy} = \frac{1}{2x^2} + c.$

38. $\frac{siny}{1+x} = e^x + c.$

39. $\frac{e^{-y}}{x} = \frac{1}{2x^2} + c.$

40. $secy\ secx = sinx + c.$

41. $\frac{1}{y} + \left[1 - \log\left(\frac{1}{x}\right)\right] \ = cx.$

42. $e^x = xy\ (c - e^x)$

43. $e^{-x^2} = y^2\left(e^{-x^2} + c\right).$

44. $\left(\frac{x}{y}\right)^3 = 3\ sinx + c.$