Christ University
Department of Mathematics

B.Sc. I Semester- MAT 131

Practice Problems: Set- II (Leibniz' Theorem)

1. Find the n^{th} derivative of the following:

i.	$\frac{1}{2x^2-3x-2}$	xi.	$\cos^2 x$
ii.	$\frac{1}{6x^2-5x+1}$	xii.	$\cos^3 x$
iii.	$\frac{x^2}{(x-1)^2(x-2)}$	xiii.	$\sin^2 x$
iv.	$\frac{2x-1}{x^2-x-2}$	xiv.	$\sin^4 x$
v.	$\frac{x}{2x^2+3x+1}$	xv.	$\sin^3 x \cos^2 x$
vi.	$\frac{1}{(x+2)(x-1)}$	xvi.	$e^x \sin x$
vii.	$\sin^2 5x \cos 3x$	xvii.	$e^{2x} \cos^2 x$
viii.	$\cos^2 4x \sin 2x$	xviii.	$\log \sqrt{\frac{2x+1}{3x-2}}$
ix.	$\sin x \sin 2x \sin 3x$	xix.	$\log (x-3x^2)$
x.	$\cos x \cos 2x \cos 3x$	xx.	$\log (x^2-a^2)$

2. Find the n^{th} derivative of the following using Leibniz's theorem:
   a. $x^2 \log x$
   b. $x \sin^2 x$
   c. $x^3 e^{ax}$
   d. $x^2 e^x \cos x$
   e. $x^4 \log (x+4)$
   f. $x^3 \sin 5x \cos 2x$
3. If $y = \sin^{-1}x$, prove that $(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} - n^2 y_n = 0$.
4. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + (n^2+1)y_n = 0$.
5. If $y = e^{m\sin^{-1}x}$, show that $(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} - (m^2+n^2)y_n = 0$.

6. If x= *sin t* and y = *cos pt*, show that $(1-x^2)y_{n+2} - (2n+1)x\, y_{n+1} - (n^2 - p^2)y_n = 0$.

7. If y = $\left(x+\sqrt{1+x^2}\right)^m$, prove that $(1+x^2)y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$.

8. If y = $\tan^{-1}$ x, show that $(1+x^2)y_{n+1} - 2nx\, y_n + n(n-1)y_{n-1} = 0$.

9. If y = $(\sinh^{-1} x)^2$, show that $(1+x^2)y_{n+2} + (2n+1)x\, y_{n+1} + n^2 y_n = 0$.

10. If y = $(x^2 - 1)^n$, show that $(x^2 - 1)y_{n+2} + 2x\, y_{n+1} - n(n+1)y_n = 0$.

11. If $y^{-1/m} + y^{1/m} = 2x$, show that $(x^2-1)y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$.

12. If x = tan (log y) , show that $(1+x^2)y_{n+2} + [2(n+1)x-1\}\, y_{n+1} + n(n+1)y_n = 0$.

13. If $\cos^{-1}(y/b) = \log(x/n)^n$, show that $x^2 y_{n+2} + (2n+1)x\, y_{n+1} + 2n^2 y_n = 0$.

14. If y = $[\log(x+\sqrt{1+x^2})]^2$, show that $(1+x^2)y_{n+2} + (2n+1)x\, y_{n+1} + n^2 y_n = 0$.

15. If y = cos (log ($x^2 - 2x + 1$)), show that $(x-1)^2 y_{n+2} + (2n+1)(x-1)\, y_{n+1} + (n^2+4) = 0$.

16. If y = x^n log x, show that $y_{n+1} = \frac{n!}{x}$.

17. If y = x^{n-1} log x, show that $y_n = \frac{(n-1)!}{x}$.

18. Find $y_n(0)$ for the following functions:

(a) y = Cos (m $\sin^{-1}$x) (b) $y = e^{m\sin^{-1}x}$ (c) y = $(\sin^{-1}x)^2$ (d) y = $\log(x+\sqrt{1+x^2})$

Challenge questions:

19. If x + y = 1, show that $D^n(x^n y^n) = n!\,[\,y^n - (nc_1)^2 y^{n-1}x + -(nc_2)^2 y^{n-2}x^2 + \ldots\ldots\ldots + (-1)^n x^n]$.

20. If y = a cos $\sqrt{x}$ + b sin $\sqrt{x}$, show that $4x\, y_{n+2} + (4n+2)\, y_{n+1} + y_n = 0$.

21. If y = cos (m $\sin^{-1}\sqrt{x}$), show that $\frac{y_{n+1}}{y_n} = \frac{4n^2 - m^2}{4n+2}$ at x =0.

22. If y = A $[(x+\sqrt{x^2-1}]^m$ + B $[(x-\sqrt{x^2-1}]^m$, show that

$(x^2-1)y_{n+2} + (2n+1)x\, y_{n+1} + (n^2 - m^2)y_n = 0$.

24. If y = $x^2 e^x$, show that $y_n = \frac{1}{2} n(n-1)\, y_2 - n(n-2)\, y_1 + \frac{1}{2}(n-1)(n-2)\, y$.

25. Prove that $D^n(x^n \log x) = n!\,\{\log x + 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots + \frac{1}{n}\}$.