

# CHRIST UNIVERSITY,BENGALURU - 560029

End Semester Examination March - 2016

Bachelor of Technology VI SEMESTER

**Code: CS635 / IT635**

**Subject: NUMERICAL METHODS**

**Max.Marks: 100**

**Duration: 3Hrs**

## SECTION A

**Answer ALL the questions. Assume any missing data. Precisely mention all formula for each numerical method.**

**5X20=100**

- 1 a) Apply Newton-Raphson Method to solve  $3x - \cos x - 1 = 0$ . (10)

**[OR]**

- 2 b) Give the accuracy and precision of the following numbers (a) 95.763 (b) 0.008472 (c) 0.0456000 (d) 36 (e) 57.396 (f) 3600.00. Represent the decimal numbers 0.1 and 0.4 in binary form with an accuracy of 8 binary digits. Add them and then convert the result back to the decimal form. Estimate the error .

- 3 a) Find the positive root of the equation  $x^3 + x^2 - 1 = 0$ , correct to six decimal places by Fixed point iteration method, assuming the initial approximation as  $x_0 = 0.75$ . (10)

**[OR]**

- 4 b) Given  $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ , find the inverse by Gauss – Jordan method. (10)

Given  $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ , find the inverse by Gauss – Jordan method.

- 5 a) The population of a town in the decennial census was as below. Estimate the population for the year 1895. (10)

Year x:	1891	1901	1911	1921	1931
Population y:(in thousands)	46	66	81	93	101

**[OR]**

- 6 b) Given,  $\log_{10} 654 = 2.8156$ ,  $\log_{10} 658 = 2.8182$ ,  $\log_{10} 659 = 2.8189$ ,  $\log_{10} 661 = 2.8202$ , find the value of  $\log_{10} 656$  by Newton's divided difference. (10)

- 7 a) The following table gives the percentage number of patients for different age groups (10)

Age over years	30	35	45	55
% number of patients	148	96	68	34

Use Lagrange's formula to find the percentage number of patients over 40 years. What are the methods available for interpolation?

**[OR]**

- 8 b) The following table gives the population of a town during the last six censuses. Estimate using Newton's interpolation formula, the increase in the population during the period 1946 to 1948. (10)

Year	1911	1921	1931	1941	1951	1961
Population (in thousands)	12	15	20	27	39	52

- 9 a) The population of a certain town is given below in the table. Find the rate of growth of the population in 1931, 1941 and 1971. (10)

Year X	1931	1941	1951	1961	1971
Population in thousands	40.62	60.80	79.95	103.56	132.65

**[OR]**

10 b) Use Simpsons 3/8 th rule by dividing the given interval into six equal parts to evaluate (10)

$$\int_1^2 \frac{dx}{\sqrt{3 + 2x - x^2}}.$$

11 a) Evaluate  $I = \int_0^1 \frac{dx}{1+x}$  by using Romberg's method correct to three decimals. Hence evaluate  $\log_e 2$ . (10)

[OR]

12 b) Evaluate  $I = \int_2^3 \frac{\cos 2x}{1+\sin x} dx$ , using two and three point Gaussian quadrature formula. (10)

13 a) Use Taylors series method, find  $y(0.1)$ ,  $y(0.2)$  and  $y(0.3)$ . correct to four decimal places (10) given that  $\frac{dy}{dx} = x^2 + y^2$  and  $y(0) = 1$ .

[OR]

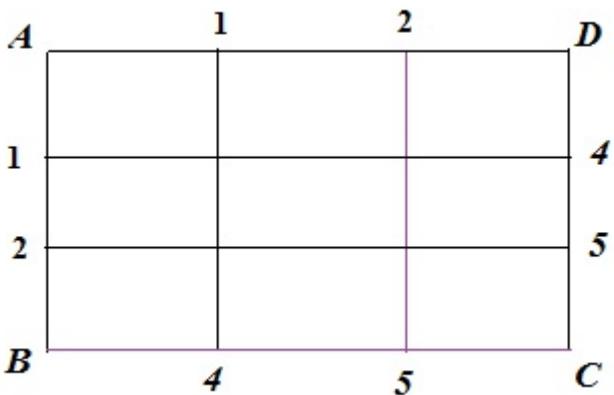
14 b) Given  $\frac{dy}{dx} = x + \sin y$ ,  $y(0) = 1$  Compute  $y(0.2)$  and  $y(0.4)$  using modified Euler's method. (10)

15 a) Use the classical Rungee Kutta method to estimate  $y(0.4)$  when  $y'(x) = x^2 + y^2$  (10) with  $y(0) = 0$ . Assume 1)  $h=0.2$ . and 2)  $h=0.1$ .

[OR]

16 b) Use the Adams – Bashforth method, find  $y(1.4)$ , if  $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$  and  $y(1) = 1$ ,  $y(1.1) = 0.996$ ,  $y(1.2) = 0.986$ ,  $y(1.3) = 0.972$ . (10)

17 a) Solve the equation  $u_{xx} + u_{yy} = 0$  for the following square ABCD mesh with boundary values (10) as shown in figure below:



[OR]

18 b) Use Crank – Nicholson's scheme, solve  $\frac{\partial^2 u}{\partial x^2} = 16 \frac{\partial u}{\partial t}$ ,  $0 < x < 1$ ,  $t > 0$  given  $u(x, 0) = 0$ ,  $u(0, t) = 0$ ,  $u(1, t) = 100t$ . Compute  $u$  for one step in  $t$  direction taking  $h = 1 / 4$ . (10)

19 a) Solve  $\nabla^2 u = -10(x^2 + y^2 + 10)$  over the square mesh of length 1 unit with sides  $x = 0$ ,  $y = 0$ ,  $x = 3$ ,  $y = 3$  with  $u = 0$  on the boundary. (10)

[OR]

**20 b)** Evaluate the pivotal values of the equation  $\frac{\partial^2 u}{\partial t^2} = 16 \frac{\partial^2 u}{\partial x^2}$ , taking  $h = 1$  upto  $t = 1.25$ . The **(10)** boundary conditions are  $u(0, t) = u(5, t) = 0$ ,  $u_i(x, 0) = 0$  and  $u(x, 0) = x^2(5 - x)$ ,

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Bachelor of Technology VI SEMESTER

**Code: CS635**

**Subject: NUMERICAL METHODS**

**Max.Marks: 100**

**Duration: 3Hrs**

## SECTION A

**Answer ALL the questions. Assume any missing data. Precisely mention all formula for each numerical method.**

**5X20=100**

- 1 a) Deduce a formula for absolute error in chopping method and symmetric round off method. (10)  
Find the round off error in storing the number 852.6835 using a four digit mantissa. Use both Chopping method and Symmetric round off method.

**[OR]**

- 2 b) Solve by Gauss elimination method: (10)

$$3x+4y+5z=18, 2x-y+8z=13, 5x-2y+7z=20,$$

- 3 a) Solve for a positive root of  $x - \cos x = 0$  by Regula Falsi method. Find the root between 0 and 1. (10)

**[OR]**

- 4 b) Given  $A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ , find the inverse by Gauss – Jordan method. (10)

$$\text{Given } A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}, \text{ find the inverse by Gauss – Jordan method.}$$

- 5 a) Calculate the number of students who obtained marks between 40 and 45 from the data given (10) below:

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

**[OR]**

- 6 b) Using Newton's divided difference formula, find the values of  $f(2)$ ,  $f(8)$  and  $f(15)$  given the (10) following table:

X.	4.	5.	7.	10.	11.	13.
$f(X)$	48.	100.	294.	900.	1210.	2028.

- 7 a) The following table gives the percentage number of patients for different age groups (10)

Age over years	30	35	45	55
% number of patients	148	96	68	34

Use Lagrange's formula to find the percentage number of patients over 40 years. What are the methods available for interpolation?

**[OR]**

- 8 b) The following table gives the values of density of saturated water for various temperatures of (10) saturated steam.

Temperature $^{\circ}\text{C}$	100	150	200	250	300
Density $\text{kg/m}^3$	958	917	865	799	712

Find by interpolation the density when the temperature is  $275^{\circ}\text{C}$ .

- 9 a) Find the derivatives at  $x = 5$  and  $x = 6$  of the function tabulated below: (10)

$x$	5	6	7	8	9	10

**[OR]**

- 10 b)** Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by dividing into ten equal parts using Simpson's 1/3 rd rule. (10)
- 11 a)** Compute  $\int_0^1 \frac{dt}{\sqrt{1+t^4}}$  correct to three decimals using Romberg's method. Hence compare the theoretical value. (10)

**[OR]**

- 12 b)** Evaluate  $I = \int_2^3 \frac{\cos 2x}{1+\sin x} dx$ , using two and three point Gaussian quadrature formula. (10)
- 13 a)** Compute  $y(1.1)$  and  $y(1.2)$  using Taylors series method for  $\frac{dy}{dx} = y + x^3, y(1) = 1$  considering the term up to 4<sup>th</sup> order. (10)

**[OR]**

- 14 b)** Given that  $\frac{dy}{dx} = x^2 + y, y(0) = 1$  Find an approximate value of  $y(0.1)$  taking step size  $h = 0.05$  by modified Euler's method. (10)
- 15 a)** Give the solution by 4th order Rungee Kutta Method with initial value of  $x$  and  $y$  as 1 and 2 respectively. It is required to compute  $y$  at  $x=2$ . The input step size  $h$  is 0.25. Compute values of  $y$  till  $x=3.5000000$ . (10)

**[OR]**

- 16 b)** Given  $\frac{dy}{dx} = x^2 - y$  and  $y(0) = 1, y(0.1) = 0.90516, y(0.2) = 0.82127, y(0.3) = 0.74918$ , evaluate  $y(0.4)$  using Adams – Bashforth method. (10)
- 17 a)** By iteration method solve the Laplace equation  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$  over the square region satisfying the following boundary conditions:

$$u(0, y) = 0 \quad 0 \leq y \leq 3$$

$$u(3, y) = 9 + y \quad 0 \leq y \leq 3$$

$$u(x, 0) = 3x \quad 0 \leq x \leq 3$$

$$u(x, 3) = 4x \quad 0 \leq x \leq 3$$

**[OR]**

- 18 b)** Solve by Crank – Nicholson's method the equation  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$  subject to  $u(x, 0) = 0, u(0, t) = 0$  and  $u(1, t) = t$  for two time steps taking  $h = 1 / 4$ . (10)
- 19 a)** Solve  $\nabla^2 u = -10(x^2 + y^2 + 10)$  over the square mesh of length 1 unit with sides  $x = 0, y = 0, x = 3, y = 3$  with  $u = 0$  on the boundary. (10)

**[OR]**

- 20 b)** The transverse displacement  $u$  of a point at a distance  $x$  from one end and at any time  $t$  of a vibrating string satisfies the equation  $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$ , with boundary conditions  $u = 0$  at  $x = 0, t > 0$  and  $u = 0$  at  $x = 4, t > 0$  and the initial conditions  $u = x(4-x)$  and  $\frac{\partial u}{\partial t} = 0$  at  $t = 0, 0 \leq x \leq 4$ . Solve this equation numerically for one half period of vibration, taking  $h = 1$  and  $k = 1 / 2$ . (10)

# CHRIST UNIVERSITY,BENGALURU - 560029

End Semester Examination March - 2017

Bachelor of Technology VI SEMESTER

**Code: EC631**

**Subject: NUMERICAL METHODS**

**Max.Marks: 100**

**Duration: 3Hrs**

## SECTION A

**Answer ALL the questions.**

**5X20=100**

- 1 a) (i) Use Newton – Raphson method to find a real root of  $x \sin x + \cos x = 0$  near  $x = \pi$  (10)  
Calculate upto four decimal places of accuracy.[5]

(ii) Solve the following system of equations by Gauss Elimination Method  
 $10x - 7y + 3z + 5u = 6; -6x + 8y - z - 4u = 3x; y + 4z + 11u = 2; 5x - 9y - 2z + 4u = 7$   
[5]

**[OR]**

- 2 b) Solve the following system of equations by Gauss - Seidel method: (10)  
 $20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25.$

- 3 a) Use the Regula Falsi method to obtain a real root of the equation  $x^4 + 2x^2 - 16x + 5 = 0$  (10)  
correct to three decimal places.

**[OR]**

- 4 b) Apply Gauss – Jordan method to find the inverse of (10)  

$$\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}.$$

- 5 a) The values of  $x$  and  $y$  are given below: (10)

$x:$	1	2	3	4
$y:$	1	2	5	11

Find the cubic splines and evaluate  $y(1.5)$  and  $y'(3)$ , assuming that and  $M_0 = M_3 = 0$

**[OR]**

- 6 b) Calculate the number of students who obtained marks between 40 to 45 from the data given (10)  
below:

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

- 7 a) Find the cubic polynomial which passes through the points (2, 4) (4, 56) (9, 711) (10, 980) (10)  
and hence estimate the dependent variable corresponding to the values of the independent  
variable 3, 5, 7, 11.

**[OR]**

- 8 b) Find the interpolating polynomial  $f(x)$  satisfying  $f(0) = 0, f(2) = 4, f(4) = 56, f(6) = 204, f(8) = 496, f(10) = 980$ , and hence find  $f(7)$  and  $f(9)$ . (10)

- 9 a) Obtain the derivatives at  $x = 5$  and  $x = 6$  of the function tabulated below: (10)

x	5	6	7	8	9	10
y	196	394	686	1090	1624	2306

**[OR]**

- 10 b) Compute the value of  $\int_{0.2}^{1.4} (\sin x - \ln x + e^x) dx$  taking  $h = 0.2$  using Simpsons 3/8 rule. (10)  
Also compare your results by integration in analytical method.

- 11 a) Evaluate  $\int_{2}^3 \frac{\cos 2x}{1+\sin 2x} dx$ , using two and three point Gaussian quadrature formula. (10)

**[OR]**

- 12 b) Compute  $\int_0^1 \frac{dx}{1+x^2}$  correct to three decimal places using Romberg's method. Hence obtain the (10)  
approximate value for  $\pi$ .

- 13 a) Using Taylor's series method, find  $y(0.1), y(0.2)$  and  $y(0.3)$ , correct to three decimal places (10)

given that  $\frac{dy}{dx} = x^2 + y^2$  and  $y(0) = 1$ .

[OR]

- 14 b) Find approximate value of  $y(0.25)$  and  $y(0.5)$  given that  $\frac{dy}{dx} = 3x^2 + y$ ,  $y(0) = 1$  taking  $h = 0.25$ .
- 15 a) Use fourth order Runge – Kutta method to find  $y$  at  $x = 0.2$ , given that  $\frac{dy}{dx} = 3e^x + 2y$ ,  $y(0) = 0$  and  $h = 0.1$ .

[OR]

- 16 b) Given  $\frac{dy}{dx} = \frac{1}{x+y}$   $y(0) = 2$ ,  $y(0.2) = 2.0933$ ,  $y(0.4) = 2.1755$ ,  $y(0.6) = 2.2493$ , find  $y(0.8)$  (10) by Milne's predictor – corrector method.
- 17 a) Solve  $u_{xx} + u_{yy} = 0$  over the square of side 4 units satisfying the following boundary conditions:  $u(0, y) = 0$ ,  $0 \leq y \leq 4$ ;  $u(4, y) = 12 + y$ ,  $0 \leq y \leq 4$ ;  $u(x, 0) = 3x$ ,  $0 \leq x \leq 4$ ,  $u(x, 4) = x^2$ ,  $0 \leq x \leq 4$

[OR]

- 18 b) Solve by Crank – Nicholson's method the equation  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$  subject to  $u(x, 0) = 0$ ,  $u(0, t) = 0$  (10) and  $u(1, t) = t$  for two time steps taking  $h = 1/4$ .
- 19 a) Solve  $\nabla^2 u = -10(x^2 + y^2 + 10)$  over the square mesh of length 1 unit with sides  $x = 0$ ,  $y = 0$ ,  $x = 3$ ,  $y = 3$  with  $u = 0$  on the boundary.

[OR]

- 20 b) Solve  $\frac{\partial^2 u}{\partial x^2} = 0.04 \frac{\partial^2 u}{\partial t^2}$  at the pivotal points given that  $u(0, t) = u(5, t) = u_t(x, 0) = 0$  and (10)  $u(x, 0) = \begin{cases} 20x, & 0 \leq x \leq 2.5 \\ 10 - 2x, & 2.5 \leq x \leq 5 \end{cases}$  with  $h = 1$ ,  $k = 1/5$ .

**Code: EC631/ EC833D**

**Subject: NUMERICAL METHODS**

**Max.Marks: 100**

**Duration: 3Hrs**

**SECTION A**

**Answer ALL the questions.**

**5X20=100**

- 1 a) (i) Use Newton – Raphson Method to find a real root of the equation  $x^3 - 2x - 5 = 0$  and correct to three decimal places.[5]

(ii) Solve the following system of equations by Gauss Elimination Method:[5]

$$4x_1 + x_2 + x_3 = 4; x_1 + 4x_2 - 2x_3 = 4; 3x_1 + 2x_2 - 4x_3 = 6$$

**[OR]**

- 2 b) Solve the following system of equations by Gauss-Seidel method: [10]

$$x + y + 54z = 110; 27x + 6y - z = 85; 6x + 15y + 2z = 72.$$

- 3 a) (i) Use the regula falsi method to find the fourth root of 12 correct to three decimal places.[5] (10)

(ii) Show that the equation  $\log x - x + 2 = 0$  has a root between 3 and 4. Solve using the fixed point iterative method.[5]

**[OR]**

- 4 b) Apply Gauss – Jordan method to find the inverse of 
$$\begin{bmatrix} 5 & 8 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$
 [10]

- 5 a) Given the data points [10]

$i$	0	1	2
$x_i$	4	9	16
$f_i$	2	3	4

Estimate the function value  $f$  at  $x=7$  using cubic spline

**[OR]**

- 6 b) Calculate the number of students who obtained marks between 40 to 45 from the data given below: [10]

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of students	31	42	51	35	31

- 7 a) (i) Apply Lagrange's formula to find a root of the equation  $f(x) = 0$  given that  $f(30) = -30$ ,  $f(34) = -13$ ,  $f(38) = 3$  and  $f(42) = 18$ .[5]

(ii) By means of Newton's divided difference formula, find the value of  $f(8)$  and  $f(12)$  from the following table :[5]

$x$	4	5	7	10	11	13
$f(x)$	48	100	294	900	1210	2028

**[OR]**

- 8 b) The following data gives the melting point of an alloy of lead and zinc, where  $t$  is the temperature in  $^{\circ}\text{C}$  and  $P$  is the percentage of lead in the alloy. [10]

$P$	40	50	60	70	80	90
$t$	184	204	226	250	276	304

Using suitable Newton's interpolation formula, find the melting point of the alloy containing

84% of lead.

- 9 a) Given that: (10)

$X$	1.0	1.1	1.2	1.3	1.4	1.5	1.6
$Y$	7.989	8.403	8.781	9.129	9.451	9.750	10.031

Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$  at (i)  $x = 1.1$  (ii)  $x = 1.6$ .

[OR]

- 10 b) By using Simpsons 3/8 th rule by dividing the given interval into six equal parts to evaluate (10)

$$\int_1^2 \frac{dx}{\sqrt{3 + 2x - x^2}}. \text{ Hence deduce the value of } \pi.$$

- 11 a) (10)

(i) Evaluate using Gaussian two-point formula  $\int_{-2}^2 e^x dx$ .

(ii).Evaluate using Gaussian three-point formula  $\int_{-2}^{1.5} e^{-t^2} dt$ .  
[OR]

- 12 b) (10)  
Evaluate  $\int_1^2 \int_3^4 \frac{1}{(x+y)^2} dxdy$  by taking  $h=k=0.5$  using both Trapezoidal and Simpson's rule.

- 13 a) Using Taylors series method, find  $y(0.1)$ ,  $y(0.2)$  and  $y(0.3)$ .correct to three decimal places (10)

given that  $\frac{dy}{dx} = x^2 + y^2$  and  $y(0) = 1$ .

[OR]

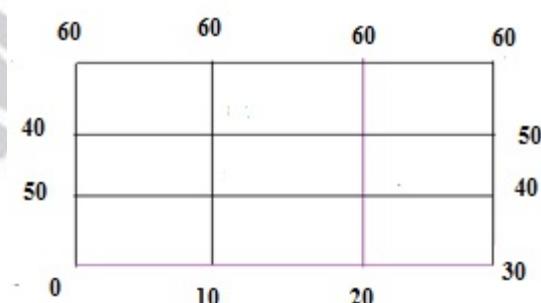
- 14 b) (10)  
Given that  $\frac{dy}{dx} = \log(x+y)$ ,  $y(0) = 1$ . Compute  $y(0.2)$  using modified Euler's method, by taking  $h=0.2$ .

- 15 a) (10)  
 $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ ,  $y(0) = 1$ ,  $\frac{dy}{dx} = 0$   
Given  $\frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ ,  $y(0) = 1$ ,  $\frac{dy}{dx} = 0$  at  $x = 0$ , find the value of  $y(0.1)$  using Runge - Kutta method of fourth order.

[OR]

- 16 b) (10)  
Using Adam-Bashforth method determine  $y(0.4)$  and  $y(0.5)$  correct to 3 decimals, given that  
 $\frac{dy}{dx} = 0.5xy$  and  $y(0)$ ,  $y(0.1)$ ,  $y(0.2)$  and  $y(0.3)$  have values 1.0, 1.0025, 1.0101 and 1.0228 respectively.

- 17 a) Solve at the nodal points of the square grid of the given figure, using the given boundary values (10)



[OR]

- 18 b) (10)  
Find the values of  $u(x, t)$  satisfying the heat equation  $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$  and the boundary

conditions  $u(0,t) = 0 = u(8,t)$  and  $u(x,0) = 4x - \frac{x^2}{2}$  at the points  $x = i$ ,  $i = 0, 1, 2, \dots, 8$  and

$$t = \frac{1}{8}j, \quad j = 0, 1, 2, \dots, 5.$$

- 19 a) Solve  $\nabla^2 u = -10(x^2 + y^2 + 10)$  over the square mesh of length 1 unit with sides  $x = 0, y = 0, (10)$   
 $x = 3, y = 3$  with  $u = 0$  on the boundary.

[OR]

- 20 b) Evaluate the pivotal values of the equation  $25 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$  for one half period of vibration  $(10)$   
given  $u(0,t) = u(5,t) = 0$ ,  
 $u(x,0) = 2x \quad \text{for } 0 \leq x \leq 2.5$   
 $= 10 - 2x \quad \text{for } 2.5 \leq x \leq 5 \text{ and } u_t(x,0) = 0$ .

**Code: MAT531**

**Max.Marks: 100**

**Subject: NUMERICAL METHODS**

**Duration: 3Hrs**

**SECTION A**

**Answer any EIGHT questions:**

**8X3=24**

- 1 If 0.5555 is the approximate value of  $5/9$ , find the absolute, relative and percentage error.
- 2 Compute the absolute error and relative error with the exact value determined to at least five digits for  $\pi = \frac{22}{7}$ .
- 3 Let  $p = 0.54617$  and  $q = 0.54601$ . Use four-digit arithmetic to approximate  $p - q$  and determine the absolute and relative errors using chopping.
- 4 What is the difference between algebraic and transcendental equations? Give two examples for each.
- 5 Show that (i)  $\Delta - \nabla = \Delta \nabla$  (ii)  $(1 + \Delta)(1 - \nabla) = 1$  (iii)  $\nabla = 1 - E^{-1}$ .
- 6 Evaluate  $(2 \Delta + 3)(E + 2)(3x^2 + 4)$  by taking  $h = 1$ .
- 7 Evaluate  $\Delta^2 Ex^3$ .
- 8 Given:  $U_0 = 148$ ,  $U_1 = 192$ ,  $U_2 = 241$  &  $U_4 = 374$  find  $U_3$ .
- 9 Use the trapezoidal rule with  $n = 4$  to estimate  $\int_0^4 \frac{1}{1+x^2} dx$ . Compare the estimate with the exact value of the integral.
- 10 Construct a table of divided difference for the data

1	3	6	11
5	34	321	5541

- 11 Explain Modified Euler's method to find the solution of differential equation.
- 12 Use Picard's method to compute  $y(0.1)$ , from the differential equation  $\frac{dy}{dx} = \frac{y-x}{y+x}$ ;  $y(0) = 1$  upto two approximations.

**SECTION B**

**Answer any SEVEN questions:**

**7X8=56**

- 13 When trying to find the acidity of a solution of magnesium hydroxide in hydrochloric acid, we obtain the following equation,  $A(x) = x^3 + 3.6x^2 - 36.4$  where  $x$  is the hydronium ion concentration. Find the hydronium ion concentration for a saturated solution(acidity equals zero) using any of the known iterative numerical method.
- 14 The following methods are proposed to compute. Rank them in order, based on their apparent speed of convergence, assuming  $p_0 = 1$  (i)  $P_n = P_{n-1} \left( 1 + \frac{7-P_{n-1}^5}{P_{n-1}^2} \right)^3$  (ii)  $P_n = P_{n-1} - \frac{P_{n-1}^5 - 7}{P_{n-1}^2}$ .
- 15 Solve the system of equations :  $x + y + 54z = 110$ ;  $27x + 6y - z = 85$ ;  $6x + 15y + 2z = 72$  by Gauss-Seidel method to obtain the final solution correct to three places of decimal.
- 16 Find the 7<sup>th</sup> term and the general term of the series 3,9,20,38,65,--- .
- 17 Determine  $f(x)$  as a polynomial in  $x$  for the following data using Newton's divided difference formula

X	-4	-1	0	2	5
Y	1245	33	5	9	1335

- 18 A function  $y = f(x)$  is given in the following table:

x	1	1.2	1.6	1.8	2
y	0	0.128	1.296	2.432	4

Find the approximate values of  $f'(1.6)$  and  $f''(1.6)$ .

- 19 Evaluate  $\int_0^1 \frac{dx}{1+x^2}$  by using (a) Simpson's 1/3<sup>rd</sup> rule taking four equal strips and hence deduce an approximate value of  $\pi$  (b) Simpson's 3/8<sup>th</sup> rule.

- 20** If  $\frac{dy}{dx} = x(1 + x^3y)$ ;  $y(0) = 3$  where  $y(4) = 4$ , compute the values of  $y(4.1)$ ,  $y(4.2)$  and  $y(4.3)$  by Taylor series method.
- 21** Given that  $\frac{dy}{dx} - \sqrt{xy} = 2$ ,  $y(1) = 1$  find the value of  $y(2)$  in steps of 0.2 using Euler's modified method.

### SECTION C

**Answer any TWO questions:**

**2X10=20**

- 22** Solve the system of equations by LU decomposition method:

$$x_1 + x_2 + 3x_4 = 8; 2x_1 + x_2 - x_3 + 2x_4 = 7; 3x_1 - x_2 - x_3 + 2x_4 = 14; -x_1 + 2x_2 + 3x_3 - x_4 = -7$$

- 23** Use method of separation of symbols to prove the identity

$$v_0 - v_1 + v_2 - v_4 + \dots = \frac{1}{2}v_0 - \frac{1}{4}\Delta v_0 + \frac{1}{8}\Delta^2 v_0 - \frac{1}{16}\Delta^3 v_0 + \dots$$

- 24** Given the initial value problem  $\frac{dy}{dx} = x + y$ ;  $y(0) = 0$  Find the value of  $y$  approximately for  $x = 1$  by fourth-order Runge-Kutta method in two steps. Compare the result with the exact value.

**Code: MAT532**

**Max.Marks: 100**

**Subject: NUMERICAL METHODS**

**Duration: 3Hrs**

**SECTION A**

**Answer any TEN Questions:**

**10X3=30**

- 1 If 0.333 is the approximate value of  $1/3$ , find the absolute, relative and percentage error.
- 2 How many decimal digits must be taken in the number  $\sqrt{52}$  for the error not to exceed the percentage 0.2%.
- 3 Approximate values of  $\sqrt{5.5}$  and  $\sqrt{6.1}$  correct to four significant figures places are respectively 2.345 and 2.470. Find the relative error in taking the difference of these numbers.
- 4 Compute the root of  $\log x = \cos x$  correct to three decimal places which lie between 1 and 2, by any of the known methods.
- 5 Explain Jacobi Iteration method to solve non-homogeneous equation involving three variables.
- 6 Explain forward difference, backward difference and shift operator.
- 7 If  $f(x)$  be a polynomial of nth degree in  $x$ , then  $\Delta^n f(x)$  is a constant and  $\Delta^{n+1} f(x) = 0$ . And hence evaluate:  $\Delta^8[(1+2x)(1-x^2)(1-8x)(1+2x^4)]$  by taking  $h = 2$ .
- 8 Given  $v_0 = 4, v_1 = 8, v_2 = 21, v_3 = 75, v_4 = 32, v_5 = 16$  and  $v_6 = 10$ , find the value of  $\Delta^6 v_0$ .
- 9 Estimate the value of  $\tan 32^\circ$  from the data:

x	$30^\circ$	$35^\circ$	$45^\circ$	$50^\circ$	$60^\circ$
$\tan x$	0.5773	0.7002	1	1.1918	1.7320

Using Lagrange's formula.

- 10 Explain Picard's method to find solution of differential equation.
- 11 Use Picard's method to compute  $y(0.2)$ , for the solution of  $\frac{dy}{dx} = x + y$  with the condition  $y = 1$  when  $x = 0$  upto third approximation.
- 12 Use Euler's Method to evaluate  $y(2)$ , from  $\frac{dy}{dx} = \frac{1}{2}(x + y); y(0) = 2$  taking  $h = 0.1$ .

**SECTION B**

**Answer any TEN Questions:**

**10X5=50**

- 13 Find the real root of the equation  $2x - \log_{10} x - 7 = 0$ , by Newton-Raphson method, which lies between 3 and 4, correct to three places of decimal places.
- 14 Find the Cube root of 15, correct to four significant figures by Iteration method.
- 15 Solve the following system of equations by Gauss-elimination method  
 $x + 2y + 3z = 10; x + 3y - 2z = 7; 2x - y + z = 5.$
- 16 Solve the following equations by employing the method of partial pivoting  
 $x + 2y + z = 8; 2x + 3y + 4z = 20; 4x + 3y + 2z = 16.$
- 17 Solve the following equations by Jacobi iteration method  
 $x + y + 4z = 9; 8x - 3y + 2z = 20; 4x + 11y - z = 33.$
- 18 Solve the following equations by Gauss-Seidel iteration method  
 $25x + 2y + 2z = 69; 2x + 10y + z = 63; x + y + z = 43.$
- 19 Find a polynomial of degree two which takes the values

x	1	2	3	4
$f(x)$	2	2	4	8

- 20 Find the missing values in the following table:

x	45	50	55	60	65
y	3.0	--	2.0	---	-2.4

- 21 From the following table find the value of  $\tan 17^\circ$

$\theta^\circ$	0	4	8	12	16	20	24
$\tan \theta^\circ$	0	0.0699	0.1405	0.2126	0.2867	0.3630	0.4452

- 22** Evaluate  $\int_0^1 \frac{x}{1+x} dx$ , correct upto 3 significant figures, taking 6 intervals, by using (i) Trapezoidal Rule (ii) Simpson's 1/3 rule (iii) Simpson's 3/8 rule.

- 23** A function  $y = f(x)$  is given in the following table:

x	1	1.2	1.4	1.6	1.8	2.0
y	0.0000	0.128	0.544	1.296	2.432	4.00

Find the approximate values of  $f'(1.6)$  and  $f''(1.6)$ .

- 24** Solve  $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$  at  $x = 0.1(0.1)0.4$ , using Taylor series method, correct to three decimal places.
- 25** Using Euler's modified method, compute the solution of  $\frac{dy}{dx} = x^2 + y, y(0) = 1$  at  $x = 0.02(0.01)0.04$  correct to four decimal places.
- 26** Solve the initial-value problem  $\frac{dy}{dx} = x + y, y(0) = 1$  using the Runge-Kutta method of second order with  $h=0.5$  to estimate the value of  $y$  when  $x=1$ .

### SECTION C

**Answer any TWO Questions:**

**2X10=20**

- 27** Solve the following equations by LU decomposition method  
 $2x + 2y + z = 12; 3x + 2y + 2z = 8; 5x + 10y - 8z = 10.$
- 28** Use method of separation of symbols to prove the identity  
 $\nu_0 - \nu_1 + \nu_2 - \nu_3 + \dots = \frac{1}{2}\nu_0 - \frac{1}{4}\Delta\nu_0 + \frac{1}{8}\Delta^2\nu_0 - \frac{1}{16}\Delta^3\nu_0 + \dots$
- 29** Solve  $\frac{dy}{dx} = x^2(1+y), y(1) = 1$  by Runge Kutta 4<sup>th</sup> order method with  $h = 0.1$ , find  $y(1.3)$ .

**Code: MAT532**

**Max.Marks: 100**

**Subject: NUMERICAL METHODS**

**Duration: 3Hrs**

**SECTION A**

**Answer any TEN Questions:**

**10X3=30**

- 1 If 0.42857 is the approximate value of  $3/7$ , find the absolute, relative and percentage error.
- 2 If  $x_1, x_2, x_3, \dots, x_n$  are approximations to  $X_1, X_2, X_3, \dots, X_n$  and that in each case the maximum possible error is E. Show that the maximum possible error in the sum  $X_1 + X_2 + X_3 + \dots + X_n$  is  $nE$ .
- 3 Approximate values of  $2/7$  and  $1/3$  correct to four decimal places are respectively 0.2857 and 0.3333 .Find the possible relative and absolute errors in the sum of the approximate values.
- 4 Explain Bisection Method to find root of the equation  $f(x) = 0$ . Hence find the real root of the equation  $x^3 - x - 1 = 0$  up to three stages, correct to three places of decimals.
- 5 What do you mean by diagonally dominant system of equations? Explain with two examples.
- 6 Show that (i)  $E\Delta = \Delta E$ (ii)  $E = 1 + \Delta$ (iii)  $\nabla = 1 - E^{-1}$ .
- 7 If  $f(x)$  be a polynomial of nth degree in x, then  $\Delta^n f(x)$  is a constant and  $\Delta^{n+1} f(x) = 0$ . And hence evaluate:  $\Delta^7(3 - 4x)(1 - x^2)(1 + 2x^2)(1 + 5x^2)$  by taking  $h = 3$ .
- 8 Evaluate : (i)  $\Delta^2 E x^3$  (ii)  $\Delta^2(e^{ax+b})$ .
- 9 Estimate the value of  $\tan 32^\circ$  from the data:

x	$30^\circ$	$35^\circ$	$45^\circ$	$50^\circ$	$60^\circ$
$\tan x$	0.5773	0.7002	1	1.1918	1.7320

Using Lagrange's formula.

- 10 Explain Runge-Kutta method to find the solution of differential equation.
- 11 Use Picard's method to compute  $y(0.1)$ , for the solution of  $\frac{dy}{dx} = x - y^2$ , with the condition  $y = 1$  when  $x = 0$  upto third approximation.
- 12 Find  $y(0.2)$ , by Euler's Method given that  $\frac{dy}{dx} = x^2 + y^2; y(0) = 0$  taking step length  $h = 0.1$ .

**SECTION B**

**Answer any TEN Questions:**

**10X5=50**

- 13 Find a root of the equation  $\sin x + \cos x - 1 = 0$  by Newton-Raphson method, correct to four decimal points.
- 14 Find the Cube root of 15, correct to four significant figures by Iteration method.
- 15 Solve the following equations by Gauss-elimination method  

$$x + 2y + z = 8; 2x + 3y + 4z = 20; 4x + 3y + 2z = 16.$$
- 16 Where do the curves  $y = \cos x$  and  $y = x^3 - 1$  intersect? Solve by any of the known numerical method.
- 17 Solve the following equations by Gauss-Jordan method  

$$2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16.$$
- 18 Solve the following equations by Gauss-Seidel iteration method  $5x - y = 9; x - 5y + z = -4; y - 5z = 6$ .
- 19 Find a polynomial of degree three which takes the values

x	3	4	5	6	7
$f(x)$	6	24	60	120	210

- 20 Estimate the missing value from the following table

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
$y_x$	.135	--	.111	.100	--	.82	.074

- 21 Using Newton's formula, find the values of  $f(9)$  and  $f(12)$  from the following table:

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

Evaluate  $\int_0^1 \frac{x}{1+x} dx$ , correct upto 3 significant figures, taking 6 intervals, by using (i) Trapezoidal Rule (ii) Simpson's 1/3 rule (iii) Simpson's 3/8 rule.

- 23 A function  $y = f(x)$  is given in the following table:

x	1	1.2	1.4	1.6	1.8	2.0
y	0.0000	0.128	0.544	1.296	2.432	4.00

Find the approximate values of  $f'(1.6)$  and  $f''(1.6)$ .

- 24 Solve  $\frac{dy}{dx} = x^2y - 1, y(0) = 1$  at  $x = 0.2(0.1)0.4$ , using Taylor series method, correct to three decimal places.
- 25 Using Euler's modified method, compute the solution of  $\frac{dy}{dx} = x^2 + y, y(0) = 1$  at  $x = 0.02(0.01)0.04$  correct to four decimal places.
- 26 Solve the initial-value problem  $\frac{dy}{dx} = 1 + \frac{y}{x}, y(2) = 2$  using the Runge-Kutta method of second order with  $h=0.1$  to estimate the value of  $y$  when  $x = 2.2$ .

### SECTION C

Answer any TWO Questions:

2X10=20

- 27 Solve the following equations by LU decomposition method  
 $5x - 2y + z = 4; 7x + y - 5z = 8; 3x + 7y - 4z = 10.$
- 28 Use method of separation of symbols prove that  
 $u_0 + x_{c_1}\Delta u_1 + x_{c_2}\Delta^2 u_2 + x_{c_3}\Delta^3 u_3 + \dots = u_x + x_{c_2}\Delta^2 u_{x-1} + x_{c_3}\Delta^4 u_{x-2} + \dots$
- 29 Solve  $\frac{dy}{dx} = x^2 + y^2$  with initial conditions  $y = 1.5$  when  $x = 1$  for  $x = 1.2$  using Runge-Kutta 4<sup>th</sup> order Method using  $h = 0.1$ .

**Code: MAT541B****Course: NUMERICAL METHODS****Max. Ma****Duratio****SECTION A****Answer any EIGHT questions:**

- 1** If 1.285714 is the approximate value of  $9/7$ , find the absolute and percentage error.
- 2** Find the relative error of the approximate number  $N = 437.4$ , if all its digits are valid.
- 3** Find the relative error in taking the difference of the approximate values of the numbers correct to four significant figures.
- 4** Explain Jacobi Iteration method to solve non-homogeneous equation involving three variables.
- 5** Define Forward, Backward and Shift operators with an example.
- 6** Evaluate :  $\Delta^{14} [(1 - 2x^2)(3x^3 - 1)(4x^4 - 1)(5x^5 + 1)]$  by taking  $h=1$ .
- 7** Evaluate:  $\Delta \left( \frac{3^x}{\log x} \right)$ .
- 8** Prove the identity  $U_4 = U_3 + \Delta U_2 + \Delta^2 U_1 + \Delta^3 U_1$ .
- 9** Use the trapezoidal rule with  $n = 4$  to estimate  $\int_1^2 \frac{1}{x} dx$ . Compare the estimate with the exact value of the integral.
- 10** Derive a method of finding the first derivative of  $y = f(x)$  at any value of  $x$  where  $y$  is specified by a set of discrete data points using backward difference formula.
- 11** Derive Euler's method to find the solution of the equation  $\frac{dy}{dx} = f(x, y)$ , with initial condition  $y = y_0$  at  $x = x_0$ .
- 12** Use Picard's method to compute  $y(0.2)$ , find the solution  $\frac{dy}{dx} = x + y$ , with the condition  $y(0) = 0$  upto third approximation.

**SECTION B****Answer any SEVEN questions:**

- 13** Use a fixed-point iteration method to determine a solution accurate to within  $10^{-2}$  for  $x^4 - 3x^2 - 3 = 0$  on  $[1, 2]$ . Use  $p_0 = 1$ .
- 14** Find a real root correct to two decimal places of the equation  $xe^x = \cos x$ , using the method of successive approximation.
- 15** Solve the system of equations :  $x + 2y + z = 8; 2x - y + 2z = 6; 3x + 2y - z = 4$ , by matrix method to obtain the final solution correct to three places of decimal.
- 16** In the following table, the values of  $y$  are consecutive terms of a series of which the number of terms is 5. Find the first and tenth term of the series. Find also the polynomial which approximates the values:

X	3	4	5	6	7	8	9
y	13	21	31	43	57	73	91

- 17** The population of a town in decennial census were as under. Estimate the population for the year 1946.

Year	1921	1931	1941	1951	1961
Population (in thousands)	46	66	81	93	101

- 18** A function  $y = f(x)$  is given in the following table:

x	1	1.2	1.6	1.8	2
y	0	0.128	1.296	2.432	4

Find the approximate values of  $f'(1.6)$  and  $f''(1.6)$ .

19

Evaluate  $\int_0^6 3x^2 dx$  dividing the interval  $[0, 6]$  into six equal parts by applying

(a) Simpson's  $1/3$ rd rule (b) Simpson's  $3/8$ th rule (c) Weddle's rule.

20

Given that  $\frac{dy}{dx} = \frac{y-x}{y+x}$ ;  $y(0) = 1$ , use Picard's method to find  $y(x)$  at  $x = 0.02(0.02)0.06$ .

21

Solve by Euler's modified method, the equation  $\frac{dy}{dx} = x + y$ ,  $y(0) = 0$ . Choose  $h = 0.2$  and  $y(0.4)$ .

## SECTION C

**Answer any TWO questions:**

22 Solve the system of equations by LU decomposition  $2x + 3y + z = 9$ ;  $x + 2y + 3z = 6$ ;  $3x + y - z = 1$

23 Use method of separation of symbols prove that

$$(i) e^x [U_0 + x \Delta U_0 + \frac{x^2}{2!} \Delta^2 U_0 + \frac{x^3}{3!} \Delta^3 U_0 + \dots] = U_0 + \frac{x}{1!} E U_0 + \frac{x^2}{2!} E^2 U_0 + \dots$$

$$(ii) U_0 + U_1 + U_2 + U_3 + \dots + U_n = (n+1)C_1 U_0 + (n+1)C_2 \Delta U_0 + (n+1)C_3 \Delta^2 U_0 + \dots$$

24 Given that  $\frac{dy}{dx} = 1 + y^2$ ;  $y(0) = 0$ , find  $y(0.2)$  and  $y(0.4)$  using fourth order Runge-Kutta method.

**Code: MAT632**

**Max.Marks: 100**

**Subject: NUMERICAL METHODS**

**Duration: 3Hrs**

**SECTION A**

**Answer ALL the following questions:**

**10X1=10**

- 1 If 0.333 is the approximate value of  $1/3$ , find the absolute error.
- 2 Define an error.
- 3 Find the interval in which roots of the equation  $2x^2 - x - 1 = 0$  lies.
- 4 Find the interval in which the root of  $2\sin x - 1 = 0$  lies.
- 5 Write Newton-Gregory forward interpolation formula.
- 6 Show that  $\nabla = 1 - E^{-1}$ .
- 7 Evaluate:  $\Delta^2(x^2 + 5x + 16)$  by taking  $h = 1$ .
- 8 Write down the formula for first derivative of  $y$  w.r.t.  $x$  at  $x=x_0$  by using forward difference formula.
- 9 Write Simpson's 1/3 rule for evaluating definite integral.
- 10 Explain Taylor's series method to find the solution of differential equation.

**SECTION B**

**Answer any NINE questions:**

**9X2=18**

- 11 The absolute error of the number 2681.5579 is 0.3. Find which of the digit of the number is valid and round off the number having only valid digits.
- 12 Calculate the relative error of the difference between the given approximate numbers 6.312 & 6.297.
- 13 Perform four iterations of Bisection Method to obtain the smallest positive root of the equation  $x^3 - 5x + 1 = 0$  correct to three places of decimals.
- 14 Evaluate :  $\Delta^2 Ex^3$ .
- 15 Given  $v_0 = 4$ ,  $v_1 = 8$ ,  $v_2 = 21$ ,  $v_3 = 75$ ,  $v_4 = 32$  find the value of  $\Delta^4 v_0$ .
- 16 By constructing a difference table find the 10<sup>th</sup> term of the sequence 3, 14, 39, 84, 155, 258, .....
- 17 Estimate the missing term in the following table.

x	1	2	3	4	5	6	7
y	2	4	8	—	32	64	128

- 18 Evaluate  $\int_0^{\pi/2} \sqrt{\sin x} dx$ , taking 6 intervals, by Trapezoidal Rule.
- 19 Use Picard's method to compute  $y(0.1)$ , find the solution  $\frac{dy}{dx} = x - y^2$ , with the condition  $y = 1$  when  $x = 0$  upto third approximation.
- 20 Find  $y(0.6)$ , by Euler's Method , from the differential equation  $\frac{dy}{dx} = -\frac{y}{1+x}$ ;  $y(0.3) = 2$ , taking step length  $h = 0.1$ .

**SECTION C**

**Answer any EIGHT of following:**

**8X6=48**

- 21 Find a root of the equation  $\sin x + \cos x - 1 = 0$ , by Regula-Falsi Method correct to four decimal points.
- 22 Find the Cube root of 15, Correct to four significant figures by Iteration method.
- 23 Solve the following equations by Gauss- Jordon Method  $x + 2y + z = 8$ ;  $2x + 3y + 4z = 20$ ;  $4x + 3y + 2z$

$$= 16.$$

- 24 Solve the following equations by Gauss-Seidel iteration method  $x + y + 4z = 9; 8x - 3y + 2z = 20; 4x + 11y - z = 33$ .

- 25 Use method of separation of symbols prove that  $e^x \left[ u_0 + x\Delta u_0 + \frac{x^2}{2!} \Delta^2 u_0 + \frac{x^3}{3!} \Delta^3 u_0 + \dots \right] = u_0 + \frac{x}{1!} Eu_0 + \frac{x^2}{2!} E^2 u_0 + \dots$

- 26 Find the number of students from the following data who secured marks not more than 45

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	35	48	70	40	22

Estimate the increase in population during the period 1955 to 1961.

- 27 The following table gives corresponding values of pressure p and specific volume v of a superheated steam

v	2	4	6	8	10
p	105	42.7	25.3	16.7	13

Find the rate of change with respect to v at v = 2.

- 28 Derive the general quadrature formula for numerical integration to evaluate the definite integral and hence obtain the Simpson's three-eighth rule.

- 29 Applying (i) Simpson's one-third rule and (ii) Simpson's three-eighth rule, evaluate

$$\int_0^{0.3} \sqrt{2x - x^2} dx$$

by dividing the interval into ten equal parts.

- 30 Solve  $\frac{dy}{dx} = x + y$  with  $y(1) = 0$  using Taylor's series method and carrying up to  $x=1.2$  taking  $h = 0.1$ .

#### SECTION D

Answer any THREE of the following questions:

3X8=24

- 31 Solve the following equations by LU decomposition method  $5x - 2y + z = 4; 7x + y - 5z = 8; 3x + 7y - 4z = 10$ .
- 32 Obtain the Newton's bivariate interpolating polynomial that fits the following data and hence find  $f(1.5, 2.5)$ .

y/x	1	2	3
1	4	18	56
2	11	25	63
3	30	44	82

- 33 Construct the Hermit's interpolation polynomial for the given data and Estimate the value of  $f(1.5)$ .

x	f(x)	f'(x)
1	7.389	14.778
2	54.598	109.198

- 34 Using Runge – Kutta method find  $y(0.2)$  for the equation  $\frac{dy}{dx} = \frac{y-x}{y+x}$ ,  $y(0) = 1$  taking  $h = 0.2$ .



**Code: MTH332****Max.Marks: 100****Course: COMPUTER ORIENTED NUMERICAL METHODS USING MATLAB****Duration: 3Hrs****SECTION A****Answer ALL questions:****5X2=10**

- 1** Find a real root of  $x + \log x = 2$  using the Newton-Raphson method. Perform two iterations.

- 2** For approximating a polynomial an iterative scheme is given by,

$$x_{n+1} = \frac{1}{2}x_n\left(1 + \frac{a}{x_n^2}\right). \text{ Find the rate of convergence.}$$

- 3** Evaluate  $y(0.1)$  correct to six places of decimals by Taylor's series method if  $y(x)$  satisfies  $y' = xy + 1, y(0) = 1$ .

- 4** What is the output for the following MATLAB code?

```
function z = fun(x,y)
```

```
u = 3*x;
```

```
z = u + 6*y.^2;
```

```
end
```

```
>> x = 3; y = 7;
```

```
>>z = fun(x,y)
```

- 5** Given the array  $A = [2 \ 7 \ 9 \ 7 ; 3 \ 1 \ 5 \ 6 ; 8 \ 1 \ 2 \ 5]$ , explain the results of the following commands:

a)  $A(1:3,:)$

b)  $[A ; A(1:2,:)]$

c)  $\text{sum}(A,2)$

d)  $[[A ; \text{sum}(A)] [\text{sum}(A,2) ; \text{sum}(A(:))]]$

**SECTION B****Answer any SIX questions:****6X5=30**

- 6** Use the fixed-point iterative method to find a real root of  $\sin x = 10(x - 1)$  correct to three decimal places.

- 7** Find a real root of  $x - e^{-x} = 0$  correct to five decimal places using the Aitken's  $\Delta^2$  method and  $x_0 = 1$ .

- 8** Perform three iterations of the Muller method to find the smallest positive root of the equation  $\cos x - xe^x = 0$  using initial approximations  $x_0 = -1, x_1 = 0, x_2 = 1$ .

- 9** Solve the initial-value problem  $\frac{dy}{dx} = -2xy^2, y(0) = 1$  using the Runge-Kutta method of second order with  $h=0.25$  to estimate the value of  $y$  when  $x=0.5$ .

- 10** Use the Runge-Kutta method of fourth order to estimate the solution of the initial-value problem  $\frac{d^2y}{dt^2} + 4y = \cos t, y(0) = y'(0) = 0$  at  $t = 0.5$ .

- 11** Solve the initial value problem  $\frac{dy}{dx} = xy^2, y(0) = 2$  by modified Euler's method and obtain  $y$  at  $x=0.2$  using  $h=0.1$ .

- 12** Rewrite the following code, using a while loop to avoid using the break command.

```
for k = 1:10
```

```
    x = 50 - k^2;
```

```
    if x < 0
```

```
        break
```

```
    end
```

```

y = sqrt(x)
end

```

- 13 Develop an M—file to implement Gauss-Seidel method to solve system of equations.

### SECTION C

**Answer any SIX questions:**

**6X10=60**

- 14 Use initial approximation  $p_0 = 2, q_0 = 2$  to find a quadratic factor of the form  $x^2 + px + q$  of the polynomial equation  $x^4 - 3x^3 + 20x^2 + 44x + 54 = 0$ .

15 Solve the system

$$\begin{bmatrix} 2 & 3 & 0 & 0 \\ 6 & 3 & 9 & 0 \\ 0 & 2 & 5 & 2 \\ 0 & 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 21 \\ 69 \\ 34 \\ 22 \end{bmatrix}.$$

- 16 Use Graeffe's root squaring method to find the roots of  $f(x) = -50 + 45x + 13x^2 - 9x^3 + x^4$ .

- 17 Solve the boundary-value problem  $\frac{d^2y}{dx^2} - y = 0, y(0) = 0, y(1) = 1.1752$  using the shooting method with  $h=0.5$  to estimate the value of  $y$  when  $x=0.5$ . Take the initial guesses for  $y'(0)$  to be 0.7 and 0.8.

- 18 Given  $\frac{dy}{dx} = x^2(1 + y), y(1) = 1$ , evaluate  $y(x)$  at  $x=1.4$  using Adams - Moulton predictor-corrector method with  $h=0.1$ .

- 19 Solve the boundary value problem  $\frac{d^2u}{dx^2} = xu + 1, u(0) + u'(0) = 1, u(2) = 2$ , by using finite difference method .(Use  $h=0.5$ )

- 20 Water is flowing in a trapezoidal channel at a rate of  $Q = 20m^3/s$ . The critical depth  $y$  for such a channel must satisfy the equation  $0 = 1 - \frac{Q^2 B}{g A_c^3}$  where  $g = 9.81m/s^2$ ,  $A_c$  = the cross-sectional area ( $m^2$ ), and  $B$ =the width of the channel at the surface (m). For this case, the width and the cross-sectional area can be related to depth  $y$  by  $B=3+y$  and  $A_c = 3y + \frac{y^2}{2}$ . Write a M-file to solve the critical depth using bisection method.

- 21 Develop a M-file to implement the Euler's method for solution of an initial value problem.

**Code: MTH332**

**Max.Marks: 100**

**Subject: COMPUTER ORIENTED NUMERICAL METHODS USING MATLAB**

**Duration: 3Hrs**

**SECTION A**

**Answer ALL questions:**

**5X2=10**

- 1 Find a real root of  $e^x - 4x = 0$  using the Newton-Raphson method.
- 2 Find the number of real and complex roots of the polynomial equations  $x^4 - 3x^2 + x + 1 = 0$ .
- 3 Employ Taylor's method to obtain approximate value of y at x=0.2 for the differential equation  $\frac{dy}{dx} = 2y + 3e^x, y(0) = 0$ . Compare the numerical solution obtained with the exact solution.
- 4 Write the output of the following:

```
>>sqrtsum = @(x,y) sqrt(x.^2 + y.^2);
>>sqrtsum(3, 4)
>>A = 6; B = 4;
>>plane = @(x,y) A*x + B*y;
>>z = plane(2,8)
```

- 5 Given the array  $A = [2 \ 7 \ 9 \ 7 ; 3 \ 1 \ 5 \ 6 ; 8 \ 1 \ 2 \ 5]$ , explain the results of the following commands:
  - a)  $A(1:3,:)$
  - b)  $[A ; A(1:2,:)]$
  - c)  $\text{sum}(A,2)$
  - d)  $[ [ A ; \text{sum}(A) ] [ \text{sum}(A,2) ; \text{sum}(A,:) ] ]$

**SECTION B**

**Answer any SIX questions:**

**6X5=30**

- 6 Use the fixed-point iterative method to find a real root of  $\sin x = 10(x - 1)$  correct to three decimal places.
- 7 Find a real root of  $x^3 + x - 5 = 0$  correct to five decimal places using the Aitken's  $\Delta^2$  method.
- 8 Use the Birge-Vieta method to find a real root correct to three decimal places of  $x^6 - x^4 - x^3 - 1 = 0$  with  $p_0 = 0.5$ .
- 9 Solve the initial-value problem  $\frac{dy}{dx} = 3x + \frac{y}{2}, y(0) = 1$  using the Runge-Kutta method of fourth order with h=0.5 to estimate the value of y when x=1.
- 10 Use the Runge-Kutta method of fourth order with h=0.5 to estimate the solution of the system  $\frac{dx}{dt} = y - t, \frac{dy}{dt} = x + t$  with x(0)=1 and y(0)=1 at t=0.5.
- 11 Use the modified Euler's method to solve the initial value problem  $\frac{dy}{dx} = x + \sqrt{y}, y(0) = 1$  for the range  $0 < x \leq 0.4$  using h=0.2.
- 12 Use a while loop to determine how many terms in the series  $3k^2 - 2k, k = 1, 2, 3, \dots$ , are required for the sum of the terms to exceed 2000. What is the sum for this number of terms?
- 13 Develop an M—file to implement Gauss-Elimination method to solve system of equations.

**SECTION C**

**Answer any SIX questions:**

**6X10=60**

- 14 Find all roots of  $x^3 - 4x^2 + 5x - 2 = 0$  using the Graffe's root squaring method. Perform the squaring three times.
- 15 Solve the following system of equations by Cholesky's method:  

$$4x_1 - x_2 = 1; -x_1 + 4x_2 - x_3 = 0; -x_2 + 4x_3 = 0$$
- 16 Find all roots of  $x^3 - 6x^2 - 11x + 6 = 0$  using the Graffe's root squaring method. Perform the squaring three times.
- 17 Solve the boundary-value problem  $\frac{d^2y}{dx^2} - y = 0, y(0) = 0, y(1) = 1$  using the shooting method with h=0.5 to estimate the value of y when x=0.5. Take the initial guesses for  $y'(0)$  to be 0.7 and 0.8.
- 18 Use the Adams-Moulton predictor-corrector method to estimate the solution of the initial-value problem

$$\frac{dy}{dx} = x - y^2, y(0) = \text{at } x=0.8 \text{ with } h=0.2.$$

- 19** Solve the boundary value problem  $\frac{d^2y}{dx^2} = u, u(0) = 0, u(1) = 1 - e^{-2}$  by converting to tridiagonal system of equations.(Use h=0.2)
- 20** Write a script file to plot the functions  $y = 4\sqrt{6x+1}$  and  $z = 5e^{0.3x} - 2x$  over the interval  $0 \leq x \leq 1.5$ .  
The variables z and y represents force in newtons; the variable x represents distance in meters.
- 21** Develop a M-file to implement the shooting method for a linear second-order ODE.