Numerical Methods Questions

3 markers:

1. What is the difference between algebraic and transcendental equations? Give two examples for each.
2. Show that (i) $\Delta-\nabla=\Delta\nabla$ (ii) $(1+\Delta)(1-\nabla)=1$ (iii) $\nabla=1-E^{-1}$.
3. Evaluate $(2\triangle+3)(E+2)(3x^2+4)$ by taking h = 1.
4. Evaluate $\triangle^2 Ex^3$.
5. Given: $U_0=148, U_1=192, U_2=241 \& U_4=374$ find U_3
6. If 0.5555 is the approximate value of 5/9, find the absolute, relative and percentage error.
7. If 0.333 is the approximate value of 1/3, find the absolute, relative and percentage error.
8. Compute the root of $logx = cosx$ correct to three decimal places which lie between 1 and 2, by any of the known methods.
9. Explain Jacobi Iteration method to solve non-homogeneous equation involving three variables.
10. Explain forward difference, backward difference and shift operator.
11. If $f(x)$ be a polynomial of nth degree in x, then $\Delta^n f(x)$ is a constant and $\Delta^{n+1} f(x)=0$. And hence evaluate: $\Delta^8[(1+2x)(1-x^2)(1-8x)(1+2x^4)]$ by taking h = 2.
12. Given $\nu_0=4, \nu_1=8, \nu_2=21, \nu_3=75, \nu_4=32, \upsilon_5=16$ and $\nu_6=16$, find the value of $\Delta^6\nu_0$.
13. If 0.42857 is the approximate value of 3/7, find the absolute, relative and percentage error.
14. Explain Bisection Method to find root of the equation f(x) = 0. Hence find the real root of the equation $x^3-x-1=0$ up to three stages, correct to three places of decimals.
15. What do you mean by diagonally dominant system of equations? Explain with two examples.
16. Show that (i) $E\Delta=\Delta E$ (ii) $E=1+\Delta$ (iii) $\nabla=1-E^{-1}$.
17. If $f(x)$ be a polynomial of nth degree in x, then $\Delta^n f(x)$ is a constant and $\Delta^{n+1} f(x)=0$. And hence evaluate: $\Delta^7(3-4x)(1-x^2)(1+2x^2)(1+5x^2)$ by taking h = 3.
18. Evaluate : (i) $\Delta^2 Ex^3$ (ii) $\Delta^2(e^{ax+b})$.

19. Estimate the value of $\tan 32^0$ from the data:

x	30^0	35^0	45^0	50^0	60^0
$\tan x$	0.5773	0.7002	1	1.1918	1.7320

20. If 1.285714 is the approximate value of 9/7, find the absolute and percentage error.

21. Explain Jacobi Iteration method to solve non-homogeneous equation involving three varia

22. Define Forward, Backward and Shift operators with an example.

23. Evaluate : $\triangle^{14}\left[(1-2x^2)(3x^3-1)(4x^4-1)(5x^5+1)\right]$ by taking h=1.

24. Evaluate: $\triangle\left(\frac{3^x}{logx}\right)$.

25. Prove the identity $U_4 = U_3 + \Delta U_2 + \Delta^2 U_1 + \Delta^3 U_1$.

26. Derive a method of finding the first derivative of y = f(x) at any value of x where y is spe backward difference formula.

27. Find the interval in which roots of the equation $2x^2-x-1=0$ lies.

28. Find the interval in which the root of 2Sinx -1=0 lies.

29. Show that $\nabla = 1 - E^{-1}$.

30. Evaluate: $\Delta^2\left(x^2+5x+16\right)$ by taking h = 1.

31. Write down the formula for first derivative of y w.r.t. x at $x=x_0$ by using forward difference formula.

32. Find a real root of $x + logx = 2$ using the Newton-Raphson method. Perform two iterations.

7 markers:

1. When trying to find the acidity of a solution of magnesium hydroxide in hydrochloric acid, we obtain the following equation, $A(x) = x^3 + 3.6x^2 - 36.4$ where x is the hydronium ion concentration. Find the hydronium ion concentration for a saturated solution(acidity equals zero) using any of the known iterative numerical method.

2. Solve the system of equations : $x + y + 54z = 110; 27x + 6y - z = 85; 6x + 15y + 2z = 72$, by Gauss-Seidel method to obtain the final solution correct to three places of decimal.

3. Find the real root of the equation $2x - \log_{10} x - 7 = 0$, by Newton-Raphson method, which lies between 3 and 4 , correct to three places of decimal places.

4. Find a polynomial of degree two which takes the values

x	1	2	3	4
f(x)	2	2	4	8

5. Find the Cube root of 15, correct to four significant figures by Iteration method.

6. Solve the following system of equations by Gauss-elimination method $x + 2y + 3z = 10; x + 3y - 2z = 7; 2x - y + z = 5.$

7. Find the missing values in the following table:

x	45	50	55	60	65
y	3.0	__	2.0	___	-2.4

8. Find the cubic polynomial which passes through the points (2, 4) (4, 56) (9, 711) (10, 980) and hence estimate the dependent variable corresponding to the values of the independent variable 3, 5, 7, 11.

9. Solve the following equations by employing the method of partial pivoting $x + 2y + z = 8; 2x + 3y + 4z = 20; 4x + 3y + 2z = 16.$

10. Find a polynomial of degree three which takes the values

x	3	4	5	6	7
f(x)	6	24	60	120	210

11. Solve the following equations by Jacobi iteration method $x + y + 4z = 9; 8x - 3y + 2z = 20; 4x + 11y - z = 33.$

12. Solve the following equations by Gauss-Seidel iteration method $25x + 2y + 2z = 69; 2x + 10y + z = 63; x + y + z = 43.$

13. Find a root of the equation $\sin x + \cos x - 1 = 0$ by Newton-Raphson method, correct to four decimal points.

14. Estimate the missing value from the following table

x	2.0	2.1	2.2	2.3	2.4	2.5	2.6
y_x	.135	__	.111	.100	__	.82	.074

15. Find the interpolating polynomial f(x) satisfying f (0) = 0, f (2) = 4, f (4) = 56, f (6) = 204, f (8) = 496, f (10) = 980, and hence find f (7) and f (9).

16. Solve the following equations by Gauss-elimination method $x + 2y + z = 8; 2x + 3y + 4z = 20; 4x + 3y + 2z = 16.$

17. Where do the curves $y = \cos x$ and $y = x^3 - 1$ intersect? Solve by any of the known numerical method.

18. (ii) By means of Newton's divided difference formula, find the value of f(8) and f(12) from the following table :[5]

x	4	5	7	10	11	13
f(x)	48	100	294	900	1210	2028

19. Solve the following equations by Gauss-Jordan method $2x + y + z = 10; 3x + 2y + 3z = 18; x + 4y + 9z = 16.$

20. Solve the following equations by Gauss-Seidel iteration method $5x - y = 9; x - 5y + z = -4; y - 5z = 6.$

21. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1, 2]. Use $p_0 = 1$.

The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in ^{o}C and P is the percentage of lead in the alloy.

P	40	50	60	70	80	90
t	184	204	226	250	276	304

Using suitable Newton's interpolation formula, find the melting point of the alloy containing

22. 84% of lead.

Find a real root correct to two decimal places of the equation $xe^x = cosx$, using the me

23. position.

Solve the system of equations : $x + 2y + z = 8; 2x - y + 2z = 6; 3x + 2y - z = 4$, by

24. method to obtain the final solution correct to three places of decimal.

Perform four iterations of Bisection Method to obtain the smallest positive root of the equation

25. $x^3 - 5x + 1 = 0$ correct to three places of decimals.

Evaluate : $\Delta^2 Ex^3$.

26.

A function y = f(x) is given in the following table:

x	1	1.2	1.6	1.8	2
y	0	0.128	1.296	2.432	4

27. Find the approximate values of f′(1.6) and f″(1.6).

28. Given $\upsilon_0 = 4$, $\upsilon_1 = 8$, $\upsilon_2 = 21$, $\upsilon_3 = 75$, $\upsilon_4 = 32$ find the value of $\Delta^4 \upsilon_0$.

29. By constructing a difference table find the 10th term of the sequence 3, 14, 39, 84, 155, 258, ...

30. Find a root of the equation sinx + cosx – 1 = 0, by Regula-Falsi Method correct to four decimal points.

Solve the following equations by Gauss- Jordon Method x + 2y + z = 8; 2x + 3y + 4z =20; 4x +3y + 2z

31. = 16.

Solve the following equations by Gauss-Seidel iteration method x + y + 4z = 9; 8x – 3y +2z = 20; 4x +

32. 11y – z =33.

33. Using Newton's formula, find the values of f(9) and f(12) from the following table:

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

34. Use method of separation of symbols prove that $e^x\left[u_0+x\Delta u_0+\frac{x^2}{2!}\Delta^2 u_0+\frac{x^3}{3!}\Delta^3 u_0+......\right]=$ $u_0+\frac{x}{1!}Eu_0+\frac{x^2}{2!}E^2u_0+......$

35. Use the fixed-point iterative method to find a real root of $\sin x = 10(x-1)$ correct to three decimal places.

36. Find a real root of $e^x - 4x = 0$using the Newton-Raphson method.

37. Use the fixed-point iterative method to find a real root of $\sin x = 10(x-1)$correct to three decimal places.

38. Estimate the missing term in the following table.

x	1	2	3	4	5	6	7
y	2	4	8	__	32	64	128

39.

10 markers:

1. Apply Newton-Raphson Method to solve $3x-\cos x-1=0$.

2. Find the positive root of the equation $x^3+x^2-1=0$, correct to six decimal places by Fixed point iteration method, assuming the initial approximation as $x_0=0.75$.

3. Given $A=\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$, find the inverse by Gauss – Jordan method.

4. Solve by Gauss elimination method:
3x+4y+5z=18, 2x-y+8z=13, 5x-2y+7z=20

5. Solve for a positive root of x - cos x = 0 by Regula Falsi method. Find the root between 0 and 1.

6. (i) Use Newton – Raphson method to find a real root of $x \sin x + \cos x = 0$ near $x = \pi$. **(10)**
Calculate upto four decimal places of accuracy.[5]

(ii) Solve the following system of equations by Gauss Elimination Method
$10x - 7y + 3z + 5u = 6; -6x + 8y - z - 4u = 3; 5x + y + 4z + 11u = 2; 5x - 9y - 2z + 4u = 7$

7. Solve the following system of equations by Gauss - Seidel method:
$20x + y - 2z = 17; 3x + 20y - z = -18; 2x - 3y + 20z = 25.$
Use the Regula Falsi method to obtain a real root of the equation $x^4 + 2x^2 - 16x + 5 = 0$ correct to three decimal places.

8. Apply Gauss – Jordan method to find the inverse of $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$

9. (i) Use Newton – Raphson Method to find a real root of the equation $x^3 - 2x - 5 = 0$ and correct to three decimal places.[5]

(ii) Solve the following system of equations by Gauss Elimination Method:[5]
$4x_1 + x_2 + x_3 = 4;\ x_1 + 4x_2 - 2x_3 = 4;\ 3x_1 + 2x_2 - 4x_3 = 6$

10. Solve the following system of equations by Gauss-Seidel method:
x + y + 54z = 110; 27x + 6y - z = 85; 6x + 15y + 2z = 72.
(i) Use the regula falsi method to find the fourth root of 12 correct to three decimal places.[5]
(ii) Show that the equation log x - x + 2 = 0 has a root between 3 and 4. Solve using the fixed point iterative method.[5]

11. Apply Gauss – Jordan method to find the inverse of $\begin{bmatrix} 5 & 8 & 4 \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

12. Solve the system of equations by LU decomposition method:
$x_1 + x_2 + 3x_4 = 8; 2x_1 + x_2 - x_3 + 2x_4 = 7; 3x_1 - x_2 - x_3 + 2x_4 = 14; -x_1 + 2x_2 + 3x_3 - x_4 = -7$

13. Use method of separation of symbols to prove the identity
$v_0 - v_1 + v_2 - v_4 + --- = \frac{1}{2} v_0 - \frac{1}{4} \triangle v_0 + \frac{1}{8} \triangle^2 v_0 - \frac{1}{16} \triangle^3 v_0 + ---$

14. Solve the following equations by LU decomposition method $2x + 2y + z = 12; 3x + 2y + 2z = 8; 5x + 10y - 8z = 10.$

15. Solve the following equations by LU decomposition method $5x - 2y + z = 4; 7x + y - 5z = 8; 3x + 7y - 4z = 10.$

16. Use method of separation of symbols prove that $u_0 + x_{c_1}\Delta u_1 + x_{c_2}\Delta^2 u_2 + x_{c_3}\Delta^3 u_3 + \cdots = u_x + x_{c_2}\Delta^2 u_{x-1} + x_{c_3}\Delta^4 u_{x-2} + \ldots$

17. $e^x[U_0 + x \triangle U_0 + \frac{x^2}{2!} \triangle^2 U_0 + \frac{x^3}{3!} \triangle^3 U_0 + - - - - - -] = U_0 + \frac{x}{1!} E U_0 + \frac{x^2}{2!} E^2 U_0 + -$

$U_0 + U_1 + U_2 + U_3 + - - - - + U_n = (n+1)C_1 U_0 + (n+1)C_2 \Delta U_0 + (n+1)C_3 \Delta^2 U_0$

18. Solve the following equations by LU decomposition method 5x – 2y + z = 4; 7x + y - 5z = 8; 3x + 7y - 4z = 10.