# Equation for Logistic Population Growth ¶

$$\frac{dN}{dt} = rN\left(\frac{K-N}{K}\right)$$

We can also look at logistic growth as a mathematical equation. Population growth rate is measured in number of individuals in a population (N) over time (t). The term for population growth rate is written as (dN/dt). The d just means change. K represents the carrying capacity per individual for a population. The logistic growth equation growth equation assumes that K and r do not chnage over time in a population.

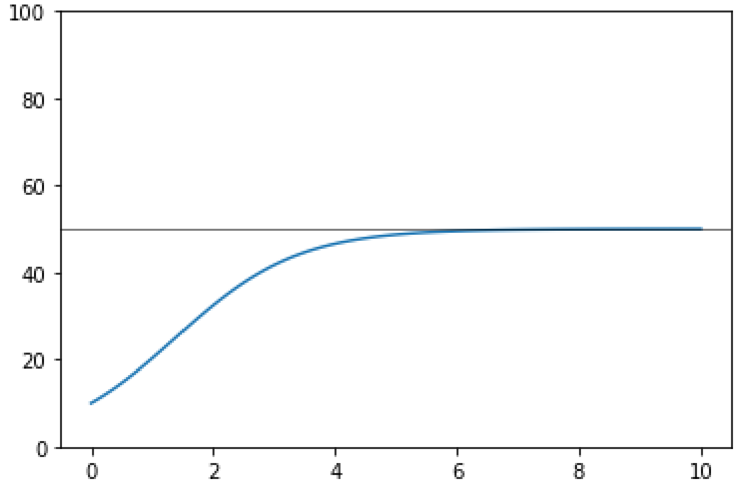
## Logistic Growth

In [26]:

```
import numpy as np
import math
from numpy import *
from sympy import *
from pylab import *
import matplotlib.pyplot as plt
from scipy.integrate import odeint
from sympy.interactive import printing
printing.init_printing(use_latex=True)
```

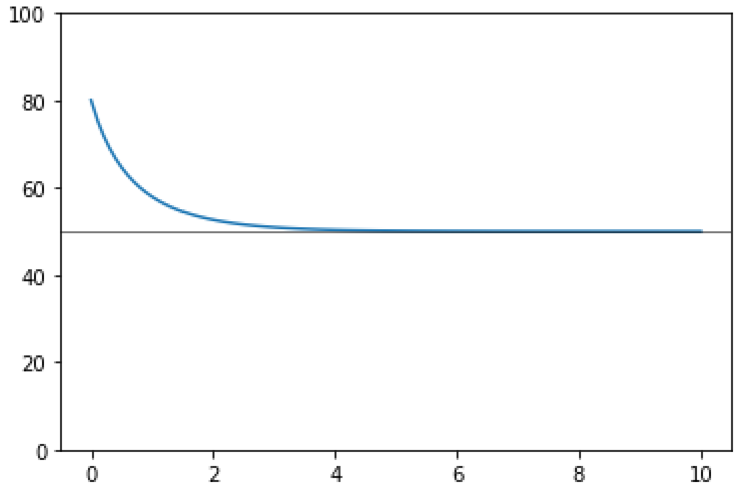
In [3]:

```
def f(N,t,r,K):
    return r * N * (1-N/K)

r = 1
K = 50
N0 = 10
t = np.linspace(0,10,100)

N = odeint(f,N0,t,args = (r,K))
plt.plot(t,N)
plt.ylim([0,100])
plt.axhline(y=K,color='k',linewidth=0.5)
plt.show()
```



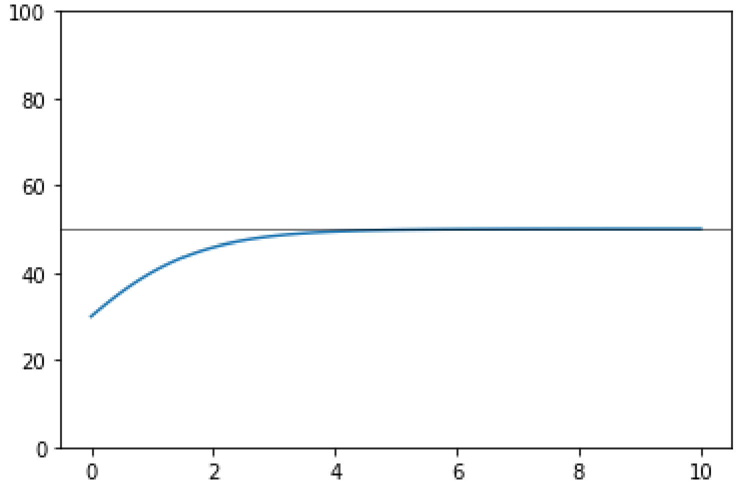

In [5]:

```
N0 = 80
t = np.linspace(0,10,100)

N = odeint(f, N0, t, args = (r,K))
plt.plot(t,N)
plt.ylim([0,100])
plt.axhline(y=K,color='k',linewidth=0.5)
plt.show()
```



In [6]:

```
N0 = 30
t = np.linspace(0,10,100)

N = odeint(f, N0, t, args = (r,K))
plt.plot(t,N)
plt.ylim([0,100])
plt.axhline(y=K,color='k',linewidth=0.5)
plt.show()
```



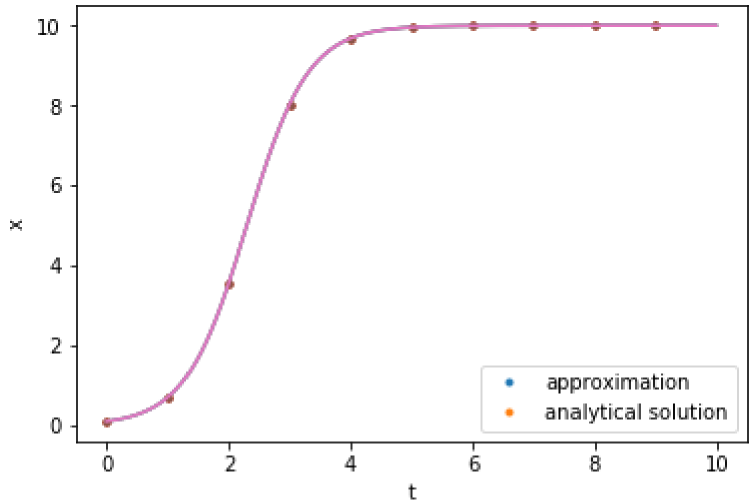

# Example 02

## Let's start with the logistic equation, now with any parameters for growth rate and carrying capacity:

$\frac{dx}{dt} = rx(1 - \frac{x}{K})$ **with** $r = 2$, $K = 10$ **and** $x0 = 0.1$

In [13]:

```
t = arange(0,10,1)
# parameters
r = 2
K = 10
# initial condition
x0 = 0.1

# Let's define the right - hand side of the differential equation
# It must be a function of the dependent variable (x) and of the time (t), even if time
# this is how you define a function:
def f(x,t,r,K):
    # in python, there are no curling braces '{}' to start or end a function, nor any speci
    # the block is defined by leading spaces (usually 4)
    # arithmetic is done the same as in other languages: +, -, *, /
    return r*x*(1-x/K)

# call the function that performs intergration
# the order of the arguments as below: the derivative function, the initial condition, the
# and the list of the parameters
x = odeint(f,x0,t,(r,K))

#plot the solution
plt.plot(t,x,'.')
plt.xlabel('t')
plt.ylabel('x')
t = arange(0,10,0.01)
# plot analytical solution
# notice that 't' is an array when you do an arithmetic operation
# with an array, it is the same as doing it for each element.
plt.plot(t,K*x0*exp(r*t)/(K+x0*(exp(r*t)-1)))
plt.legend(['approximation','analytical solution'],loc = 'best') # draw legend
plt.show()
```



In [16]:

```
t = arange(0,10,1)
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r = 2
K = 10
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x0 = 0.1

# Let's define the right - hand side of the differential equation
# It must be a function of the dependendent variable (x) and of the time (t), even if time
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plt.axhline(y=K,color='k',linewidth=0.5)
plt.legend(['approximation','analytical'],loc = 'best') # draw legend
plt.show()
```
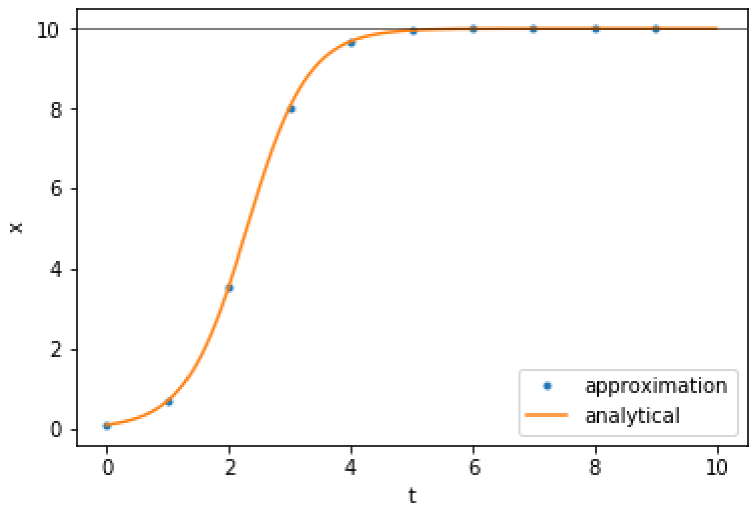




# BACTERIAL GROWTH

### Example 01: A culture has $P_0$ number of bacteria. At $t = 1$ h the number of bacteria is measured to be $\frac{3}{2}P_0$. If the rate of growth is proportional to the

**number of bacteria $Pt$ present at the time $t$, the determine the time necessary for the number of bacterial to triple.**

In [21]:

```
t,k = Symbol('t'),Symbol('k')
P = Function('P')(t)
diffeq = Eq(P.diff(t)-k*P,0)
sol = dsolve(diffeq)
P0 = Symbol('P0')
constants = solve([sol.subs([(P,P0),(t,t)]),sol.subs([(P,3/2*P0),(t,t+1)])])
sol = sol.subs(k,constants[1][k])
sol
```

Out[21]:

$$P(t) = C_1 e^{0.405465108108164t}$$

In [22]:

```
c1 = solve([sol.subs([(P,P0),(t,0)]),sol.subs([(P,3/2*P0),(t,1)])])
c1
```

Out[22]:

$$\{C_1 : P_0\}$$

In [24]:

```
sol = sol.subs(c1)
sol
```

Out[24]:

$$P(t) = P_0 e^{0.405465108108164t}$$

In [25]:

```
solve([sol.subs(P,3*P0)],t)
```

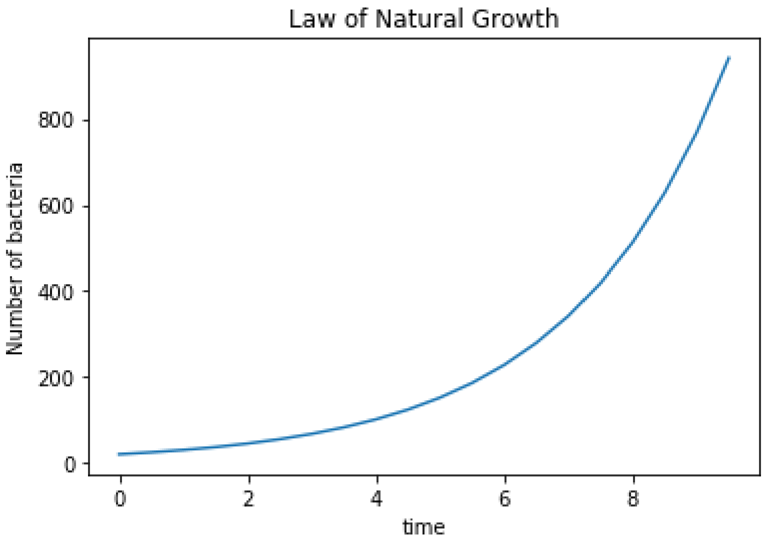
Out[25]:

$$\{t : 2.70951129135146\}$$

## Example 02: Plot the function $y = P_0 e^{0.405465108164}$,$y_0 = 20$

In [27]:

```
t = arange(0,10,0.5)
P0 = 20
y = 20*exp(0.405465108108164*t)
plt.plot(t,y)
plt.xlabel('time')
plt.ylabel('Number of bacteria')
plt.title('Law of Natural Growth')
plt.show()
```




# Mathematical model: Simple Pendulum

$\theta'' + (b/m)\theta' + (g/L). Sin\theta = 0$

$\theta''$ - Angular Acceleration

$\theta'$ - Angular Velocity

$\theta$ - Angular Displacement
$b$ - Damping factor
$g$ - Acceleration due to gravity = 9.81
$L$ - Length of the pendulum
$m$ - Mass of the bob

## Converting this 2nd order differential equation to two 1st order differential equations -

Let $\theta_1 = 0$
and $\theta_2 = \frac{d\theta}{dt} = \frac{d\theta_1}{dt}$
$\frac{d\theta_2}{dt} = \frac{d^2\theta}{dt} = \frac{d^2\theta_1}{dt}$

Thus, the 1st order differential equations are,

$\frac{d\theta_2}{dt} = (-\frac{b}{m})\theta_1 + (-fracgL)sin\theta_1$

$\frac{d\theta_1}{dt} = \theta_2$

In [20]:

```
# importing libraries

import math
import numpy as np
from scipy.integrate import solve_ivp
import matplotlib.pyplot as plt
import os

# Initial and end values
st = 0              # Start time(s)
et = 20.4           # End time (s)
ts = 0.1            # Time step(s)
g = 9.81            # Acceleration due to gravity (m/s^2)
L = 1               # Length of the pendulum (m)
b = 0.5             # Damping factor(kg/s)
m = 1               # Mass of bob (kg) (14-03-2019 CMS)

# 1st order equations to solve in a fraction

"""
theta1 is angular displacement at current time instant
theta2 is angular velocity at current time instant
dtheta2_dt is angular acceleration at current time instant
dtheta1_dt is rate of change of angular displacement at current time instant
"""
def sim_pen_eq(t,theta):
   dtheta2_dt=(-b/m)*theta[1]+(-g/L)*np.sin(theta[0])
   dtheta1_dt=theta[1]
   return [dtheta1_dt,dtheta2_dt]

theta1_ini=0
theta2_ini=3
theta_ini=[theta1_ini,theta2_ini]
t_span=[st,et+ts]
t=np.arange(st,et+ts,ts)
sim_points=len(t)
l=np.arange(0,sim_points,1)
theta12= solve_ivp(sim_pen_eq,t_span,theta_ini,t_eval=t)
theta1=theta12.y[0,:]
theta2=theta12.y[1,:]
plt.plot(t,theta1,label='Angular displacement(rad)')
plt.plot(t,theta2,label='Angular velocity(rad/s)')
plt.xlabel('Time(s)')
plt.ylabel('Angular Disp. (rad) and Angular velocity. (rad/s)')
plt.legend()
plt.show()
```

```
---------------------------------------------------------------------------
ImportError                               Traceback (most recent call last)
<ipython-input-20-7fc8c193a0a0> in <module>()
      3 import math
      4 import numpy as np
----> 5 from scipy.integrate import solve_ivp
      6 import matplotlib.pyplot as plt
      7 import os

ImportError: cannot import name 'solve_ivp'
```

In [ ]: