Mean and Variance of F- distribution:

$$\mu_r' (\text{about origin}) = E(F^r) = \int_0^\infty F^r f(F)\, dF$$

$$= \frac{(\nu_1/\nu_2)^{\nu_1/2}}{B\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \int_0^\infty F^r \cdot \frac{F^{(\nu_1/2)-1}}{\left(1+\frac{\nu_1}{\nu_2}F\right)^{(\nu_1+\nu_2)/2}}\, dF$$

To evaluate the integral, Put $\frac{\nu_1}{\nu_2} F = y$, So that

$$dF = \frac{\nu_2}{\nu_1} dy. \qquad \Rightarrow F = \frac{\nu_2}{\nu_1} y.$$

$$\therefore \mu_r' = \frac{(\nu_1/\nu_2)^{\nu_1/2}}{B\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \int_0^\infty \left(\frac{\nu_2}{\nu_1} y\right)^r \cdot \frac{\left(\frac{\nu_2}{\nu_1} y\right)^{(\nu_1/2)-1}}{(1+y)^{(\nu_1+\nu_2)/2}} \left(\frac{\nu_2}{\nu_1}\right) dy$$

$$= \frac{(\nu_1/\nu_2)^{\nu_1/2}}{B\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \int_0^\infty \left(\frac{\nu_2}{\nu_1}\right)^{r+(\nu_1/2)-1} \frac{y^{r+(\nu_1/2)-1}}{(1+y)^{(\nu_1+\nu_2)/2}} \left(\frac{\nu_2}{\nu_1}\right) dy$$

$$= \frac{\left(\frac{\nu_2}{\nu_1}\right)^r}{B\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \int_0^\infty \frac{y^{r+(\nu_1/2)-1}}{(1+y)^{\left(\frac{\nu_1}{2}+r+\frac{\nu_2}{2}-r\right)}}\, dy$$

$$\boxed{B(x,y) = \int_0^\infty t^{x-1} (1+t)^{y-1}\, dt}$$

$$= \left(\frac{\nu_2}{\nu_1}\right)^r \cdot \frac{1}{B\left(\frac{\nu_1}{2}, \frac{\nu_2}{2}\right)} \cdot B\left(r + \frac{\nu_1}{2}, \frac{\nu_2}{2} - r\right), \quad \nu_2 > 2r$$

$$\boxed{B(x,y) = \frac{\Gamma x \, \Gamma y}{\Gamma(x+y)}}$$

$$= \left(\frac{\nu_2}{\nu_1}\right)^r \cdot \frac{\Gamma\left(\frac{\nu_1}{2} + \frac{\nu_2}{2}\right)}{\Gamma\frac{\nu_1}{2} \, \Gamma\frac{\nu_2}{2}} \cdot \frac{\Gamma\left(r + \frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2} - r\right)}{\Gamma\left(r + \frac{\nu_1}{2} + \frac{\nu_2}{2} - r\right)}, \quad r < \frac{\nu_2}{2}.$$

$$= \left(\frac{\nu_2}{\nu_1}\right)^r \frac{\cancel{\Gamma\left(\frac{\nu_1}{2} + \frac{\nu_2}{2}\right)}}{\Gamma\frac{\nu_1}{2} \, \Gamma\frac{\nu_2}{2}} \cdot \frac{\Gamma\left(r + \frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2} - r\right)}{\cancel{\Gamma\left(\frac{\nu_1}{2} + \frac{\nu_2}{2}\right)}}$$

$$\boxed{\Gamma r = (r-1)\,\Gamma(r-1)}$$

$$\therefore \mu_r' = \left(\frac{\nu_2}{\nu_1}\right)^r \frac{\Gamma\left(r + \frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2} - r\right)}{\Gamma\frac{\nu_1}{2} \, \Gamma\frac{\nu_2}{2}}, \quad r < \frac{\nu_2}{2} \rightarrow (1)$$

Put $r = 1$ we get,

$$\mu_1' = \left(\frac{\nu_2}{\nu_1}\right) \frac{\Gamma\left(1 + \frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2} - 1\right)}{\Gamma\frac{\nu_1}{2} \, \Gamma\frac{\nu_2}{2}}, \quad r < \frac{\nu_2}{2}$$

$$\Gamma\left(1 + \frac{\nu_1}{2}\right) = \left(1 + \frac{\nu_1}{2} - 1\right) \Gamma\left(1 + \frac{\nu_1}{2} - 1\right)$$

$$= \frac{\nu_1}{2} \, \Gamma\frac{\nu_1}{2}$$

$$\because \Gamma(r-1) = \frac{\Gamma r}{r-1}$$

$$= \left(\frac{\nu_2}{\nu_1}\right) \frac{\frac{\nu_1}{2} \cancel{\Gamma\frac{\nu_1}{2}} \cdot \cancel{\Gamma\frac{\nu_2}{2}} \Big/ \left(\frac{\nu_2}{2} - 1\right)}{\cancel{\Gamma\frac{\nu_1}{2}}\ \cancel{\Gamma\frac{\nu_2}{2}}}$$

$$= \frac{\frac{\nu_2}{\cancel{\nu_1}} \frac{\cancel{\nu_1}}{2}}{\left(\frac{\nu_2}{2} - 1\right)} = \frac{\nu_2}{\cancel{2}} \times \frac{\cancel{2}}{(\nu_2 - 2)} = \frac{\nu_2}{\nu_2 - 2}.$$

$$\therefore \mu_1' = \frac{\nu_2}{\nu_2 - 2}.$$

Put $r = 2$ in (1) we get,

$$\mu_2' = \left(\frac{\nu_2}{\nu_1}\right)^2 \frac{\Gamma\left(2 + \frac{\nu_1}{2}\right) \Gamma\left(\frac{\nu_2}{2} - 2\right)}{\Gamma\frac{\nu_1}{2}\ \Gamma\frac{\nu_2}{2}}, \quad r < \frac{\nu_2}{2} \longrightarrow (2)$$

Consider,

$$\Gamma\left(2 + \frac{\nu_1}{2}\right) = \left(2 + \frac{\nu_1}{2} - 1\right) \Gamma\left(2 + \frac{\nu_1}{2} - 1\right)$$

$$= \left(\frac{\nu_1}{2} + 1\right) \Gamma\left(\frac{\nu_1}{2} + 1\right)$$

$$= \left(\frac{\nu_1}{2} + 1\right) \frac{\nu_1}{2} \Gamma\frac{\nu_1}{2}$$

$$\Gamma\left(\frac{\nu_2}{2}-2\right) = \Gamma\left(\frac{\nu_2}{2}-1-1\right) = \frac{\Gamma\left(\frac{\nu_2}{2}-1\right)}{\left(\frac{\nu_2}{2}-2\right)}$$

$$= \frac{\Gamma\frac{\nu_2}{2}}{\left(\frac{\nu_2}{2}-2\right)\left(\frac{\nu_2}{2}-1\right)}$$

Subt. these in (2) we get,

$$\mu_2' = \left(\frac{\nu_2}{\nu_1}\right)^2 \frac{\left(\frac{\nu_1}{2}+1\right)\cdot\frac{\nu_1}{2}\cdot\cancel{\Gamma\frac{\nu_1}{2}}\cdot\cancel{\Gamma\frac{\nu_2}{2}}}{\cancel{\Gamma\frac{\nu_1}{2}}\,\cancel{\Gamma\frac{\nu_2}{2}}\left(\frac{\nu_2}{2}-2\right)\left(\frac{\nu_2}{2}-1\right)}$$

$$= \frac{\frac{\nu_2^2}{\nu_1}\left(\frac{\nu_1+2}{\cancel{2}}\right)\frac{1}{\cancel{2}}}{\left(\frac{\nu_2-4}{\cancel{2}}\right)\left(\frac{\nu_2-2}{\cancel{2}}\right)}$$

$$\mu_2' = \frac{\nu_2^2(\nu_1+2)}{\nu_1(\nu_2-4)(\nu_2-2)}, \quad \nu_2 > 4.$$

$$\therefore \mu_2 = \mu_2' - (\mu_1')^2$$
$$= \frac{\nu_2^2(\nu_1+2)}{\nu_1(\nu_2-4)(\nu_2-2)} - \left(\frac{\nu_2}{\nu_2-2}\right)^2$$

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$\therefore$ Mean $= \dfrac{\nu_2}{\nu_2 - 2}$ $\quad \nu_2 > 2.$

$$Var = \frac{\nu_2^2 (\nu_1 + 2)}{\nu_1 (\nu_2 - 4)(\nu_2 - 2)} - \left(\frac{\nu_2}{\nu_2 - 2}\right)^2, \quad \nu_2 > 4$$