CHRIST UNIVERSITY,BENGALURU - 560029

End Semester Examination March - 2015

Bachelor of Science II SEMESTER CMS / EMS

Code: STA231 **Max.Marks: 100**

Subject: RANDOM VARIABLES AND PROBABILITY DISTRIBUTIONS **Duration: 3Hrs**

SECTION A

Answer any TEN questions. **10X2=20**

1 State addition theorem as well as multiplication theorem of expectations for 'n' random variables.

2 Define marginal distribution in a bivariate probability table.

3 Give an expression for variance of a linear combination of a random variable X.

4 What are the conditions to be satisfied for a function to be a joint pdf in case of continuous random variables?

5 The mean and variance of a binomial distribution are 4 and 4/3 respectively. Write the pmf.

6 Under what conditions will a binomial distribution tend to Poisson distribution?

7 For a Poisson distribution with parameter 3.8 what are the mean and variance?

8 Prove that geometric distribution is a special case of negative binomial distribution.

9 Derive the rth order raw moment for a uniform distribution in (a, b).

10 Define a beta distribution of second kind.

11 Write the mgf of a standard normal distribution.

12 In modeling a simulation random numbers from 00 to 99 are assigned with the intervals determined from frequency distributions for each behavior occurrence. If there are two behaviors, X and Y, out of 50 tallies, on behavior X you record 20 tallies and on behavior Y you record 30 tallies. Which of the following is a correct random number interval for behavior X?

A) 00 to 19
B) 01 to 20
C) 00 to 39
D) 41 to 61
E) 45 to 100

SECTION B

Answer any FOUR questions. **4X6=24**

13 (a) The independent probabilities that the three sections of a costing department wil encounter a computer error are 0.1, 0.3 and 0.3 each week respectively. Calculate the probability that there will be:
(i) at least one computer error and (ii) one and only one error encountered by costing department next week.
(b) Let A and B be two events such that P(A) = 1/2, P(B) = 1/3 and P(AB) = 1/4. Obtain the probabilities P(A/B), P(AUB) and $P(A^cB^c)$.

14 Check whether the following function is a valid probability density function. Also examine the independence of Y_1 and Y_2.

$$f(y_1, y_2)=\begin{cases} 6(1-y_2), & 0 \le y_1 \le y_2 \le 1 \\ 0 & otherwise \end{cases}$$

15 Explain the additive property of binomial distribution.

16 Define the mean and variance of a hyper geometric distribution.

17 Derive the mgf of gamma distribution and hence obtain the mean and variance.

18 Generate a random sample of size 8 for gamma distribution with papremeter 3.

SECTION C

Answer any FOUR questions **4X14=56**

19 a) If A, Band C are mutually independent, prove that AUB and C are also independent.

b) If A, B, C are pair wise independent and A is independent of BUC, then A, B and C are mutually independent.

20 (a) Let X and Y be independent gamma variables with $X \sim G(\alpha_1, \beta)$ and $Y \sim G(\alpha_2, \beta)$. Find the joint density and marginal density of $U = X/\{X+Y\}$ and $V = X+Y$.

(b) Define mgf of a random variables and state and prove its effect by change of origin and scale.

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a. Prove that for a Poisson distribution $\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right)$ and hence obtain the coefficients of skewness and kurtosis.

b. State and prove the additive property of Poisson distribution.

22 a. Show that Poisson distribution is a limiting case of binomial distribution, stating the necessary conditions.

b. Obtain the mgf of a negative binomial distribution.

23 a. Derive the mgf of two parameter gamma distribution and hence obtain the mean and variance. Show the relationship between the mean and variance.

b. Show that beta distribution of second kind will be transformed to beta distribution of first kind by the transformation $1 + x = \frac{1}{y}$.

24 a. Show that exponential distribution lacks memory.

b. If X1, X2, …Xn are independent random variables, Xi having an exponential distribution with parameter then Z=min(X1, X2, …Xn) has exponential distribution with parameter - Prove.