Assignment 2

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1. (a) $h \wedge w \wedge \sim s$
   (b) $\sim w \wedge h \wedge s$
   (c) $\sim h \wedge \sim w \wedge \sim s$
   (d) $\sim w \wedge \sim s \wedge h$
   (e) $w \wedge \sim (h \wedge s)$

2. (a)

p	q	$\sim (p \wedge q) \vee (p \vee q)$
T	T	T
T	F	T
F	T	T
F	F	T

   (b)

p	q	r	$p \wedge (\sim q \vee r)$
T	T	T	T
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	F

3. (a) $p \vee (p \wedge q)$ and q are not logically equivalent because their truth tables are not the same.
   (b) $\sim (p \wedge q)$ and $\sim p \wedge \sim q$ are logically equivalent because their truth tables are the same.
   (c) $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$ are logically equivalent because their truth tables are the same.

4. (a) $\sim (-2 < x < 6) \rightarrow x \leq -2 \vee x \geq 6$
   (b) $\sim (-9 < x < 2) \rightarrow x \leq -9 \vee x \geq 2$
   (c) $\sim (x < 2 \vee x > 6) \rightarrow x \geq 2 \wedge x \leq 6$
   (d) $\sim (x \leq -1 \vee x > 1) \rightarrow x > -1 \wedge x \leq 1$

(e) $\sim (0 > x \geq -4) \rightarrow x \leq 0 \wedge x < -4$

5. (a) $(p \wedge q) \vee (\sim p \vee (p \wedge \sim q))$ is a tautology.
   (b) $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a contradiction.
   (c) $((\sim p \wedge q) \wedge (q \wedge r)) \vee \sim q$ is neither a tautology nor a contradiction.

6. (a) Let b represent "Bob is a double math and computer science major" and a represent "Ann is a math major". Then the statement is $b \wedge a \wedge \sim a$.
   (b) The statement is $\sim (b \wedge a) \wedge a \wedge b$. The two statements are not logically equivalent.

7. $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$ by the associative law of exclusive or.

8. (a) $(p \wedge \sim q) \vee (p \wedge q) \equiv p \wedge (\sim q \vee q)$ by distributive law.
   (b) $p \wedge (\sim q \vee q) \equiv p \wedge t$ by law of excluded middle.
   (c) $p \wedge t \equiv p$ by identity law.

9. $(p \wedge \sim q) \vee p \equiv p$ by absorption law.

10. $\sim ((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$ by De Morgan's law and law of excluded middle.