Exam 2

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1. Truth Table for $(p \wedge r) \leftrightarrow (q \wedge r)$

p	q	r	$p \wedge r$	$q \wedge r$	$(p \wedge r) \leftrightarrow (q \wedge r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	F	F
T	F	F	F	F	T
F	T	T	F	T	F
F	T	F	F	F	T
F	F	T	F	F	T
F	F	F	F	F	T

2. Rewriting Statement Forms

For $p \vee \neg q \rightarrow r$:

(a) Using $p \rightarrow q \equiv \neg p \vee q$: $\neg(p \vee \neg q) \vee r$

(b) Using $p \vee q \equiv \neg(\neg p \wedge \neg q)$: $\neg(\neg(\neg p \wedge q) \wedge \neg r)$

3. Negations, Contrapositives, Converses, and Inverses

(a) Original: If P is a triangle, then P is an equilateral triangle.

Negation: P is a triangle and P is not an equilateral triangle.

Contrapositive: If P is not an equilateral triangle, then P is not a triangle.

Converse: If P is an equilateral triangle, then P is a triangle.

Inverse: If P is not a triangle, then P is not an equilateral triangle.

(b) Original: If John is Mike's brother, then Sarah is his sister and Paul is his cousin.

Negation: John is Mike's brother and either Sarah is not his sister or Paul is not his cousin.

Contrapositive: If Sarah is not his sister or Paul is not his cousin, then John is not Mike's brother.

Converse: If Sarah is his sister and Paul is his cousin, then John is Mike's brother.

Inverse: If John is not Mike's brother, then Sarah is not his sister or Paul is not his cousin.

4. Identifying Converse Error

The argument form that exhibits the converse error is (a):

If Alice finished her project, then she will present it. Alice presented her project. Therefore, Alice finished her project.

This is the converse of the original implication and is not a valid logical conclusion.

5. Validity of Argument Forms

(a) $(p \vee q) \to \neg r$, $\neg p \wedge q$, $q \to p \therefore \neg r$

p	q	r	$(p \vee q) \to \neg r$	$\neg p \wedge q$	$q \to p$	$\neg r$
T	T	T	F	F	T	F
T	T	F	T	F	T	T
T	F	T	T	F	T	F
T	F	F	T	F	T	T
F	T	T	F	T	F	F
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	F	T	T

This argument form is invalid. There is a row (F, T, T) where all premises are true but the conclusion is false.

(b) $p \to q$, $r \to q \therefore (p \vee r) \to q$

p	q	r	$p \to q$	$r \to q$	$(p \vee r) \to q$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	F	F
F	F	F	T	T	T

This argument form is valid. In every row where both premises are true, the conclusion is also true.

6. Predicate Q(x): $x^2 < 2x$

(a) Q(1): $1^2 < 2(1)$ is true (1 ¡ 2) Q(2): $2^2 < 2(2)$ is false (4 ¿ 4) Q(0): $0^2 < 2(0)$ is false (0 = 0) Q(-1): $(-1)^2 < 2(-1)$ is false (1 ¿ -2) Q(3): $3^2 < 2(3)$ is false (9 ¿ 6)

(b) Truth set for domain R: (0, 2)

(c) Truth set for domain R+: (0, 2)

7. Truth Set for R(x): $x^2 - 5x + 6 = 0$

The correct answer is (a) 2, 3.

Explanation: We can factor the equation as $(x-2)(x-3) = 0$. The solutions are x = 2 and x = 3.

8. Formal Negations

(a) ∃ birds x such that x cannot fly. (b) ∃ cars c such that c does not have wheels. (c) ∀ buildings b, b is not over 100 stories tall. (d) ∀ trees t, t is not over 100 years old.

9. Representation of "Every engineering major is also a physics major"

The correct answer is (a) $\forall s \in D, M(s) \Rightarrow S(s)$

This statement reads as "For all students s in D, if s is an engineering major, then s is a physics major," which correctly represents the given statement.