Assignment 5, Questions Chosen: 3, 13, 22, 29, 31

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1 Question 3

$$\begin{aligned}\text{Smallest class size} &= 26 \times 26 + 1 \\ &= 676 + 1 \\ &= 677\end{aligned}$$

Where:

- 26 = number of possible first initials
- 26 = number of possible last initials
- 1 is added to exceed the maximum unique combinations

2 Question 13

$$\begin{aligned}\text{Number of integers} &= 9 \times 9 \times 9 \\ &= 9^3 \\ &= 729\end{aligned}$$

Where:

- 9 choices for hundreds place (1-9, excluding 7)
- 9 choices for tens place (0-9, excluding 7)
- 9 choices for ones place (0-9, excluding 7)

3 Question 22

$$\begin{aligned}\text{Number of ways} &= 1000 \times 999 \times 998 \\ &= 1000 \times 999 \times 998 \\ &= 997,002,000\end{aligned}$$

This is equivalent to the permutation:

$$P(1000, 3) = \frac{1000!}{(1000-3)!} = \frac{1000!}{997!} \tag{1}$$

4 Question 29

$$\begin{aligned} \text{Number of seating arrangements} &= 5! \times 2 \times 3! \times 3! \\ &= 120 \times 2 \times 6 \times 6 \\ &= 8,640 \end{aligned}$$

Where:

- $5!$: arrangements of the other 5 people relative to the president
- 2 : ways to alternate math and CS majors (MCMCMC or CMCMCM)
- $3!$: permutations of math majors among their positions
- $3!$: permutations of CS majors among their positions

5 Question 31

$$\begin{aligned} \text{Number of possible committees} &= \binom{10}{3} \times \binom{25}{4} \\ &= \frac{10!}{3!(10-3)!} \times \frac{25!}{4!(25-4)!} \\ &= \frac{10!}{3!7!} \times \frac{25!}{4!21!} \\ &= 120 \times 12,650 \\ &= 1,518,000 \end{aligned}$$

Where:

- $\binom{10}{3}$: ways to choose 3 faculty members from 10
- $\binom{25}{4}$: ways to choose 4 students from 25