Assignment 3

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1 Question 1

1.1 Part (a)

This loop will repeat exactly N times if it does not contain a stop or a go to.

- If this loop does not contain a stop or a go to, then it will repeat exactly N times.

1.2 Part (b)

Freeze or I'll shoot.

- If you do not freeze, then I'll shoot.

1.3 Part (c)

Fix my ceiling or I won't pay my rent.

- If you do not fix my ceiling, then I won't pay my rent.

2 Question 2

2.1 Part (a)

$\sim p \vee q \rightarrow \sim q$

p	q	$\sim p$	$\sim p \vee q$	$(\sim p \vee q) \rightarrow \sim q$
T	T	F	T	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

2.2 Part (b)

$(p \vee q) \vee (\sim p \wedge q) \rightarrow q$

p	q	$\sim p$	$\sim p \wedge q$	$(p \vee q) \vee (\sim p \wedge q)$	$((p \vee q) \vee (\sim p \wedge q)) \rightarrow q$
T	T	F	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	F	T	F	F	T

2.3 Part (c)

$(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

p	q	r	$p \rightarrow r$	$q \rightarrow r$	$(p \rightarrow r) \leftrightarrow (q \rightarrow r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T
F	F	F	T	T	T

3 Question 3

3.1 Part (a)

$\sim p \rightarrow q$

- If $p \rightarrow q$ is false, then p is true and q is false. Therefore, $\sim p$ is false and $\sim p \rightarrow q$ is true.

3.2 Part (b)

$p \vee q$

- Since p is true and q is false, $p \vee q$ is true.

3.3 Part (c)

$q \rightarrow p$

- Since q is false and p is true, $q \rightarrow p$ is true.

4 Question 4

4.1 Part (a)

$p \wedge \sim q \rightarrow r$

- Using $p \rightarrow q \equiv \sim p \vee q$:

- $\sim (p \wedge \sim q) \vee r$
- $\equiv (\sim p \vee q) \vee r$

4.2 Part (b)

$p \vee \sim q \rightarrow r \vee q$

- Using $p \rightarrow q \equiv \sim p \vee q$:
- $\sim (p \vee \sim q) \vee (r \vee q)$
- $\equiv (\sim p \wedge q) \vee (r \vee q)$

4.3 Part (c)

$(p \rightarrow r) \leftrightarrow (q \rightarrow r)$

- Using $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$:
- $(\sim p \vee r) \wedge (\sim r \vee p) \equiv (\sim q \vee r) \wedge (\sim r \vee q)$

4.4 Part (d)

$(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

- Using $p \rightarrow q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$:
- $(\sim p \vee (\sim q \vee r)) \equiv (\sim (p \wedge q) \vee r)$
- $\equiv (\sim p \vee \sim q \vee r) \equiv (\sim p \vee \sim q \vee r)$

5 Question 5

5.1 Part (a)

If P is a rectangle, then P is a square.

- Negation: P is a rectangle and P is not a square.
- Contrapositive: If P is not a square, then P is not a rectangle.
- Converse: If P is a square, then P is a rectangle.
- Inverse: If P is not a rectangle, then P is not a square.

6 Question 6

(a) Let p be "Jules solved this problem correctly" and q be "Jules obtained the answer 2". The argument is $p \rightarrow q, q \therefore p$. This is the converse error.

(b) Let p be "I go to the movies", q be "I won't finish my homework", and r be "I won't do well on the exam tomorrow". The argument is $p \rightarrow q, q \rightarrow r \therefore p \rightarrow r$. This is a valid argument by the rule of Hypothetical Syllogism.

(c) Let p be "At least one of these two numbers is divisible by 6" and q be "The product of these two numbers is divisible by 6". The argument is $p \rightarrow q, \sim p \therefore \sim q$. This is a valid argument by the rule of Modus Tollens.

(d) Let p be "This computer program is correct" and q be "It produces the correct output when run with the test data my teacher gave me". The argument is $p \rightarrow q, q \therefore p$. This is the converse error.

(e) Let p be "Sandra knows Java" and q be "Sandra knows C++". The argument is $p \wedge q \therefore q$. This is a valid argument by the rule of Simplification.