Assignment 5, Questions Chosen: 3, 13, 22, 29, 31

Amarnath Patel

July 19, 2024

1 Question 3

Smallest class size =
$$26 \times 26 + 1$$

= $676 + 1$
= 677

Where:

- 26 = number of possible first initials
- 26 = number of possible last initials
- 1 is added to exceed the maximum unique combinations

2 Question 13

Number of integers =
$$9 \times 9 \times 9$$

= 9^3
= 729

Where:

- 9 choices for hundreds place (1-9, excluding 7)
- 9 choices for tens place (0-9, excluding 7)
- 9 choices for ones place (0-9, excluding 7)

3 Question 22

Number of ways =
$$1000 \times 999 \times 998$$

= $1000 \times 999 \times 998$
= $997,002,000$

This is equivalent to the permutation:

$$P(1000,3) = \frac{1000!}{(1000-3)!} = \frac{1000!}{997!} \tag{1}$$

4 Question 29

Number of seating arrangements = $5! \times 2 \times 3! \times 3!$ = $120 \times 2 \times 6 \times 6$ = 8,640

Where:

• 5! : arrangements of the other 5 people relative to the president

• 2 : ways to alternate math and CS majors (MCMCMC or CMCMCM)

• 3! : permutations of math majors among their positions

• 3! : permutations of CS majors among their positions

5 Question 31

Number of possible committees =
$$\binom{10}{3} \times \binom{25}{4}$$

= $\frac{10!}{3!(10-3)!} \times \frac{25!}{4!(25-4)!}$
= $\frac{10!}{3!7!} \times \frac{25!}{4!21!}$
= $120 \times 12,650$
= $1,518,000$

Where:

• $\binom{10}{3}$: ways to choose 3 faculty members from 10

• $\binom{25}{4}$: ways to choose 4 students from 25