COT2000 Exam 4

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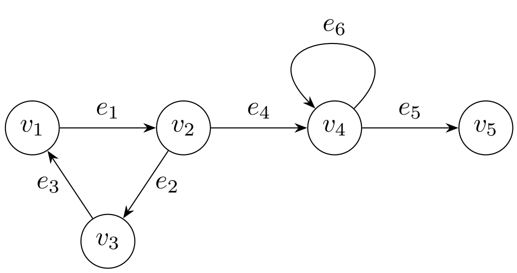
Question 1

(a)

Yes, there are two circuits in G:

$$C_1 : v_1 \to v_2 \to v_3 \to v_1$$

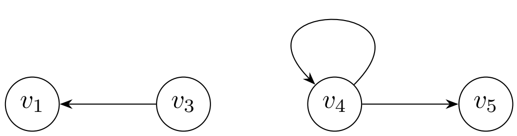
$$C_2 : v_4 \to v_4 \text{ (self-loop)}$$



(b)

(c)

Vertex	In-degreeOut-degree
v_1	11
v_2	12
v_3	11
v_4	22
v_5	10



(d)

(e)

Yes, F has two connected components:

$$\{v_1, v_3\} \text{ and } \{v_4, v_5\}$$

Question 2

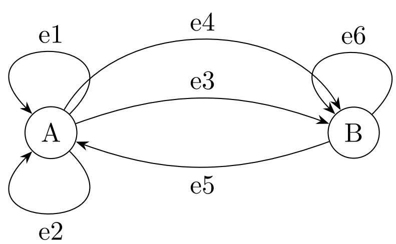
A multigraph is a graph that allows:

- Multiple edges (parallel edges) between the same pair of vertices
- Loops (edges that connect a vertex to itself)

Differences from a simple graph:

- Simple graphs do not allow parallel edges or loops
- In a simple graph, each edge is unique
- Multigraphs can have multiple edges with the same endpoints

Example Multigraph



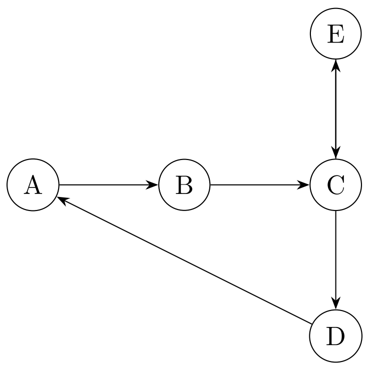
Real-World Scenario

This multigraph could represent a transportation network between two cities:

- Vertices A and B represent two cities
- Edges represent different transportation routes:
  - e1, e2: Local bus routes within city A
  - e3: Highway from A to B
  - e4: Train line from A to B
  - e5: Flight from B to A
  - e6: Local bus route within city B

The multiple edges between A and B represent different modes of transportation, while the loops represent local transportation within each city.

Question 3



(a)

(b) Circuits in the graph:

- $A \to B \to C \to D \to A$ (Simple circuit)
- $C \to E \to C$ (Simple circuit)

(c) This graph does not have an Euler path or circuit.
Justification:

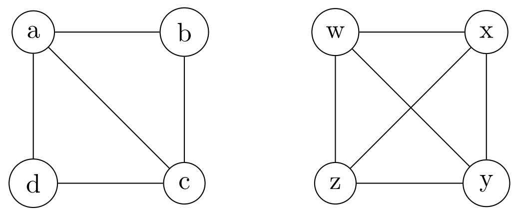
- For a directed graph to have an Euler circuit, every vertex must have equal in-degree and out-degree.
- For an Euler path, at most two vertices can have unequal in-degree and out-degree, differing by 1.
- In this graph:

Vertex	In-degreeOut-degree
A	11
B	11
C	22
D	11
E	11

- While the in-degrees and out-degrees are equal for each vertex, there are 6 edges but not all vertices are connected in a single circuit.
- The graph consists of two separate circuits that cannot be combined into a single Euler path or circuit.

Question 4

(a) Graphs G1 and G2:



G1 G2

(b) Isomorphism analysis:
i. Vertex mapping function $f : V(G1) \to V(G2)$:

$$
\begin{aligned}
f(a) &= w \\
f(b) &= x \\
f(c) &= y \\
f(d) &= z
\end{aligned}
$$

ii. Edge mapping function $h : E(G1) \to E(G2)$:

$$
\begin{aligned}
h(a,b) &= (w,x) \\
h(b,c) &= (x,y) \\
h(c,d) &= (y,z) \\
h(d,a) &= (z,w) \\
h(a,c) &= (w,y)
\end{aligned}
$$

iii. Verification of one-to-one correspondence:
For each edge $e = (u,v)$ in $E(G1)$, we confirm $h(e) = (f(u), f(v))$ in $E(G2)$:

$$
\begin{aligned}
h(a,b) &= (f(a), f(b)) = (w,x) \in E(G2) \\
h(b,c) &= (f(b), f(c)) = (x,y) \in E(G2) \\
h(c,d) &= (f(c), f(d)) = (y,z) \in E(G2) \\
h(d,a) &= (f(d), f(a)) = (z,w) \in E(G2) \\
h(a,c) &= (f(a), f(c)) = (w,y) \in E(G2)
\end{aligned}
$$

However, G1 and G2 are not isomorphic because:

- G2 has an additional edge (x,z) that doesn't correspond to any edge in G1.
- The degree sequences differ: G1 has [3,2,3,2] and G2 has [3,3,3,3].

Therefore, while we can define functions f and h, they do not establish an isomorphism between G1 and G2.

Question 5

a) Isomorphism

G1 and G2 are not isomorphic:

- Vertices: G1 (6), G2 (6)
- Edges: G1 (7), G2 (7)
- Degree sequences differ:
  - G1: [2, 3, 5, 1, 1, 2]
  - G2: [2, 1, 5, 2, 3, 1]

b) Adjacency Matrix G1

$$\begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}$$

c) Adjacency Matrix G2

$$\begin{pmatrix} 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

d) Vertex Degrees

G1: $\{2, 3, 5, 1, 1, 2\}$
G2: $\{2, 1, 5, 2, 3, 1\}$

Question 6

(a) Isomorphic Subgraphs

G1 adjacency matrix:

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

G2 adjacency matrix:

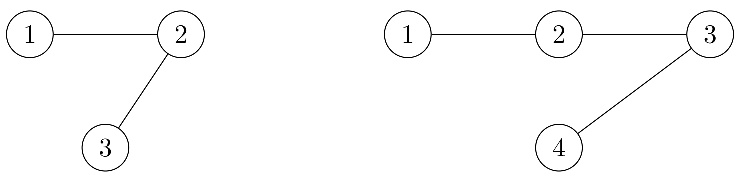
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Process:

- Examine all 3x3 submatrices of G2
- Compare each to G1's matrix
- Count isomorphic subgraphs

Result: 2 isomorphic subgraphs

(b) Graph Drawings



G1 G2