Homework 1 by Amarnath Patel

1. (10 points) Write in words how to read each of the following out loud:

   (a) $\{x \in \mathbb{R}^+ \mid -1 < x < 1\}$

   The set of all positive real numbers x such that $-1 < x < 1$.

   _____

   (b) $\{x \in \mathbb{R} \mid x \leq -3 \text{ or } x \geq 1\}$

   The set of all real numbers x such that $x \leq -3$ or $x \geq 1$.

   _____

   (c) $\{n \in \mathbb{Z} \mid n \text{ is a factor of 3}\}$

   The set of all integers n such that n is a factor of 3.

   _____

   (d) $\{n \in \mathbb{Z}^+ \mid n \text{ is a factor of 5}\}$

   The set of all positive integers n such that n is a factor of 5.

   _____

2. (10 points)

   (a) Is $3 \in \{3\}$?
       Yes.

   _____

   (b) How many elements are in the set $\{2, 2, 2, 5\}$?
       Two elements.

   _____

   (c) How many elements are in the set $\{0, 0, \{0\}\}$?
       Two elements.

   _____

   (d) Is $\{0\} \in \{\{0\}, 0, \{1\}\}$?
       Yes.

   _____

(e) Is $0 \in \{\{0\}, \{1\}\}$?

No.

3. (10 points) Which of the following sets are equal?

$$A = \{0, 1, 2, 3\}$$
$$B = \{x \in \mathbb{R} \mid -1 \leq x < 4\}$$
$$C = \{x \in \mathbb{R} \mid -1 < x < 4\}$$
$$D = \{x \in \mathbb{Z} \mid -1 < x < 4\}$$
$$E = \{x \in \mathbb{Z}^+ \mid -1 < x < 4\}$$

$A = D$ and $D = E$

4. (10 points) Use the set-roster notation to indicate the elements in each of the following sets.

(a) $S = \{n \in \mathbb{Z} \mid n = (-1)^k, \text{ for some integer } k\}$

$S = \{-1, 1\}$

(b) $T = \{m \in \mathbb{Z} \mid m = 2 + (-1)^i, \text{ for some integer } i\}$

$T = \{1, 3\}$

(c) $U = \{r \in \mathbb{Z} \mid 3 \leq r \leq -3\}$

$U = \emptyset$

(d) $V = \{s \in \mathbb{Z} \mid s > 2 \text{ or } s < 3\}$

$V = \{\ldots, -1, 0, 1, 2, 3, 4, \ldots\}$

(e) $W = \{t \in \mathbb{Z} \mid -3 < t < 3\}$

$W = \{-2, -1, 0, 1, 2\}$

(f) $X = \{u \in \mathbb{Z} \mid u \leq 4 \text{ or } u \geq 1\}$
$X = \{\ldots, -2, -1, 0, 1, 2, 3, 4, 5, \ldots\}$

5. (10 points)

(a) Is $3 \in \{1, 2, 3\}$?
Yes.

(b) Is $1 \subseteq \{1\}$?
No.

(c) Is $\{2\} \in \{1, 2\}$?
No.

(d) Is $\{3\} \in \{1, \{2\}, \{3\}\}$?
Yes.

(e) Is $1 \in \{1\}$?
Yes.

(f) Is $\{2\} \subseteq \{1, \{2\}, \{3\}\}$?
No.

(g) Is $\{1\} \subseteq \{1, 2\}$?
Yes.

(h) Is $1 \in \{\{1\}, 2\}$?
No.

(i) Is $\{1\} \subseteq \{1, \{2\}\}$?

No.

(j) Is $\{1\} \subseteq \{1\}$?

Yes.

6. (10 points) Let $A = \{w, x, y, z\}$ and $B = \{e, f\}$. Use the set-roster notation to write each of the following sets, and indicate the number of elements that are in each set:

(a) $A \times B$

$A \times B = \{(w, e), (w, f), (x, e), (x, f), (y, e), (y, f), (z, e), (z, f)\}$

Number of elements: 8

(b) $B \times A$

$B \times A = \{(e, w), (e, x), (e, y), (e, z), (f, w), (f, x), (f, y), (f, z)\}$

Number of elements: 8

(c) $A \times A$

$A \times A = \{(w, w), (w, x), (w, y), (w, z), (x, w), (x, x), (x, y), (x, z), (y, w), (y, x), (y, y), (y, z), (z, w), (z, x), (z, y), (z, z)\}$

Number of elements: 16

(d) $B \times B$

$B \times B = \{(e, e), (e, f), (f, e), (f, f)\}$

Number of elements: 4

7. (10 points) Define the set using set-builder notation:

(a) $S = \{2, 4, 6, 8, 10, 12\}$

$S = \{n \in \mathbb{Z}^+ \mid n \text{ is an even number and } 2 \leq n \leq 12\}$

(b) $T = \{1, 4, 9, 16, 25, 36\}$
   $T = \{n^2 \mid n \in \mathbb{Z}^+ \text{ and } 1 \leq n \leq 6\}$

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8. (10 points) Let $A = \{1, 3, 4\}$ and $B = \{6, 8, 10\}$. Define a relation R from A to B as follows: For all $(x, y) \in A \times B$, $(x, y) \in R$ if and only if $\frac{y}{x}$ is an integer.

   (a) Determine the validity of the following:
   - Is $4 \, R \, 6$?
     No.

   - Is $4 \, R \, 8$?
     Yes.

   - Is $(3, 8) \in R$?
     No.

   - Is $(1, 10) \in R$?
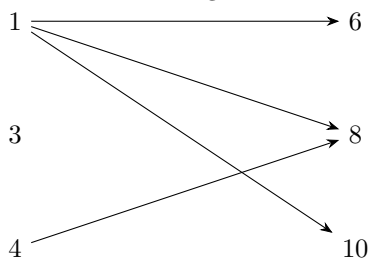   Yes.

   (b) Write R as a set of ordered pairs.
      $R = \{(1, 6), (1, 8), (1, 10), (4, 8)\}$

   (c) Identify the domain and co-domain of R.
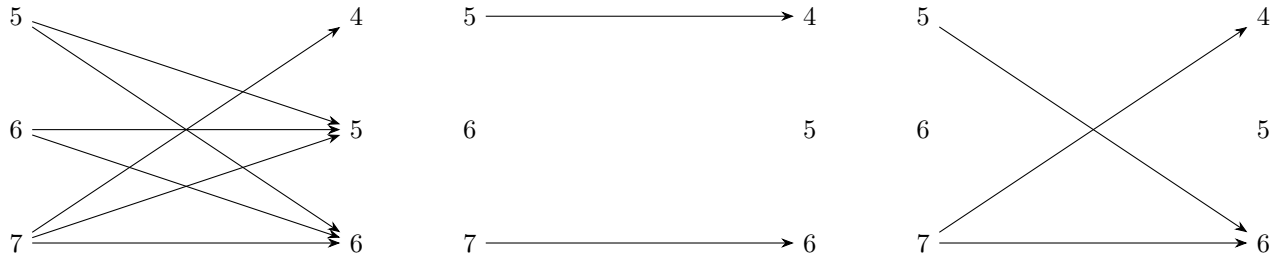   Domain: $\{1, 3, 4\}$, Co-domain: $\{6, 8, 10\}$

   (d) Draw an arrow diagram for R.

9. (10 points) Let $B = \{4,5,6\}$ and $A = \{5,6,7\}$ and define relations R, S, and T from A to B as follows:

- For all $(x, y) \in A \times B$, $(x, y) \in R$ means that $x \geq y$.
- $(x, y) \in S$ means that $\frac{x-y}{2}$ is an integer.
- $T = \{(7,4),(5,6),(7,6)\}$.

(a) Draw arrow diagrams for R, S, and T.

(b) Indicate whether any of the relations R, S, and T are functions.
R is not a function.

S is not a function.
T is a function.

10. (10 points) Define a relation T from $\mathbb{R}$ to $\mathbb{R}$ as follows: For all real numbers x and y, $(x, y) \in T$ means that $y^2 - x^2 = 1$.
Is T a function? Explain.

No, T is not a function because for a given x, there can be two different values of y (one positive and one negative) that satisfy the equation $y^2 - x^2 = 1$.