Variable task times 2

In[1]:=
```
F[t_, R_, p_] := R + (1 - E^(-p t)) (1 - R)
R[F_, τ_, μ_] := F E^(-μ τ)
```

General function of time... prove that **L < R for rest anticipation**

In[3]:= $\left(\left(1 - E^{-p\,\theta p}\right) / \left(1 - E^{-p\,\theta}\right) < (1 - \text{Fprev}) / \left(E^{\mu\,\tau} - \text{Fprev}\right)\right)$ /. θp → g[τ]

Out[3]= $$\frac{1 - e^{-p\,g[\tau]}}{1 - e^{-p\,\theta}} < \frac{1 - \text{Fprev}}{e^{\mu\,\tau} - \text{Fprev}}$$

For $\tau = 0$ both sides are 1. For $\tau = \infty$ L = c, R = 0 (**suggesting no anticipation for large rests?**)

In[4]:= $$R = \frac{1 - \text{Fprev}}{e^{\mu\,\tau} - \text{Fprev}};$$

In[5]:= $$\text{dR} = \text{Simplify@D}\left[\frac{1 - \text{Fprev}}{e^{\mu\,\tau} - \text{Fprev}}, \tau\right]$$

Out[5]= $$\frac{e^{\mu\,\tau}\,(-1 + \text{Fprev})\,\mu}{\left(e^{\mu\,\tau} - \text{Fprev}\right)^2}$$

In[6]:= $$\frac{e^{\mu\,\tau}\,(-1 + \text{Fprev})\,\mu}{\left(e^{\mu\,\tau} - \text{Fprev}\right)^2}$$

Out[6]= $$\frac{e^{\mu\,\tau}\,(-1 + \text{Fprev})\,\mu}{\left(e^{\mu\,\tau} - \text{Fprev}\right)^2}$$

g[τ] is $\theta\text{th}\left(1 + \lambda\,\text{Log}\left[1 + e^{-\mu\,\tau}\,\text{Fprev}\right]\right)$ and θ is $\theta\text{th}\left(1 + \lambda\,\text{Log}[1 + \text{Fprev}]\right)$

LHS becomes

In[7]:= $$L = \frac{E^{p\,T} - \left(1 + \text{Fprev}\,E^{-\mu\,\tau}\right)^{-p\,T\,\lambda}}{E^{p\,T} - (1 + \text{Fprev})^{-p\,T\,\lambda}};$$

In[8]:= $$\texttt{dL = Simplify@D}\left[\frac{e^{p\,T}-\left(1+e^{-\mu\,\tau}\,\text{Fprev}\right)^{-p\,T\,\lambda}}{e^{p\,T}-(1+\text{Fprev})^{-p\,T\,\lambda}},\ \tau\right]$$

Out[8]= $$-\frac{e^{-\mu\,\tau}\,\text{Fprev}\,\left(1+e^{-\mu\,\tau}\,\text{Fprev}\right)^{-1-p\,T\,\lambda}\,p\,T\,\lambda\,\mu}{e^{p\,T}-(1+\text{Fprev})^{-p\,T\,\lambda}}$$

In[9]:= $$-\frac{e^{-\mu\,\tau}\,\text{Fprev}\,\left(1+e^{-\mu\,\tau}\,\text{Fprev}\right)^{-1-p\,T\,\lambda}\,p\,T\,\lambda\,\mu}{e^{p\,T}-(1+\text{Fprev})^{-p\,T\,\lambda}}$$

Out[9]= $$-\frac{e^{-\mu\,\tau}\,\text{Fprev}\,\left(1+e^{-\mu\,\tau}\,\text{Fprev}\right)^{-1-p\,T\,\lambda}\,p\,T\,\lambda\,\mu}{e^{p\,T}-(1+\text{Fprev})^{-p\,T\,\lambda}}$$

Both derivatives are always negative for any τ: **if dL < dR anticipation is better**

Derivatives in $\tau = 0$

In[10]:= `Simplify[dR /. τ → 0]`

Out[10]= $$\frac{\mu}{-1+\text{Fprev}}$$

In[11]:= `Simplify[dL /. τ → 0]`

Out[11]= $$-\frac{\text{Fprev}\,p\,T\,\lambda\,\mu}{(1+\text{Fprev})\left(-1+e^{p\,T}\,(1+\text{Fprev})^{p\,T\,\lambda}\right)}$$

In[12]:= $$\texttt{FullSimplify}\left[\frac{\mu}{-1+\text{Fprev}}>-\frac{\text{Fprev}\,p\,T\,\lambda\,\mu}{(1+\text{Fprev})\left(-1+e^{p\,T}\,(1+\text{Fprev})^{p\,T\,\lambda}\right)}\right]$$

Out[12]= $$\left(\frac{1}{-1+\text{Fprev}}+\frac{\text{Fprev}\,p\,T\,\lambda}{(1+\text{Fprev})\left(-1+e^{p\,T}\,(1+\text{Fprev})^{p\,T\,\lambda}\right)}\right)\mu>0$$

Analyze the numerator of $\frac{\text{Fprev}\,p\,T\,\lambda*(1-\text{Fprev})-(1+\text{Fprev})\left(-1+e^{p\,T}\,(1+\text{Fprev})^{p\,T\,\lambda}\right)}{(1-\text{Fprev})\,(1+\text{Fprev})\left(-1+e^{p\,T}\,(1+\text{Fprev})^{p\,T\,\lambda}\right)}>0$

In[13]:= $$\texttt{FullSimplify}\left[-\left((1+\text{Fprev})\left(-1+e^{p\,T}\,(1+\text{Fprev})^{p\,T\,\lambda}\right)\right)+(1-\text{Fprev})\,\text{Fprev}\,p\,T\,\lambda>0\right]$$

Out[13]= $$e^{p\,T}\,(1+\text{Fprev})^{1+p\,T\,\lambda}+\text{Fprev}^2\,p\,T\,\lambda<1+\text{Fprev}+\text{Fprev}\,p\,T\,\lambda$$

In[14]:= $$\left(e^{p\,T}\,(1+\text{Fprev})^{1+p\,T\,\lambda}<1+\text{Fprev}+\text{Fprev}\,p\,T\,\lambda-\text{Fprev}^2\,p\,T\,\lambda\right)\ /.\ \text{Fprev}\to 0$$

Out[14]= $$e^{p\,T}<1$$

In[15]:=
```
(e^(p T) (1 + Fprev)^(1+p T λ) < 1 + Fprev + Fprev p T λ - Fprev^2 p T λ) /. Fprev → 1
```

Out[15]= $2^{1+p\,T\,\lambda}\, e^{p\,T} < 2$

This suggests that in $\tau = 0$ the RHS decreases faster than LHS (for limit F cases)

Derivatives in $\tau = \infty$

In[16]:=
```
Limit[dR, τ → ∞]
```

Out[16]= 0 if Fprev ∈ ℝ && $\mu > 0$

In[17]:=
```
Limit[dL, τ → ∞]
```

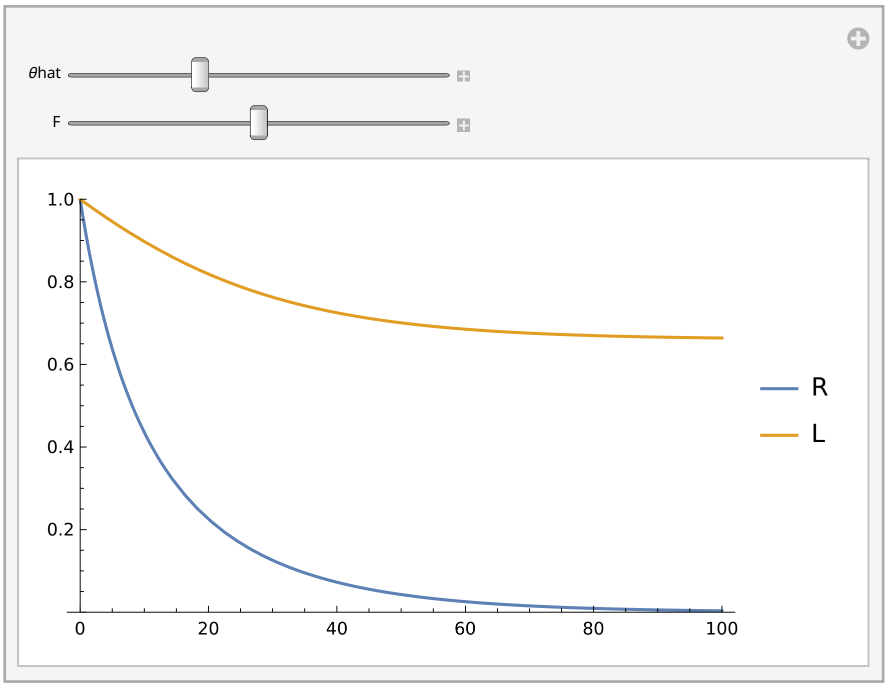
Out[17]= 0 if *condition* +

Second derivatives

Numerical examples

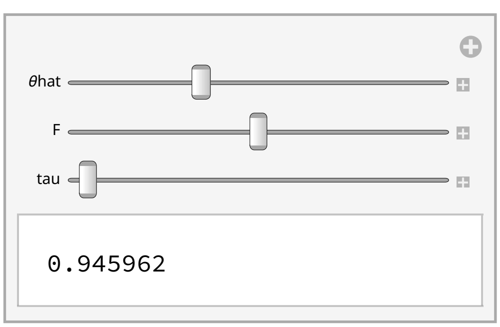
In[22]:=
```
Manipulate[
 Plot[{R /. {p → 0.1, μ → 0.05, λ → 2, T → θhat, Fprev → F, τ → tau},
   L /. {p → 0.1, μ → 0.05, λ → 2, T → θhat, Fprev → F, τ → tau}},
  {tau, 0, 100}, PlotRange → {Automatic, {0, 1}}, PlotLegends → {"R", "L"}],
 {{θhat, 5}, 0.1, 15},
 {{F, 0.5}, 0, 1}
]
```

Out[22]=



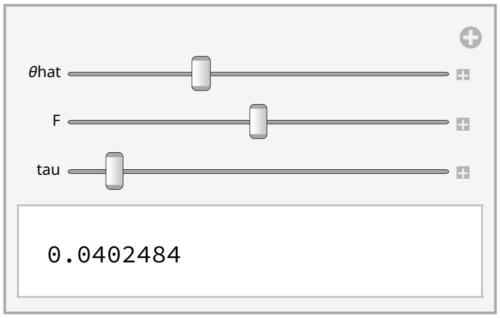
In[23]:=
```
Manipulate[
 (L) /. {p → 0.1, μ → 0.05, λ → 2, T → θhat, Fprev → F, τ → tau},
 {{θhat, 5}, 0, 15},
 {{F, 0.5}, 0, 1},
 {{tau, 5}, 0, 600}
]
```

Out[23]=



In[24]:=
```
Manipulate[
  (dL - dR) /. {p → 0.1, μ → 0.05, λ → 5, T → θhat, Fprev → F, τ → tau},
  {{θhat, 5}, 0, 15},
  {{F, 0.5}, 0, 1},
  {{tau, 5}, 0, 60}
]
```

Out[24]=



Alternative function of time

Old