

Atmiya Institute of Technology & Science, RAJKOT
Department of M.C.A.

M.C.A. Semester - II
2620004 – Computer Oriented Numerical Methods
Assignment – 3 (Open Methods)
Solution

1. Use simple fixed point iteration to locate the root of $f(x) = 2\sin(\sqrt{x}) - x$. Use an initial guess of $x_0 = 0.5$ and iterate until $e_a \leq 0.001\%$.

Fixed point iteration :

Enter initial value : 0.5

Iteration	x_i	x_{i+1}	e_a
1	0.500000	1.299274	-----
2	1.299274	1.817147	61.516964
3	1.817147	1.950574	28.499264
4	1.950574	1.969743	6.840368
5	1.969743	1.972069	0.973153
6	1.972069	1.972344	0.117966
7	1.972344	1.972377	0.013956
8	1.972377	1.972381	0.001650
9	1.972381	1.972381	0.000193

Ans: Fixed point iteration : 1.972381.

2. Determine the highest real root of $f(x) = 2x^3 - 11.7x^2 + 17.7x - 5$. Using,
- Fixed point iteration method (three iterations $x_0=3$).
 - Newton – Raphson method (three iterations $x_0=3$).
 - Secant method (three iterations $x_{-1}=3, x_0=4$).

(a) Fixed point iteration :

Enter initial value : 3

Iteration	x_i	x_{i+1}
1	3.000000	3.180791
2	3.180791	3.333959
3	3.333959	3.442543
4	3.442543	3.506330

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(b) Newton Raphson Method :

Enter initial value : 3

Iteration	xi	xi+1
1	3.000000	5.133333
2	5.133333	4.269750
3	4.269750	3.792934

Ans: (a) Fixed point iteration: 3.442543.
(b) Newton Raphson Method: 3.792934.

3. Use (a) Fixed point iteration and (b) Newton raphson method to determine a root of $f(x) = -x^2 + 1.8x + 2.5$ using $x_0=5$. Perform the computation until e_a is less than $e_s = 0.05\%$.

Ans: (a) Fixed point iteration: Ans : 2.71962 (Hint : $g(x) = \sqrt{1.8x + 2.5}$)
(b) Newton Raphson method: Divergence case.

4. Determine the real roots of $f(x) = -1 + 5.5x - 4x^2 + 0.5x^3$ using Newton raphson method
(a) within $e_s = 0.01\%$. (b) Initial guess $x_0 = 4.52$ (c) initial guess $x_0 = 4.54$.

(a) within $e_s = 0.01\%$

Enter initial value : 0.1 ($f(0)<0$ and $f(1)>0$ so, taking initial value $x_0 = 0.1$ and $f(0.1)<0$)

Iteration	xi	fxi	dfxi	nxixi	ea
1	0.100000	-0.489500	4.715000	0.203818	100.000000
2	0.203818	-0.040936	3.931772	0.214229	50.936523
3	0.214229	-0.000400	3.855007	0.214333	4.860044
4	0.214333	-0.000000	3.854244	0.214333	0.048402

(b) Initial guess $x_0 = 4.52$.

Enter initial value: 4.52

Iteration	xi	xi+1	ea
1	4.520000	-807.202942	100.000000

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2	-807.202942	-537.249207	100.559959
3	-537.249207	-357.281494	50.247395
4	-357.281494	-237.305115	50.371407
5	-237.305115	-157.324036	50.557858
6	-157.324036	-104.008018	50.838436
7	-104.008018	-68.471016	51.261448
8	-68.471016	-44.790058	51.900795
9	-44.790058	-29.018042	52.871014
10	-29.018042	-18.525667	54.352451
11	-18.525667	-11.562826	56.636959
12	-11.562826	-6.966090	60.217468
13	-6.966090	-3.963226	65.987312
14	-3.963226	-2.041908	75.768166
15	-2.041908	-0.861218	94.094284
16	-0.861218	-0.192971	137.095291
17	-0.192971	0.118860	346.294281
18	0.118860	0.206806	262.351135
19	0.206806	0.214280	42.525776
20	0.214280	0.214333	3.487691
21	0.214333	0.214333	0.024917

(c) initial guess $x_0=4.54$.

Enter initial value : 4.54

Iteration	x_i	x_{i+1}	ea
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1	4.540000	124.540962	100.000000
2	124.540962	83.935280	96.354614
3	83.935280	56.874546	48.377373
4	56.874546	38.848866	47.579693
5	38.848866	26.854469	46.399502
6	26.854469	18.893438	44.664433
7	18.893438	13.641490	42.136486
8	13.641490	10.228786	38.499813
9	10.228786	8.096901	33.363731
10	8.096901	6.901203	26.329636
11	6.901203	6.404632	17.325941
12	6.404632	6.309328	7.753312
13	6.309328	6.305902	1.510518

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14	6.305902	6.305898	0.054331
15	6.305898	6.305898	0.000068

Ans : (a) Within $e_s = 0.01\%$ $\rightarrow 0.214333$.
(b) Initial guess $x_0 = 4.52$. $\rightarrow 0.214333$.
(c) Initial guess $x_0 = 4.54$. $\rightarrow 6.305898$.

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5. Determine the lowest real root of $f(x) = -12 - 21x + 18x^2 - 2.4x^3$ using the secant method to a value of e_s corresponding to three significant figures.

Ans : Secant Method : Ans: 2.0536

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6. Locate the first positive root of $f(x) = \sin x + \cos(1+x^2) - 1$ where x is in radians. Use four iterations of the secant method with initial guesses of
- $x_{i-1} = 1.0$ and $x_i = 3.0$.
 - $x_{i-1} = 1.5$ and $x_i = 2.5$.
 - $x_{i-1} = 1.5$ and $x_i = 2.25$.

Ans: Secant Method:
(a) Ans : 0.3963
(b) Ans : 2.5324
(c) Ans : 1.9445

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7. Determine the highest real root of $f(x) = 0.95x^3 - 5.9x^2 + 10.9x - 6$.
- Using Newton Raphson method (three iterations, $x_i = 3.5$)
 - Using secant method (three iterations $x_{i-1} = 2.5$ and $x_i = 3.5$).

(a) Newton Raphson Method:

Enter initial value : 3.5

Iteration	x_i	x_{i+1}
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1	3.500000	3.365651
2	3.365651	3.345112
3	3.345112	3.344646
4	3.344646	3.344645

Ans : (a) Newton Raphson Method: 3.344646.
(b) Secant Method : 3.3670

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8. Determine the lowest positive root of $f(x) = 8\sin(x)e^x - 1$ using,
- Newton raphson method (three iterations, $x_i=0.3$)
 - Secant method (three iterations $x_{i-1} = 0.5$ and $x_i=0.4$).

Ans: (a) Newton Raphson Method: 0.111991
(b) Secant Method: 0.1129

9. The polynomial $f(x) = 0.007x^4 - 0.28x^3 + 3.35x^2 - 12.183x + 5$ has a real root between 15 and 20. Apply the Newton raphson method to this function using an initial guess of $x_0 = 16.15$.

Ans : (a) Newton Raphson Method: 14.489699

10. Find the root of the equations $x^3 + 2x^2 + 10x - 20 = 0$ that is near 1 using Birge Vieta method, correct to 4 places of decimals.

Ans: 2.71932