

# CHAPTER 1

## INTRODUCTION

Fuzzy sets are generalizations of conventional set theory introduced by Zadeh [85] as a mathematical way to represent vagueness in everyday life. A fuzzy set assigns to each possible individual in the universe of discourse, a value representing its grade of membership in the set. It is concerned with the degree to which events occur rather than the likelihood of their occurrence. Fuzzy logic is most successful in situations with very complex models, where understanding is strictly limited and where human reasoning, human perception, human decision making are inextricably involved. Fuzzy sets play an important role in human thinking, particularly in the domains of pattern recognition, communication of information, decision making and abstraction. Applications of fuzzy sets in various fields are discussed in Timothy [69] and George J. Klir and Bo Yuan [24].

### 1.1 Guided tour of fuzzy set theory

In conventional set theory, elements of a set satisfy precise properties. In crisp sets an element  $x$  in the universe  $X$  is either a member or not a member of some crisp set  $A$ . This binary issue of membership can be represented mathematically with a function called characteristic function  $\psi_A : X \rightarrow \{0,1\}$  where

$$\psi_A(x) = \begin{cases} 1 & x \in A \\ 0 & x \notin A \end{cases}$$

Crisp sets handle black and white concepts. However, everyday life abounds in innumerable vague concepts like ‘young’, ‘old’, ‘hot’, ‘intelligent’ and linguistic terms like ‘few’, ‘very few’, ‘almost all’, etc.,. The major limitation of classical set theory concept is that it fails to define such vague concepts which are favorably addressed by the fuzzy set theory. A unique advantage of the fuzzy set

theory is that its ability to generalize 0 and 1 membership values of a crisp set to a membership function of a fuzzy set.

### 1.1.1 Basic Definitions

#### Fuzzy set

A **fuzzy set** is a set containing elements that have varying degrees of membership in the set. Each element is mapped to  $[0, 1]$  by membership function  $\mu_A : X \rightarrow [0,1]$  where  $X$  is the universal set.

A fuzzy set is fully characterized by its membership function. There are two kinds of representations of fuzzy set one discrete and the other continuous.

A fuzzy set  $A$  in  $X$  may be represented as a set of ordered pairs of generic element  $x$  and its grade of membership function is given below

$$A = \{(x, \mu_A(x)) / x \in X\}$$

When  $X$  is discrete,  $A$  is commonly written as,  $A = \sum_x \mu_A(x) / x$

When  $X$  is continuous,  $A = \int_x \mu_A(x) / x$

In both the notations, the symbol “/” is not a quotient but rather a delimiter. The numerator in each term is the membership value in set  $A$  associated with the element of the universe indicated in the denominator. The symbol “ $\sum$ ” in the first notation is not algebraic “add” but are a function-theoretic union and the symbol “ $\int$ ” is not algebraic integral but a continuous function theoretic union notation for continuous variables.

### Core

The **core** of a fuzzy set  $A$  is the set of all points with unit membership degree in  $A$  and is represented as  $\text{core}(A) = \{x \in X / \mu_A(x) = 1\}$ .

### Normal

A fuzzy set  $A$  of  $X$  is called **normal** if there exists at least one element  $x$  in  $X$  such that  $\mu_A(x) = 1$ . A fuzzy set  $A$  is normal if its core is nonempty. A fuzzy set that is not normal is called subnormal.

### Height

The **height** of a fuzzy set  $A$  is the largest membership grade of any element in  $A$ .

$$\text{Height}(A) = \text{Max } \mu_A(x) \text{ for all } x \text{ in } X.$$

### Support

The **support** of a fuzzy set  $A$  is the set of all points with nonzero membership degree in  $A$ . It is denoted by  $\text{Supp}(A) = \{x \in X / \mu_A(x) > 0\}$ .

### Crossover point

The **crossover points** of a membership function are defined as the elements in the universe for which a particular fuzzy set  $A$  has values equal to 0.5.

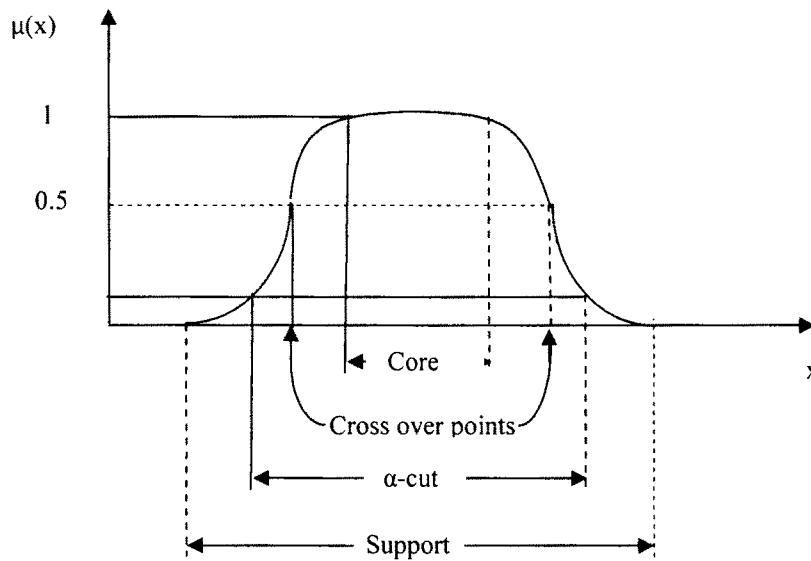
### Alpha Cut

It is one of the most important concepts of fuzzy sets.

For a fuzzy set  $A$  and for  $\alpha \in [0,1]$  the  **$\alpha$ -cut**  $A_\alpha = \{x \in X / \mu_A(x) \geq \alpha\}$

The strong  **$\alpha$ -cut**  $A_{\alpha^+} = \{x \in X / \mu_A(x) > \alpha\}$ .

The  $\alpha$ -cut set is a crisp set. This threshold cut restricts the domain of the fuzzy set. Two main reasons why  $\alpha$ -cuts are important are (i) The alpha level set describes a power or strength that is used by fuzzy models to decide whether or not a truth value should be considered equivalent to zero. This is a facility that controls the execution of fuzzy rules as well as intersection of multiple fuzzy sets. (ii) The strong  $\alpha$ -cut at zero defines the support set for a fuzzy set. Fig 1.1 illustrates the regions in the universe comprising the core, support, crossover points and alpha cut of a typical fuzzy set.

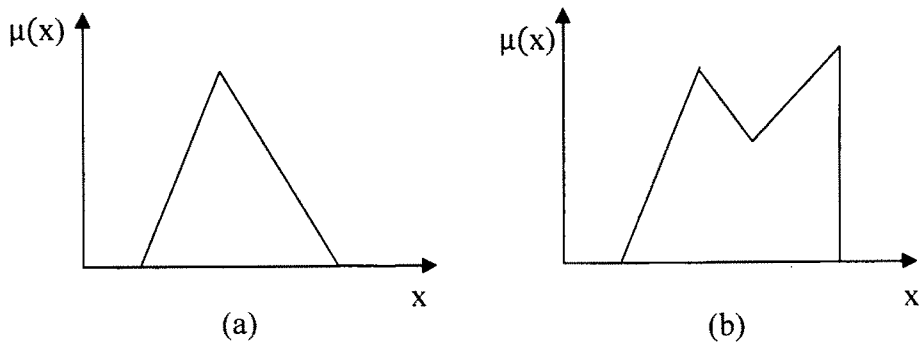


**Fig 1.1 Support, Alpha cut, Crossover and Core of a fuzzy set**

### **Convex fuzzy set**

A **convex fuzzy set** is described by a membership function whose values are strictly monotonically increasing, or whose membership values are strictly monotonically decreasing or whose membership values are strictly monotonically increasing then strictly monotonically decreasing with increasing values for elements in the universe. Fig 1.2 represents a convex and a non convex fuzzy set.

In other words for  $x, y, z$  in the fuzzy set  $A$  with  $x < y < z$  implies  $\mu_A(y) \geq \min[\mu_A(x), \mu_A(z)]$  then  $A$  is said to be a convex fuzzy set.



**Fig 1.2 a) convex fuzzy set b) non convex fuzzy set**

## **1.2 Representation of fuzzy sets**

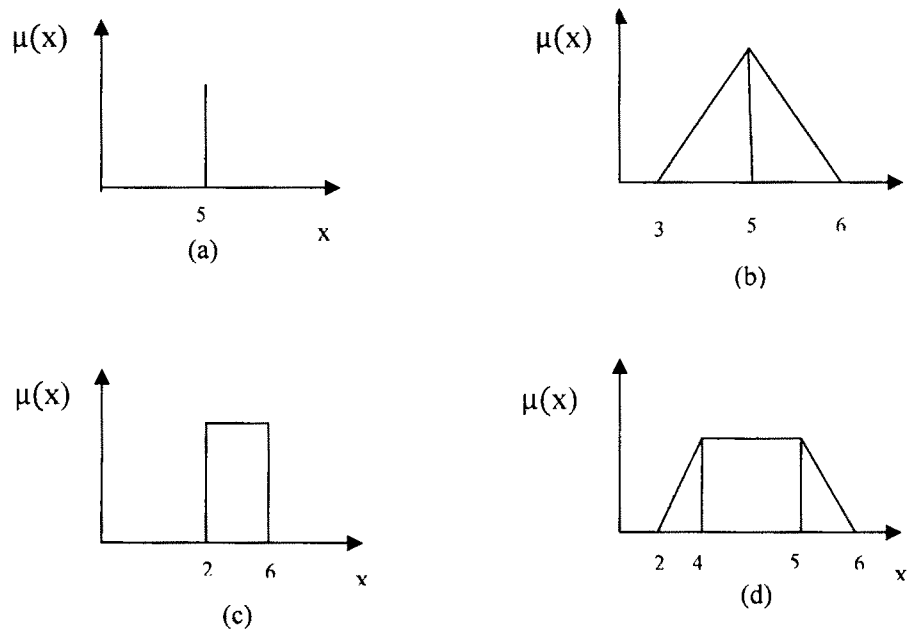
The method of recognizing fuzzy attributes and drafting the fuzzy set is an important technique. While dealing with uncertainties, decision makers are commonly provided with information characterized by vague linguistic descriptions such as “high risk”, “low profit”, “high annual interest rate” etc. The objective of fuzzy set theory is primarily concerned with the quantification of such vague descriptions. The more the object fits the vague predicate, the larger is its grade of membership. The membership function may be viewed as representing an opinion poll of human thought or an expert’s opinion.

### **1. 2 .1 Fuzzy number**

A fuzzy number  $A$  is a fuzzy set in the real line that satisfies the conditions of both normality and convexity. Most fuzzy sets used in the literature satisfy the conditions of normality and convexity. Hence fuzzy numbers are considered to be the most basic type of fuzzy sets.

A fuzzy number (fuzzy set) represents a real number interval whose boundary is fuzzy. Membership functions of fuzzy numbers need be symmetrical. Fuzzy sets can have a variety of shapes. However, a triangle or a trapezoid often provides an

adequate representation of the expert knowledge, simultaneously simplifying the process of computation to a significant level. So the triangular and trapezoidal shapes of membership functions are used most often for representing fuzzy numbers. Representation and application of fuzzy numbers are described in Heilpern [25]. Special cases of real numbers include ordinary real numbers and interval of real numbers. Fig 1.3 (a) is a real number 5, Fig 1.3 (c) is a closed interval [2,6], Fig 1.3 (b) and Fig 1.3 (d) are a triangular fuzzy number and a trapezoidal fuzzy number expressing the concept “numbers close to 5”, respectively.



**Fig 1.3 Comparison between a real number and fuzzy number**

### 1.2.2 Triangular and trapezoidal fuzzy numbers

A more convenient and concise way of defining a membership function is to express it as a mathematical formula. Triangular membership functions and trapezoidal membership functions are two well known classes of commonly parameterized membership functions. Due to the advantages of simple formulae and computational efficiency, they have been used extensively in real-time applications. Fig 1.4 and Fig 1.5 represent the graph of triangular and trapezoidal fuzzy number respectively.

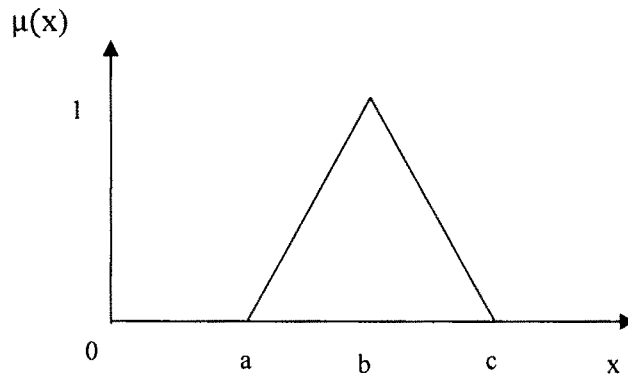
### ***Triangular fuzzy number***

A **triangular fuzzy number** is specified by three parameters  $a, b, c$  (with  $a < b < c$ ) as follows:

$$\text{Triangle}(x; a, b, c) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ \frac{c-x}{c-b} & b \leq x \leq c \\ 0 & c \leq x \end{cases}$$

An alternative expression for the preceding equation is

$$\text{Triangle}(x; a, b, c) = \max\left(\min\left(\frac{x-a}{b-a}, \frac{c-x}{c-b}\right), 0\right)$$



**Fig 1.4 A typical triangular membership function Triangle (  $x; a, b, c$  )**

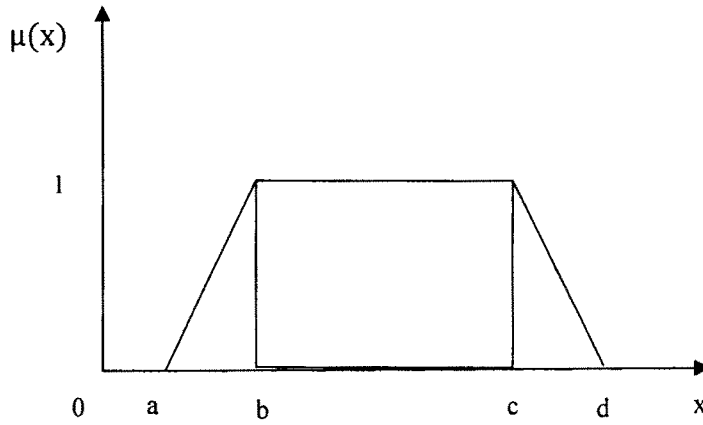
### ***Trapezoidal fuzzy number***

A **trapezoidal fuzzy number** (TFN) is specified by four parameters  $a, b, c, d$  (with  $a < b \leq c < d$ ) as follows:

$$\text{Trapezoid} (x; a, b, c, d) = \begin{cases} 0 & x \leq a \\ \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & d \leq x \end{cases}$$

An alternative expression for the preceding equation is

$$\text{Trapezoid} (x; a, b, c, d) = \max \left( \min \left( \frac{x-a}{b-a}, 1, \frac{d-x}{d-c} \right), 0 \right)$$



**Fig 1.5 A trapezoidal membership function defined by trapezoid ( x; a, b, c, d )**

### 1.2.3 Non linear representations of membership functions

Triangular membership functions and trapezoidal membership functions are composed of straight line segments and their corner points are not smooth. Membership functions can be defined both by smooth and nonlinear functions. Popularly used non linear membership functions are Gaussian membership functions and Cauchy membership functions.

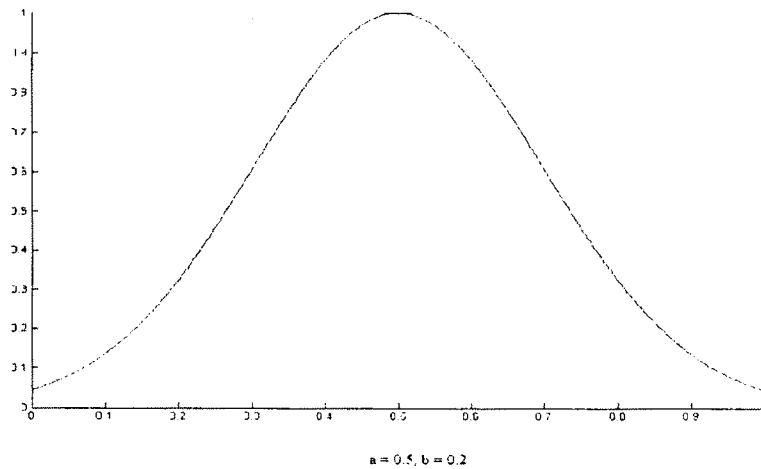


### ***Gaussian membership function***

A Gaussian membership function is specified by two parameters  $a$ ,  $b$  where  $a$  represents the membership functions center and  $b$  determines the membership functions width.

$$\text{Gaussian} (x; a, b) = e^{-\left(\frac{x-a}{b}\right)^2}$$

$a, b, x, \alpha \in \mathbb{R}$  and  $b > 0, \alpha > 0$ . Fig 1.6 depicts the membership function of Gaussian function  $(x; 0.5, 0.2)$ .  $\alpha$  determines the smoothness of the function. For fixed  $b$ , the grade of fuzziness increases as  $\alpha$  decreases.



**Fig 1.6 Gaussian membership function  $G(x; .5, .2)$**

### ***Cauchy membership function***

A Cauchy membership function or generalized bell membership function is specified by three parameters  $a, b, c$  where  $a$  represents the width,  $c$  center and  $b$  controls the slopes at the crossover points.

$$\text{Cauchy} (x; a, b, c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}} \quad a, c, x \in \mathbb{R} \text{ and } b > 0$$

The mathematical implementation of the TFN is straightforward and most importantly, it represents a rational basis for the quantification of the vague knowledge associated with most of the decision making issues. To approximate human perception of shapes of the objects in the images is a difficult task. Fuzzy numbers can very well represent the geometric shape of objects. Consequently measuring the degree of similarity of fuzzy numbers plays an important role in pattern recognition, information fusion and fuzzy decision making.

### 1.3 Aggregation operators

Information fusion techniques and aggregation operators produce the most comprehensive specific datum, from different sources thus enabling noise reduction, increased accuracy, summarization and extraction of information and finally decision making. These techniques are applied in fields such as economics, biology, knowledge based systems robotics, data mining etc. Modeling decisions, information fusion and aggregation operators are studied in detail in Vience Torra and Yasua Narukawa [74].

Decision making often involves aggregation of several pieces of information coming from different sources. Such an aggregation could either be aggregation of preferences given by several individuals of a group or it could be aggregation of multiple rules into a single rule in the case of rule based systems.

**Aggregation operator** is a function which assigns a real number  $y$  to any  $n$ -tuple  $(x_1, x_2, \dots, x_n)$  of real numbers.

$$A_g : [0,1]^n \rightarrow [0,1] \text{ such that } A_g(x_1, x_2, \dots, x_n) = y.$$

### 1.3.1 Fundamental properties

In order to qualify as an intuitively meaningful aggregation function,  $A_g$  must satisfy at least three fundamental properties which express the essence of the notion of aggregation namely,

a) Identity when unary  $A_g(x) = x$

b) Boundary Conditions  $A_g(0, 0, \dots, 0) = 0$   $A_g(1, 1, \dots, 1) = 1$

c) Non Decreasing

$$A_g(x_1, x_2, \dots, x_n) \leq A_g(y_1, y_2, \dots, y_n) \text{ if } (x_1, x_2, \dots, x_n) \leq (y_1, y_2, \dots, y_n).$$

### 1.3.2 Mathematical properties

The mathematical properties expected from an aggregation operator are:

**Continuity:** The aggregation function is continuous with respect to each of its variables. This property guarantees a certain robustness, consistency and non-chaotic behavior in the process.

**Associativity:**  $A_g(x_1, x_2, x_3) = A_g(A_g(x_1, x_2), x_3) = A_g(x_1, A_g(x_2, x_3))$  - the order in which we perform the operation is not significant.

**Commutativity:**  $A_g(x_1, x_2) = A_g(x_2, x_1)$ , the ordering of the aggregated sets is not significant.

**Monotonic:** For  $x \geq y$ ,  $A_g(x) \geq A_g(y)$ ,  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$  when the preference in individual alternate increases, the overall value should also increase.

**Comonotonic:** For  $x = (x_1, x_2, \dots, x_n)$  and  $y = (y_1, y_2, \dots, y_n)$ ,  $x \geq y$  or  $x \leq y$ . That is if  $x_i \leq y_i$  for some  $i$  then  $x \leq y$  must hold. A comonotonic set is simultaneously non decreasing in each component.

**Compensative:**  $\min ( \mathbf{x} ) \leq A_g( \mathbf{x} ) \leq \max ( \mathbf{x} )$  , the results of aggregation is lower than the highest element aggregated and higher than the lowest element aggregated.

**Idempotent:**  $A_g(x_1, x_1, \dots, x_1) = x_1$ , aggregation of same value  $n$  times results in the same value.

**Strongly idempotent:**  $A_g(x_1, x_2, \dots, x_n, x_1, x_2, \dots, x_n, \dots, x_1, x_2, \dots, x_n)$   
 $= A_g(x_1, x_2, \dots, x_n),$

aggregation of same set of values  $k$  times is same as aggregation of that set Tomasa et al., [71].

### 1.3.3 t-norms and t-conorms

In human reasoning, different statements are combined by using different logical connectives. To be able to adequately deal with such logical combinations it is essential to estimate the degrees of belief in these logical combinations. The classical two valued logic to compute the degree of belief in the composite statements “*A and B*” and “*A or B*” are either absolutely sure or absolutely no belief of the statements. In situations where 100% strong conclusion is not possible the degree of belief of the statements can be estimated using the t-norm and t-conorm operators. For example since ‘*A and B*’ mean intuitively the same as ‘*B and A*’, the operator should also satisfy this condition.

The concept of a triangular norm was introduced by Menger [50] in order to generalize the triangular inequality of a metric. The current notion of a t-norm and its dual operation (t-conorm) is due to Schweizer and Sklar [60]. Both these operations can also be used as generalizations of the Boolean logic connectives to multi-valued logic. The t-norms generalize the conjunctive 'AND' operator and the t-conorms

generalize the disjunctive 'OR' operator. This situation allows them to be used to define the intersection and union operation in fuzzy logic.

### ***t-norm T***

A fuzzy intersection t-norm for all  $x, y, z \in [0, 1]$  is a function  $T: [0, 1] \times [0, 1] \rightarrow [0, 1]$ , having the following properties

$T(x, y) = T(y, x)$	Commutativity
$T(x, y) \leq T(u, v)$ if $x \leq u$ and $y \leq v$	Monotonicity
$T(x, T(y, z)) = T(T(x, y), z)$	Associativity
$T(x, 1) = x$	One as neutral element

### ***t-conorm S***

A fuzzy union t-conorm or s-norm for all  $x, y, z \in [0, 1]$  is a function

$S: [0, 1] \times [0, 1] \rightarrow [0, 1]$  having the following properties

$S(x, y) = S(y, x)$	Commutativity
$S(x, y) \leq S(u, v)$ if $x \leq u$ and $y \leq v$	Monotonicity
$S(x, S(y, z)) = S(S(x, y), z)$	Associativity
$S(x, 0) = x$	Zero as neutral element

### ***Duality***

A t-norm  $T$  and t-conorm  $S$  are dual if they satisfy the DeMorgan's law

$$\overline{T(x, y)} = S(\overline{x}, \overline{y})$$

where  $\overline{x} = 1 - x$  is negation of the expression.

### ***Commonly used t-norms and its dual t- conorms***

Since the t-norms and t- conorms are associative, for the purpose of illustration only the definitions of t-norms and t-conorms are given for two elements. The definition, however could be extended to n elements. The commonly used t- norms and t-conorms are given in Table1.1

**Table 1.1 Commonly used t-norms and t-conorms**

<b>Operator</b>	<b>t-norm</b>	<b>Dual t-conorm</b>
Min-Max	$\text{Min} ( x , y )$	$\text{Max} ( x , y )$
Probabilistic	$x \cdot y$	$x + y - x \cdot y$
Lukasiewicz	$\text{Max} ( x + y - 1, 0 )$	$\text{Min} ( x + y , 1 )$

### ***Specialties of the above norms***

#### **Min-Max**

Min is the biggest and the only idempotent t- norm. Its dual Max is the smallest and the only idempotent t-conorm.

#### **Probabilistic**

The probabilistic case is characterized by ‘smoothness’ i.e., it has a continuous derivative.

#### **Lukasiewicz**

The Lukasiewicz t- norm satisfies the classical law of non contradiction  $T(x, \bar{x}) = 0$ . Its dual Lukasiewicz t- conorm satisfies the law of excluded middle  $S(x, \bar{x}) = 1$ .

### **1. 3. 4 Compensatory operators**

The compensatory operators are special aggregation operators to model intersection and union in many valued logic. Zimmerman and Zysno [87] noticed that the standard aggregation operators t-norms and t-conorms lack the compensation behavior. If two values are aggregated by a t-norm then there is no compensation between the low and high values. The compensatory “and” is either multiplicative or an additive combination of the t-norm and t-conorm operators introduced by Alinsa et al [1 ].A large class of aggregation operators can be derived as a combination of t-norms and t-co norms.

### **1. 4 Fuzzy systems**

In the past decade, fuzzy systems have replaced conventional technology in different scientific and system engineering applications, especially in the areas of pattern recognition and control systems. Fuzzy technology, due to its advantages of powerful reasoning capacity and limited rules also supports decision making process and expert systems.

Fuzzy inference systems (FIS) are rule-based systems with concepts and operations associated with fuzzy set theory and fuzzy logic. A detailed study of fuzzy inference system can be found in Mendel [49], Dubois et al [18]. These systems are mappings from an input space to an output space; therefore, they allow constructing structures which can be used to generate responses (outputs) to certain stimulations (inputs), based on stored knowledge of how the responses and stimulations are related. The knowledge is stored in the form of a rule base, i.e. a set of rules that express the relations between inputs of the system and expected outputs. In those cases where knowledge is obtained by eliciting information from specialists, these systems are

usually known as fuzzy expert systems. Other common denominations of FIS are fuzzy knowledge-based systems and data-driven fuzzy systems.

FIS are usually divided in to two categories : the first being multiple input, multiple output (MIMO) systems, where the system returns several outputs based on the inputs it receives; and the second is the multiple input, single output (MISO) systems, where only one output is returned from multiple inputs. Since MIMO systems can be decomposed into a set of MISO systems working in parallel, the exposition in this study follows the MISO point of view.

### ***Functioning of fuzzy logic systems***

A fuzzy logic system (FLS) maps crisp inputs into crisp outputs and has four components namely rules, fuzzifier, inference engine and defuzzifier.

### ***Fuzzy rules***

Rules may either be provided by experts or could be extracted from numerical data themselves. A fuzzy rule (fuzzy If-Then rules, fuzzy implication, or fuzzy conditional statement) assumes the form

If x is A then y is B

where A and B are linguistic values defined by fuzzy sets on the universe of discourse X and Y respectively.

A fuzzy rule is abbreviated as  $A \rightarrow B$  .

“x is A” is called the antecedent or premise while “y is B” is called the consequence or conclusion. The rule has the membership function  $\mu_{A \rightarrow B}(x, y) = \varphi(\mu_A(x), \mu_B(y))$ , where  $\mu_{A \rightarrow B}(x, y) \in [0, 1]$  measures the degree of truth of the implication between x and y.

The calculus of fuzzy If-Then rules, which defines the basic inference algorithm is applicable to a decision making problem.



A membership function that defines the implication relation can be expressed in number of ways. A review of various connectives is given in Dubois et al [19]. The most widely used implication operators are given in Table 1. 2

**minimum implication** is defined as

$$\mu_{A \rightarrow B}(x, y) = \min(\mu_A(x), \mu_B(y))$$

**product implication** is defined as

$$\mu_{A \rightarrow B}(x, y) = \mu_A(x) * \mu_B(y)$$

**Table 1. 2 Fuzzy Implication Operators**

<b>Implication Operators</b>	$\varphi(\mu_A(x), \mu_B(y))$
Mamdani	$\min(\mu_A(x), \mu_A(y))$
Larsen	$\mu_A(x) * \mu_A(y)$
Lukasiewicz	$\min(1, (1 - \mu_A(x) + \mu_A(y)))$
Bounded Product	$\max(0, (\mu_A(x) + \mu_A(x) - 1))$

### 1.5 Fuzzy Decision Making

Decision making, which is a cognitive process of selecting a course of action from multiple alternatives, is an important scientific, social and economic endeavor. A key factor in the decision making process is its ability to understand and interact with the individuals or groups have with the world. The fuzzy set theory is the most flexible framework that can be applied in decision making. It simulates the ability of human beings to deal with fuzziness of human judgments quantitatively and thus contributing to a greater level of human consistency or human intelligence in decision making process.

The original idea of fuzzy decision-making was proposed by Bellman et al [2], who introduced the concepts of fuzzy constraints, fuzzy objective and fuzzy decision. Fuzzy decision-making deals with non-probabilistic uncertainties and vagueness in the decision making environment. Fuzzy set approaches to decision-making prove to be the most appropriate especially in those cases when human evaluations and the modeling of human knowledge are required. Decision making problems are classified as individual decision making, multiperson decision making, multicriteria decision making, multistage decision making, etc.,.

In multiperson decision making (MPDM) process, the aim is to rank a finite number of alternatives with respect to a finite number of experts. Tender evaluations, public procurement processes, selections of applicants for positions, decisions related to best portfolio investments are certain real-life decisions in which MPDM models can be productively used. In solving a multiperson decision problem, one needs to know the importance or weights of the not equally important alternatives and also the evaluations of the alternatives with respect to the experts. One technique, often used, is the method of pairwise comparisons a concept which is widely used. It was introduced for voting problems by using only 0 and 1 in the pairwise comparison matrices.

Given  $n$  objects (alternatives), the decision maker(s) is (are) able to compare any two of them. In preference modelling, this assumption is called comparability. For all these pairs  $(i, j)$ ,  $i, j = 1, 2, \dots, n$ , the decision maker is requested to mention how much the  $i$ -th object is preferred (or more important) to the  $j$ -th one, the result is denoted by  $p_{ij}$ .

In real-life decision problems, pairwise comparison matrices are rarely consistent. Fuzzy preference modelling for multicriteria decision making is detailed in Fodor et al., [20]. Nevertheless, decision makers are interested in the level of

consistency of the judgements, which somehow expresses the goodness or “harmony” of pairwise comparisons in totality. It is to be noted that inconsistent judgements might lead to gross insensible decisions. Hence the study of consistency of preference relation is also an issue to be addressed to arrive at correct solution.

### ***Fuzzy majority***

It is a soft majority concept expressed by a fuzzy linguistic quantifier which is manipulated using fuzzy logic based calculus of linguistic quantifier proposition. Two fuzzy majority guided choice degree defined for multiplicative preference relation are quantifier guided dominance degree and quantifier guided non dominance degree.

### ***Fuzzy Quantifier***

Quantifier can be used to represent the amount of items satisfying a given predicate. Classical logic is restricted to the use of only two quantifiers “there exists” and “for all” which are closely related to “or” and “and” connectives respectively. But human mind is much richer and more diverse in its quantification, for e.g. almost all, few, many, at least half. Zadeh introduced the flexible knowledge representation tool the “fuzzy quantifier” to bridge the gap between the formal systems and natural language. There are two types of fuzzy quantifiers, absolute quantifier and proportional quantifier.

### ***Absolute quantifier***

*Absolute quantifier* is used to represent amounts that are absolute in nature e.g. about 5, less than 4, more than 5 etc. These quantifiers are subsets of  $R^+$ . An absolute quantifier is represented by a fuzzy subset  $Q$  such that for any  $r \in R^+$ ,  $Q(r)$  indicates the degree to which the amount  $r$  is compatible with the quantifier represented by  $Q$ . In general,  $Q(r)$  is the membership degree of  $r$  in  $Q$ .

### ***Propositional quantifier***

Propositional quantifier are used to represent preference values of variables in a set e.g. at least half, almost all, .It is represented by fuzzy subsets of unit interval  $[0,1]$ . For any  $r \in [0,1]$ ,  $Q(r)$  indicates the degree to which the proportion  $r$  is compatible with the meaning of the quantifier it represents. Fuzzy quantifiers are of three types: Increasing, Decreasing and Unimodal. The quantifier under the current study is increasing quantifier.

*Increasing quantifier* is characterized by  $Q(r_1) \geq Q(r_2)$  if  $r_1 > r_2$ .

The membership function of a increasing propositional quantifier can be represented

$$\text{by } Q(r) = \begin{cases} 0 & r < a \\ \frac{r-a}{b-a} & a \leq r \leq b \\ 1 & r > b \end{cases} \quad a, b, r \in [0,1]. \quad (1.1)$$

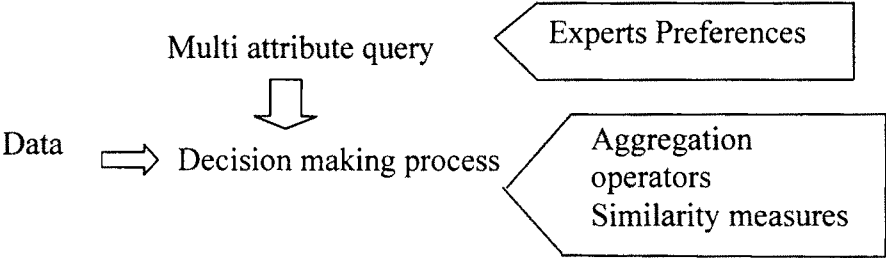
### **1.6 Objective**

Comparison of two objects or images is a fundamental property for many decision making model. Measuring meaningful image similarity is a dichotomy. The similarity of fuzzy numbers and aggregation operators characterize the imprecision built-in in a decision making problem. In this work the main ingredients namely the similarity measure and aggregation operators required for decision making are studied to obtain a more realistic decision process. The prime intent is to obtain well-suited approaches to the problems arising in decision making. Indeed, identifying a robust measure and defining a suitable aggregation operator, contributes significantly to the important goal of improving the practice of decision analysis.

While dealing with uncertainties, decision makers are commonly provided with information characterized by vague linguistic descriptor such as high risk, very close, very similar, etc. The objective of fuzzy set theory is primarily concerned with the

accurate quantification of such vague descriptions. Such vague descriptions are realized in more specific mathematical terms in fuzzy set theory. The main objective of the study is to check the effectiveness of the application of fuzzy sets in key areas of decision making.

In a decision support system related to a specific application, the decision making process has to be defined and the answers to the queries need to be extracted as shown in Fig. 1.7. Depending on the application, the decision making process is based on aggregation of information using aggregation operators and as well requires similarity measure to match the content in the information. One of the key objectives for any decision support system is to provide the required operators and measures for the decision making process. The fuzzy-engine includes several aggregators, similarity measures, and membership functions to express fuzzy attributes.

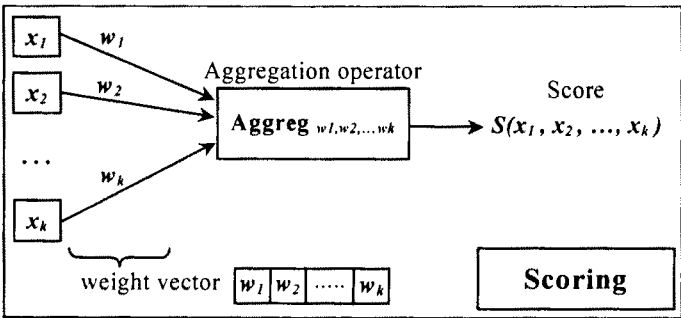


**Figure 1.7 Decision support system**

In network security, irrepressible growth of complex interconnectedness of information systems besides its obvious benefits unfortunately brought up the questions of their vulnerability. Practically universal access to computers has enabled hackers and would be terrorists to attack information systems and critical infrastructures worldwide. Fuzzy preference relation, based on fuzzy satisfaction function can be applied for comparison of attack signatures. The satisfaction degree of an arithmetic comparison relation of two fuzzy numbers is exploited in construction of fuzzy preference relation.

Fuzzy signatures are then combined by fuzzy aggregation operators. Problems that arise while comparing such signatures can be resolved using appropriate similarity measures and aggregation operators. Therefore, qualitative fuzzy decision system is achieved if the similarity measures and aggregation operators are defined properly. An agent based control for network security is modeled by Wang et al., [75].

A fuzzy query is defined on the basis of attributes or variables that are represented by fuzzy measures. However, these variables can be characterized by different degrees of importance that can be represented by weights which correspond to the user preferences. In ranking-based decision making processes, the values of attributes over the database are used for scoring each element of the database with respect to the query and the user preferences. The scores are calculated using similarity measures and aggregation operators, as shown in Fig 1.8. The scores are represented by user preferences  $(w_1, w_2, \dots, w_k)$ , fuzzy weighted aggregation coming out with a score value  $S$  for a data vector represented by  $(x_1, x_2, \dots, x_k)$ .



**Figure.1.8** Score calculation for a given query

The more the object fits the vague predicate the larger is its membership value. Fuzzy number can be viewed as representing an opinion poll of human thought or an expert's opinion. Fuzzy numbers which share the same most promising values are quite common and the study of their similarity is very essential in most decision making problems. Clearly the similarity measures largely influence the final outcome

of a decision making problem, wherein inaccurate measures might lead to totally diverse solution. The underlying premise of the work is to deeply understand the support, the difference of true membership, the difference of false-membership, to significantly distinguish the directions of difference (positive and negative), and properly identify the similarity of fuzzy numbers, so that when they are applied in areas of decision making and pattern recognition they provide more robust values. In chapter 2 a unique similarity measure is been defined to measure the similarity of fuzzy numbers.

Appropriate choice of aggregation operators is another parameter which greatly influences the consequence of decision making. It is also important to adequately describe which operator is best suited for final decision to be acceptable or consistent. It is extremely important to choose the t-norm and t-conorm operator, different operators lead to radically different results in fuzzy systems. In Chapters 3, the aggregation operators suitable for Mamdani fuzzy systems are analyzed.

In a decision making process involving more than one person where each may also have reached a different conclusion as to what is best for the organization. The impasse usually is not because of a lack of good analyses, but a lack of ability to synthesize the analyses that have been made. Any complex situation that requires measurement and/or synthesis, then pairwise comparison is a good candidate. Due to the complexity of most decision making problems, individuals preferences may not satisfy formal properties, so fuzzy preference relations are best suited and are required to verify certain fundamental properties. Consistency is one of them, and in turn it is associated with the transitivity property. Many properties have been suggested to model transitivity of fuzzy preference relations. The prime intent of latter part of the work is to define a suitable aggregation operator for a multiperson decision making problem with fuzzy preference relation as representation base. The thesis is designed

as a unified whole in which each chapter relates its contents to a decision making problem.

### **1.7 Thesis contribution and organization**

The necessary background to understand the developments of the subsequent chapters are provided in this chapter.

In chapter 2 a similarity measure is defined to calculate the degree of similarity of generalized fuzzy numbers. Since fuzzy numbers can very well represent the shape of objects, the study of their similarity measure is very crucial in areas of decision making, pattern recognition and fuzzy systems. In cases of inadequate data, most decision makers rely upon expert's knowledge to carry out simulated modelling of the problem. The generalized fuzzy numbers have been approved more flexible and more intelligent than the normalized fuzzy number since it takes the degree of confidence of the decision-makers' opinions into account. The new measure integrates the concept of center of gravity of points of fuzzy numbers and fuzzy distance between the base of fuzzy numbers, rather than the geometric distance adopted by the other existing measures.

To examine the performance of the rationality and discriminating ability of the proposed measure, a comparative study is conducted on cases used in the previous literatures Chen [5], Hsieh and Chen [31], Shi Jay Chen and Shi Meng Chen [64], Deng Yong et al., [16], Shi Jay Chen [62]. In addition to provide supportive evidence that the current measure mirrors the human expert's opinion, the similarity measure is applied to a shape recognition problem. A study of close relation between objects that are round in shape is obtained using the new measure. Lastly, the application of similarity measure of fuzzy number to a decision making problem and to a fingerprint matching problem is illustrated with an example.



In Chapter 3, a study of appropriate antecedent connector models for Mamdani fuzzy logic systems is made. The compensatory *and* and SOWA aggregation operators with different combination of the  $t$ -norms and  $t$ -conorms were taken. The  $t$ -norms considered are product and bounded difference, and the  $t$ -conorms considered are algebraic sum and bounded sum. The models are examined for separability. Mamdani product implication is taken for study.

In chapter 4, a multiperson decision making problem with fuzzy preference relation is chosen for discussion. Two weighting functions, linear weight generating function and exponential weight generating function are defined for generalized mixture operator (GMO). The mathematical properties of the GMO are studied and illustrative example from literature is taken to provide a comparative analysis of the selection process obtained by OWA operator and GMO with linear and exponential weight generating function.

In chapter 5 a transformation function is defined to transform the preference relation proposed by Saaty [58] to a fuzzy preference relation. Analytic hierarchy process (AHP) introduced by Saaty [58] is a mathematical decision making technique that allows consideration of both quantitative and qualitative aspects of decisions. In order to apply the fuzzy majority concept in decision making the multiplicative preference relation in AHP proposed by Saaty is transformed into a fuzzy preference relation using a transformation function. Various consistency properties of the fuzzy preference obtained through the transformation function is studied. The newly defined transformation function is applied to a decision model for choosing the highly preferred electives by a set of students in a class, where they provide their preference to the electives in the form of Saaty's multiplicative preference relation. The resulting

multiplicative preference relation is then aggregated to a collective preference relation using the OWA and GMO's that are discussed in chapter 4.