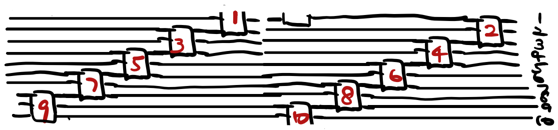
Clifford Channel Simulation

Architecture 1:



$$\mathcal{E}_{even} = \mathcal{E}_{2N} \circ \mathcal{E}_{2N-2} \circ \cdots \circ \mathcal{E}_2$$

$$\mathcal{E}_{odd} = \mathcal{E}_{2N-1} \circ \mathcal{E}_{2N-3} \circ \cdots \circ \mathcal{E}_1 .$$

$$\mathcal{E}_{odd} \circ \mathcal{E}_{even} := \mathcal{E}$$

- $j \in$ even: measure $Z_{j-1}X_jZ_{j+1}$, if outcome $= +$, do nothing.
  outcome $= -$, operate X_{j+1}

$$\mathcal{E}_j(\cdot) = \tfrac{1}{2}(\mathbb{I}+Z_{j-1}X_jZ_{j+1})(\cdot)\tfrac{1}{2}(\mathbb{I}+Z_{j-1}X_jZ_{j+1})$$
$$+ X_{j+1}\tfrac{1}{2}(\mathbb{I}-Z_{j-1}X_jZ_{j+1})(\cdot)\tfrac{1}{2}(\mathbb{I}-Z_{j-1}X_jZ_{j+1})X_{j+1}$$

- $j \in$ odd: measure Z_j. operate $Z_{j-1}X_jZ_{j+1}$

$$\mathcal{E}_j(\cdot) = Z_{j-1}X_jZ_{j+1}\tfrac{1}{2}(\mathbb{I}+Z_j)(\cdot)\tfrac{1}{2}(\mathbb{I}+Z_j)Z_{j-1}X_jZ_{j+1}$$
$$+ Z_{j-1}X_jZ_{j+1}\tfrac{1}{2}(\mathbb{I}-Z_j)(\cdot)\tfrac{1}{2}(\mathbb{I}-Z_j)Z_{j-1}X_jZ_{j+1}$$
$$= Z_{j-1}X_jZ_{j+1}(\cdot)Z_{j-1}X_jZ_{j+1} + Z_{j-1}(X_jZ_j)Z_{j+1}(\cdot)Z_{j-1}(Z_jX_j)Z_{j+1}$$

can also be modeled as two unitaries with equal probability.

- Perturbation mimics $\mathcal{L}_X = \sum_{j\in odd} \hat{X}_j\rho\hat{X}_j - \rho.$

$\Rightarrow$ on site $j \in$ odd, $\mathcal{E}_j(\cdot) = (1-p)\mathcal{E}_j^{(0)} + p\,\mathcal{E}_j^{(X)}$

$\mathcal{E}_j^{(X)}$: operate unitary X_j, or $\mathbb{I}_j$ with $1/2$, $1/2$ (or measure X_j).

$$\mathcal{E}_j^{(X)} = \rho + X_j\rho X_j = \tfrac{1}{2}(\mathbb{I}+X_j)\rho\tfrac{1}{2}(\mathbb{I}+X_j) + \tfrac{1}{2}(\mathbb{I}-X_j)\rho\tfrac{1}{2}(\mathbb{I}-X_j)$$

- Measure the following:

$S^{ZZ}_{n,m} := Z_{2n-1}X_{2n}\cdots X_{2m}Z_{2m+1}$ ✓

$S^{\mathbb{1}\mathbb{1}}_{n,m} := \mathbb{1}_{2n-1}X_{2n}\cdots X_{2m}\mathbb{1}_{2m+1}.$ ✓

$W^{ZZ}_{n,m} := Z_{2n}X_{2n+1}\cdots X_{2m-1}Z_{2m}$

$W^{\mathbb{1}\mathbb{1}}_{n,m} := \mathbb{1}\ X_{2n+1}\cdots X_{2m-1}Z_{2m}.$

- $\rho_t := \mathcal{E}^t(\rho_0)$ Initial states: $|++\cdots+\rangle$ & $|-++\cdots+\rangle$.

$S_1^{zz}(t, n-m=\frac{N}{2}; N) = \mathrm{Tr}(\rho_t S_{n,m}^{zz})$.

$S_1^{\mathbb{1}\mathbb{1}}(t, n-m; N) = \mathrm{Tr}(\rho_t S_{n,m}^{\mathbb{1}\mathbb{1}})$.

Purity $(t) = \mathrm{Tr}(\rho_t^2)$.

$S_2^{zz}(t, n-m; N) = \langle \rho_t | S_{n,m,L}^{zz} \otimes \mathbb{I}_R | \rho_t \rangle = \mathrm{Tr}[\rho_t S_{n,m}^{zz} \rho_t]$

$S_2^{\mathbb{1}\mathbb{1}}(t, n-m; N) = \langle \rho_t | S_{n,m,L}^{\mathbb{1}\mathbb{1}} \otimes \mathbb{I}_R | \rho_t \rangle = \mathrm{Tr}[\rho_t S_{n,m}^{\mathbb{1}\mathbb{1}} \rho_t]$

$W_2^{zz}(t, n-m; N) = \langle \rho_t | W_{n,m,L}^{zz} \otimes W_{n,m}^{zz} | \rho_t \rangle = \mathrm{Tr}[\rho_t W_{n,m}^{zz} \rho_t W_{n,m}^{zz}]$

$W_2^{\mathbb{1}\mathbb{1}}(t, n-m; N) = \langle \rho_t | W_{n,m,L}^{\mathbb{1}\mathbb{1}} \otimes W_{n,m}^{\mathbb{1}\mathbb{1}} | \rho_t \rangle = \mathrm{Tr}[\rho_t W_{n,m}^{\mathbb{1}\mathbb{1}} \rho_t W_{n,m}^{\mathbb{1}\mathbb{1}}]$

- The two-ρ quantities can require $\sim \exp(N)$ sample.

Assume at a given time, the unraveling gives

$\rho_t = \sum_i p_i |\psi_i\rangle$

$\Rightarrow \mathrm{Tr}(\rho_t^2) = \sum_{ij} p_i p_j |\langle \psi_i | \psi_j \rangle|^2$

$\mathrm{Tr}(\rho_t S \rho_t) = \sum_{ij} p_i p_j \langle \psi_i | S | \psi_j \rangle \langle \psi_j | \psi_i \rangle$

$\mathrm{Tr}(\rho_t W \rho_t W) = \sum_{ij} p_i p_j |\langle \psi_i | W | \psi_j \rangle|^2$

- Want to know how the observables approach the steady state.

$|O(t; N) - O(t\to\infty; N)| \sim c e^{-t/\tau}$, $\tau \sim N^\alpha$ (exponential)

or $c t^{-\beta}$, (powerlaw).