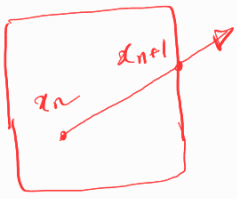
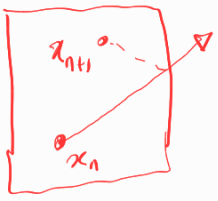
5.



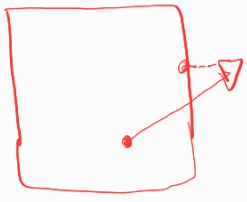
absorb
(get stuck at boundary, $v=0$)



mirror

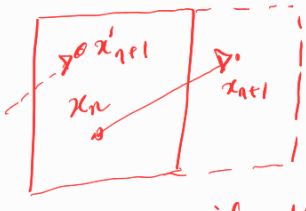
- Reset the particle to its local attractor.

→ Main cause is initial velocity of particles, which causes them to fly away!



Projection on the closest pt on Boundary.

infinity :- define fun value as -ve ∞, so we dont have to evaluate in that position.



Periodic: if the boundaries are periodic, then we can compute x_{n+1} as x'_{n+1} as the search space is periodic

6.

a) $$P[x \in S'] = \left(\frac{\text{Volume of inner}}{\text{Vol of outer}}\right)^{500} = \left(\frac{198}{200}\right)^{500} = 66 \times 10^{-3}$$

(as each dimension is independent of other, so we raise to the power 500 for 500 dim)

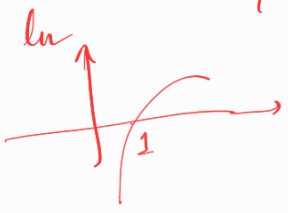
Insight: The P of x being within the inner section of S is extremely low → "Curse of Dimensionality"

b) $P\left[x \in \left[-(r-\epsilon), r-\epsilon\right]^n\right]$ (inner cube to maintain the distance to boundary)

$= \left(\frac{2(r-\epsilon)}{2r}\right)^n = \left(1-\frac{\epsilon}{r}\right)^n$ (all dimension independent of each other)

$= \exp\left(\ln\left(\left(1-\frac{\epsilon}{r}\right)^n\right)\right)$

$= \exp\left(n \underbrace{\ln\left(1-\frac{\epsilon}{r}\right)}_{<0}\right)$



so, $c = -\ln\left(1-\frac{\epsilon}{r}\right)$ as $c>0$

$= \exp(-nc)$

$\therefore P\left[x \notin \left[-(r-\epsilon), (r-\epsilon)\right]^n\right]$ = Prob of being outside the inner section and thus close to the boundary

$= 1 - \exp(-nc)$.

7 From lecture:

- per particle: its best pos so far → $p_{loc}^{(k)}$
- best position among all particles is $p_{glob}^{(k)}$
- each particle knows $p_{glob}^{(k)}$

Alternative:

each particle only knows a selected subset of

other particles (its neighbours)

— $P_{i,glob}^{(k)}$: best $P_{loc}^{(k)}$ among particles is neighbhours including the particle itself.

— different neighbourhood topologies have different effects on the swarm behaviour.

The three topologies :

① Fully Connected :

— limited exploration (as not enough space / time for each particle)

— Proper exploitation

② Ring Topology :

— few connections ⇒ best position is propagated slowly to all other, as connections can be very far away.

⇒ Good Exploration (as P_{glob} is not instantly found and every start rushing to it like FC saxum)

— But partly limited exploitation ⇒ cause it takes a lot of time for particles to reach local best position

③ Grid / von Neumann topology :

— Balance btw ① & ②

$\Rightarrow$ Topology is only in the connections between the particles