Redefine centrality measures for directed graphs:

$$C_D(e) = \frac{\deg_{in}(e)}{n-1}$$

$$C_B(e) = \frac{2}{(n-1)(n-2)} \sum_{s \neq e \neq t, t \neq s, \exists \text{ path from } s \text{ to } t} \frac{\sigma_{st}(e)}{\sigma_{st}}$$

$$C_C(e) = \frac{1}{2(n-1)} \sum_{t \neq e} \text{dist}(e, t) + \text{dist}(t, e)$$

Remarks:

- Betweenness Centrality:
  Vertex pairs (s, t) are skipped within the summation if there is no path in the graph from s to t.

- Closeness Centrality:
  The distance from vertex e to t is defined to be n if there is no path in the graph from e to t.

The analysis of the book graph with the help of the centrality measures Degree Centrality (C_D), Betweenness Centrality (C_B), Closeness Centrality (C_C) yields the following values (HITS authorities and PageRank for comparison).

ID	Name	Authority	PageRank	C_D	C_B	C_C
00	Binaere Suche	0.19846	0.099122	0.16667	0.059398	23.524
01	Sortieren durch Einfuegen	0.1383	0.099119	0.095238	0.010153	23.726
02	Schnelle Sortieralgorithmen	0.44657	0.10705	0.2619	0.032849	23.536
03	Paralleles Sortieren	0.068749	0.057515	0.047619	0	23.917
04	Texte durchsuchen	0	0.0046512	0	0	30.536
05	Tiefensuche	0.28908	0.045889	0.14286	0.39983	15.429
06	Pledge-Algorithmus	0.066658	0.013234	0.047619	0.0085559	15.976
07	Zyklensuche in Graphen	0.3604	0.030535	0.14286	0.07501	15.833
08	PageRank	1.015e-39	0.012312	0.02381	0.031359	29.667
09	Multiplikation langer Zahlen	0.088453	0.020727	0.11905	0.13806	15.988
10	Euklidischer Algorithmus	0.070814	0.0052713	0.02381	0.01083	29.679
11	Sieb des Eratosthenes	0.024136	0.01001	0.02381	0.052749	16.274
12	Einweg-Funktionen	0.11424	0.033491	0.14286	0.20192	15.869
13	One-Time-Pads	0.029311	0.017325	0.047619	0.063202	15.774
14	Public-Key-Kryptographie	0.14487	0.03326	0.16667	0.17053	15.69
15	Teilen von Geheimnissen	0.0414	0.020656	0.071429	0.041773	16.048
16	Poker per E-Mail	0	0.0046512	0	0	30.345
17	Fingerprinting	0.015353	0.01472	0.047619	0.01717	15.988
18	Hashing	0.027337	0.0085136	0.047619	0.025552	16.821
19	Zufallszahlen	0.090435	0.030107	0.11905	0.36686	15.536
20	Fehlererkennende Codes	0.031479	0.010118	0.047619	0.025358	16.202

The analysis of the book graph with the help of the centrality measures Degree Centrality (C_D), Betweenness Centrality (C_B), Closeness Centrality (C_C) yields the following values (HITS authorities and PageRank for comparison).

ID	Name	Authority	PageRank	C_D	C_B	C_C
21	Turnier- und Sportligaplanung	0	0.0046512	0	0	29.56
22	Eulertour	0.15643	0.013646	0.071429	0.060685	16.179
23	Dynamische Programmierung	0.14406	0.01217	0.071429	0	27.869
24	Gewinnstrategie fuer ein Streichholzspiel	0	0.0046512	0	0	29.988
25	Alphabeta-Algorithmus fuer Spielbaeume	0.0076262	0.0065116	0.02381	0.030197	29.702
26	Gauss-Seidel Iteration	0.008281	0.0095759	0.02381	0	29.988
27	Kreise zeichnen	0	0.0046512	0	0	29.583
28	Faires Teilen	0	0.0046512	0	0	30.202
29	Mehrheitsbestimmung	0.024136	0.01001	0.02381	0.0014518	26.226
30	Broadcasting	0	0.0046512	0	0	41.048
31	Kuerzeste Wege	0.44654	0.025877	0.16667	0.16608	15.679
32	Rucksackproblem	0.10919	0.011041	0.047619	0.027613	16.226
33	Minimale aufspannende Baeume	0.25734	0.015111	0.095238	0.023209	16.024
34	Maximale Fluesse	0.14649	0.011554	0.071429	0.046158	16.298
35	Partnerschaftsvermittlung	0.068313	0.0060742	0.02381	0	16.643
36	Travelling Salesman Problem	0.33087	0.018625	0.11905	0.042809	16.202
37	Simulated Annealing	0.010034	0.010673	0.02381	0.1374	16.107
38	Kleinster umschliessender Kreis	0.0082115	0.0058915	0.02381	0.0073558	29.738
39	Topologisches Sortieren	0	0.0046512	0	0	43
40	Zahlen auf Deutsch aussprechen	0	0.0046512	0	0	43
41	Online-Algorithmen	1.015e-39	0.023256	0.02381	0	42
42	Bin Packing	1.015e-39	0.023256	0.02381	0	42