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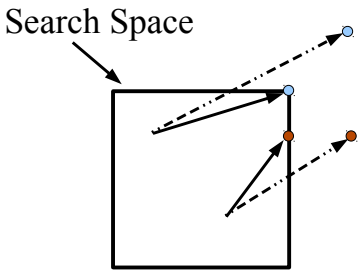
Exercises to

Swarm Intelligence

Summer 2022

Sheet 2

Problem 5:
Let $f : S \to \mathbb{R}$ be a function to be minimized in an n-dimensional bounded search space $S = [lb_1, ub_1] \times [lb_2, ub_2] \times \ldots \times [lb_n, ub_n] \subsetneq \mathbb{R}^n$. So S is a high-dimensional cuboid. When applying the PSO algorithm to such a function, particles may leave the search space. One possibility to handle this problem is to reset invalid particles to the search space boundary, as shown in the following figure:



Consider further possibilities for the treatment of invalid particles and discuss them.

Problem 6:

(a) Suppose a 500-dimensional search space is bounded by $S = [-100, 100]^{500}$. Determine the probability that a particle randomly placed (with uniform distribution) in S is in the hypercube $S' = [-99 \ldots 99]^{500}$. What does this mean?

(b) Prove: Suppose an n-dimensional search space is bounded by $S = [-r \ldots r]^n$, and let $\varepsilon > 0$ be arbitrary. Then the following is true: The probability that a particle is initialized such that the distance to the boundary of S is less than ε, is $1 - e^{-c \cdot (n)}$, with $c > 0$.

Problem 7:
In the PSO algorithm presented in the lecture, all particles of generation k access $p^{(k)}_{\text{glob}}$, the best solution found so far. This means that every particle can communicate with every other particle, and so the swarm is fully connected. Often, however, the set of communication partners (the so-called *neighbors*) of a particle is limited, and each particle has a subset of the population as its neighborhood at the beginning of the optimization. Particle i then uses the best solution $p^{(k)}_{i,\text{glob}}$ (note the additional i) in its neighborhood, instead of $p^{(k)}_{\text{glob}}$.

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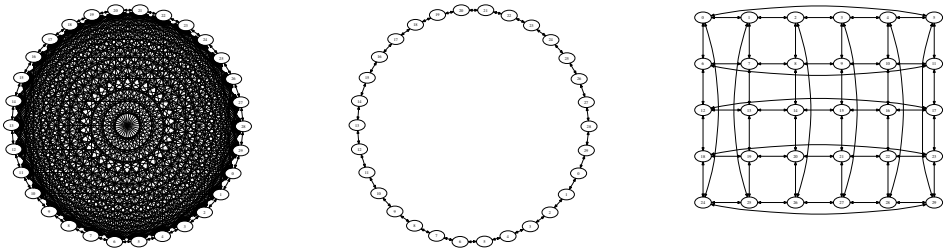
The neighborhood relationship can be represented as a graph, for example:



What influence do the different topologies have on swarm behavior? In particular, discuss the influence on exploration (the ability of the swarm to explore the search space, i.e., "to get everywhere") and exploitation (the ability of the swarm to find even better solutions in the vicinity of good ones).