

HILL CIPHER

- This multi-letter cipher is developed by the mathematician lester hill in 1929.
- An n-gram substitution may define using an invertible nxn matrix $A = a_{ij}$ as the key to map an n-character plaintext m_1, m_2, \dots, m_n to a cipher text n-gram
- $C = E_K(X) = KX \bmod 26$
- $X = D_K(C) = K^{-1}C \bmod 26$
- For $n=3$
- $C_1 = (k_{11}x_1 + k_{12}x_2 + k_{13}x_3) \bmod 26$
- $C_2 = (k_{21}x_1 + k_{22}x_2 + k_{23}x_3) \bmod 26$
- $C_3 = (k_{31}x_1 + k_{32}x_2 + k_{33}x_3) \bmod 26$

HILL CIPHER (Conti...)

- Example: encrypt 'meet b' using 2x2 hill cipher with the key $k = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix}$
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- $K^{-1} = \begin{pmatrix} 2 & -1 \\ -5 & 3 \end{pmatrix}$
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- Plain text will be written as ME ET BX
- Letters with there numerical values are as follows

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

- Numerical values for plain text letters are 12 4, 4 19, 1 23

HILL CIPHER (Conti...)

- **ENCRYPTION**

- $$\begin{pmatrix} c1 \\ c2 \end{pmatrix} = \begin{pmatrix} k11 & k12 \\ k21 & k22 \end{pmatrix} \times \begin{pmatrix} x1 \\ x2 \end{pmatrix} \mod 26$$

- $$\begin{pmatrix} c1 \\ c2 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 5 & 2 \end{pmatrix} \times \begin{pmatrix} 12 \\ 4 \end{pmatrix} \mod 26$$

- $C1 = (36 + 4) \mod 26 = 14 = o$
- $C2 = (60 + 8) \mod 26 = 16 = q$
- $C3 = (12 + 19) \mod 26 = 5 = f$
- $C4 = (20 + 38) \mod 26 = 6 = g$
- $C5 = (3 + 23) \mod 26 = 0 = a$
- $C6 = (5 + 46) \mod 26 = 25 = z$
- Encrypted text is : 'oq fg az'

HILL CIPHER (Conti...)

■ DECRYPTION

$$\begin{bmatrix} X1 \\ X2 \end{bmatrix} = \begin{bmatrix} k11 & k12 \\ k21 & k22 \end{bmatrix} \times \begin{bmatrix} C1 \\ C2 \end{bmatrix} \mod 26$$

$$\begin{bmatrix} x1 \\ x2 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} \times \begin{bmatrix} 14 \\ 16 \end{bmatrix} \mod 26$$

- $x1 = (28 - 16) \mod 26 = 12 = m$
- $x2 = (-70 + 48) \mod 26 = 4 = e$
- $x3 = (10 - 6) \mod 26 = 4 = e$
- $x4 = (-25 + 18) \mod 26 = 19 = t$
- $x5 = (0 - 25) \mod 26 = 1 = b$
- $x6 = (0 + 75) \mod 26 = 23 = x$
- Decrypted text is : 'me et bx'

HILL CIPHER (Conti...)

- Encrypt a message “CIPHER” using 3x3 hill cipher with key=[{2,1,1}, {1,1,2}, {1,0,-2}]
- $K = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}$
- $K^{-1} = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix}$
- Plain text: CIP HER
- Cipher text: BOY JIP