
PATTERN RECOGNITION

CS 669

Assignment 4

Polynomial Curve Fitting

Group 13

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1. **Basic Definitions**

1.1. Regression:

Regression is a statistical method in data science and statistics that models the connection between a dependent variable (the target) and independent variables (predictors or features). It quantifies how changes in predictors relate to changes in the target.

1.2. Univariate & Bivariate Data:

Univariate data refers to data with a single variable, while bivariate data involves two variables. In your context, Dataset 1 contains 1-dimensional data, and Dataset 2 contains 2-dimensional data.

1.3. Training & Test Data:

Training data is used to train the machine learning model, and test data is used to evaluate its performance. You are splitting the data into 70% for training and 30% for testing.

1.4. Polynomial Curve Fitting:

A polynomial curve fitting model is a type of regression model that is used to fit a polynomial function to a set of data points. The polynomial function can be of any degree, and the degree of the polynomial is determined by the complexity of the data.

1.5. Univariate Polynomial Curve Fitting

The general form of a univariate polynomial curve fitting equation is:

$$y=a_0 + a_1x + a_2x^2 +\cdots+ a_nx^n$$

where:
y is the output variable/dependent variable
x is the input variable/independent variable
$a_0,a_1,a_2,...,a_n$ are the coefficients/weights of the polynomial

1.5.1. Bivariate Polynomial Curve Fitting

The general form form of a bivariate polynomial curve fitting equation is:

$$y=a_0 + a_1x + a_2y + a_3xy + a_4x^2 + a_5y^2 + a_6xy^2 +\cdots+ a_{n+m}x^ny^m$$

where:
y is the output variable/dependent variable

x is the input variable/independent variable
$a_0, a_1, a_2, \ldots, a_{n+m}$ are the coefficients/weights of the polynomial

1.6. Linear Model with Gaussian Basis Functions:

A linear model uses linear combinations of basis functions to make predictions. Gaussian basis functions are functions that model data using Gaussian distributions. In your case, the centres of these functions will be determined by K-means clustering. The Gaussian basis function is defined as:

$$\phi(x, \mu, \sigma) = \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

Here: - x is the input variable. - μ is the mean or center of the Gaussian. - σ is the standard deviation, controlling the width of the Gaussian curve

1.7. Model Complexity:

Model complexity refers to the degree of a polynomial (for Dataset 1) or the number of basis functions (for Dataset 2). It influences the model's capacity to fit the data.

1.8. Regularization:

Regularization techniques are used to prevent overfitting by adding a penalty term to the model's loss function. It helps control the model's complexity.

1.9. Mean Squared Error (MSE):

MSE is a common metric used to measure the accuracy of a regression model. It quantifies the average squared difference between predicted and actual values.

2. Problem Description

The problem at hand involves modelling two different datasets using appropriate techniques and ensuring that the models do not overfit. The specific tasks are as follows:

- *Dataset 1:* Perform polynomial curve fitting with varying model complexity and regularization. Evaluate and compare model performance using MSE on training and test data.
- *Dataset 2:* Create a linear model with Gaussian basis functions, varying the number of basis functions and regularization. Again, evaluate and compare the model's performance using MSE on training and test data.

3. Solution Approach

3.1. For dataset 1:

- Split the data into training and test sets.
- For different training dataset sizes (10, 50, 100, and the complete training set), fit polynomial curves with degrees ranging from 2 to 9.
- Apply regularization to address overfitting and observe the weight values before and after regularization.
- Plot the approximated functions for different model complexities and regularization parameters.
- Calculate and plot MSE on training and test data for different model complexities and regularization parameters.

3.2. For Dataset 2:

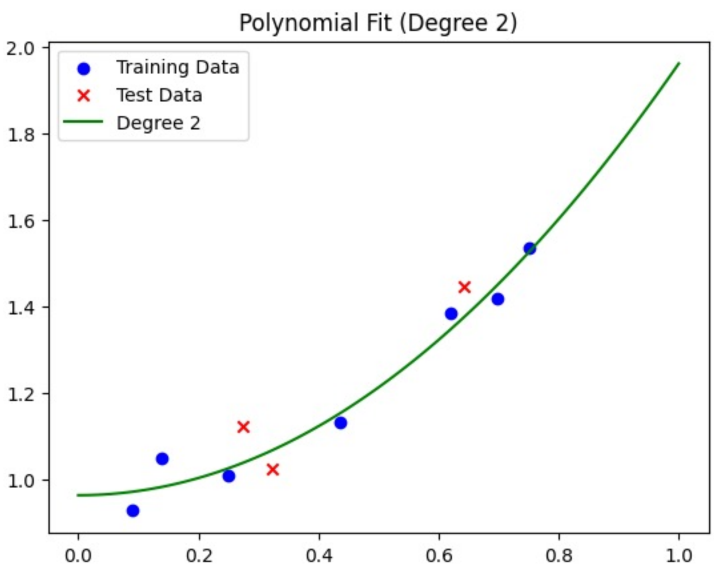
- Split the data into training and test sets.
- Use K-means clustering to determine the centres of Gaussian basis functions on the training data.
- Fit linear models with different numbers of basis functions (e.g., 2, 4, 8, 16, 32, 128, 256).
- Apply regularization to address overfitting and plot the approximated functions.
- Calculate and plot MSE on training and test data for different model complexities and regularization parameters.

4. Results

4.1. Dataset 1 (Univariate Dataset)

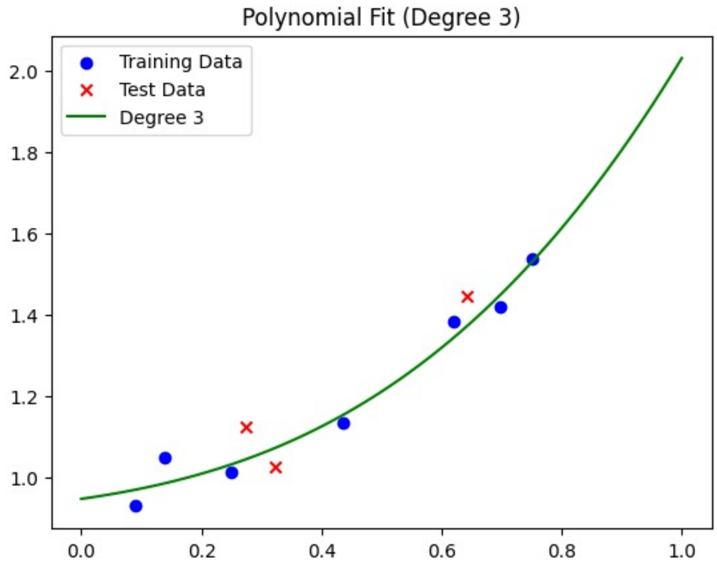
Without regularisation:

- ***No. of data points = 10***



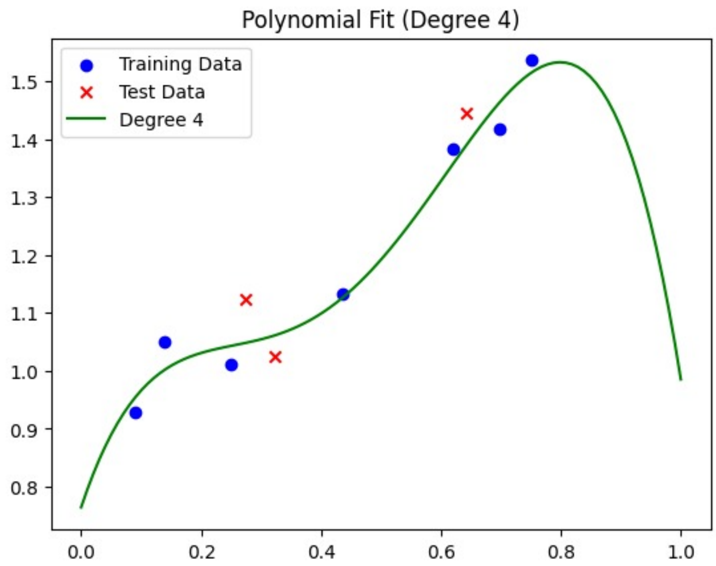
Training MSE: 0.0013, Test MSE: 0.0046

Weights: (0.996, -0.0000878, 0.9964)



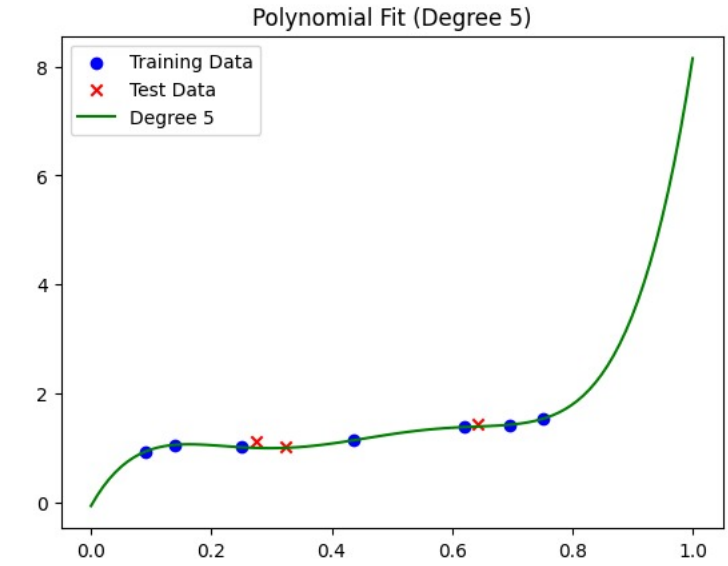
Training MSE: 0.0013, Test MSE: 0.0046

Weights: (0.4806, 0.3908, 0.2115, 0.9467)



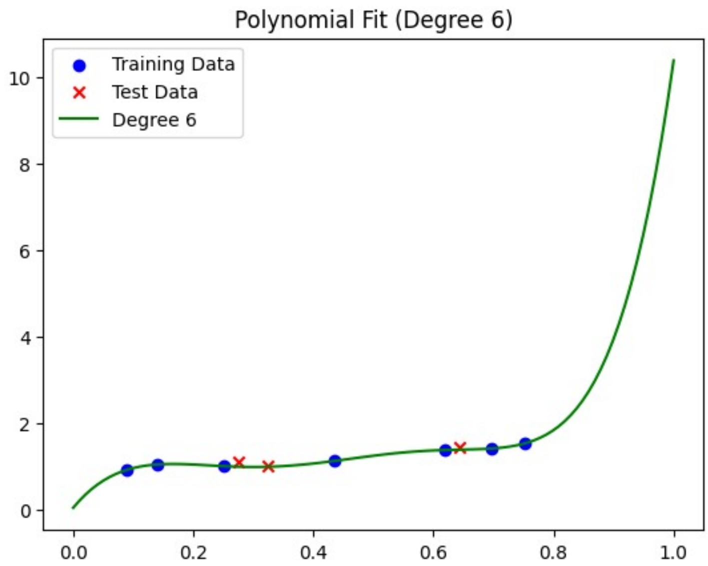
Training MSE: 0.0010, Test MSE: 0.0033

Weights: (-14.1677, 24.4874, -13.2060, 3.1072, 0.7643)



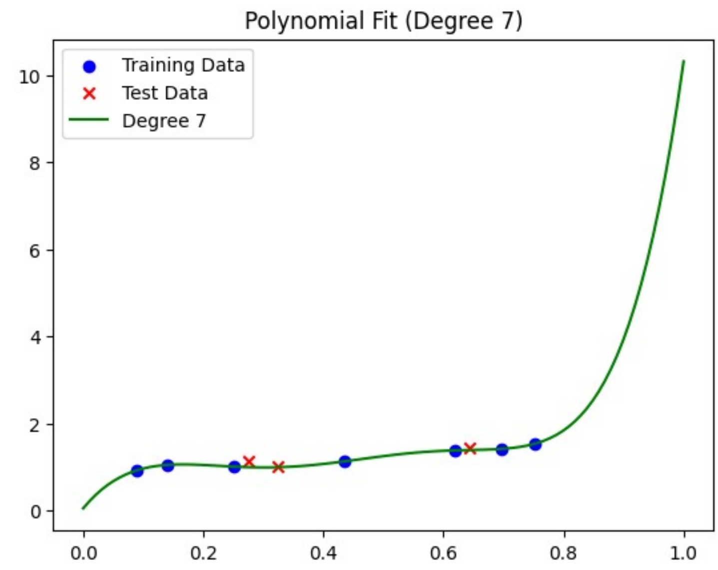
Training MSE: 0.0000115, Test MSE: 0.00684

Weights: (192.67, -423.72, 343.87, -124.38, 19.78, -0.069)



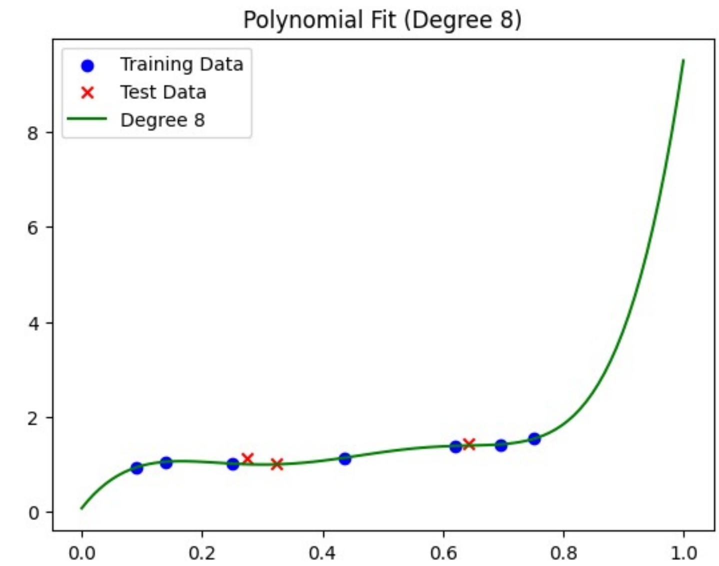
Training MSE: 0, Test MSE: 0.0065

Weights: (96.32, -45.24, -195.33, 236.32, -98.60, 16.85, 0.051)



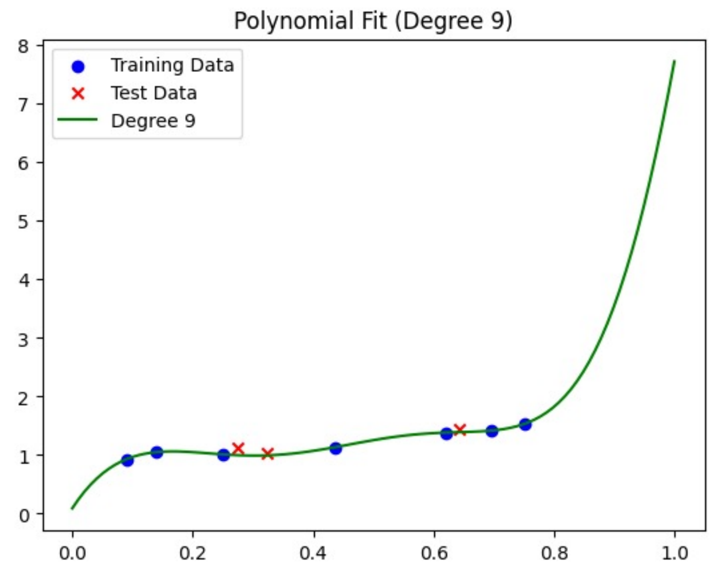
Training MSE: 0, Test MSE: 0.0065

Weights: (-44.75, 229.87, -206.10, -95.24, 202.11, -92.29, 16.28, 0.0711)



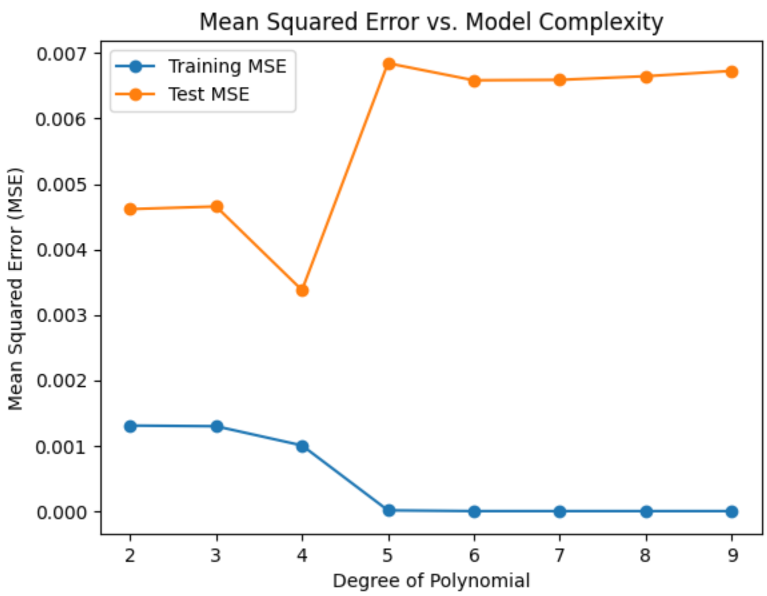
Training MSE: 0, Test MSE: 0.0066

Weights: (-148.49, 307.94, -34.08, -198.93, -6.55, 153.95, -81.44, 15.18, 0.11)

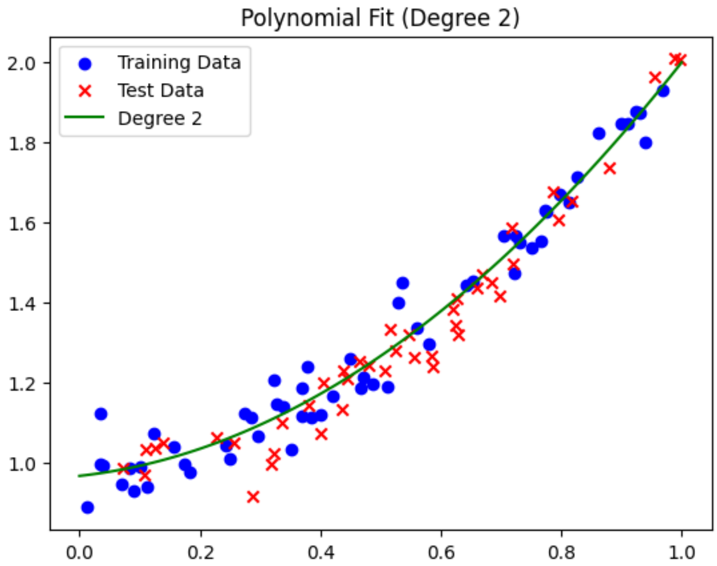


Training MSE: 0, Test MSE: 0.0067

Weights: (-238.24, 310.90, 111.87, -135.86, -142.99, 36.68, 117.36, -71.83, 14.14, 0.15)

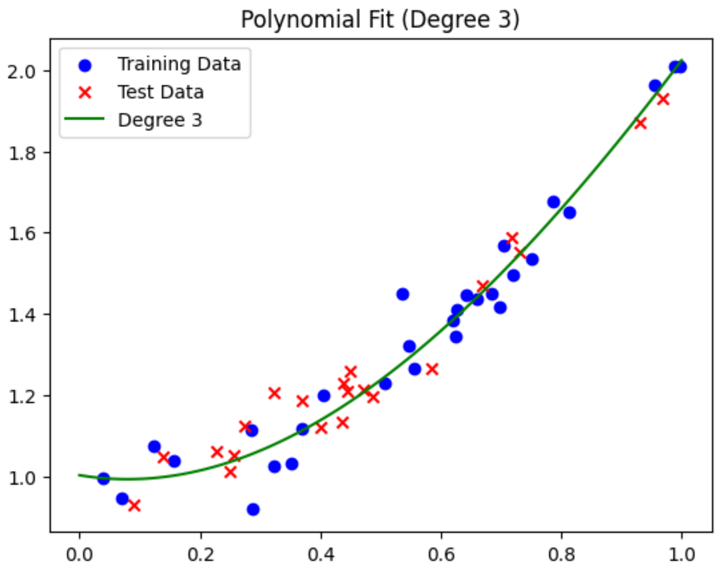


- ***No. of data points = 50***



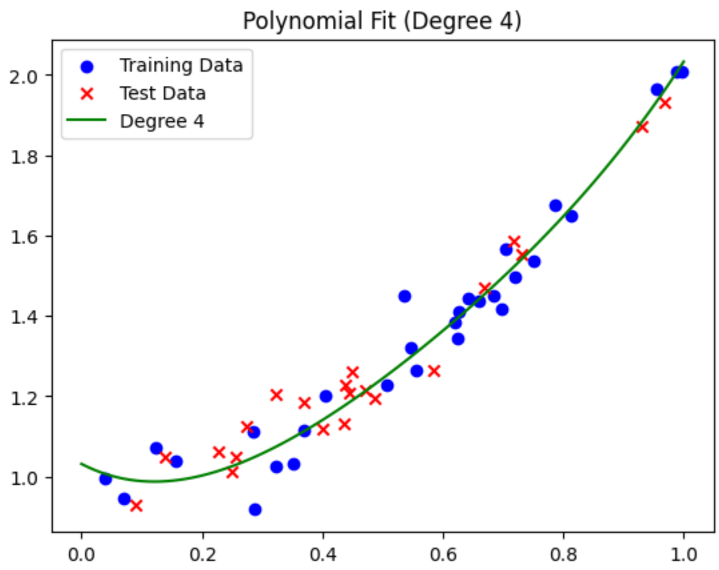
Training MSE: 0.0029, Test MSE: 0.0032

Weights: (1.072, -0.017, 0.980)



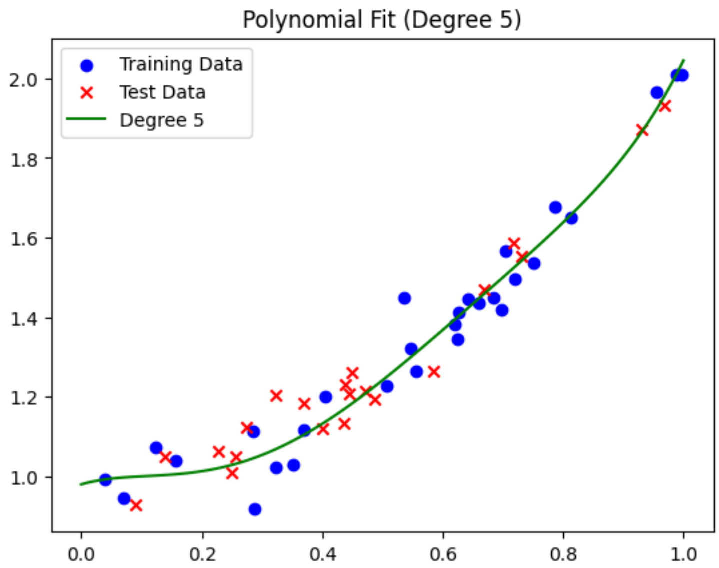
Training MSE: 0.0032, Test MSE: 0.0028

Weights: (-0.32, 1.59, -0.24, 1.00)



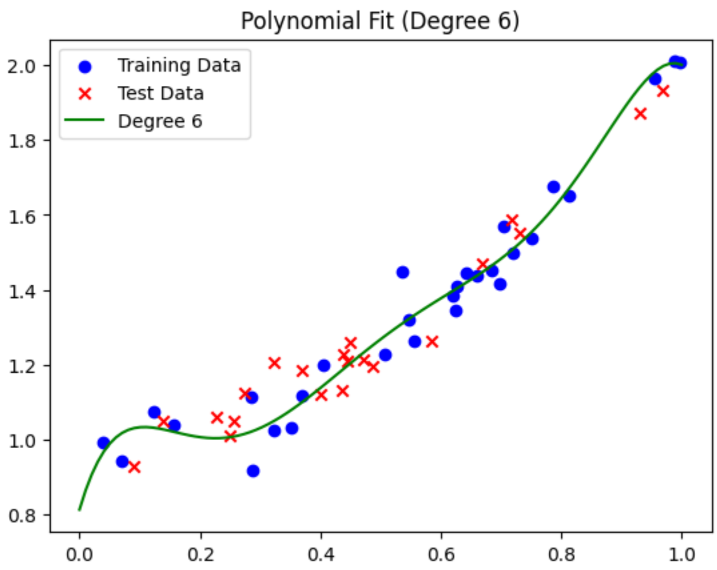
Training MSE: 0.0032, Test MSE: 0.0029

Weights: (1.57, -3.61, 3.81, -0.77, 1.03)



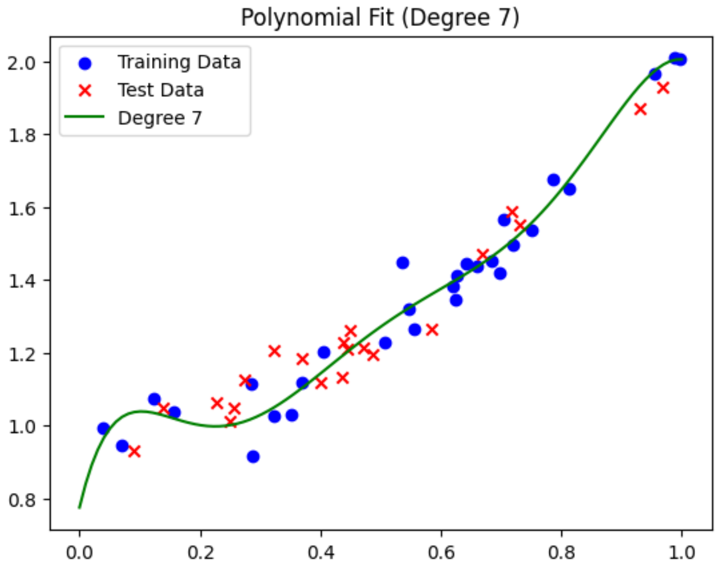
Training MSE: 0.0032, Test MSE: 0.0031

Weights: (7.71, -18.35, 14.93, -3.67, 0.43, 0.98)



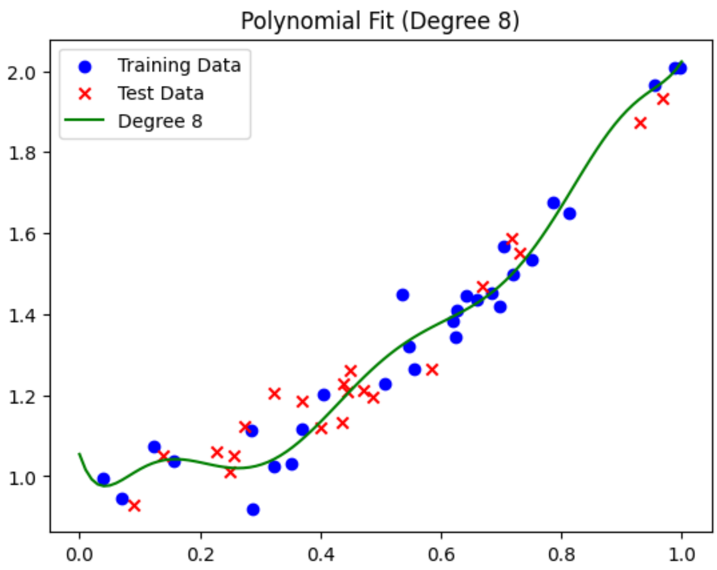
Training MSE: 0.0027, Test MSE: 0.0045

Weights: (-88.94, 277.09, -329.46, 185.85, -48.87, 5.53, 0.81)



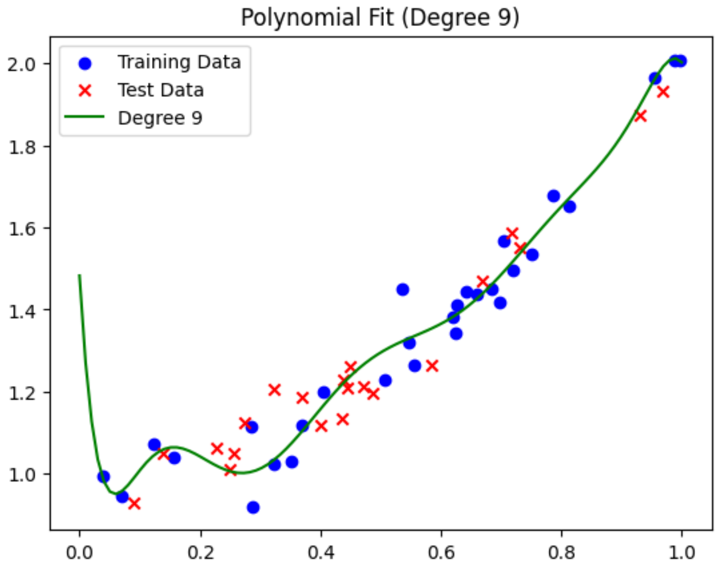
Training MSE: 0.0027, Test MSE: 0.0046

Weights: (53.36, -282.97, 558.62, -536.87, 267.34, -65.21 , 6.96, 0.77)



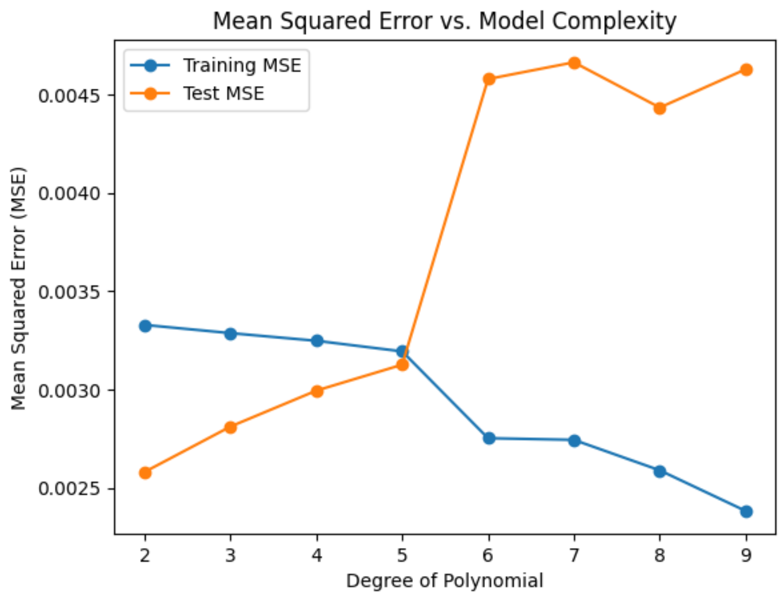
Training MSE: 0.0026, Test MSE: 0.0044

Weights: (907.59, -3754.84, 6304.36, -5502.75, 2654.17, -693.07, 90.12, -4.62, 1.05)

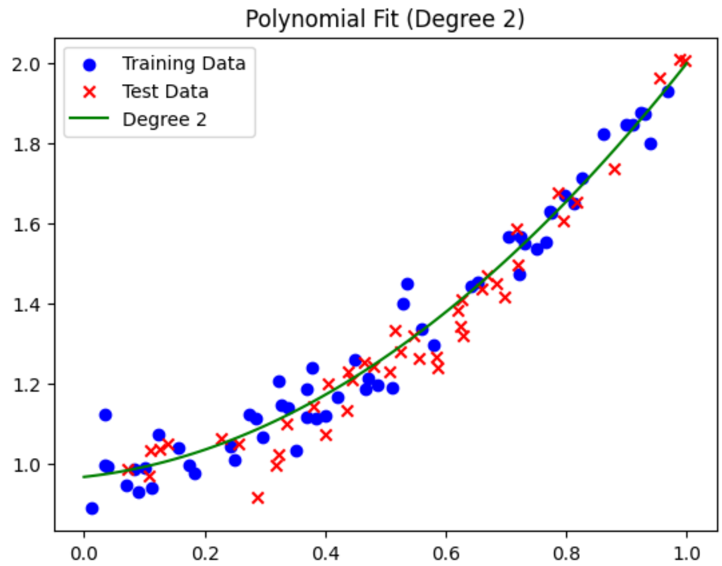


Training MSE: 0.0024, Test MSE: 0.0046

Weights: (-4856.85, 23396.79, -47554.80, 52946.68, -35061.82, 14026.74, -3291.25, 419.86, -24.82, 1.48)

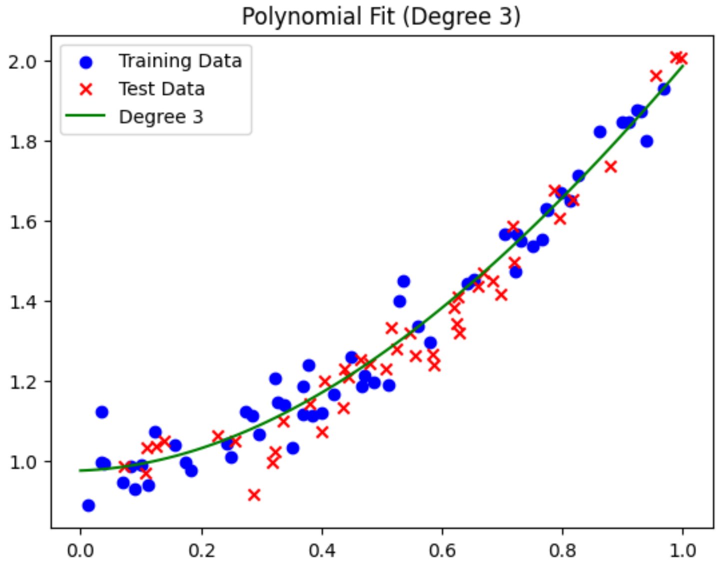


- ***No. of data points = 100***



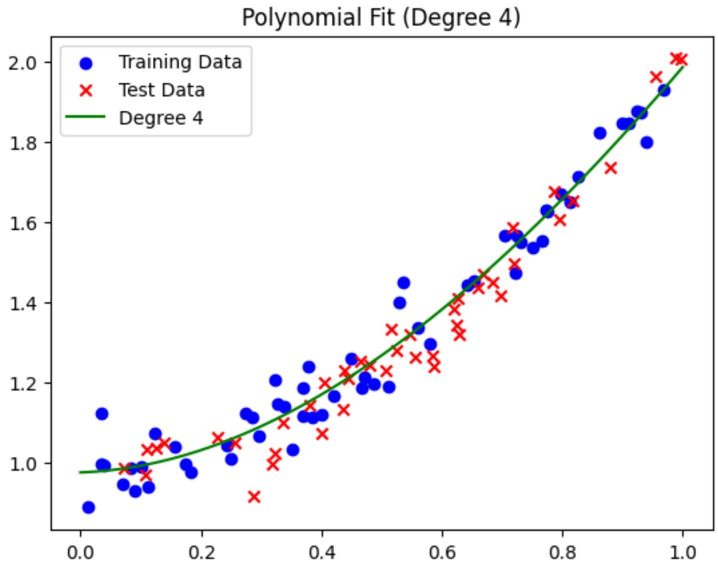
Training MSE: 0.0029, Test MSE: 0.0032

Weights: (0.86, 0.16, 0.96)



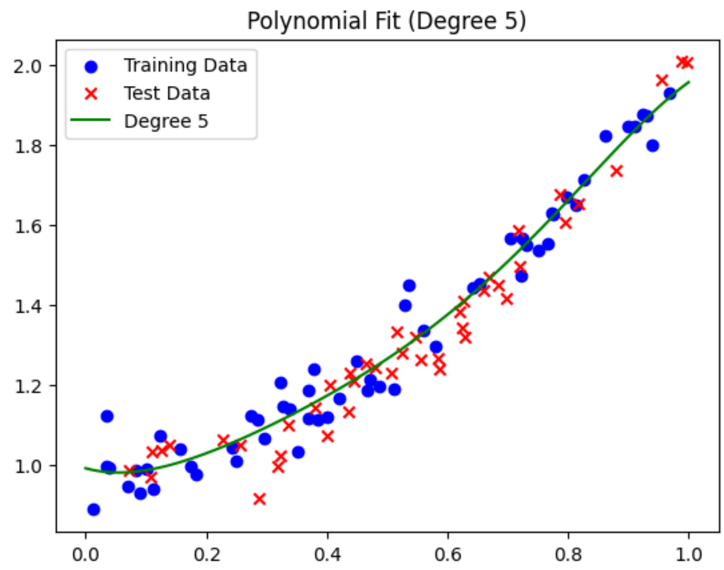
Training MSE: 0.0029, Test MSE: 0.0033

Weights: (-0.20, 1.16, 0.05, 0.97)



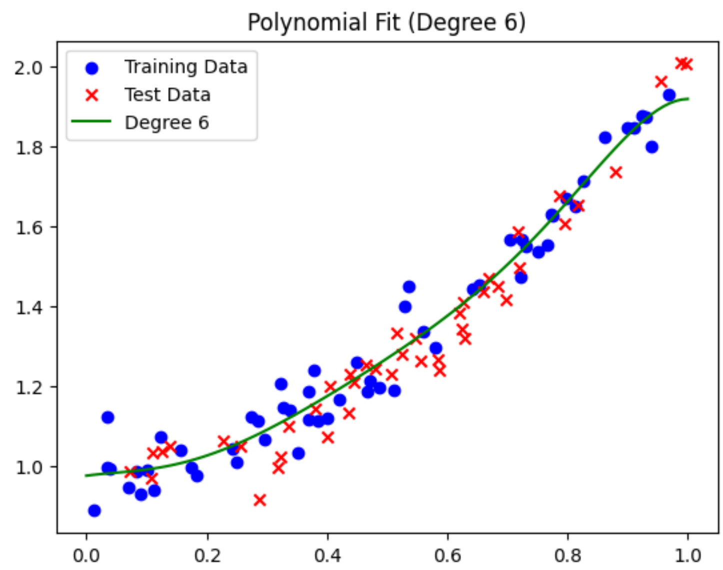
Training MSE: 0.0029, Test MSE: 0.0033

Weights: (0.0023, -0.21, 1.16, 0.05, 0.97)



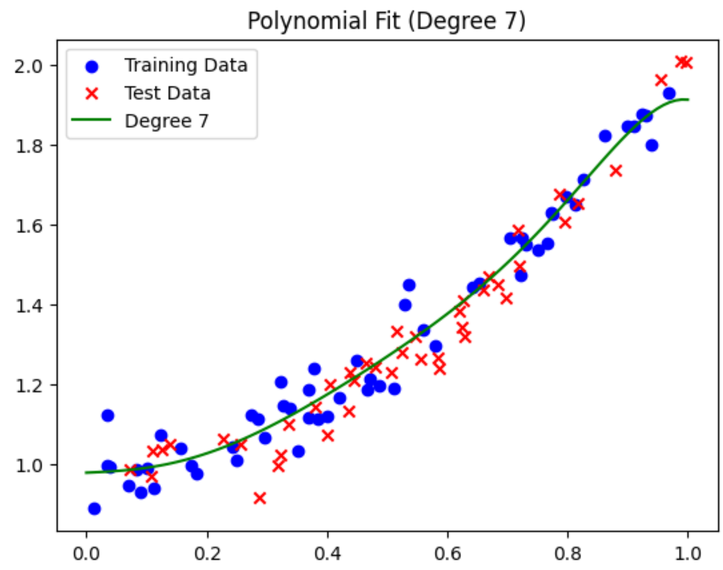
Training MSE: 0.0029, Test MSE: 0.0034

Weights: (-4.37, 10.59, -9.33, 4.48, -0.41, 0.99)



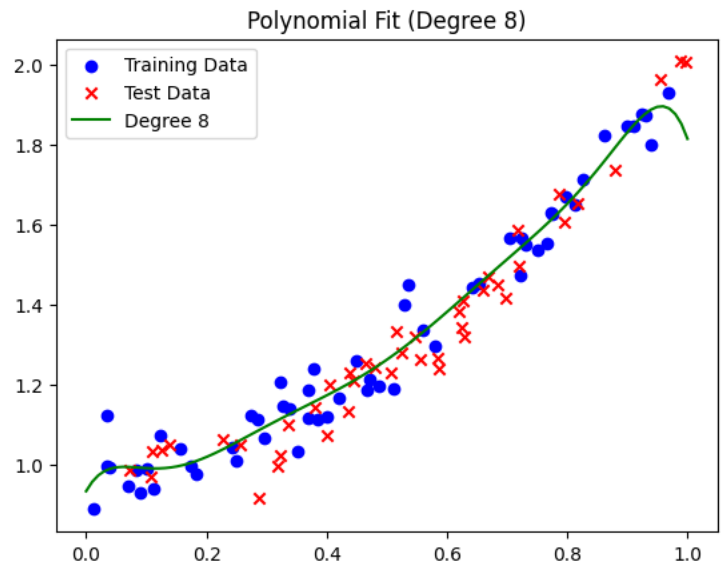
Training MSE: 0.0029, Test MSE: 0.0036

Weights: (-15.65, 41.11, -39.46, 16.55, -1.80, 0.20, 0.97)



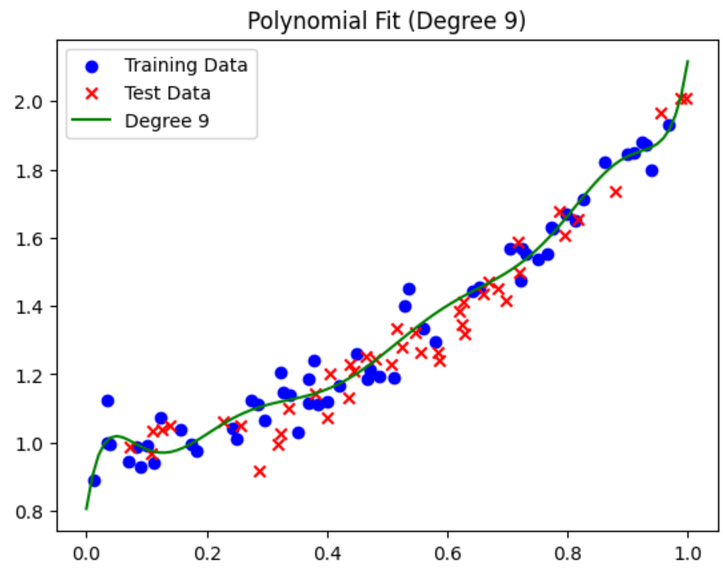
Training MSE: 0.0029, Test MSE: 0.0037

Weights: (-8.57, 13.96, 0.52, -11.51, 6.44, 0.013, 0.068, 0.98)



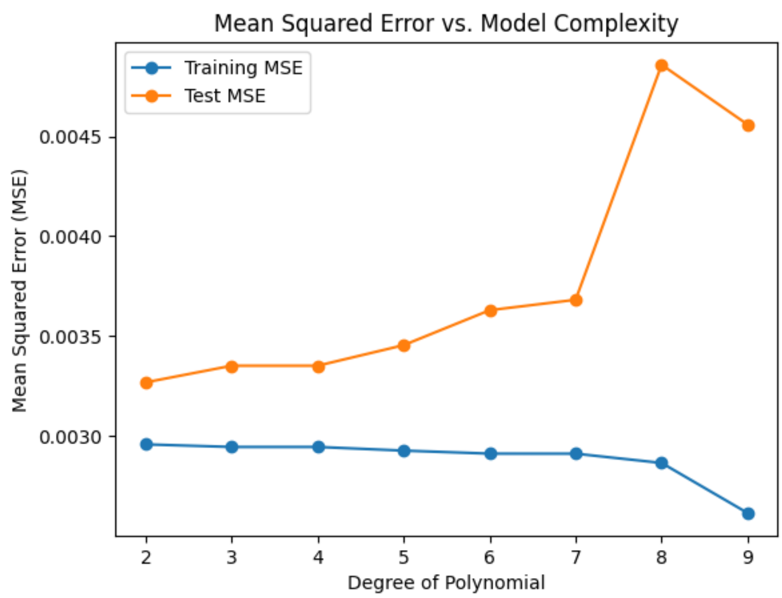
Training MSE: 0.0028, Test MSE: 0.0048

Weights: (-470.92, 1843.41, -2963.85, 2519, -1211.33, 326.05, -44.21, 2.72, 0.93)

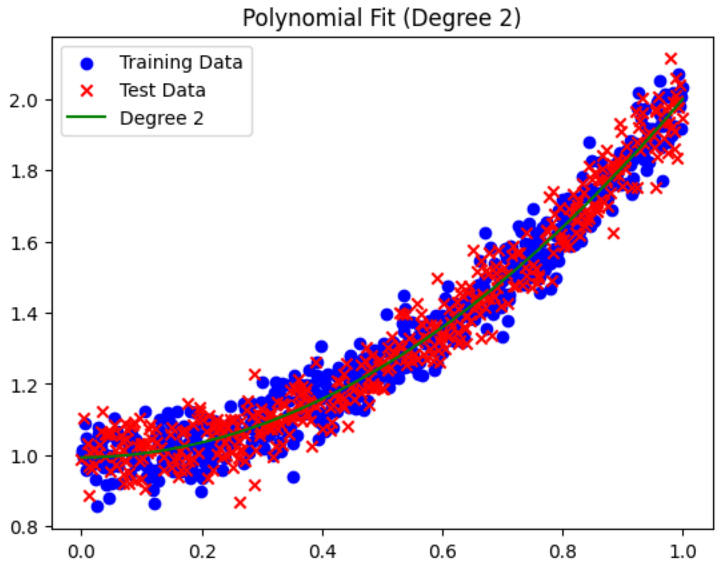


Training MSE: 0.0026, Test MSE: 0.0045

Weights: (4548.74, -20605.13, 39169.69, -40563.48, 24848.62, -9146.05, 1961.19, -223.81, 11.54, 0.80)

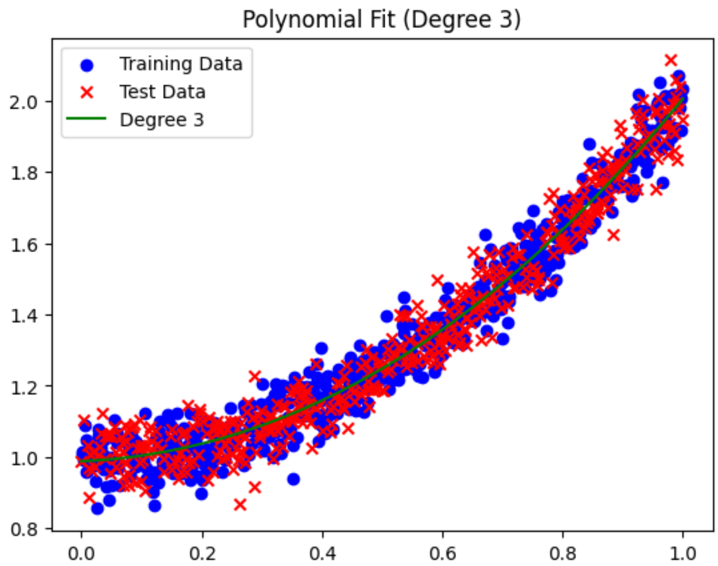


- ***No. of data points = Complete set***



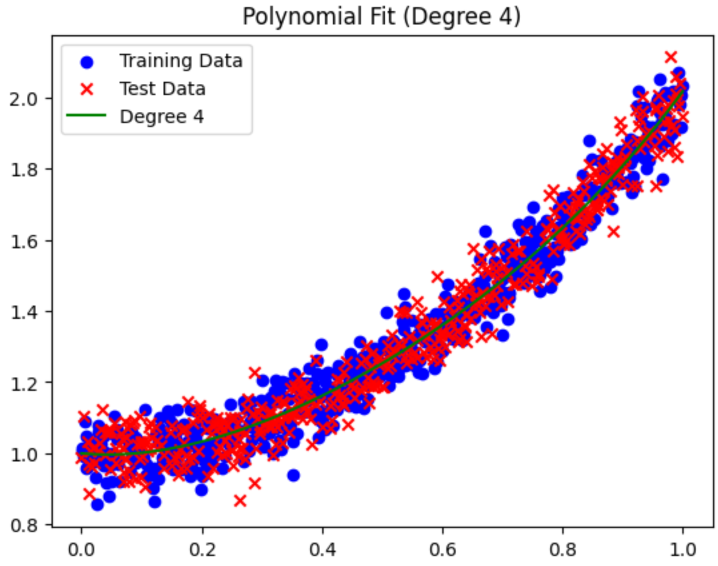
Training MSE: 0.0032, Test MSE: 0.0033

Weights: (0.99, 0.012, 0.99)



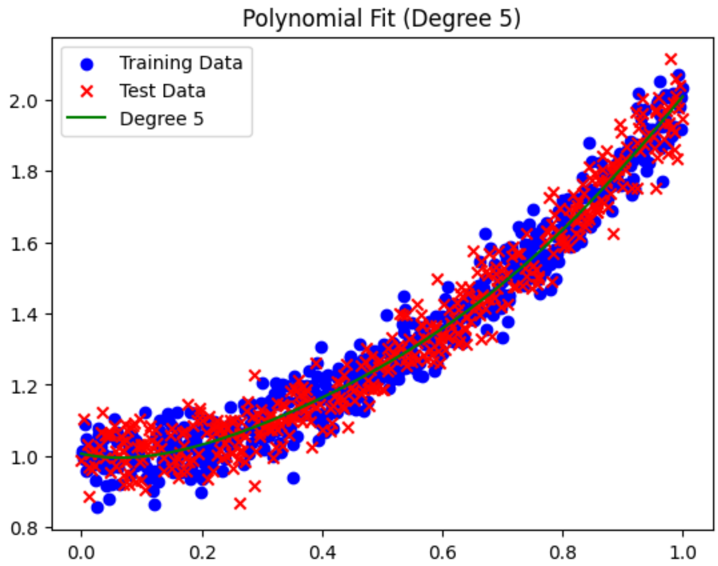
Training MSE: 0.0032, Test MSE: 0.0034

Weights: (0.10, 0.84, 0.07, 0.98)



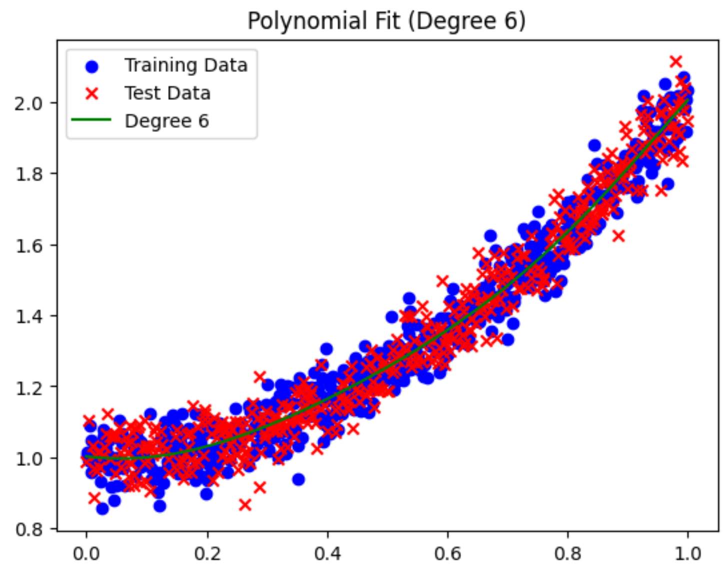
Training MSE: 0.0032, Test MSE: 0.0034

Weights: (0.82, -1.53, 1.89, -0.16, 0.99)



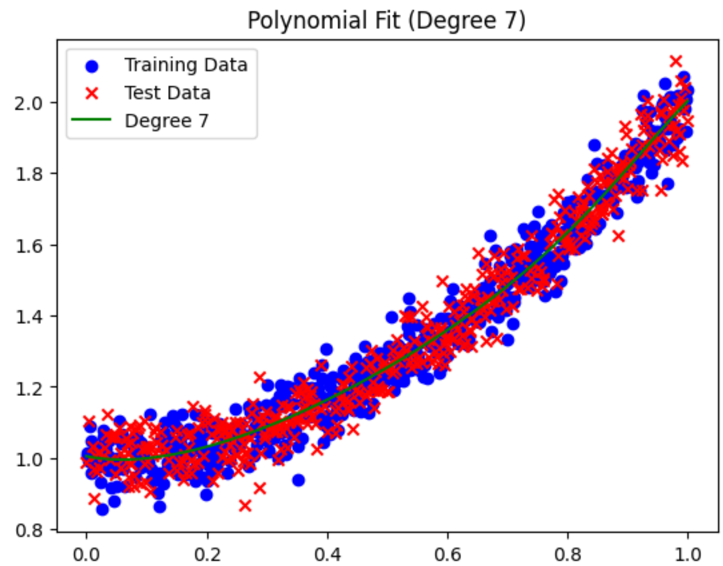
Training MSE: 0.0032, Test MSE: 0.0034

Weights: (-1.88, 5.49, -5.65, 3.41, -0.37, 1.00)



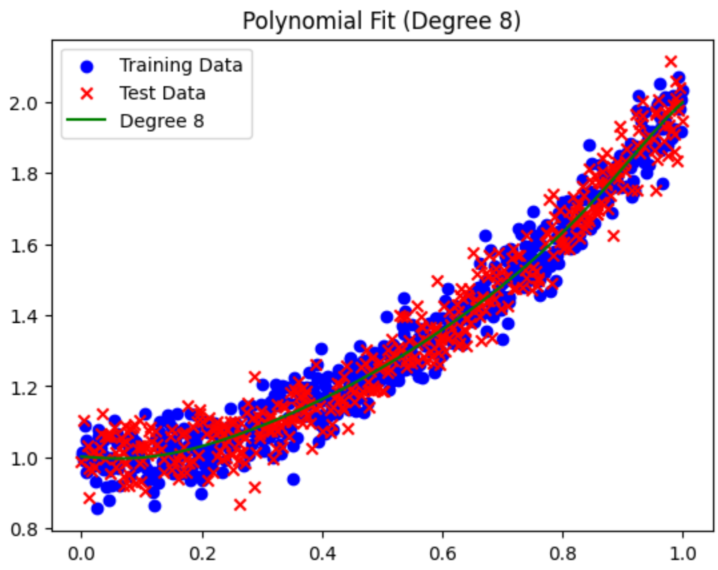
Training MSE: 0.0032, Test MSE: 0.0033

Weights: (-4.01, 10.14, -8.16, 1.63, 1.59 , -0.19, 1.00)



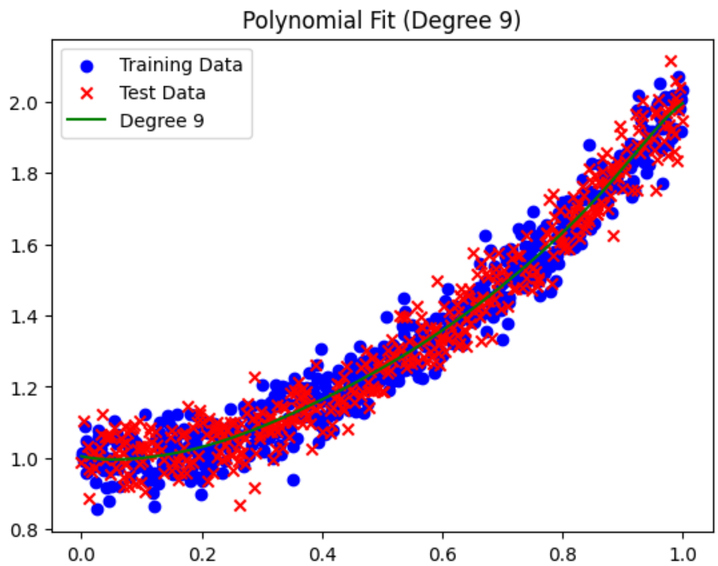
Training MSE: 0.0032, Test MSE: 0.0033

Weights: (-9.68, 29.95, -36.99, 24.65, -10.33, 3.75, -0.35, 1.00)



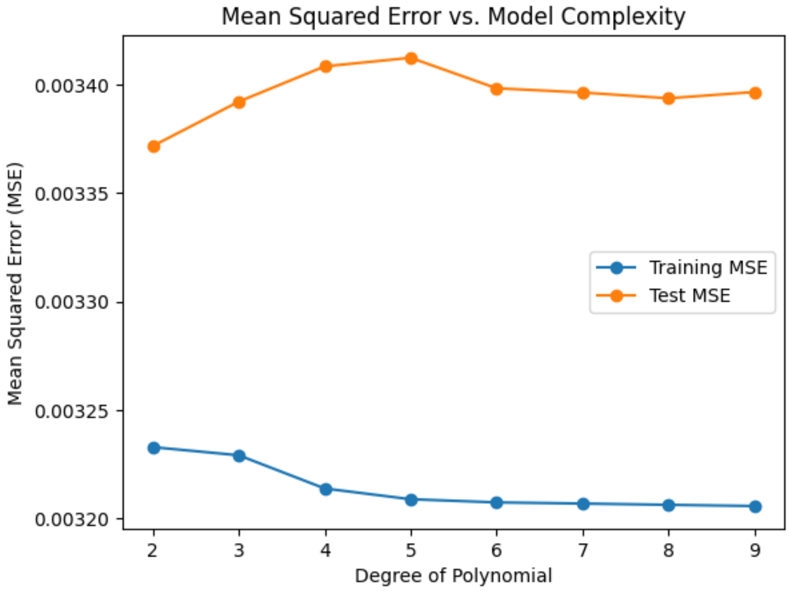
Training MSE: 0.0032, Test MSE: 0.0033

Weights: (-40.01, 150.28, -231.07, 186.43, -82.57, 18.19, -0.12, -0.13, 1.00)



Training MSE: 0.0032, Test MSE: 0.0033

Weights: (-152.07, 643.68, -1135.08, 1078.77, -597.49, 196.35, -38.69, 5.91, -0.40, 1.00)



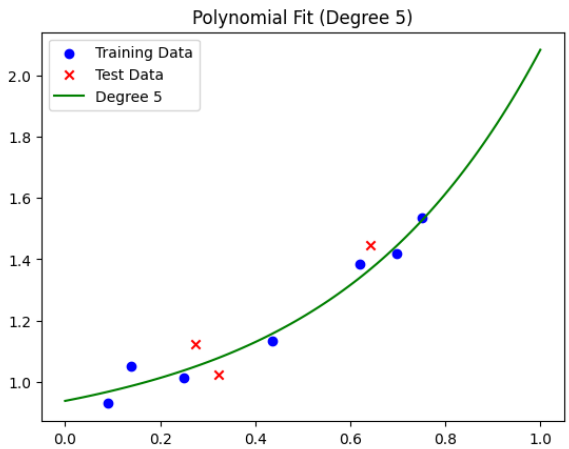
With regularization:

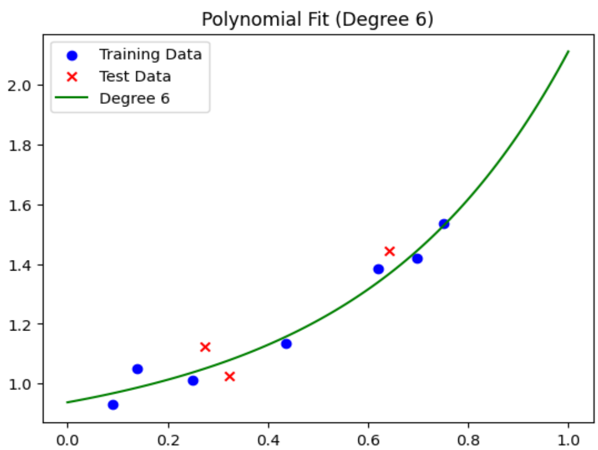
Let the regularization parameter be denoted by α. We have performed the experiment on 4 values of α = {0.01, 0.05, 0.1, 1}

- ***Dataset size = 10***

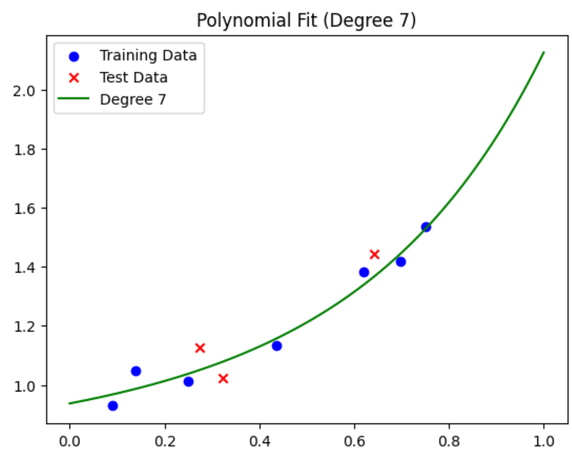
For training with *10 points in dataset*, from the MSE curve, we see overfitting from n = 5 onwards.

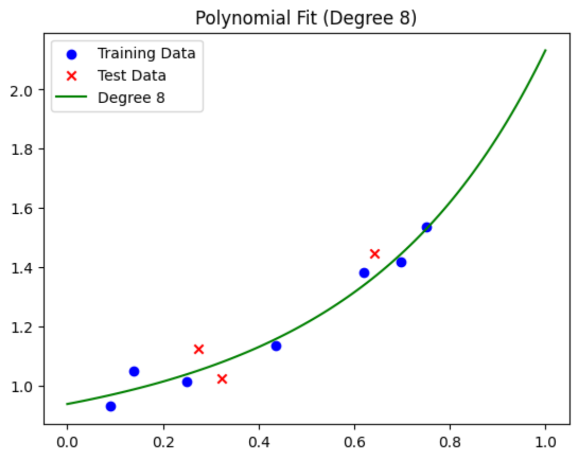
For $\boldsymbol{\alpha = 0.01}$

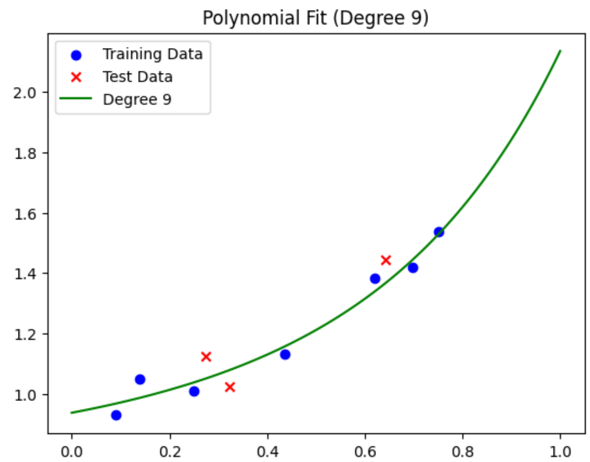


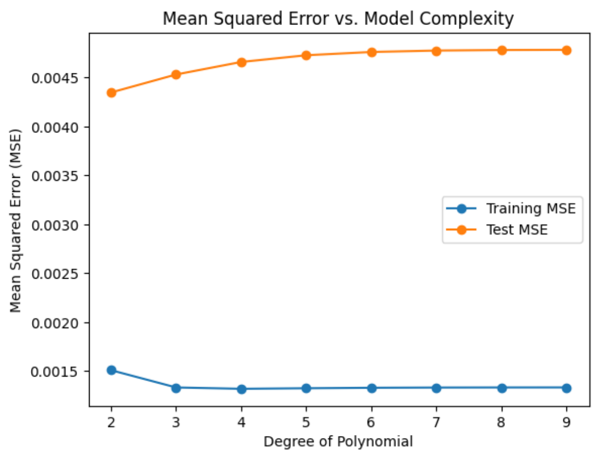


Weights: (192.67, 423.73, 343.88, -124.38, 19.78, -0.07)









Weights after regularization:

n=5 (192.67, 423.73, 343.88, -124.38, 19.78, -0.07)

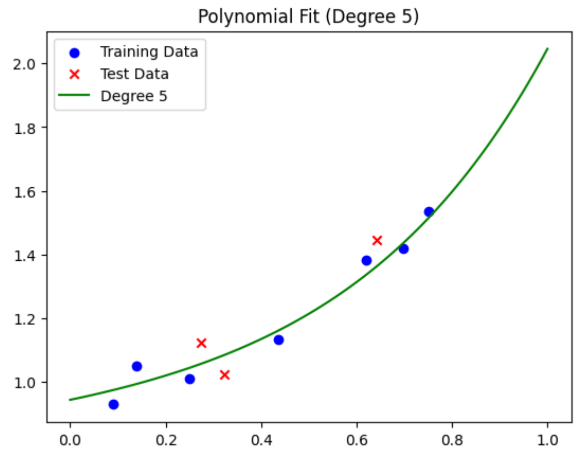
n=6 (96.33, -45.24, -195.34, 236.32, -98.61, 16.86, 0.05)

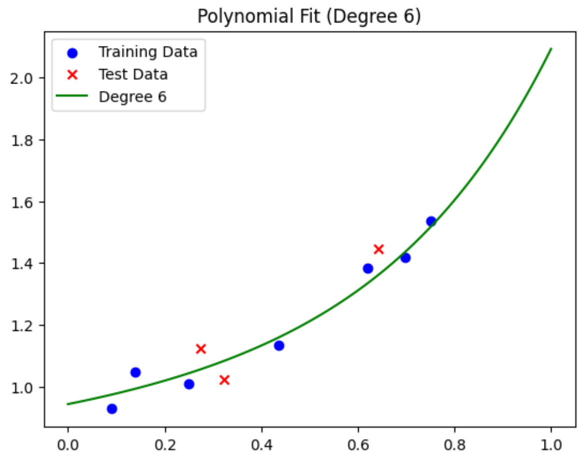
n=7 (-44.76, 229.88, -206.11, -95.24, 202.11, -92.29, 16.28, 0.07)

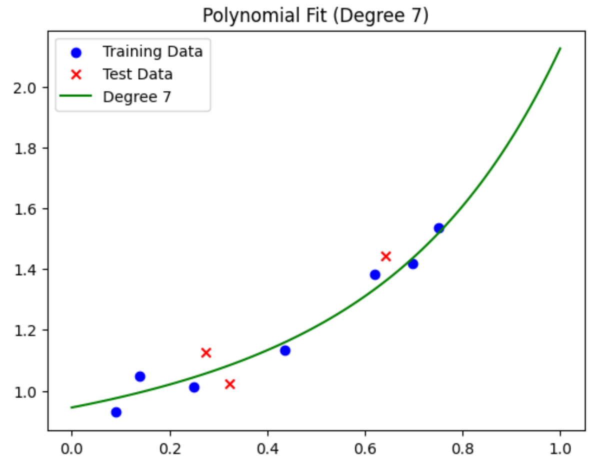
n=8 (-148.49, 307.94, -34.08, -198.93, -6.56, 153.95, -81.45, 15.19, 0.11)

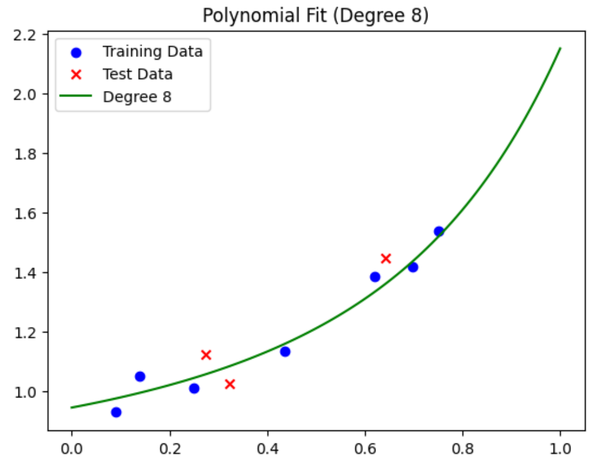
n=9 (-238.24, 310.91, 111.87, -135.86, -142.98, 36.69, 117.37, -71.83, 14.14, 0.15)

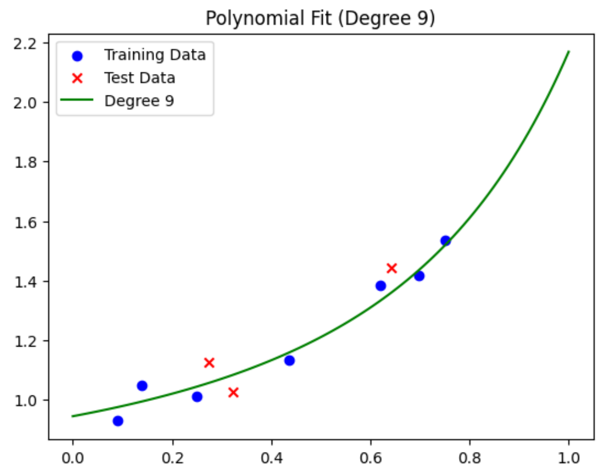
For $\alpha = 0.05$

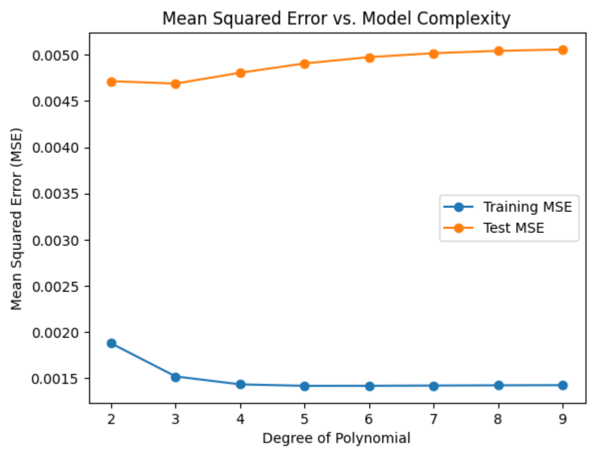








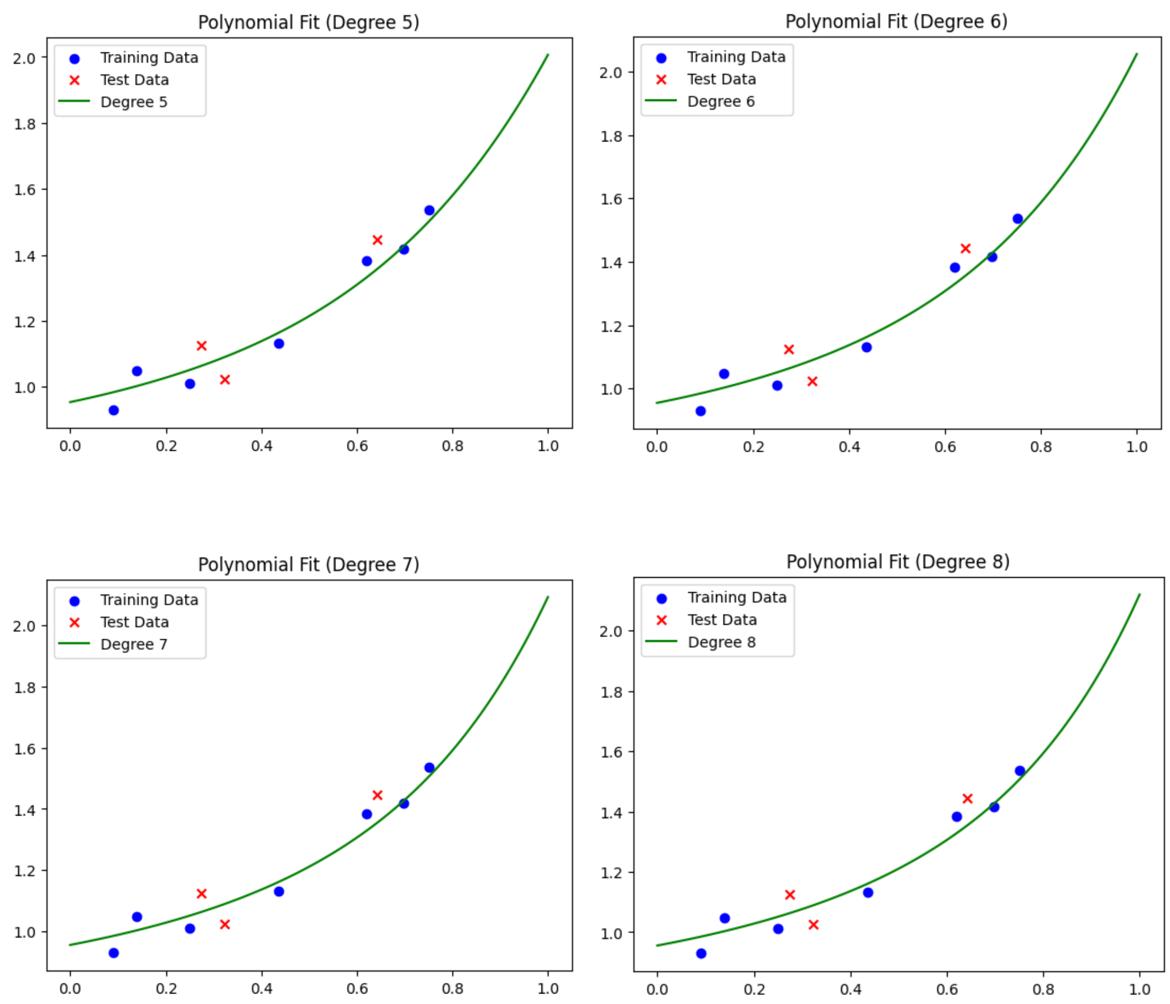


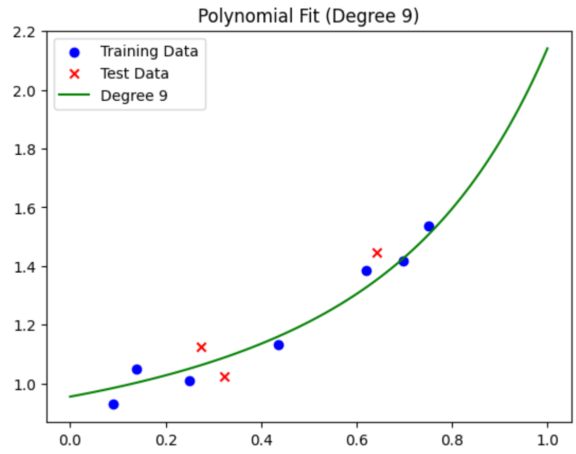


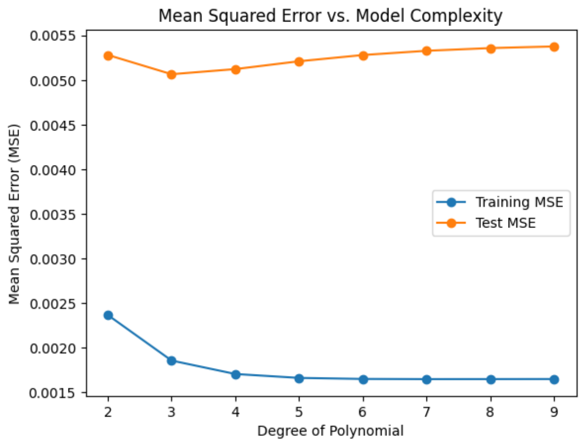
Weights after regularization:

n=5 (192.67, -423.73, 343.88, -124.38, 19.78, -0.07)
n=6 (96.33, -45.24, -195.34, 236.32, -98.61, 16.86, 0.05)
n=7 (-44.76, 229.88, -206.11, -95.24, 202.11, -92.29, 16.28, 0.07)
n=8 (-148.49, 307.94, -34.08, -198.93, -6.56, 153.95, -81.45, 15.19, 0.11)
n=9 (-238.24, 310.91, 111.87, -135.86, -142.98, 36.69, 117.37, -71.83, 14.14, 0.15)

For $\boldsymbol{\alpha = 0.1}$



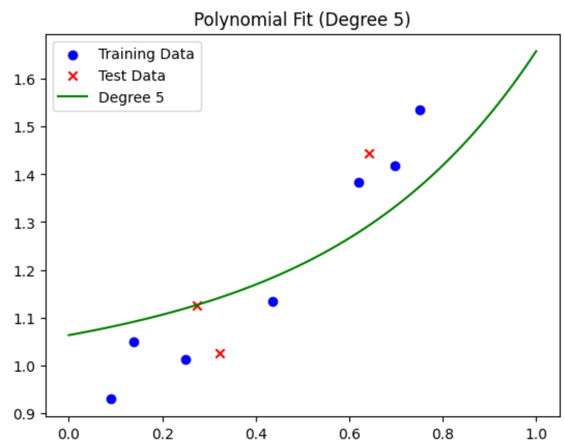


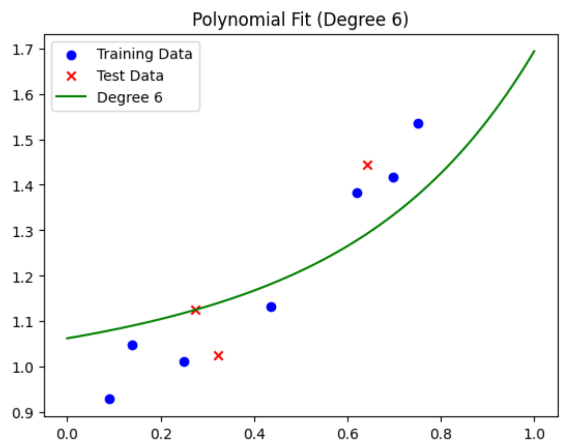


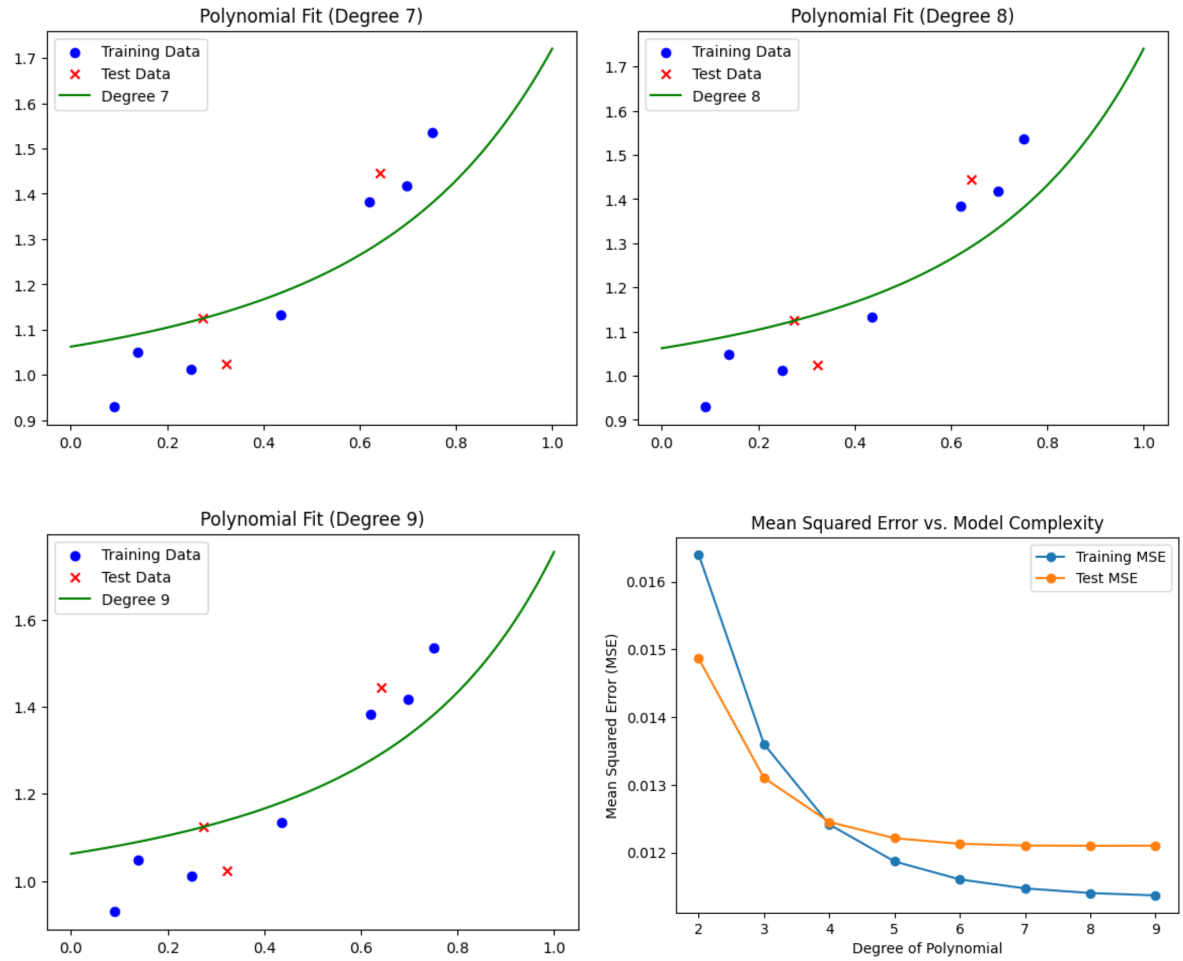
Weights after regularization:

n=5 (192.67, -423.73, 343.88, -124.38, 19.78, -0.07)
n=6 (96.33, -45.24, -195.34, 236.32, -98.61, 16.86, 0.05)
n=7 (-44.76, 229.88, -206.11, -95.24, 202.11, -92.29, 16.28, 0.07)
n=8 (-148.49, 307.94, -34.08, -198.93, -6.56, 153.95, -81.45, 15.19, 0.11)
n=9 (-238.24, 310.91, 111.87, -135.86, -142.98, 36.69, 117.37, -71.83, 14.14, 0.15)

For $\boldsymbol{\alpha} = \mathbf{1}$







Weights after regularization:

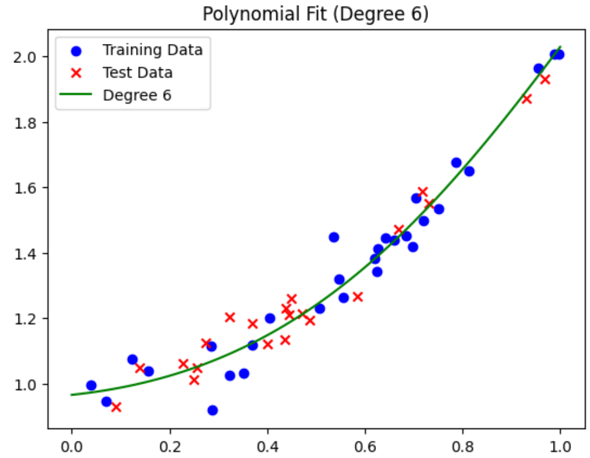
n=5 (192.67, -423.73, 343.88, -124.38, 19.78, -0.07)
n=6 (96.33, -45.24, -195.34, 236.32, -98.61, 16.86, 0.05)
n=7 (-44.76, 229.88, -206.11, -95.24, 202.11, -92.29, 16.28, 0.07)
n=8 (-148.49, 307.94, -34.08, -198.93, -6.56, 153.95, -81.45, 15.19, 0.11)
n=9 (-238.24, 310.91, 111.87, -135.86, -142.98, 36.69, 117.37, -71.83, 14.14, 0.15)

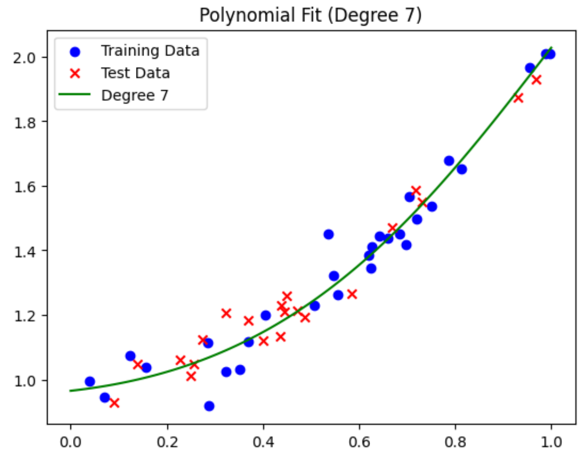
Degree	Alpha = 0.01	Alpha = 0.05	Alpha = 0.1	Alpha = 1
5	Training MSE = 0.013 Test MSE = 0.047	Training MSE = 0.014 Test MSE = 0.049	Training MSE = 0.016 Test MSE = 0.052	Training MSE = 0.013
6	Training MSE = 0.013 Test MSE = 0.048	Training MSE = 0.014 Test MSE = 0.049	Training MSE = 0.016 Test MSE = 0.052	Training MSE = 0.013 Test MSE = 0.047
7	Training MSE = 0.013 Test MSE = 0.048	Training MSE = 0.014 Test MSE = 0.050	Training MSE = 0.013 Test MSE = 0.047	Training MSE = 0.013 Test MSE = 0.047
8	Training MSE = 0.013 Test MSE = 0.048	Training MSE = 0.014 Test MSE = 0.050	Training MSE = 0.013 Test MSE = 0.047	Training MSE = 0.013 Test MSE = 0.047
9	Training MSE = 0.013 Test MSE = 0.048	Training MSE = 0.014 Test MSE = 0.050	Training MSE = 0.013 Test MSE = 0.047	Training MSE = 0.013 Test MSE = 0.047

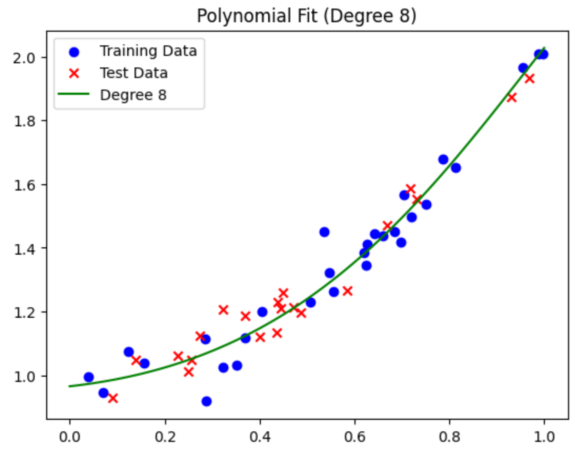
- ***Dataset size = 50***

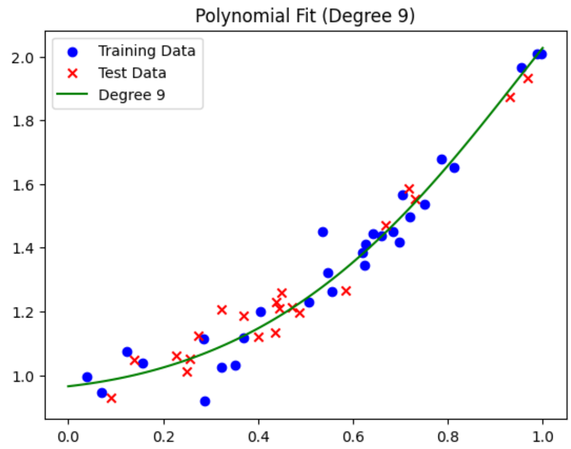
For training with *50 points in dataset*, from the MSE curve, we see overfitting from n = 6 onwards.

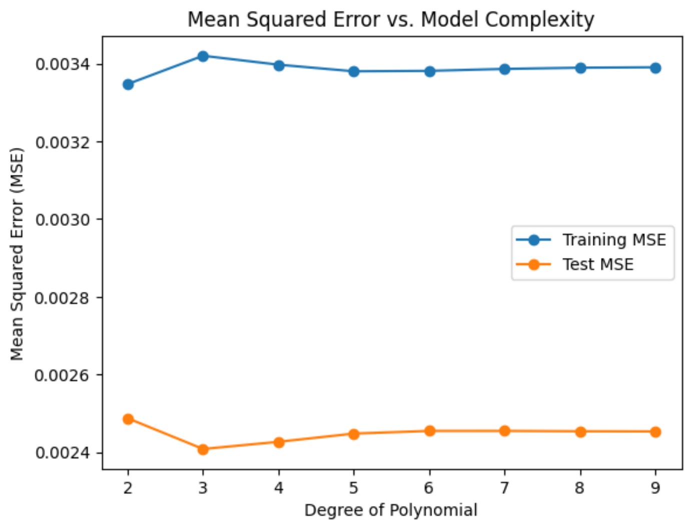
For $\alpha = \mathbf{0.01}$







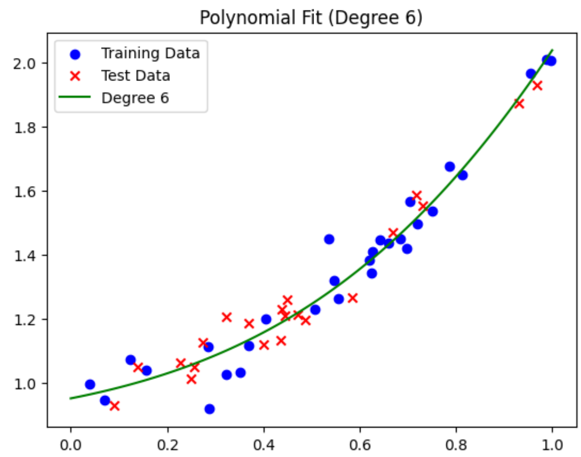


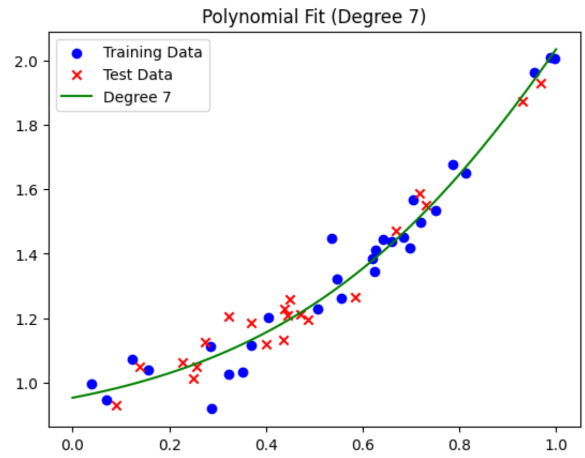


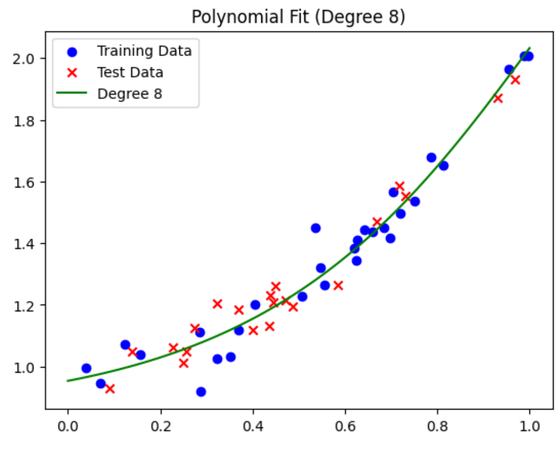
Weights after regularization:

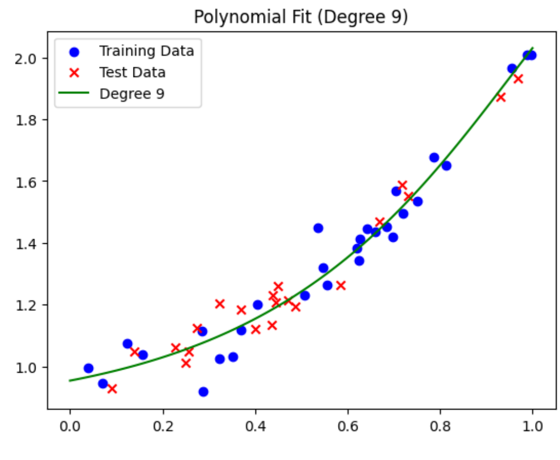
n=6 (-88.94, 277.09, -329.47, 185.85, -48.88, 5.53, 0.81)
n=7 (53.37, -282.97, 558.62, -536.88, 267.34, -65.22, 6.97, 0.77)
n=8 (907.59, -3754.84, 6304.36, -5502.75, 2654.18, -693.07, 90.12, -4.62, 1.05)
n=9 (-4856.86, 23396.80, -47554.80, 52946.68, -35061.82, 14026.74, -3291.26, 419.86, -24.83, 1.48)

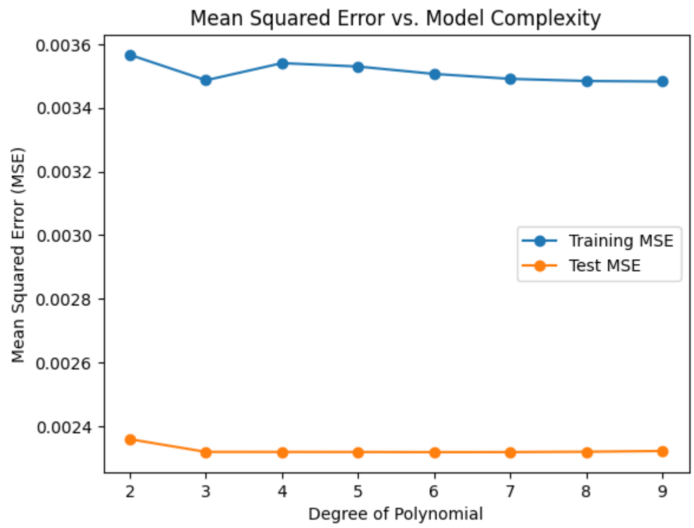
For $\alpha = 0.05$







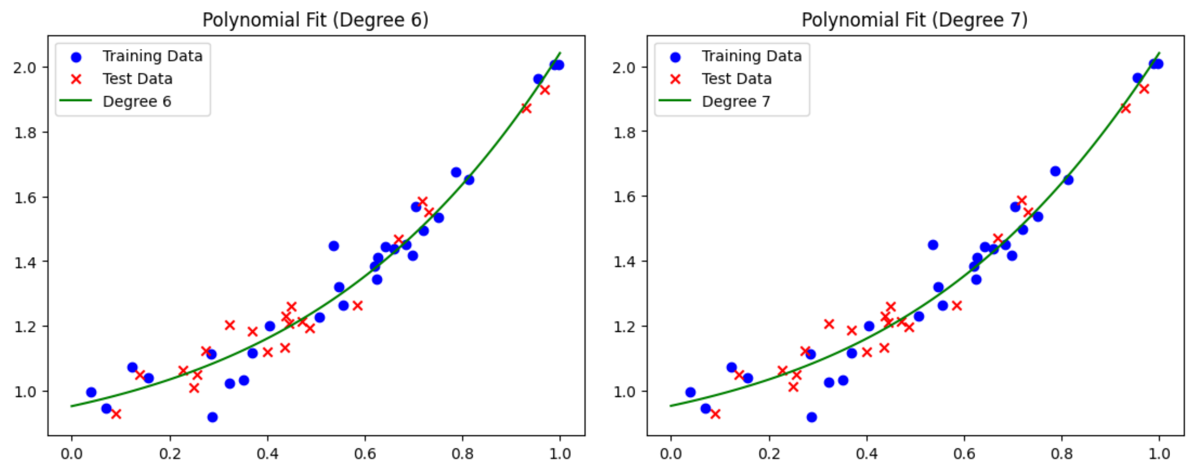


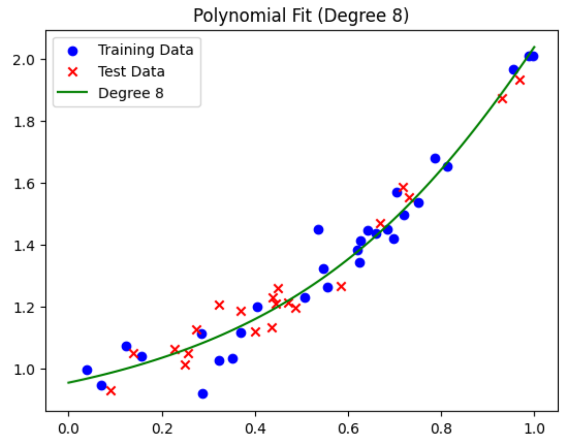


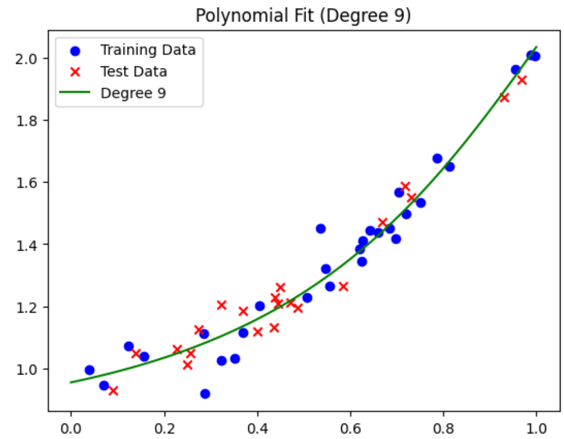
Weights after regularization:

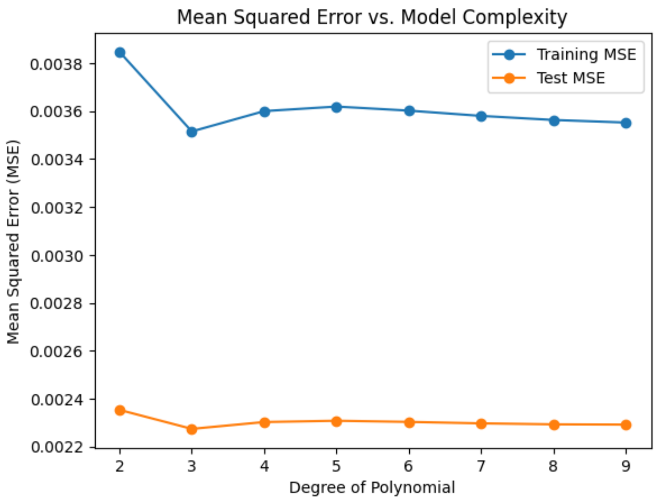
n=7 (-88.94, 277.09, -329.47, 185.85, -48.88, 5.53, 0.81)
n=8 (53.37, -282.97, 558.62, -536.88, 267.34, -65.22, 6.97, 0.77)
n=9 (907.59, -3754.84, 6304.36, -5502.75, 2654.18, -693.07, 90.12, -4.62, 1.05)
n=10 (-4856.86, 23396.80, -47554.80, 52946.68, -35061.82, 14026.74, -3291.26, 419.86, -24.83, 1.48)

For $\boldsymbol{\alpha = 0.1}$





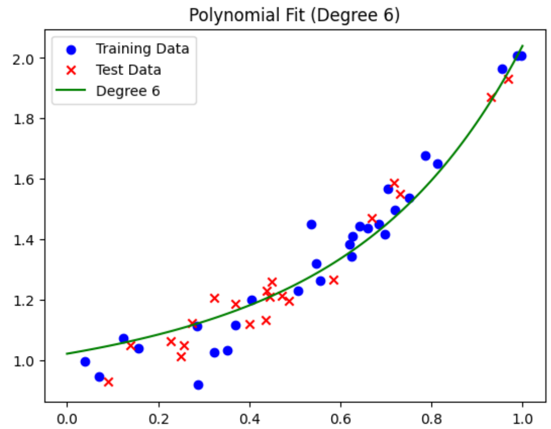




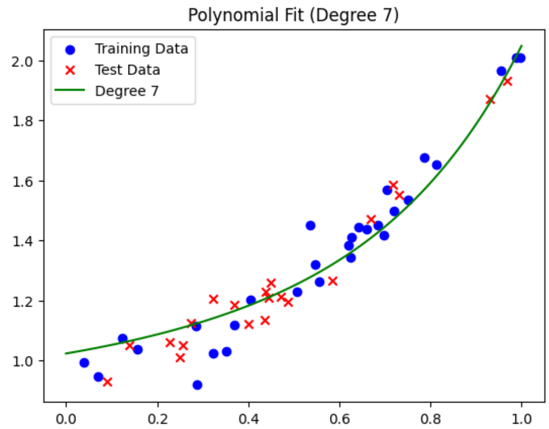
Weights after regularization:

n=6 (-88.94, 277.09, -329.47, 185.85, -48.88, 5.53, 0.81)
n=7 (53.37, -282.97, 558.62, -536.88, 267.34, -65.22, 6.97, 0.77)
n=8 (907.59, -3754.84, 6304.36, -5502.75, 2654.18, -693.07, 90.12, -4.62, 1.05)
n=9 (-4856.86, 23396.80, -47554.80, 52946.68, -35061.82, 14026.74, -3291.26, 419.86, -24.83, 1.48)

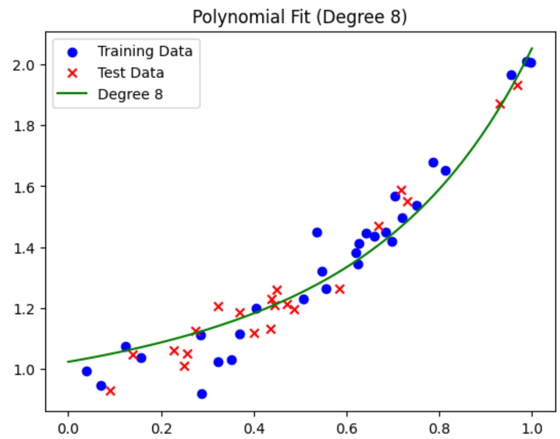
For $\alpha = 1$



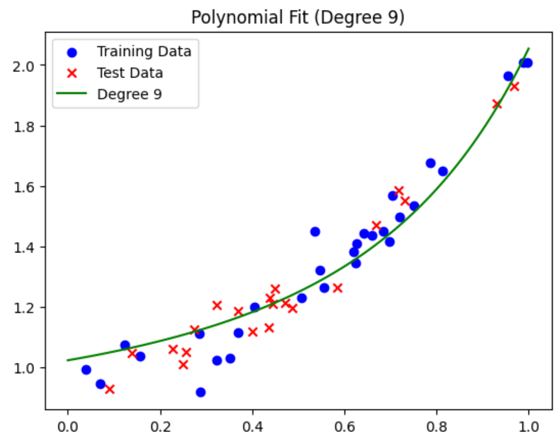
Polynomial Fit (Degree 6)
Training Data
Test Data
Degree 6



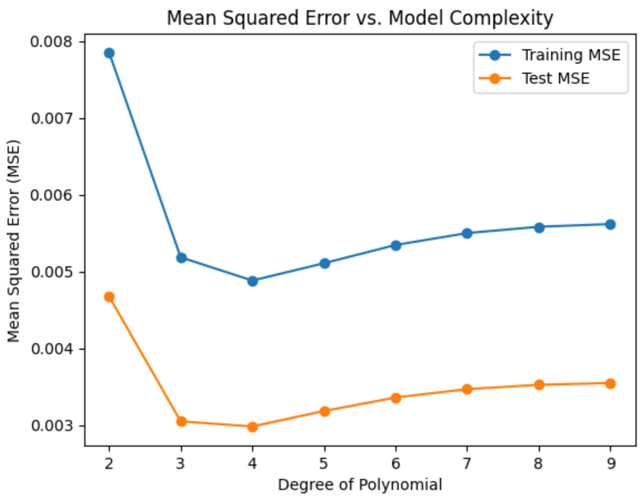
Polynomial Fit (Degree 7)
Training Data
Test Data
Degree 7



Polynomial Fit (Degree 8)
Training Data
Test Data
Degree 8



Polynomial Fit (Degree 9)
Training Data
Test Data
Degree 9



Mean Squared Error vs. Model Complexity
Training MSE
Test MSE
Mean Squared Error (MSE)
Degree of Polynomial

Weights after regularization:

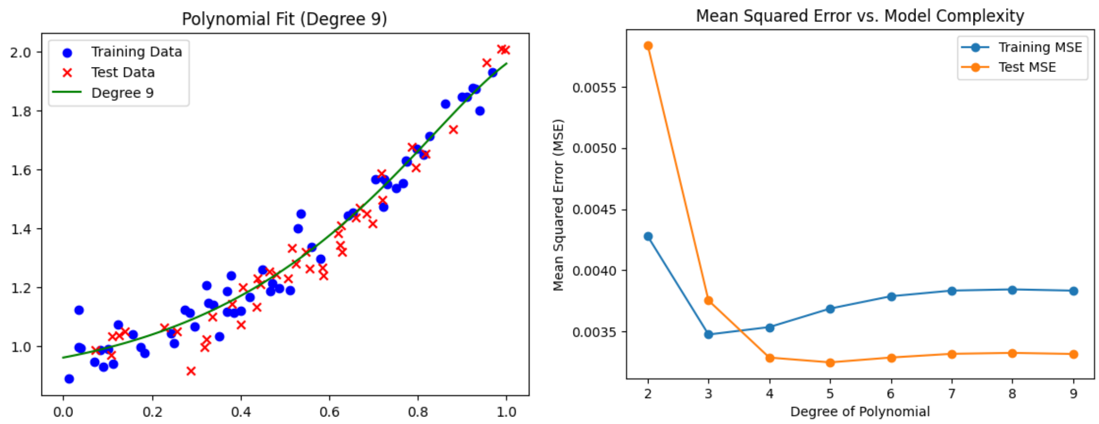
n=6 (-88.94, 277.09, -329.47, 185.85, -48.88, 5.53, 0.81)
n=7 (53.37, -282.97, 558.62, -536.88, 267.34, -65.22, 6.97, 0.77)
n=8 (907.59, -3754.84, 6304.36, -5502.75, 2654.18, -693.07, 90.12, -4.62, 1.05)
n=9 (-4856.86, 23396.80, -47554.80, 52946.68, -35061.82, 14026.74, -3291.26, 419.86, -24.83, 1.48)

6	Training MSE = 0.0033 Test MSE = 0.024	Training MSE = 0.0035 Test MSE = 0.0023	Training MSE = 0.0036 Test MSE = 0.0023	Training MSE = 0.0053 Test MSE = 0.0033
7	Training MSE = 0.0033 Test MSE = 0.0024	Training MSE = 0.0035 Test MSE = 0.0023	Training MSE = 0.0035 Test MSE = 0.0022	Training MSE = 0.0055 Test MSE = 0.0034
8	Training MSE = 0.0033 Test MSE = 0.0024	Training MSE = 0.0035 Test MSE = 0.0023	Training MSE = 0.0035 Test MSE = 0.0022	Training MSE = 0.0055 Test MSE = 0.0035
9	Training MSE = 0.0033 Test MSE = 0.0024	Training MSE = 0.0035 Test MSE = 0.0023	Training MSE = 0.0035 Test MSE = 0.022	Training MSE = 0.0056 Test MSE = 0.0035

- ***Dataset size = 100***

For training with *100 points in dataset*, from the MSE curve, we see overfitting only at n = 9.

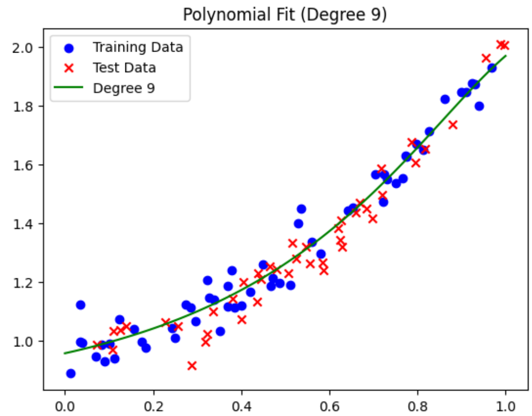
For $\alpha = \mathbf{0.01}$

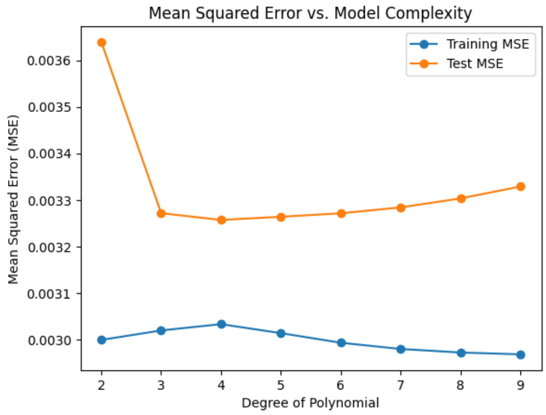


Weights after regularization:

(4548.74, -20605.12, 39169.69, -40563.48, 24848.62, -9146.05, 1961.18, -223.81, 11.54, 0.80)

For $\alpha = \mathbf{0.05}$

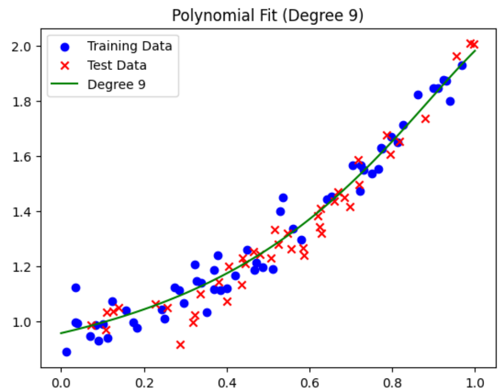


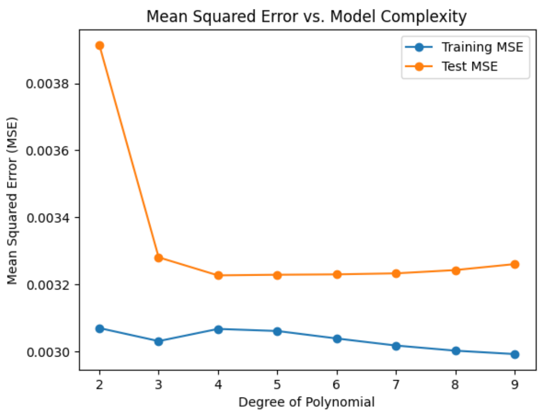


Weights after regularization:

(4548.74, -20605.12, 39169.69, -40563.48, 24848.62, -9146.05, 1961.18, -223.81, 11.54, 0.80)

For $\boldsymbol{\alpha = 0.1}$

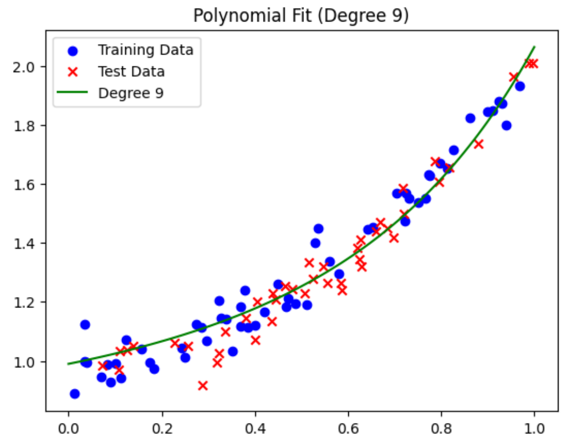


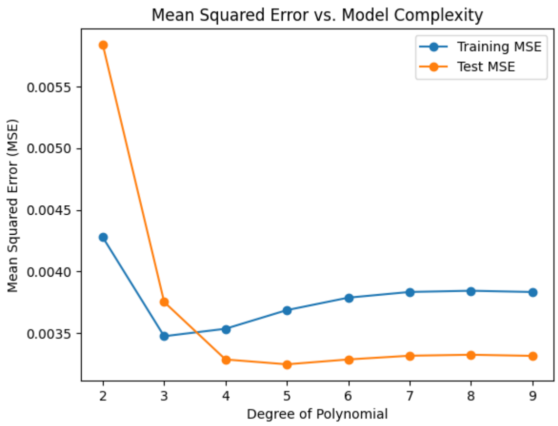


Weights after regularization:

(4548.74, -20605.12, 39169.69, -40563.48, 24848.62, -9146.05, 1961.18, -223.81, 11.54, 0.80)

For $\boldsymbol{\alpha = 1}$





Weights after regularization:

(4548.74, -20605.12, 39169.69, -40563.48, 24848.62, -9146.05, 1961.18, -223.81, 11.54, 0.80)

9	Training MSE = 0.0029 Test MSE = 0.0033	Training MSE = 0.0029 Test MSE = 0.0033	Training MSE = 0.0029 Test MSE = 0.0032	Training MSE = 0.0038 Test MSE = 0.0033

- ***Dataset size = Complete set***

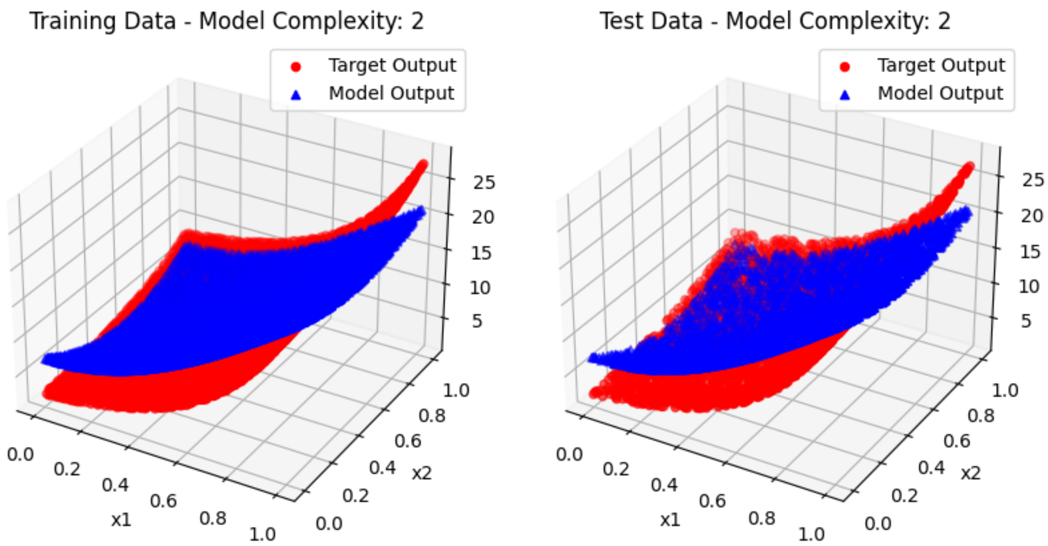
For training with *complete set of points in dataset*, from the MSE curve, we see overfitting doesn't happen at all.

4.2. Dataset 2 (Multivariate dataset)

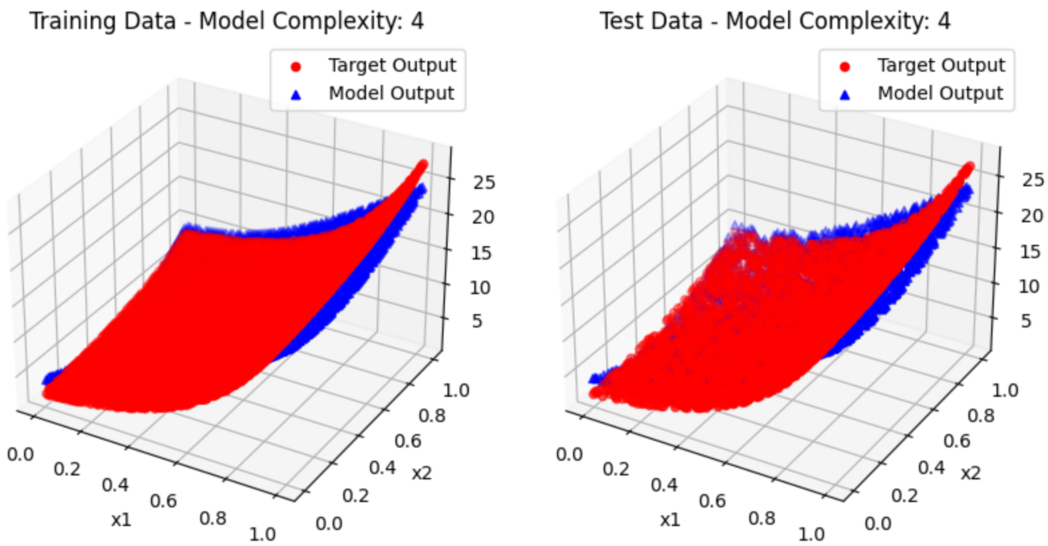
Here, the input data consists of two variables. We need to find the Linear regression curve by taking into account Gaussian basis functions. We also need to address models of different complexities. **Let *n* represent the number of gaussian basis functions (model complexity)** to be taken with the center of clusters being the centers of Gaussian basis functions.

We get the following results:

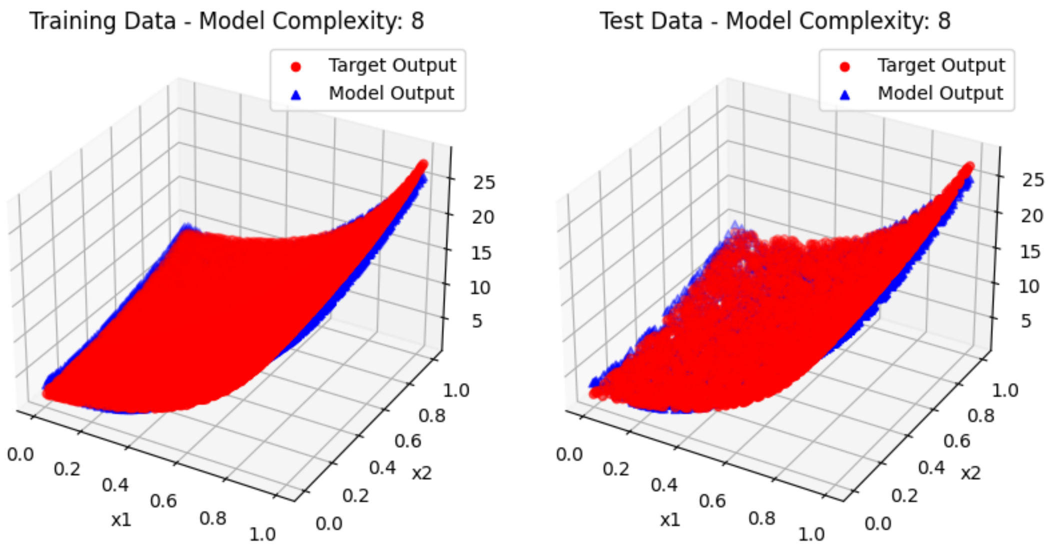
<u>**Without regularization:**</u>



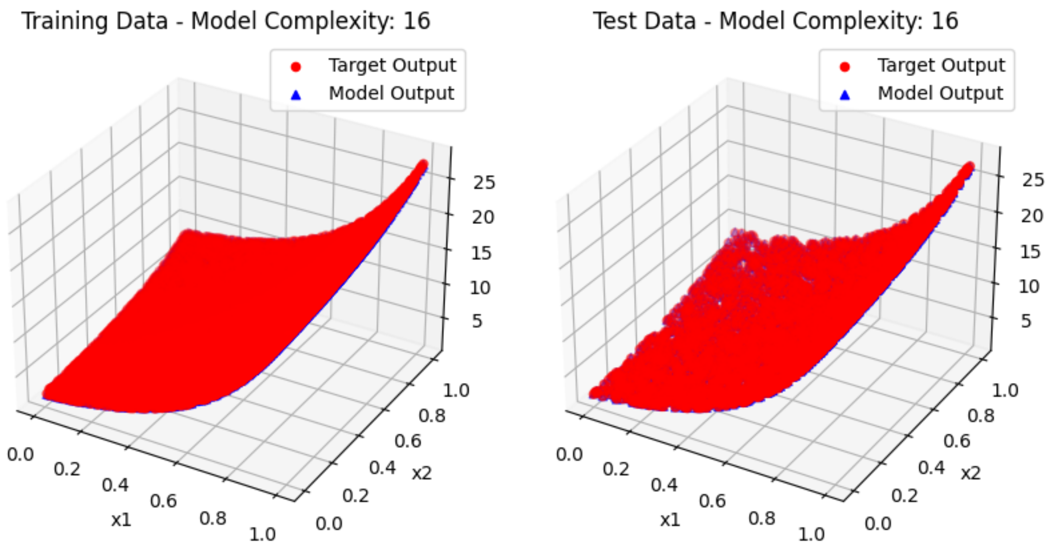
Train MSE: 5.10
Test MSE: 5.17



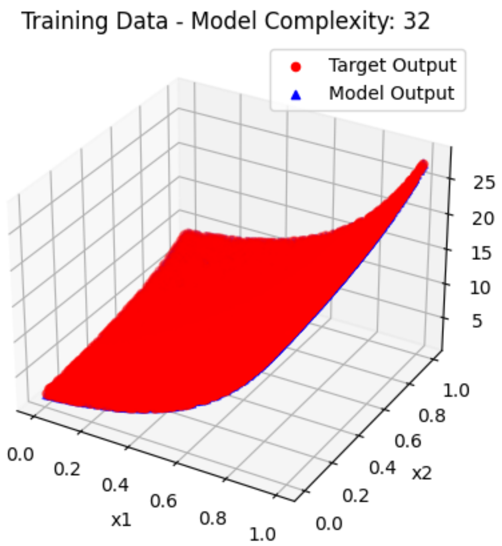
Train MSE: 1.34
Test MSE: 1.38

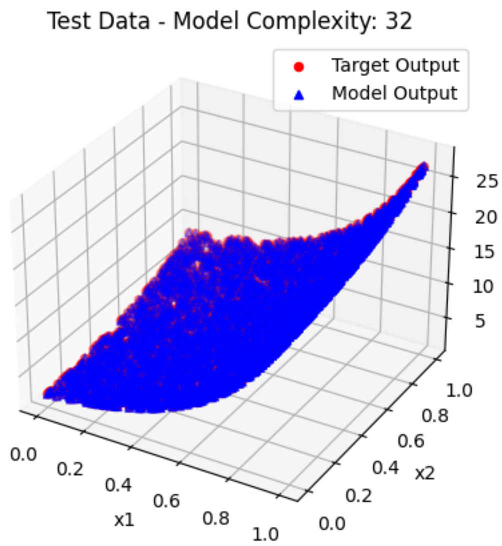


Train MSE: 0.38
Test MSE: 0.37

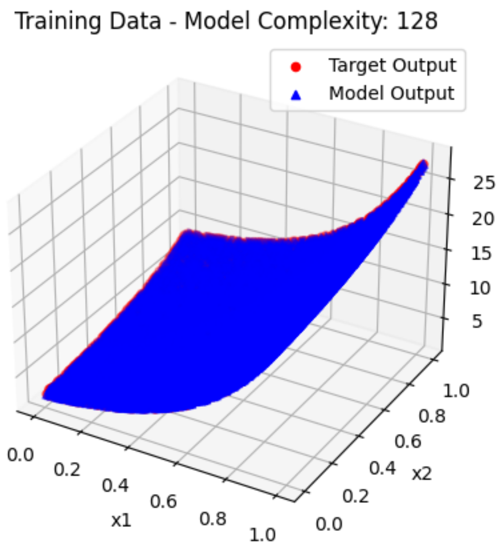


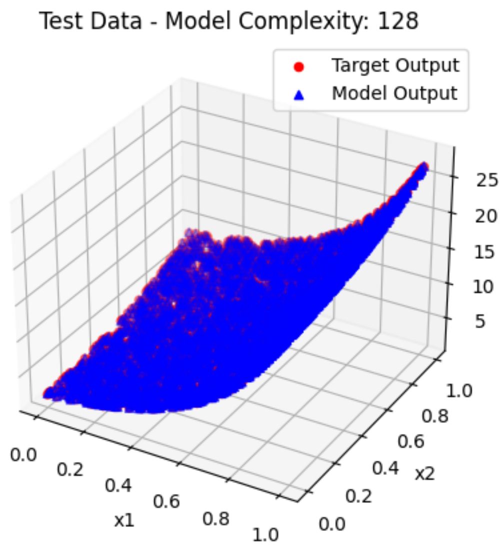
Train MSE: 0.0039
Test MSE: 0.0038



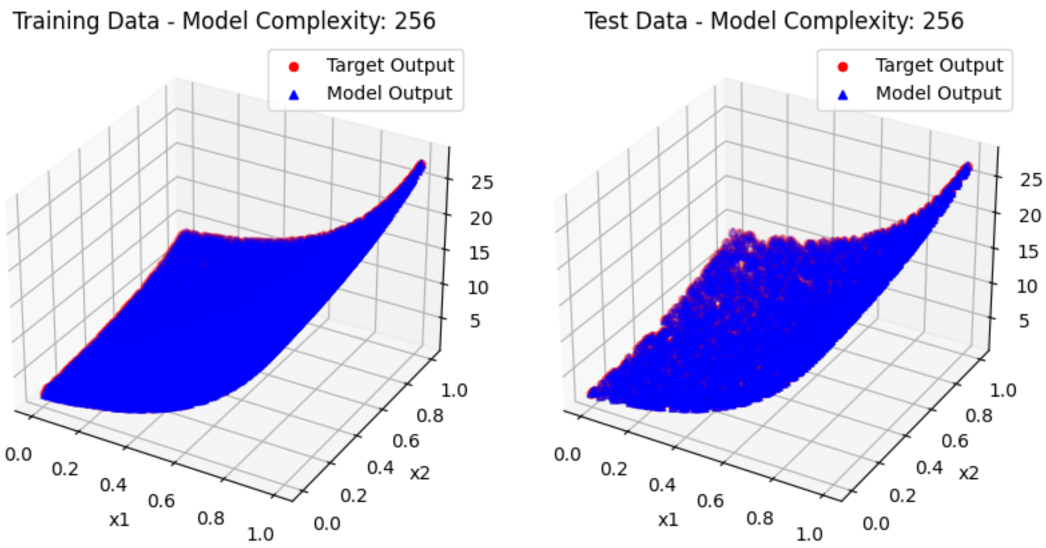


Train MSE: 0.0031
Test MSE: 0.0030

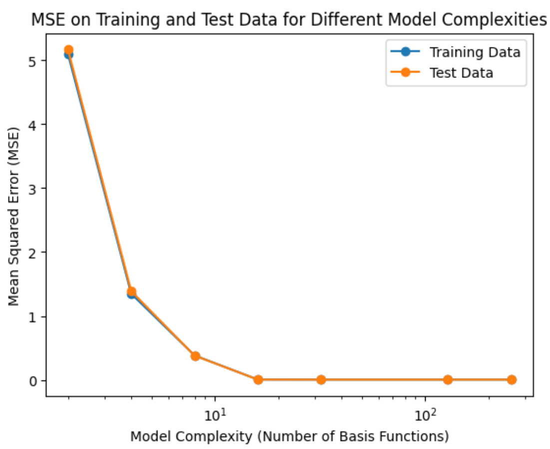




Train MSE: 0.0031
Test MSE: 0.0031



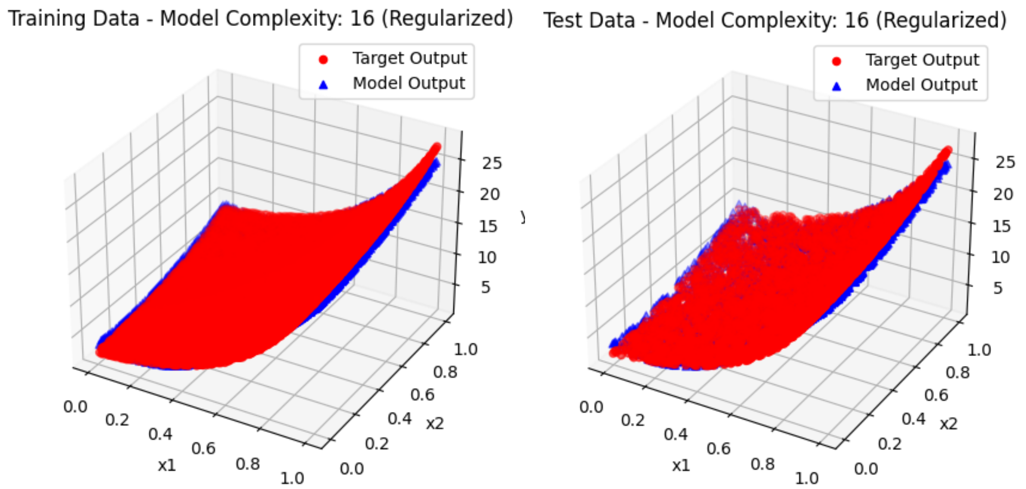
Train MSE: 0.0031
Test MSE: 0.0031



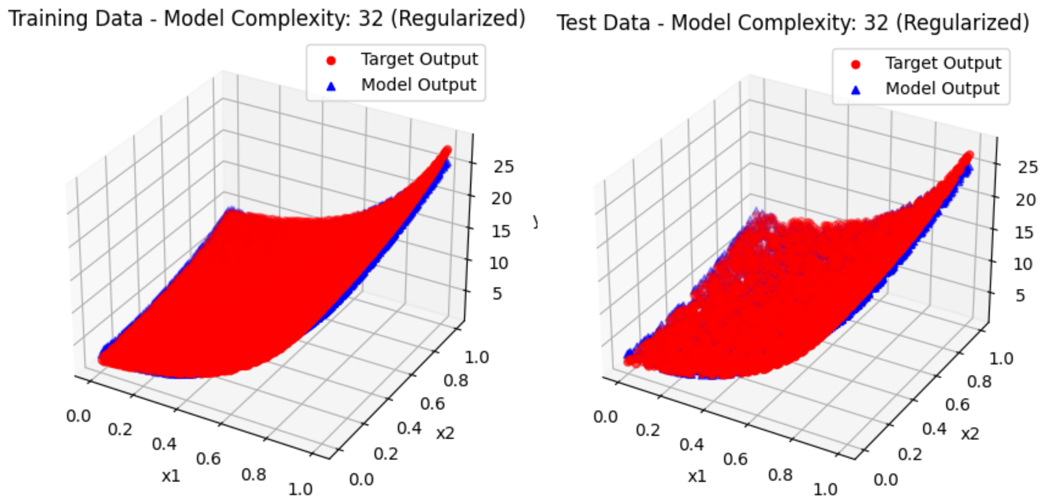
Clearly from the MSE graph, **model complexity** $\geq$ **16** leads to *overfitting*.

<u>With regularization:</u>

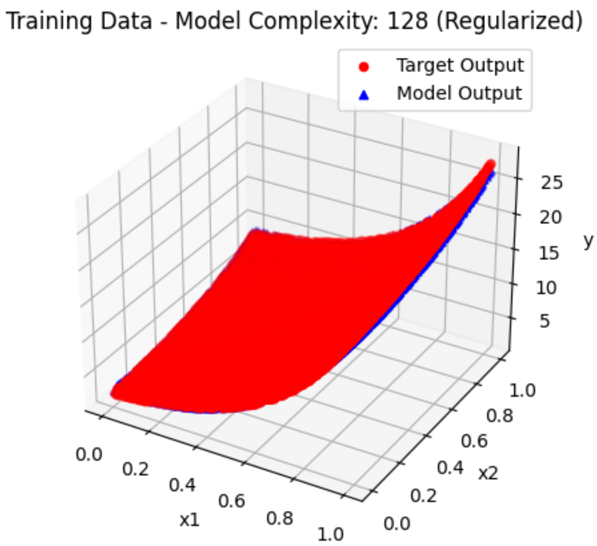
For $\boldsymbol{\alpha = 0.001}$

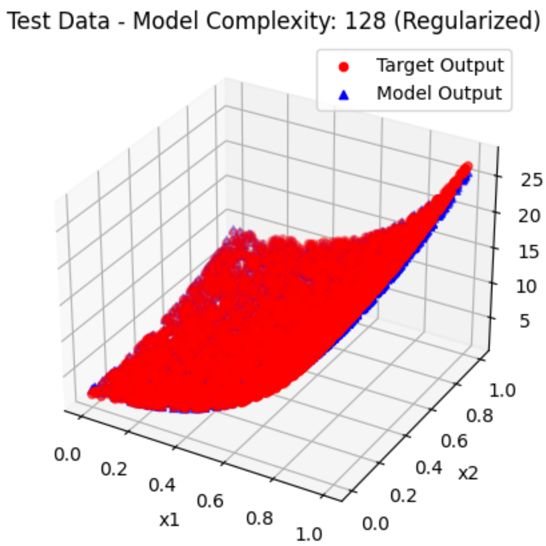


Train MSE: 0.32
Test MSE: 0.32

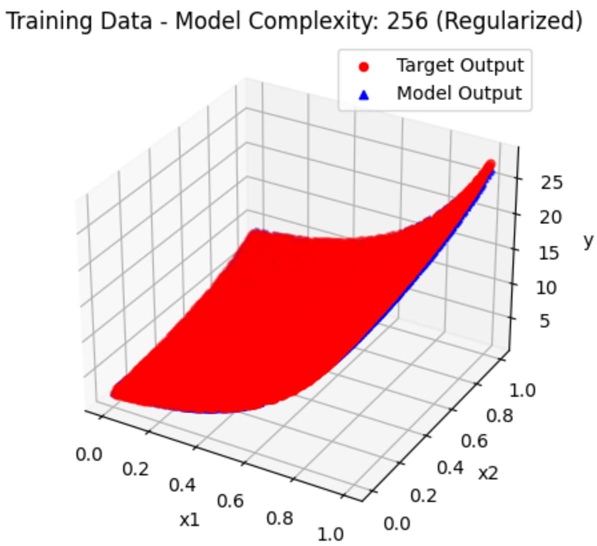


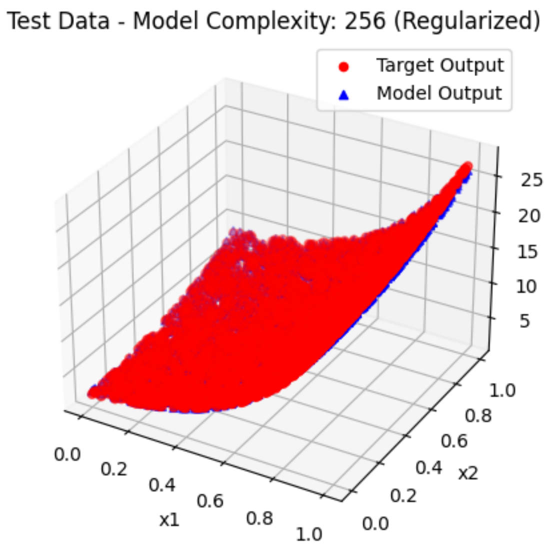
Train MSE: 0.209
Test MSE: 0.208



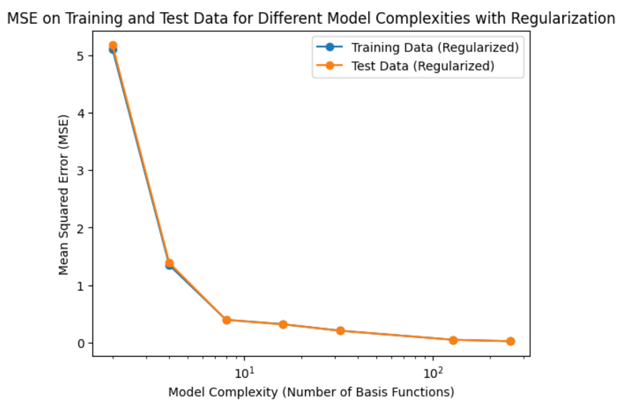


Train MSE: 0.05
Test MSE: 0.05



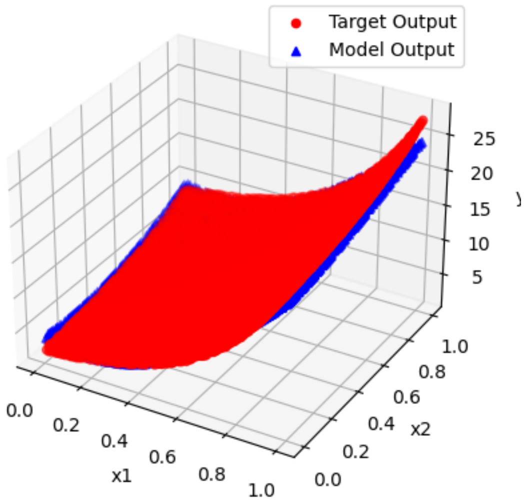


Train MSE: 0.026
Test MSE: 0.026

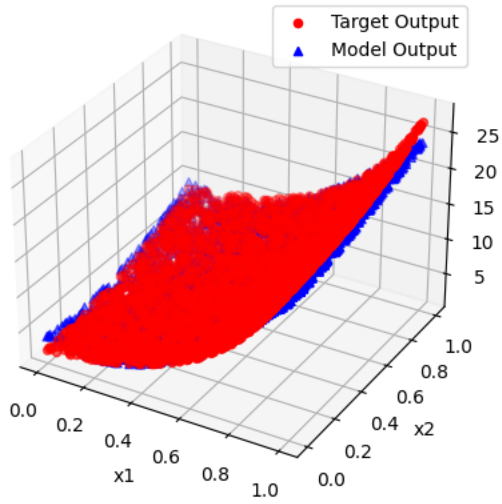


For $\alpha = \mathbf{0.01}$

Training Data - Model Complexity: 16 (Regularized)

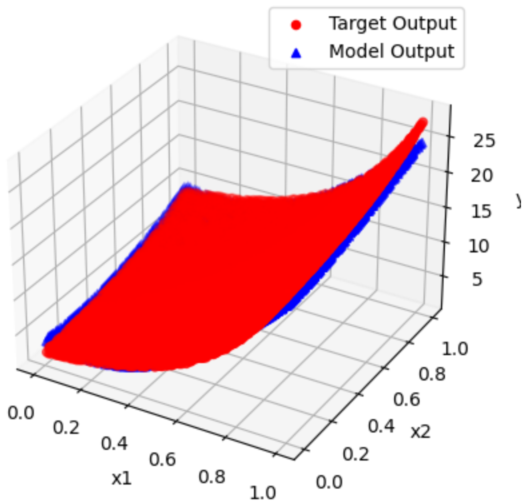


Test Data - Model Complexity: 16 (Regularized)

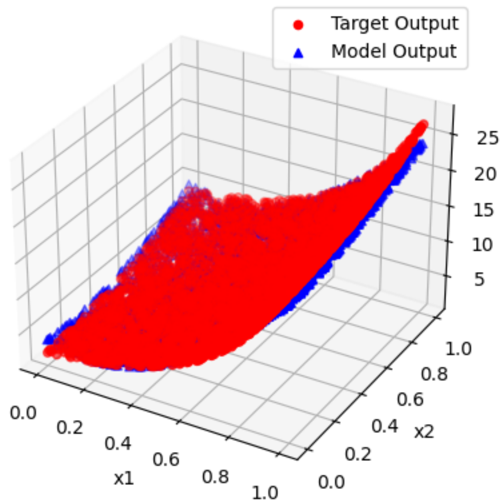


Train MSE: 0.41
Test MSE: 0.407

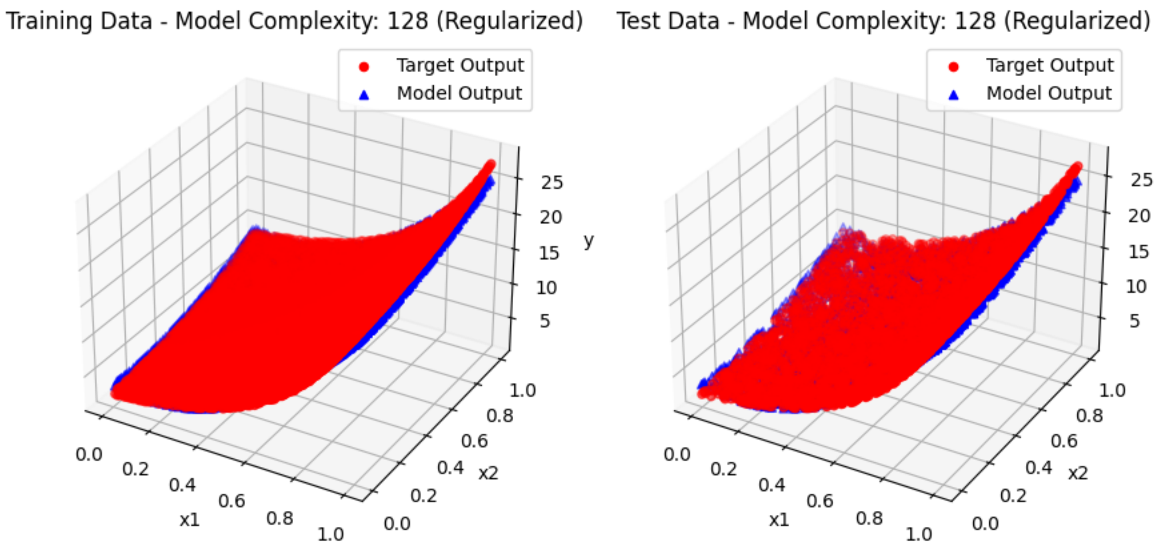
Training Data - Model Complexity: 32 (Regularized)



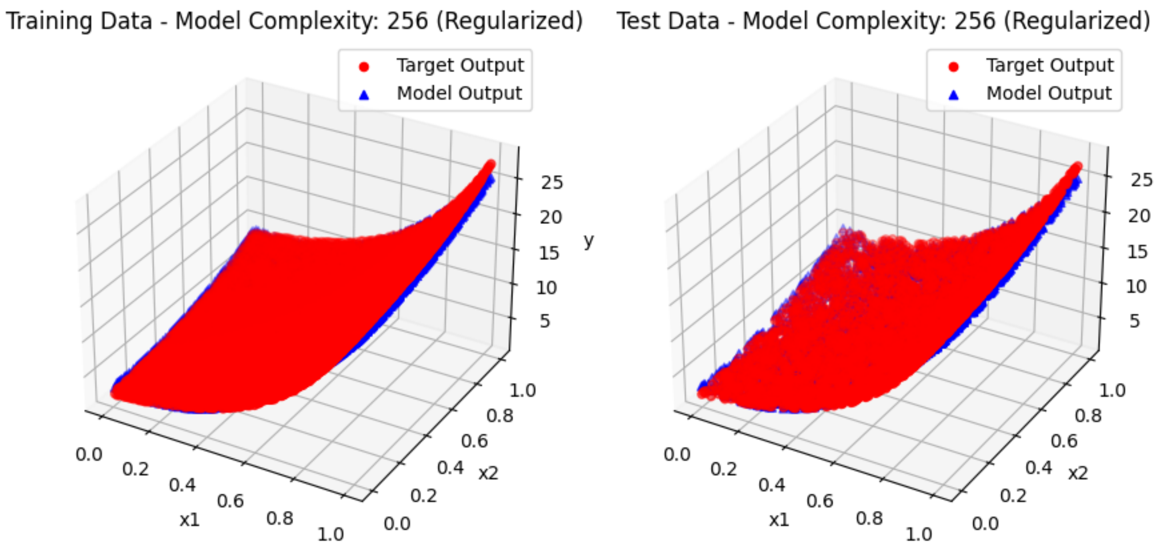
Test Data - Model Complexity: 32 (Regularized)



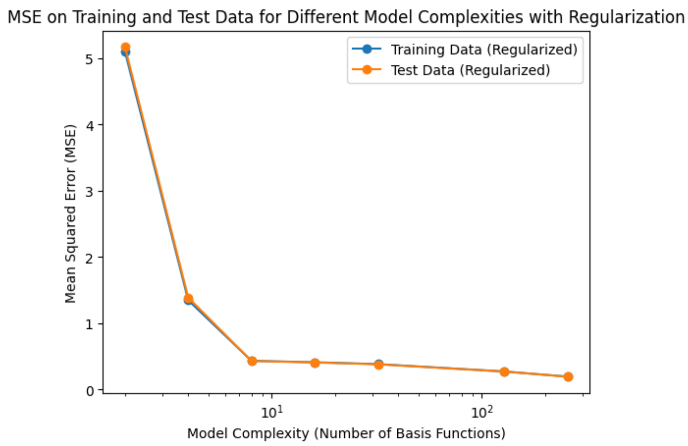
Train MSE: 0.38
Test MSE: 0.379



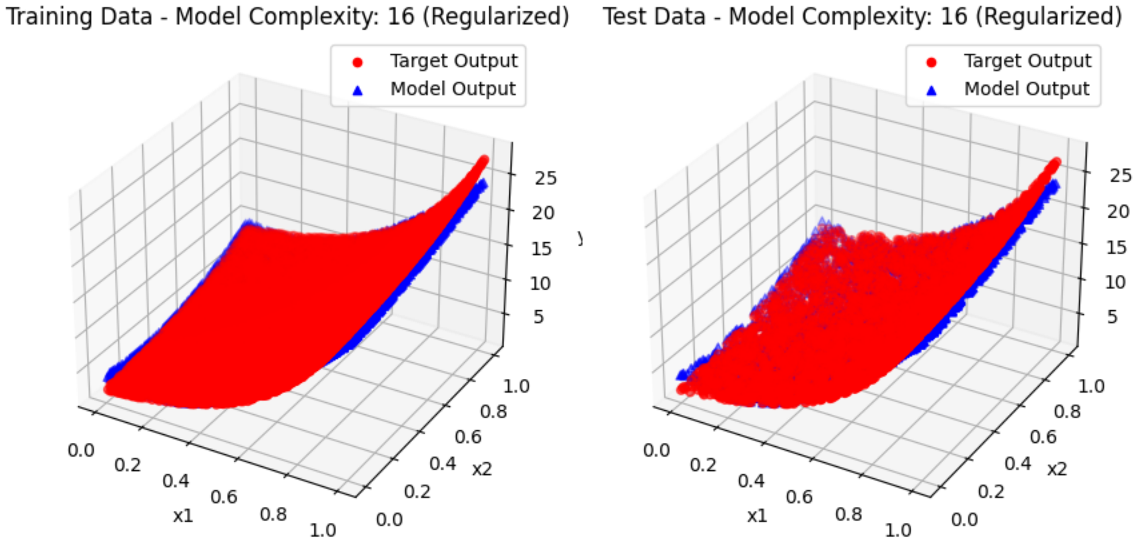
Train MSE: 0.271
Test MSE: 0.269



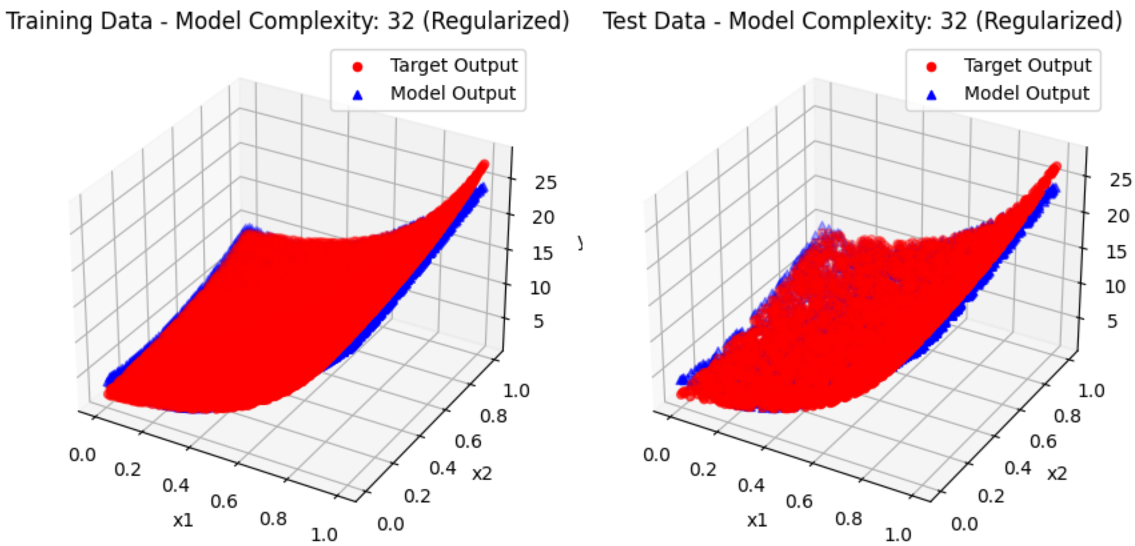
Train MSE: 0.1929
Test MSE: 0.1920



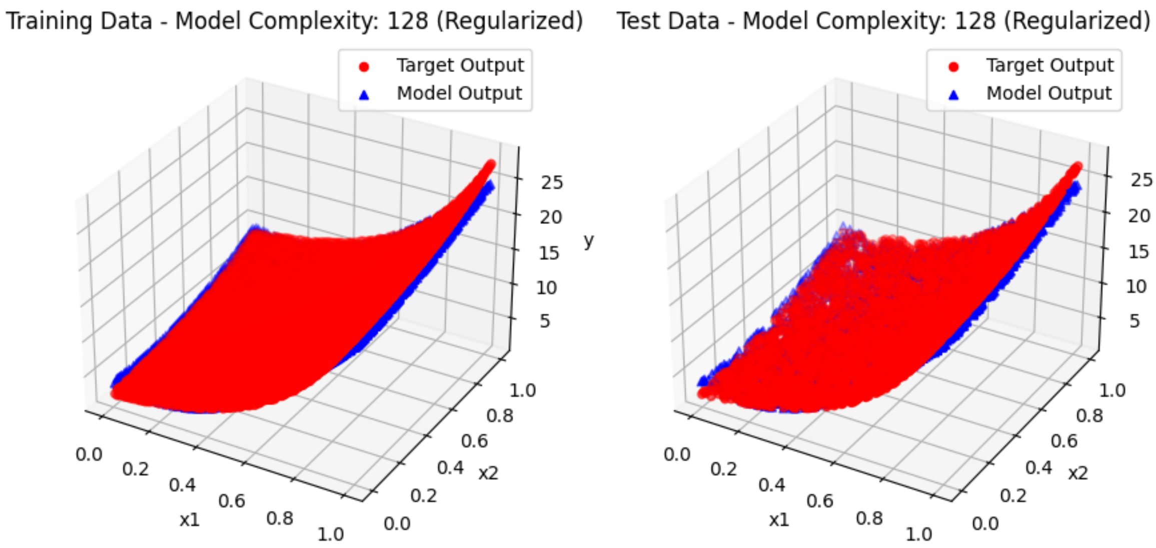
For $\alpha = \mathbf{0.05}$



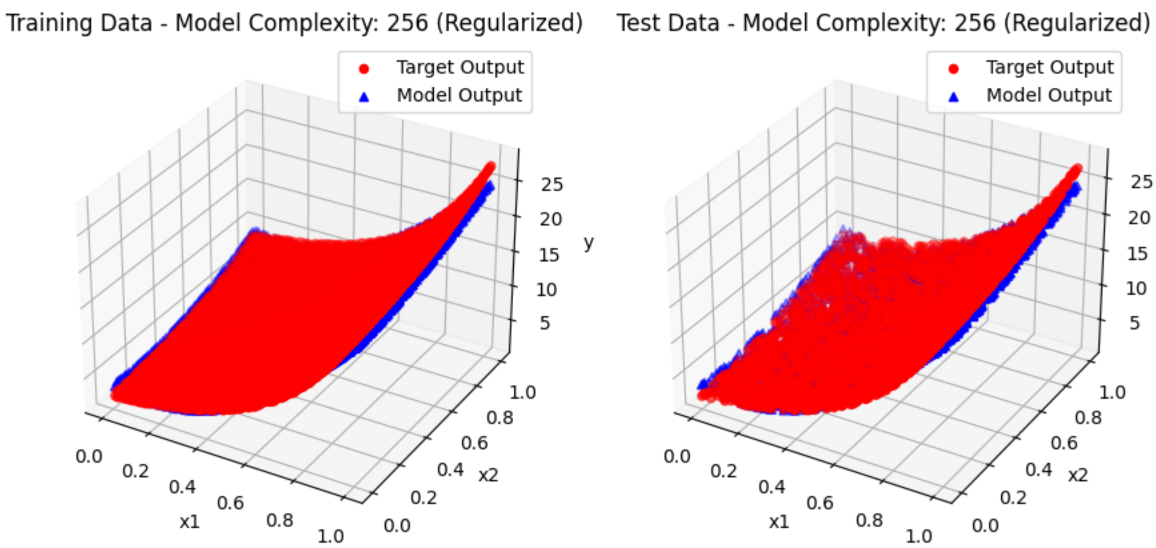
Train MSE: 0.4349
Test MSE: 0.4334



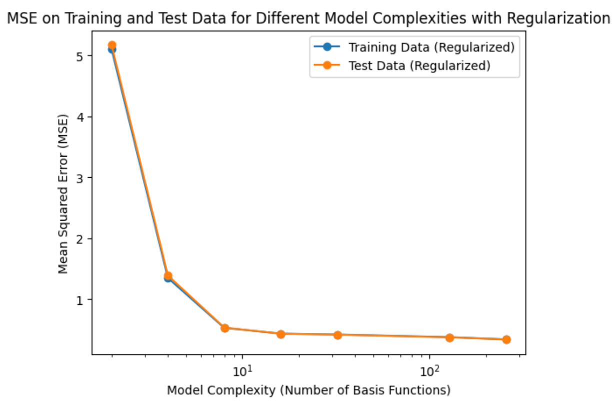
Train MSE: 0.4203
Test MSE: 0.4176



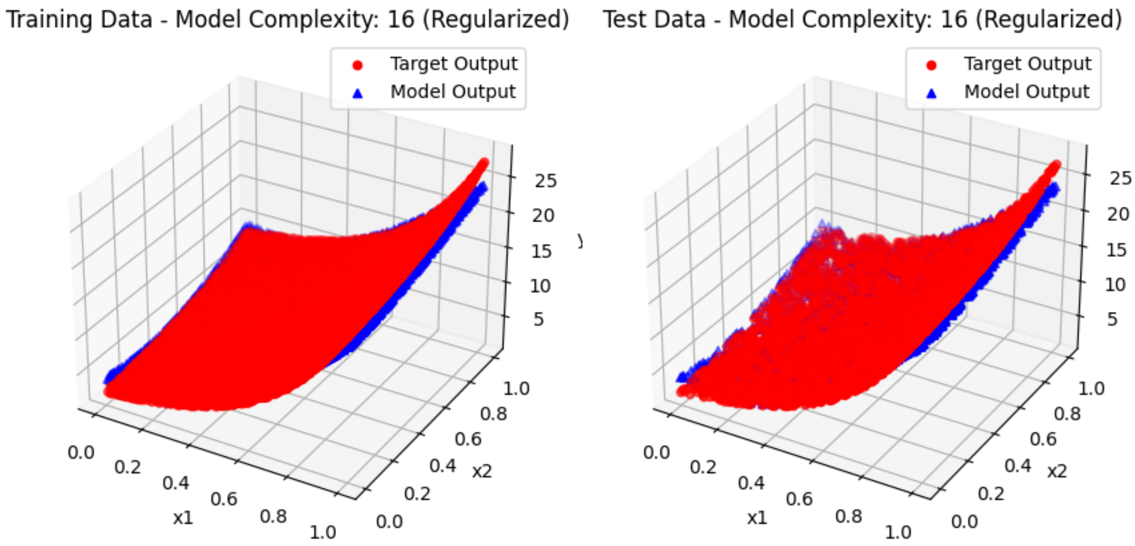
Train MSE: 0.377
Test MSE: 0.374



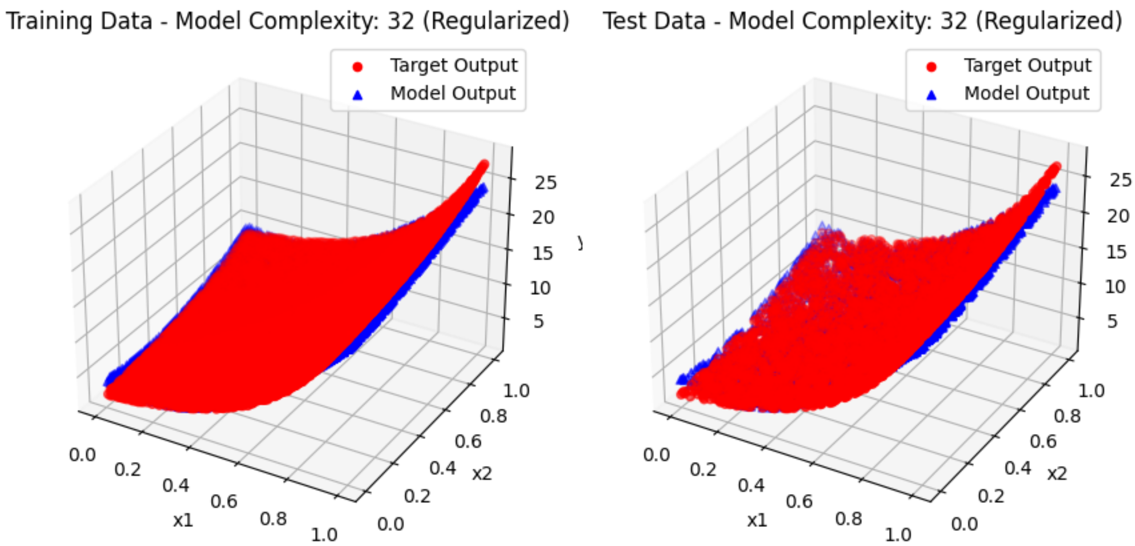
Train MSE: 0.3406
Test MSE: 0.3380



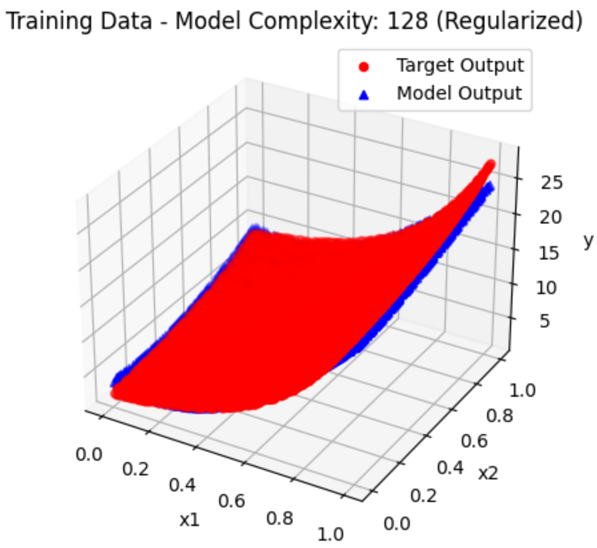
For $\boldsymbol{\alpha} = \mathbf{0.1}$

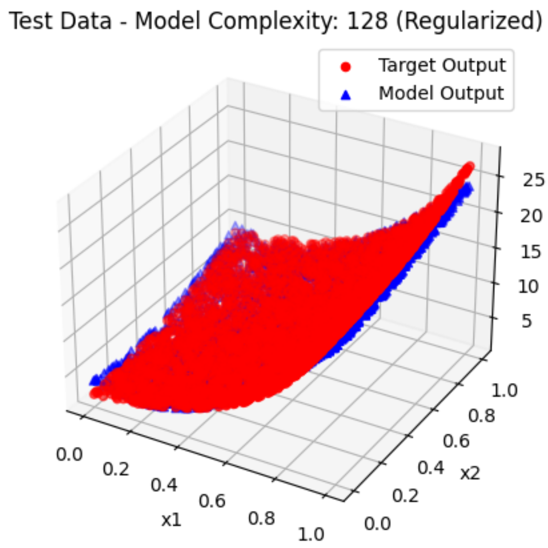


Train MSE: 0.4571
Test MSE: 0.4576

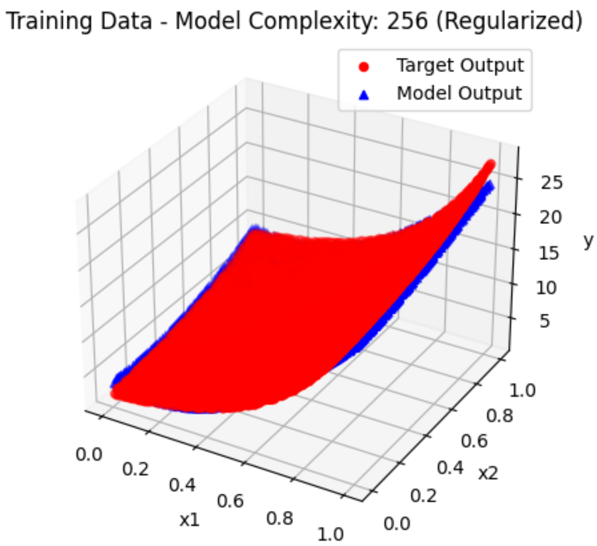


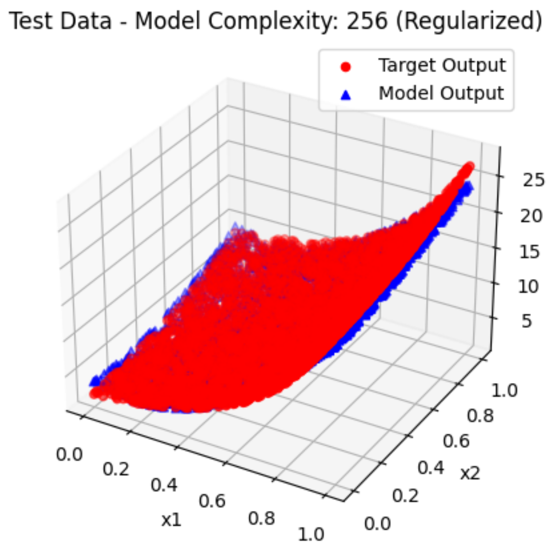
Train MSE: 0.4310
Test MSE: 0.4291



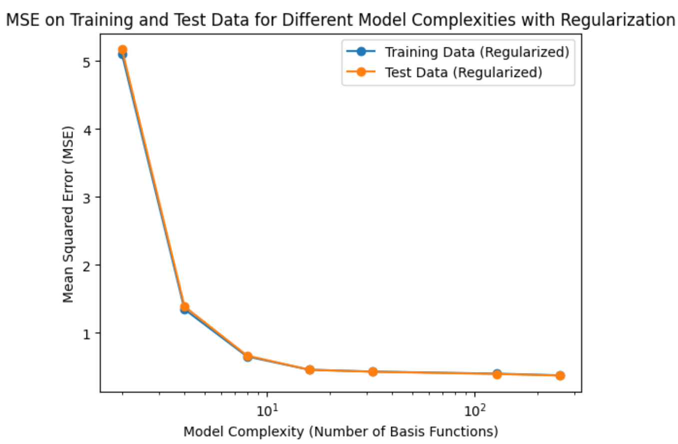


Train MSE: 0.3996
Test MSE: 0.3965





Train MSE: 0.3757
Test MSE: 0.3728



5. Conclusion

- In the univariate dataset, as the number of training points increase, the model complexity doesn't account for overfitting the model anymore. For example, when there were 10 points, degree of polynomial more than 5 resulted into overfitting. When there were 100 points, only degree 9 corresponded to a little overfitting and for the complete dataset, there was no overfitting.

- In case of the bivariate dataset, we can see an overfitting at model complexity more than or equal to 16 Gaussian basis functions. The regularization parameter 0.001 turned out to be the best regularization parameter.

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