# PATTERN RECOGNITION

**CS-669**

ASSIGNMENT-1

Bayes Classifier

**Group Number 13**

| | |
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# Contents

# 1 Introduction:

Bayesian theory is a statistical approach to making inferences about the world based on prior knowledge and new evidence. It is based on the Bayes theorem, which states that the probability of an event occurring given some evidence is equal to the probability of the evidence occurring given the event, times the probability of the event occurring.

In pattern recognition, Bayesian theory can be used to classify objects or events based on their features. The features of an object or event are the attributes that can be used to distinguish it from other objects or events. For example, the features of a face might include the shape of the eyes, nose, and mouth.

The Bayesian approach to pattern recognition starts with a prior probability distribution over the classes of objects or events. This prior probability distribution represents our belief about the likelihood of each class before we have seen any evidence. For example, if we are trying to classify a face as male or female, we might start with a prior probability of 50

Once we have seen some evidence, we can update our prior probability distribution using Bayes theorem. The new probability distribution, called the posterior probability distribution, represents our belief about the likelihood of each class after we have seen the evidence.

The posterior probability distribution can be used to make a decision about the class of an object or event. The most common decision rule is to choose the class with the highest posterior probability.

Bayesian theory is a powerful tool for pattern recognition, but it can be computationally expensive. However, there are a number of efficient algorithms that can be used to implement Bayesian methods.

# 2 Objective:

- To build a Bayes Classifier, and use it classify:
  - Linearly Separable 2D Dataset
  - Non Linearly Separable 2D Dataset
  - Real world 2D Dataset
- Plot Decision Regions for all pairs of classes, and one for all classes combined.
- Plot Contour Regions for all pairs of classes, and one for all classes combined.
- Calculate Accuracy, Precision, Mean Recall, F-measure and confusion matrix.

## 3 Procedure

1. Each dataset was partitioned into 75% training data and 25% testing data.
2. Data from each set was assumed to come from Gaussian distribution.
3. **Case 1: Common Covariance Matrix with Zero Off-Diagonal Terms ($\Sigma = \sigma^2 I$)**:
   - In this case, the mean of all the covariance matrices was calculated.
   - The off-diagonal terms for the resulting matrix were assumed to be zero.

   Mathematical Representation:

   $$\text{Mean of Covariance Matrices:} \quad \Sigma = \frac{1}{N}\sum_{i=1}^{N}\Sigma_i$$
   $$\text{Zero Off-Diagonal Terms:} \quad \Sigma_{ij} = 0, \ \text{where } i \neq j$$

   Log-Likelihood and Discriminant Function:

   $$\text{Log-Likelihood:} \quad \log P(\mathbf{x}|C_i) = -\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu}_i)^T\Sigma^{-1}(\mathbf{x}-\boldsymbol{\mu}_i) - \frac{1}{2}\log|\Sigma| + \log P(C_i)$$
   $$\text{Discriminant Function:} \quad g_i(\mathbf{x}) = \mathbf{x}^T\Sigma^{-1}\boldsymbol{\mu}_i - \frac{1}{2}\boldsymbol{\mu}_i^T\Sigma^{-1}\boldsymbol{\mu}_i + \log P(C_i)$$

4. **Case 2: Common Covariance Matrix ($\Sigma_i = \Sigma$)**:
   - In this case, the mean of the covariance matrices of all classes was calculated and used for further calculations.

   Mathematical Representation:

   $$\text{Mean of Covariance Matrices:} \quad \Sigma = \frac{1}{N}\sum_{i=1}^{N}\Sigma_i$$

   Log-Likelihood and Discriminant Function equations are the same as in Case 1.

5. **Case 3: Diagonal Covariance Matrices with Zero Off-Diagonal Terms ($\Sigma_i$ is a diagonal matrix)**:
   - Here, it was assumed that the off-diagonal terms of covariance matrices for all classes were zero.

   Mathematical Representation:

   $$\text{Diagonal Covariance Matrix:} \quad \Sigma_i = \begin{bmatrix} \sigma_{1i}^2 & 0 & 0 \\ 0 & \sigma_{2i}^2 & 0 \\ 0 & 0 & \sigma_{3i}^2 \end{bmatrix}$$

Log-Likelihood and Discriminant Function:

$$\text{Log-Likelihood:} \quad \log P(\mathbf{x}|C_i) = -\frac{1}{2}\sum_{j=1}^{D}\left(\frac{(x_j-\mu_{ij})^2}{\sigma_{ij}^2}\right) + \log P(C_i)$$

$$\text{Discriminant Function:} \quad g_i(\mathbf{x}) = \sum_{j=1}^{D}\left(\frac{x_j\mu_{ij}}{\sigma_{ij}^2} - \frac{\mu_{ij}^2}{2\sigma_{ij}^2}\right) + \log P(C_i)$$

6. **Case 4: Unique Covariance Matrices ($\Sigma_i$ is unique)**:
   - In this case, no assumptions were made about the covariance matrices for each class.
7. Based on assumptions for each case, the discriminant function ($g_i(x)$) was calculated for each class, using which, the Decision region and Contour plot were made.
8. The remaining 25% data was used for analysis on each case.
9. **Decision Region and Contour Plot**:

   Using the computed discriminant functions, decision regions and contour plots were generated. These plots help visualize how the classifier assigns data points to different classes based on the discriminant functions.
10. **Analysis with Testing Data**:

    The remaining 25% of the data, the testing data, was used to analyze the performance of the classifier under each case. Metrics like accuracy, precision, recall, or F1-score may be calculated to evaluate the classifier's performance.

    This procedure outlines the steps for analyzing different cases of data classification using Gaussian-based Bayes classifiers, considering various assumptions about the covariance matrices. The mathematical equations for log-likelihood and discriminant functions are included based on the specific assumptions made for each case.

# 4 Observations

## 4.1 Linearly Seperable Data

### 4.1.1 All Classes With Same Diagonal Matrix

Decision Regions, Training Data and Contours

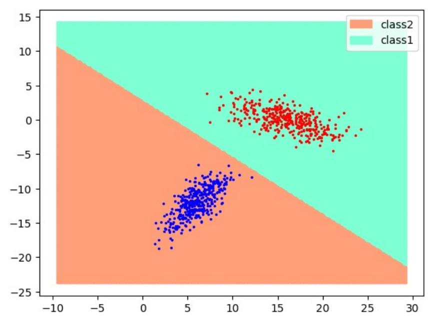



Figure 1: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 1 | 1 |
| Precision | 1 | 0.1 |
| Recall | 1 | 1 |
| F-Measure | 1 | 1 |

(b) Analysis

Table 1: LS : Class 1 and Class 2

Decision Regions, Training Data and Contours

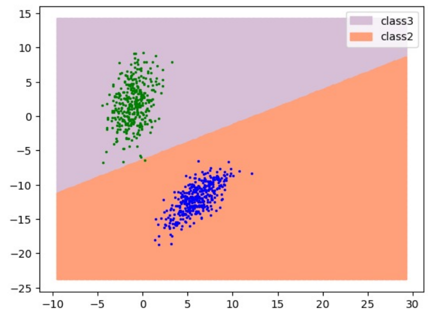



Figure 2: Decision regions and Training data Class 2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.1

| | Class 2 | Class 3 |
|---|---|---|
| Class 2 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 1. | 1. |
| Precision | 1. | 1. |
| Recall | 1. | 1. |
| F-Measure | 1. | 1. |

(b) Analysis

Table 2: LS : Class 2 and Class 3

Decision Regions, Training Data and Contours

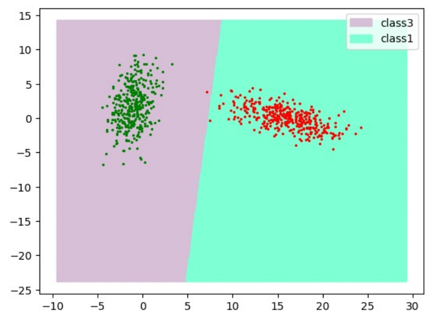



Figure 3: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9959

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 21.24e+02 | 1.00e+00 |
| Class 3 | 1.00e-10 | 1.25e+02 |

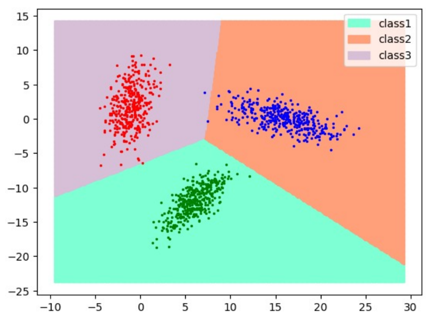
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 0.996 | 0.996 |
| Precision | 1. | 0.9920 |
| Recall | 0.992 | 1. |
| F-Measure | 0.9959 | 0.9960 |

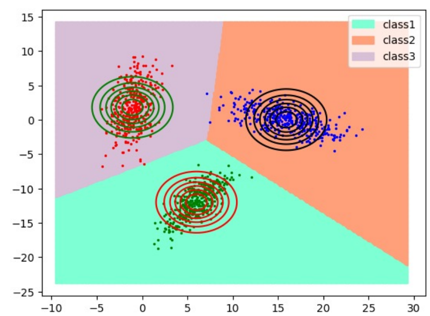
(b) Analysis

Table 3: LS : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9973332906643902

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 1.24e+02 | 1.00e-10 | 1.00e+00 |
| Class 2 | 1.00e-10 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 0.99733333 | 1. | 0.99733333 |
| Precision | 1. | 1. | 0.9920 |
| Recall | 0.992 | 1. | 1. |
| F-Measure | 0.9959 | 1. | 0.9960 |

(b) Analysis

Table 4: LS : Class 1 and Class 2 and Class 3

#### 4.1.2 All Classes With Same Covariance Matrix

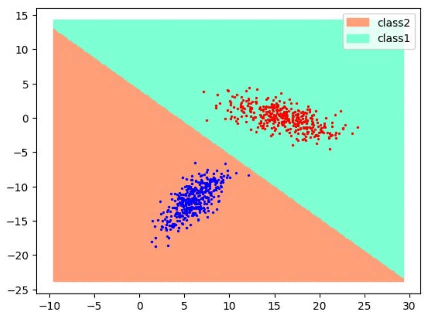



Figure 5: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 1 | 1 |
| Precision | 1 | 1. |
| Recall | 1 | 1 |
| F-Measure | 1 | 1 |

(b) Analysis

Table 5: LS : Class 1 and Class 2

Decision Regions, Training Data and Contours

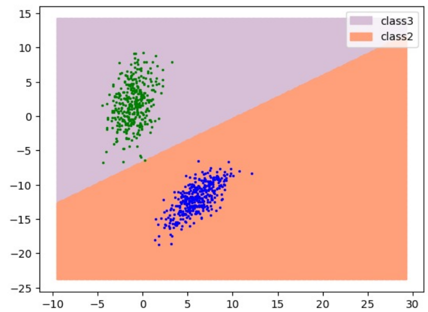



Figure 6: Decision regions and Training data Class 2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 2 | Class 3 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 1. | 1. |
| Precision | 1. | 1. |
| Recall | 1. | 1. |
| F-Measure | 1. | 1. |

(b) Analysis

Table 6: LS : Class 2 and Class 3

Decision Regions, Training Data and Contours

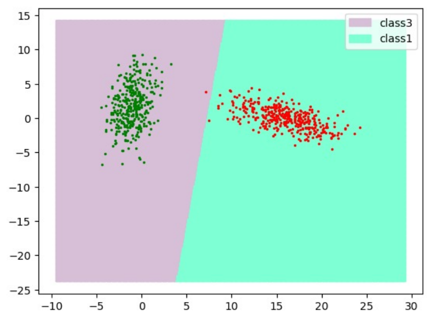



Figure 7: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9959

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 21.24e+02 | 1.00e+00 |
| Class 2 | 1.00e-10 | 1.25e+02 |

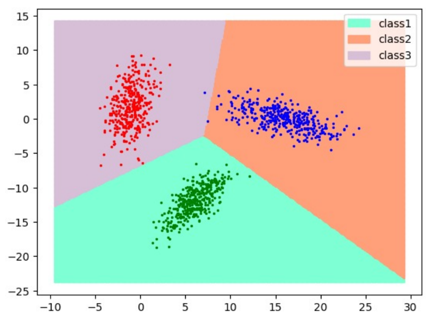
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 0.996 | 0.996 |
| Precision | 1. | 0.9920 |
| Recall | 0.992 | 1. |
| F-Measure | 0.9959 | 0.9960 |

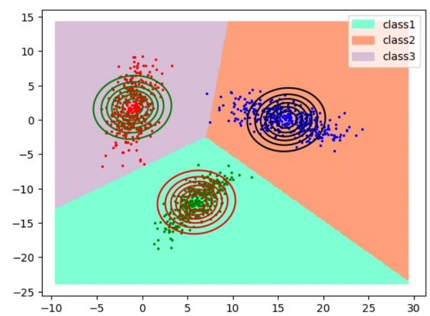
(b) Analysis

Table 7: LS : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9973

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 1.24e+02 | 1.00e-10 | 1.00e+00 |
| Class 2 | 1.00e-10 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 0.9973 | 1. | 0.9973 |
| Precision | 1. | 1. | 0.99206349 |
| Recall | 0.992 | 1. | 1. |
| F-Measure | 0.9959 | 1. | 0.9960 |

(b) Analysis

Table 8: LS : Class 1 and Class 2 and Class 3

#### 4.1.3 All Classes have different Covariance Matrix But Are Diagonal

Decision Regions, Training Data and Contours

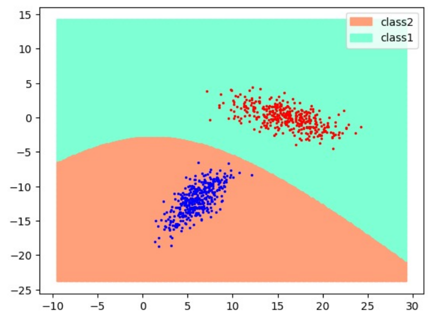



Figure 9: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 1 | 1 |
| Precision | 1 | 1. |
| Recall | 1 | 1 |
| F-Measure | 1 | 1 |

(b) Analysis

Table 9: LS : Class 1 and Class 2

Decision Regions, Training Data and Contours

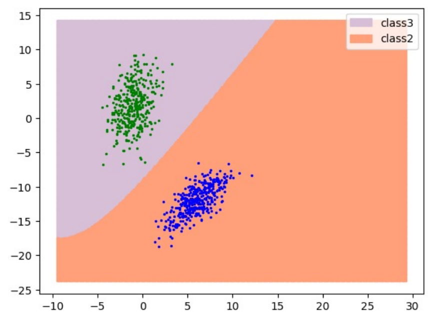



Figure 10: Decision regions and Training data Class 2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 2 | Class 3 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 1. | 1. |
| Precision | 1. | 1. |
| Recall | 1. | 1. |
| F-Measure | 1. | 1. |

(b) Analysis

Table 10: LS : Class 2 and Class 3

Decision Regions, Training Data and Contours

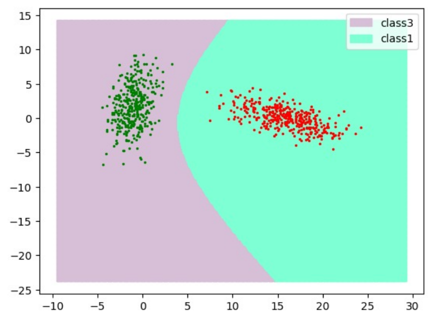



Figure 11: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

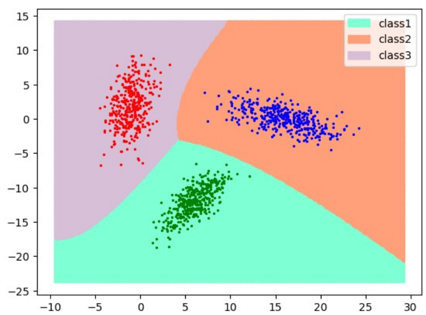
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 1 | 1 |
| Precision | 1 | 1. |
| Recall | 1 | 1 |
| F-Measure | 1 | 1 |

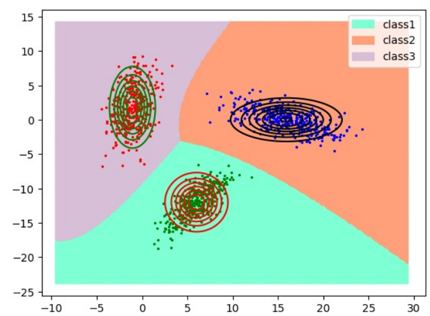
(b) Analysis

Table 11: LS : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 1. | 1. | 1. |
| Precision | 1. | 1. | 1. |
| Recall | 1. | 1. | 1. |
| F-Measure | 1. | 1. | 1. |

(b) Analysis

Table 12: LS : Class 1 , Class 2 and Class 3

### 4.1.4 All Classes With Different Covariance Matrices

Decision Regions, Training Data and Contours

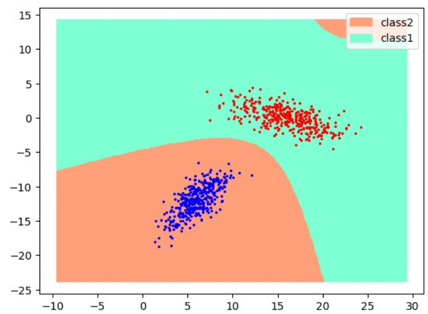



Figure 13: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 1 | 1 |
| Precision | 1 | 1. |
| Recall | 1 | 1 |
| F-Measure | 1 | 1 |

(b) Analysis

Table 13: LS : Class 1 and Class 2

Decision Regions, Training Data and Contours

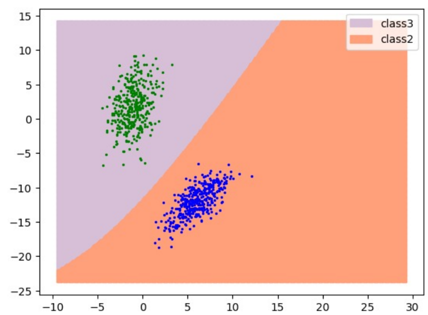



Figure 14: Decision regions and Training data Class 2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 2 | Class 3 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 1. | 1. |
| Precision | 1. | 1. |
| Recall | 1. | 1. |
| F-Measure | 1. | 1. |

(b) Analysis

Table 14: LS : Class 2 and Class 3

Decision Regions, Training Data and Contours

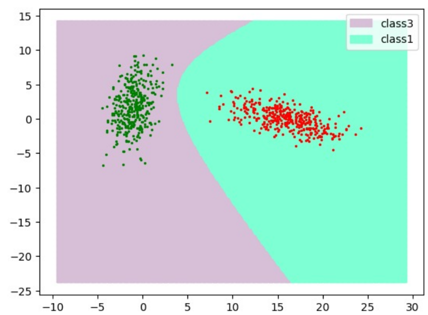



Figure 15: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.25e+02 |

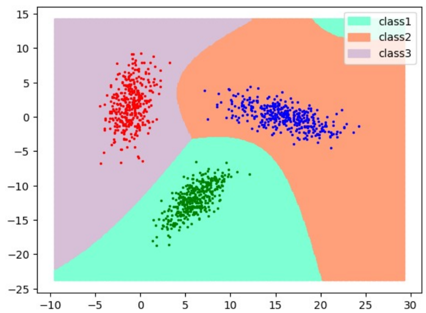
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 1 | 1 |
| Precision | 1 | 1. |
| Recall | 1 | 1 |
| F-Measure | 1 | 1 |

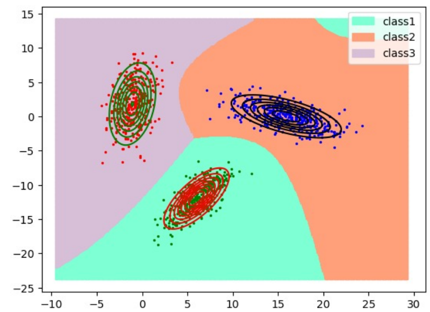
(b) Analysis

Table 15: LS : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 1.25e+02 | 1.00e-10 | 1.00e-10 |
| Class 2 | 1.00e-10 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 1. | 1. | 1. |
| Precision | 1. | 1. | 1. |
| Recall | 1. | 1. | 1. |
| F-Measure | 1. | 1. | 1. |

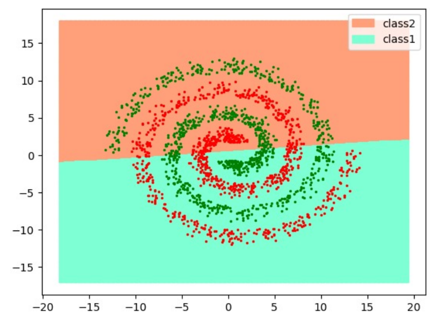
(b) Analysis

Table 16: LS : Class 1 , Class 2 and Class 3

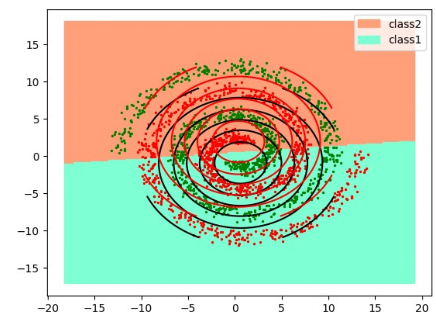
## 4.2 Non Linear Seperable Data

### 4.2.1 All Classes With Same Diagonal Matrix

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1 and 2




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.5140

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 201. | 190. |
| Class 2 | 190. | 201. |

(a) Confusion Matrix

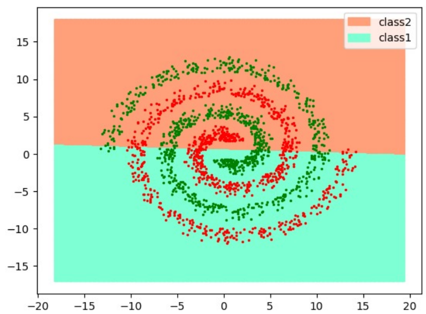
| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.5140 | 0.5140 |
| Precision | 0.5140 | 0.5140 |
| Recall | 0.5140 | 0.5140 |
| F-Measure | 0.5140 | 0.5140 |

(b) Analysis

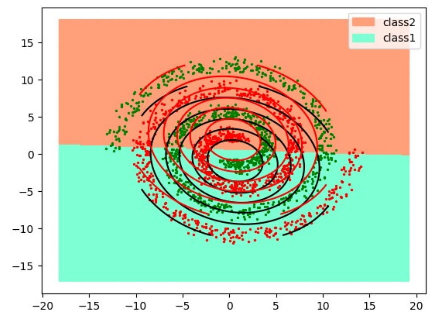
Table 17: NLS : Class 1 and Class 2

### 4.2.2 When All Have Same Covariance Matrices

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1 and 2




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.5140

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 201. | 190. |
| Class 2 | 190. | 201. |

(a) Confusion Matrix

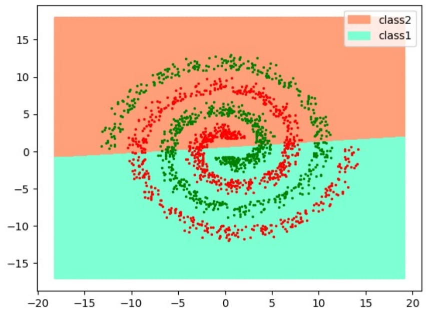
| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.5140 | 0.5140 |
| Precision | 0.5140 | 0.5140 |
| Recall | 0.5140 | 0.5140 |
| F-Measure | 0.5140 | 0.5140 |

(b) Analysis

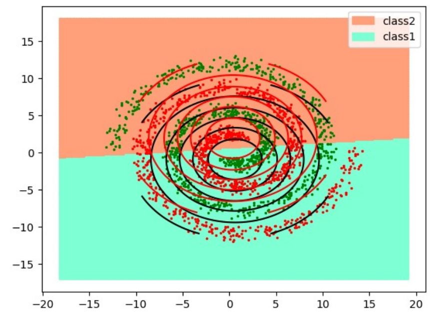
Table 18: NLS : Class 1 and Class 2

#### 4.2.3 All Classes Have Different Covariance Matrix But Are Diagonal

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1 and 2




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.5166

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 202. | 189. |
| Class 2 | 189. | 202. |

(a) Confusion Matrix

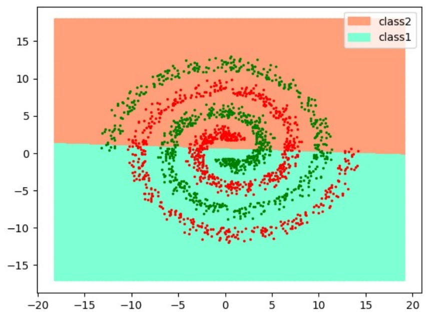
| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.5166 | 0.5166 |
| Precision | 0.5166 | 0.5166 |
| Recall | 0.5166 | 0.5166 |
| F-Measure | 0.5166 | 0.5166 |

(b) Analysis

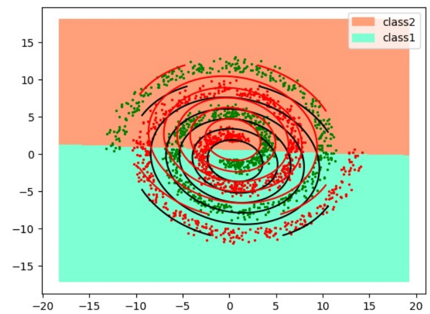
Table 19: NLS : Class 1 and Class 2

### 4.2.4 All Classes With Different Covariance Matrices

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1 and 2




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.5140

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 201. | 190. |
| Class 2 | 190. | 201. |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.5140 | 0.5140 |
| Precision | 0.5140 | 0.5140 |
| Recall | 0.5140 | 0.5140 |
| F-Measure | 0.5140 | 0.5140 |

(b) Analysis

Table 20: NLS : Class 1 and Class 2

## 4.3 Real World Data

### 4.3.1 When all Classes have same Diagonal Matrix

Decision Regions, Training Data and Contours

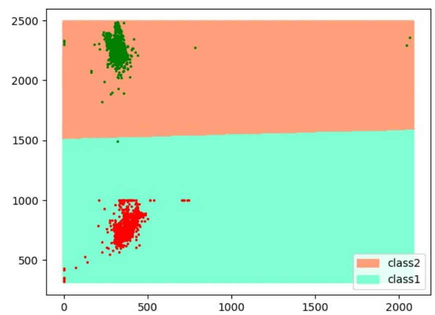



Figure 21: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9997

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 1.741e+03 | 1.000e-10 |
| Class 2 | 1.000e+00 | 1.670e+03 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.9997 | 0.9997 |
| Precision | 0.9994 | 1. |
| Recall | 1. | 0.9994 |
| F-Measure | 0.9997 | 0.9997 |

(b) Analysis

Table 21: Real World Data : Class 1 and Class 2

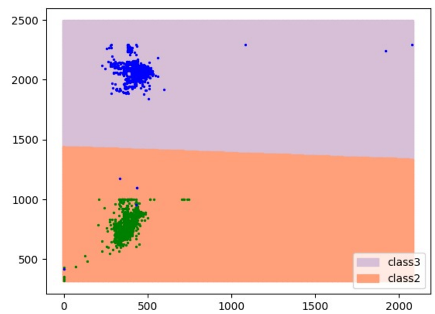



Figure 22: Decision regions and Training data Class2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.3280

| | Class 2 | Class 3 |
|---|---|---|
| Class 2 | 1.000e-10 | 1.671e+03 |
| Class 3 | 5.000e+00 | 1.598e+03 |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 0.4881 | 0.4881 |
| Precision | 2.00000000e-1 | 4.88834506e-01 |
| Recall | 5.98444045e-14 | 9.96880848e-01 |
| F-Measure | 1.19331742e-13 | 6.55993432e-01 |

(b) Analysis

Table 22: Real World Data : Class 2 and Class 3

Decision Regions, Training Data and Contours

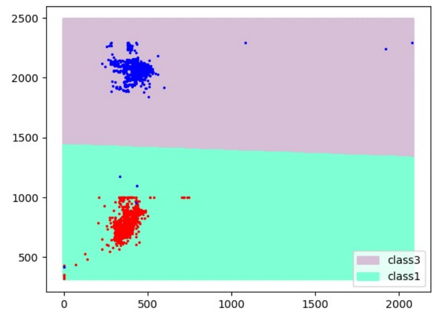



Figure 23: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9985

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 1.741e+03 | 1.000e-10 |
| Class 3 | 5.000e+00 | 1.598e+03 |

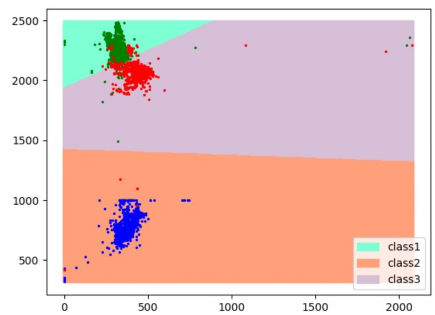
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 0.9985 | 0.9985 |
| Precision | 0.9971 | 1. |
| Recall | 1. | 0.9969 |
| F-Measure | 0.9986 | 0.9984 |

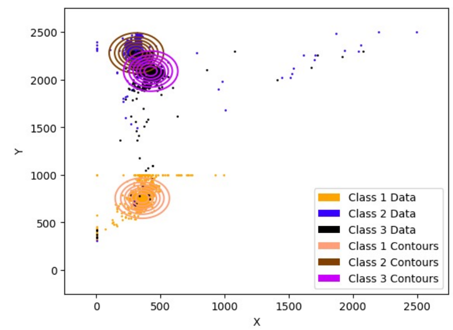
(b) Analysis

Table 23: Real World Data : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9629

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 1.741e+03 | 1.000e-10 | 1.000e-10 |
| Class 2 | 1.000e-10 | 1.615e+03 | 5.600e+01 |
| Class 3 | 5.000e+00 | 1.210e+02 | 1.477e+03 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 0.9990 | 0.9647 | 0.9637 |
| Precision | 0.9971 | 0.9303 | 0.9635 |
| Recall | 1. | 0.9665 | 0.9214 |
| F-Measure | 0.9986 | 0.9480 | 0.9420 |

(b) Analysis

Table 24: Real World Data : Class 1 , Class 2 and Class 3

### 4.3.2 When all Classes have same covariance Matrix

Decision Regions, Training Data and Contours

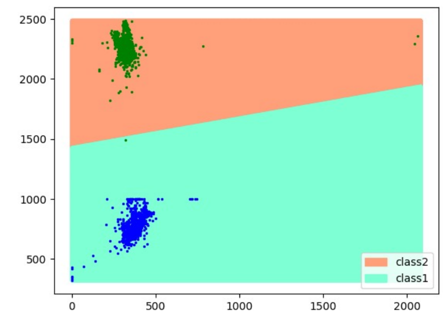



Figure 25: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9979

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 7.47e+02 | 1.00e-10 |
| Class 2 | 3.00e+00 | 7.14e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.9979 | 0.9979 |
| Precision | 0.996 | 1. |
| Recall | 1. | 0.9958 |
| F-Measure | 0.9979 | 0.9979 |

(b) Analysis

Table 25: Real World Data : Class 1 and Class 2

Decision Regions, Training Data and Contours

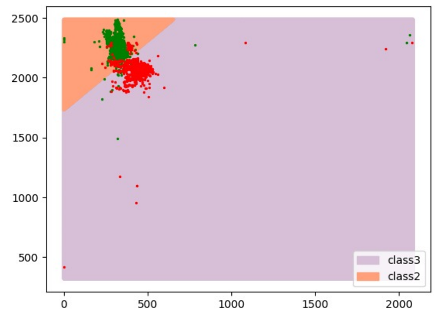



Figure 26: Decision regions and Training data Class2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.7186

| | Class 2 | Class 3 |
|---|---|---|
| Class 2 | 691. | 26. |
| Class 3 | 345. | 343. |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 0.7359 | 0.7359 |
| Precision | 0.6669 | 0.9295 |
| Recall | 0.9637 | 0.4985 |
| F-Measure | 0.7883 | 0.6490 |

(b) Analysis

Table 26: Real World Data : Class 2 and Class 3

Decision Regions, Training Data and Contours

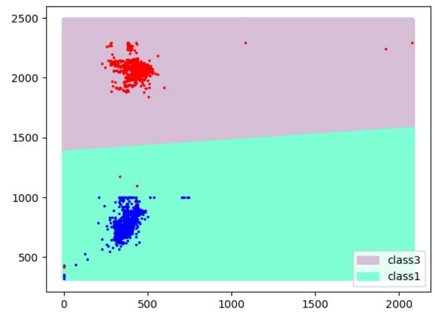



Figure 27: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9874

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 7.47e+02 | 1.00e-10 |
| Class 3 | 1.80e+01 | 6.70e+02 |

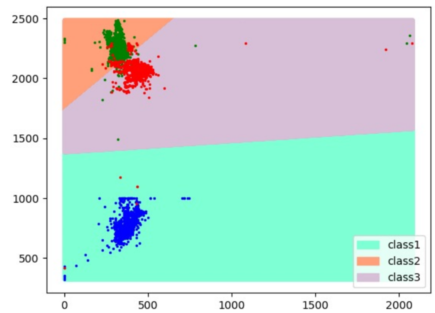
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 0.9874 | 0.9874 |
| Precision | 0.9764 | 1. |
| Recall | 1. | 0.9738 |
| F-Measure | 0.9880 | 0.9867 |

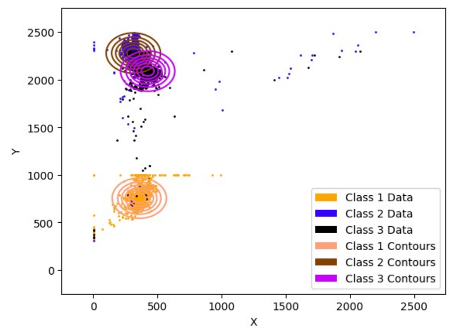
(b) Analysis

Table 27: Real World Data : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.8006381826303856

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 7.47e+02 | 1.00e-10 | 1.00e-10 |
| Class 2 | 3.00e+00 | 6.91e+02 | 2.30e+01 |
| Class 3 | 1.80e+01 | 3.45e+02 | 3.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 0.9902 | 0.8276 | 0.8206 |
| Precision | 0.9726 | 0.6669 | 0.9339 |
| Recall | 1. | 0.9637 | 0.4723 |
| F-Measure | 0.9861 | 0.7883 | 0.6274 |

(b) Analysis

Table 28: Real World Data : Class 1 , Class 2 and Class 3

### 4.3.3 When All Classes Have Different But Diagonal Covariance Matrix

Decision Regions, Training Data and Contours

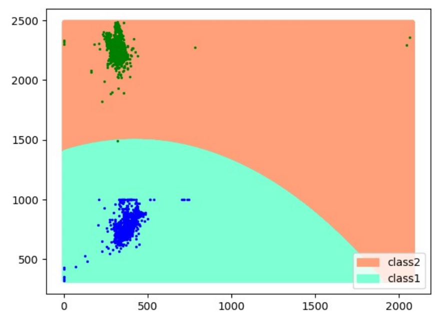



Figure 29: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9979

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 7.47e+02 | 1.00e-10 |
| Class 2 | 3.00e+00 | 7.14e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.9979 | 0.9979 |
| Precision | 0.996 | 1. |
| Recall | 1. | 0.9958 |
| F-Measure | 0.9979 | 0.9979 |

(b) Analysis

Table 29: Real World Data : Class 1 and Class 2

Decision Regions, Training Data and Contours

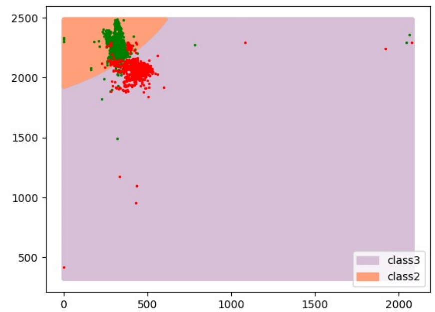



Figure 30: Decision regions and Training data Class 2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 1

| | Class 2 | Class 3 |
|---|---|---|
| Class 2 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 1. | 1. |
| Precision | 1. | 1. |
| Recall | 1. | 1. |
| F-Measure | 1. | 1. |

(b) Analysis

Table 30: Real World Data : Class 2 and Class 3

Decision Regions, Training Data and Contours

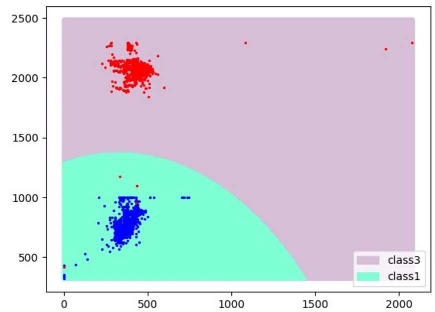



Figure 31: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9959

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 21.24e+02 | 1.00e+00 |
| Class 3 | 1.00e-10 | 1.25e+02 |

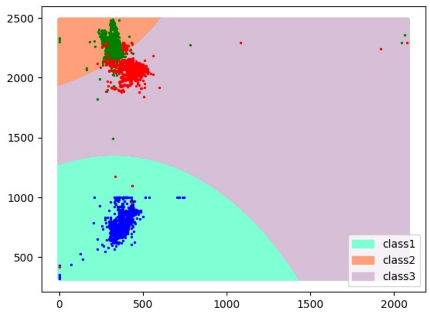
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 0.996 | 0.996 |
| Precision | 1. | 0.9920 |
| Recall | 0.992 | 1. |
| F-Measure | 0.9959 | 0.9960 |

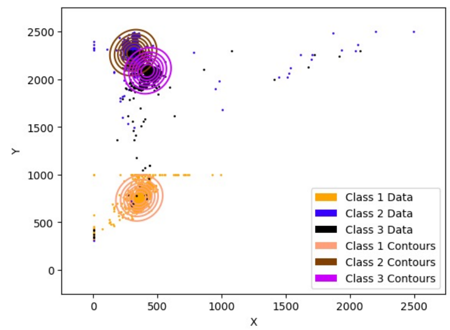
(b) Analysis

Table 31: Real World Data : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9973

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 1.24e+02 | 1.00e-10 | 1.00e+00 |
| Class 2 | 1.00e-10 | 1.25e+02 | 1.00e-10 |
| Class 3 | 1.00e-10 | 1.00e-10 | 1.25e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 0.9973 | 1. | 0.9973 |
| Precision | 1. | 1. | 0.9920 |
| Recall | 0.992 | 1. | 1. |
| F-Measure | 0.9959 | 1. | 0.9960 |

(b) Analysis

Table 32: Real world Data : Class 1, Class 2 and Class 3

### 4.3.4 When All The Classes Have Different Covariance Matrices

Decision Regions, Training Data and Contours

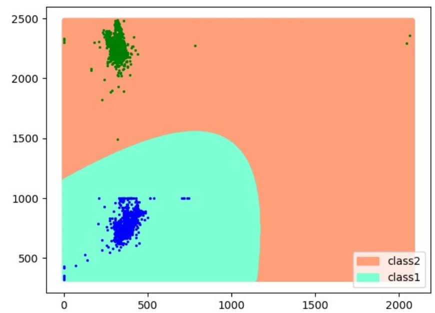



Figure 33: Decision regions and Training data Class 1 and 2

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9979

| | Class 1 | Class 2 |
|---|---|---|
| Class 1 | 7.47e+02 | 1.00e-10 |
| Class 2 | 3.00e+00 | 7.14e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 |
|---|---|---|
| Accuracy | 0.9979 | 0.9979 |
| Precision | 0.996 | 1. |
| Recall | 1. | 0.9958 |
| F-Measure | 0.9979 | 0.9979 |

(b) Analysis

Table 33: Real World Data : Class 1 and Class 2

Decision Regions, Training Data and Contours

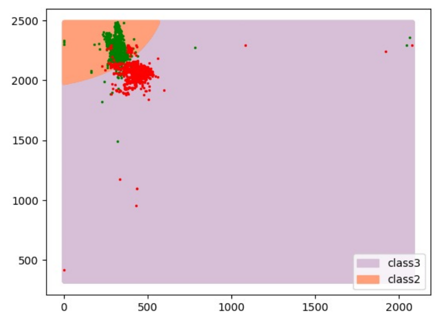



Figure 34: Decision regions and Training data Class 2 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.7225

| | Class 2 | Class 3 |
|---|---|---|
| Class 2 | 689. | 28. |
| Class 3 | 339. | 349. |

(a) Confusion Matrix

| | Class 2 | Class 3 |
|---|---|---|
| Accuracy | 0.7387 | 0.7387 |
| Precision | 0.6702 | 0.9257 |
| Recall | 0.9609 | 0.5072 |
| F-Measure | 0.7896 | 0.6553 |

(b) Analysis

Table 34: Real World Data : Class 2 and Class 3

Decision Regions, Training Data and Contours

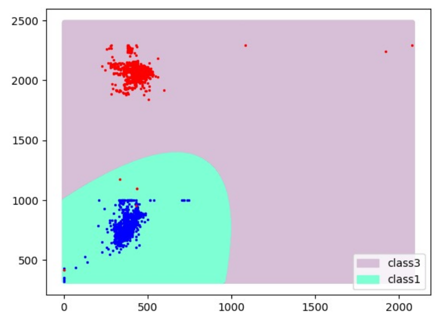



Figure 35: Decision regions and Training data Class 1 and 3

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.9888

| | Class 1 | Class 3 |
|---|---|---|
| Class 1 | 746 | 1. |
| Class 3 | 15. | 673. |

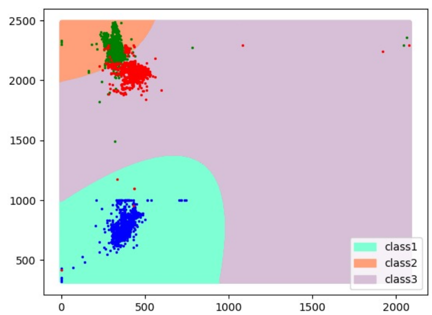
(a) Confusion Matrix

| | Class 1 | Class 3 |
|---|---|---|
| Accuracy | 0.9888 | 0.9888 |
| Precision | 0.9802 | 0.9985 |
| Recall | 0.9986 | 0.9781 |
| F-Measure | 0.9893 | 0.9882 |

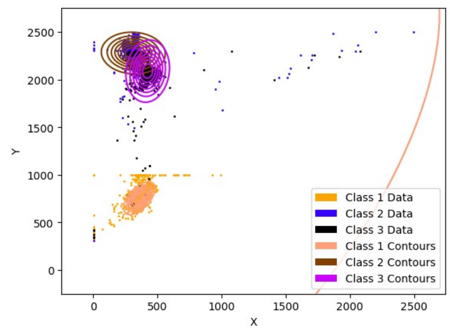
(b) Analysis

Table 35: Real World Data : Class 1 and Class 3

Decision Regions, Training Data and Contours




(a) Decision regions and Training data Class 1, 2 and 3




(b) Contours

**Confusion Matrix, Precision, Recall and F-measure**

Mean F-Measure : 0.8048

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Class 1 | 7.46e+02 | 1.00e-10 | 1.00e+00 |
| Class 2 | 3.00e+00 | 6.89e+02 | 2.50e+01 |
| Class 3 | 1.50e+01 | 3.39e+02 | 3.34e+02 |

(a) Confusion Matrix

| | Class 1 | Class 2 | Class 3 |
|---|---|---|---|
| Accuracy | 0.9911 | 0.8294 | 0.8234 |
| Precision | 0.9764 | 0.6702 | 0.9277 |
| Recall | 0.9986 | 0.9609 | 0.4854 |
| F-Measure | 0.9874 | 0.7896 | 0.6374 |

(b) Analysis

Table 36: Real World Data : Class 1 , Class 2 and Class 3

# 5 Conclusion

1. Data Separability and Bayes Classifier: Bayes Classifier excels when data is linearly separable, achieving high accuracy, but struggles with non-linearly separable data, resulting in lower accuracy.

2. Overlapping Real-World Data: Real-world data often exhibits overlapping characteristics between classes, leading to reduced accuracy when using the Bayes Classifier.

3. Decision Boundaries and Covariance Matrices: The choice of covariance matrices in the Bayes Classifier influences the decision boundary. Identical matrices produce a linear boundary, while different matrices can create quadratic or hyper-quadratic boundaries.

4. Bayes Classifier's Strengths: Despite its limitations, the Bayes Classifier is valued for its simplicity and ability to provide not just predictions but also confidence estimates, aiding decision-making.

5. Limitations of Bayes Classifier: The Bayes Classifier has limitations due to its assumptions about data distribution shapes. It may struggle with complex, non-linear decision boundaries in real-world data, which often exhibit randomness and non-linearity.