Computational Fluid Dynamics

Part 1: from high-school maths to CFD

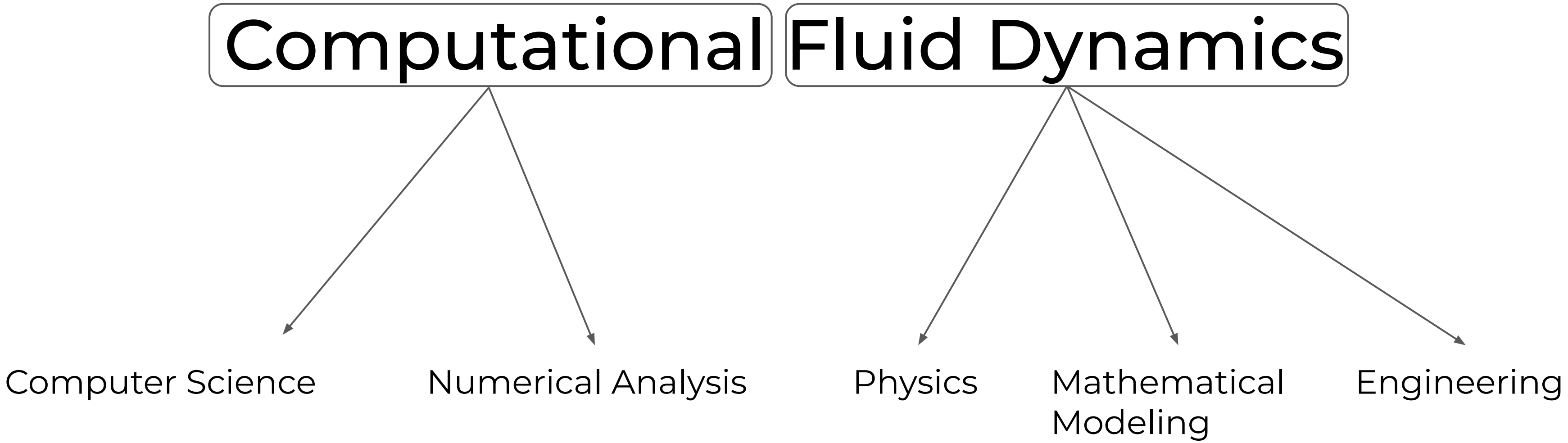
Mashy Green - Doom-conf - 14 May 2022

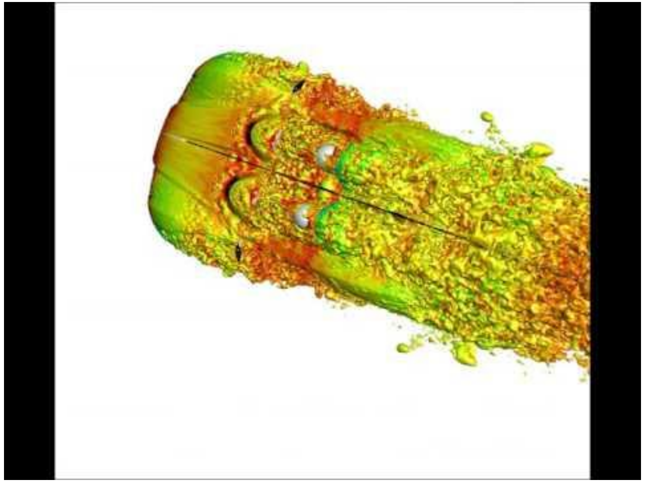
Who am I?

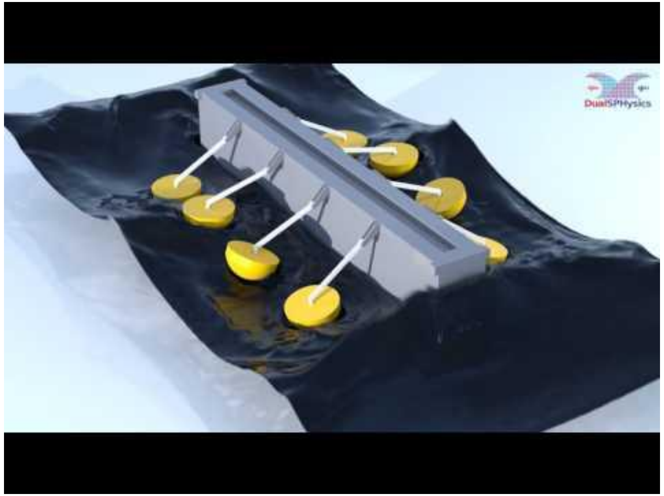
UNIVERSITY OF SUSSEX

Imperial College London

KING'S College LONDON


Computational
Fluid Dynamics
Computer Science
Numerical Analysis
Physics
Mathematical Modeling
Engineering




DualSPHysics

Layout

- Basics of differential equation
- Numerical differential equations
- Computational fluid dynamics

Basics of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x}$$

Basics of differential equations

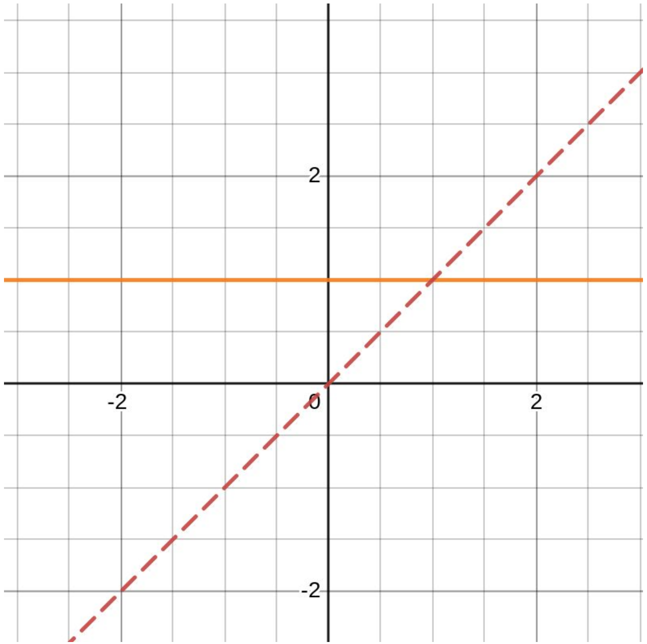
$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$$

$$y = ?$$

Basics of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = a$$

$$y = \int a \, dx = ax + c$$

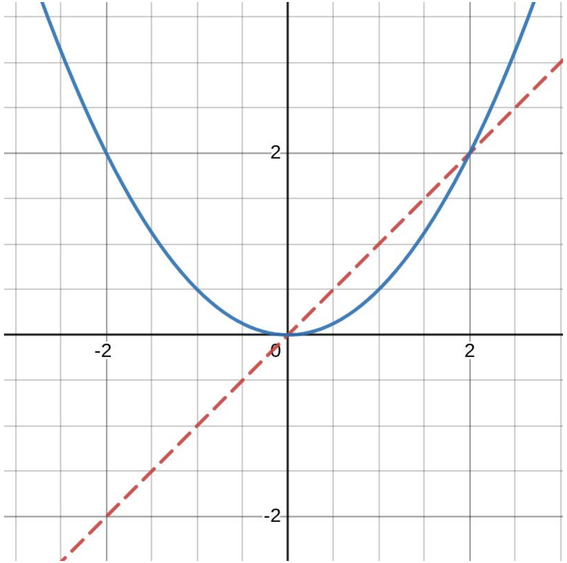


$$y(0) = c$$

Initial value

Basics of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = ax + b$$



$$y = \int (ax + b)\ dx = \frac{1}{2}ax^2 + bx + c$$

Basics of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \cos(x)$$

$$y = \int \cos(x)\ dx = \sin(x) + c$$

Basics of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\,ay$$

$$y = \int -\,ay\ dx\ =\ e^{-\,ay}\ +c$$

Basics of differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}x} \equiv \text{CHANGE}$$

Basics of differential equations

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{u}$$ Velocity

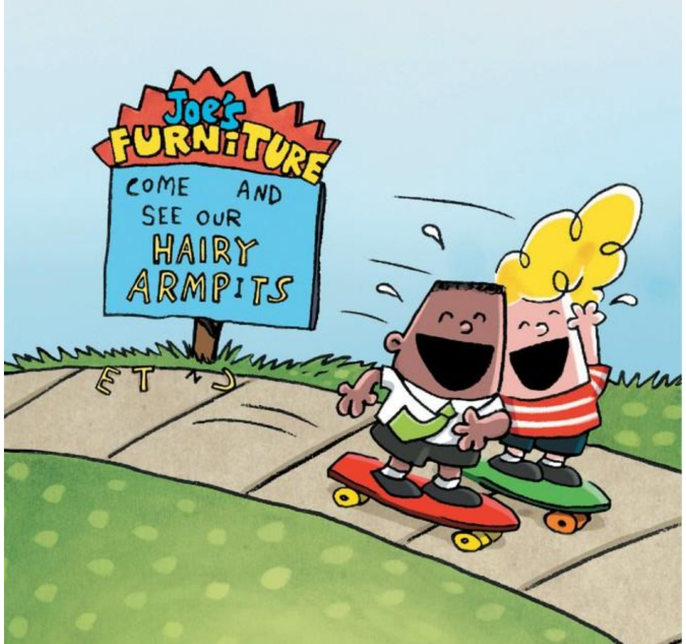
Basics of differential equations

$$\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} = \boldsymbol{u}$$ Velocity

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t} = \frac{\mathrm{d}\frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t}}{\mathrm{d}t} = \frac{\mathrm{d}^2\boldsymbol{x}}{\mathrm{d}t^2} = a$$ Acceleration

Basics of differential equations

$$\boldsymbol{F} = m\boldsymbol{a}$$



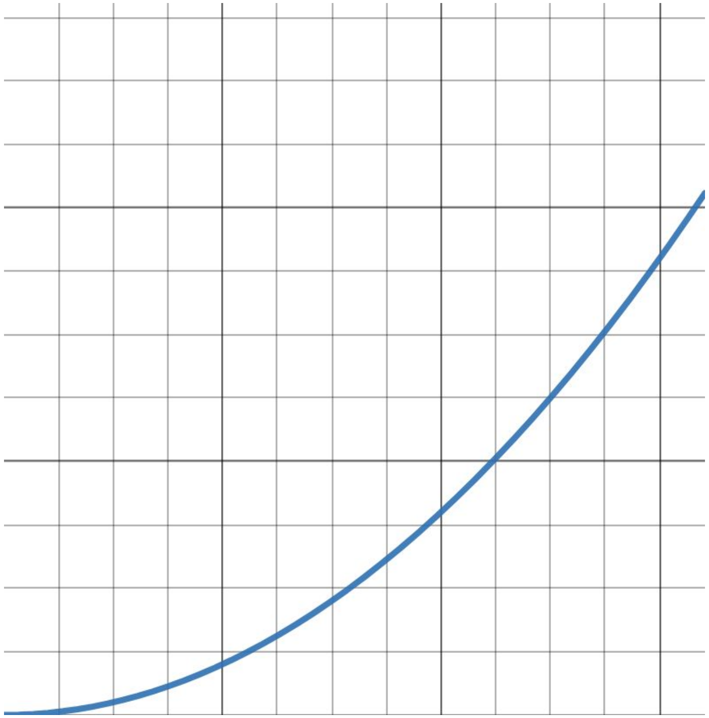
But even so, their silliness *did* get them into a lot of trouble, sometimes. In fact, it once got them into so much trouble, they accidentally almost destroyed the whole planet with an army of evil, vicious talking toilets!

But before I can tell you that story, I have to tell you *this* story. . . .

Numerical differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

$$y = ?$$



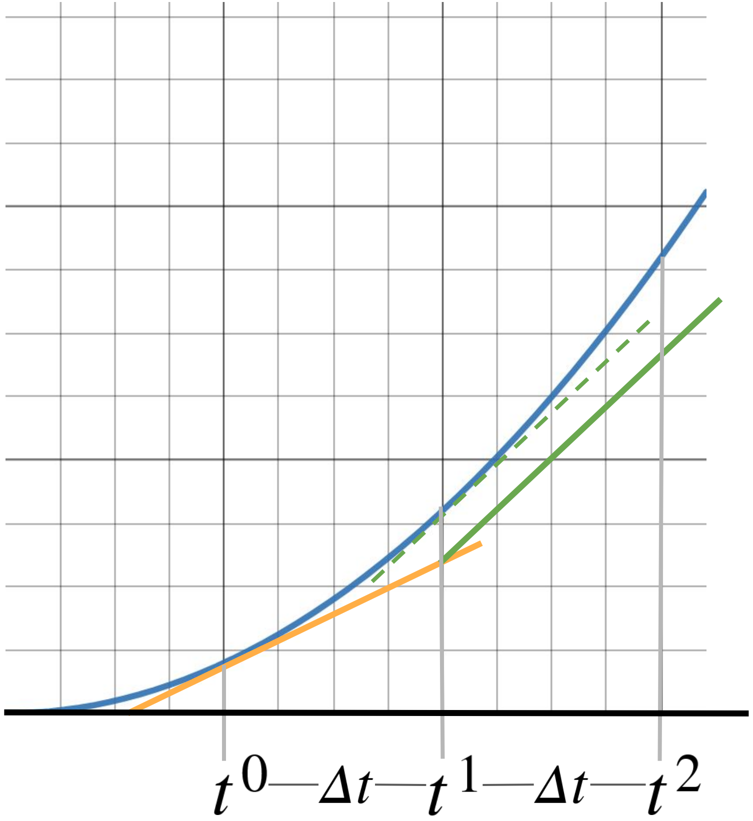
Numerical differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

Euler Method

$$y^{n+1} = y^n + \Delta t f(t^n, y^n)$$

$$y(t^0) = c$$



Numerical differential equations

To org-mode for some examples

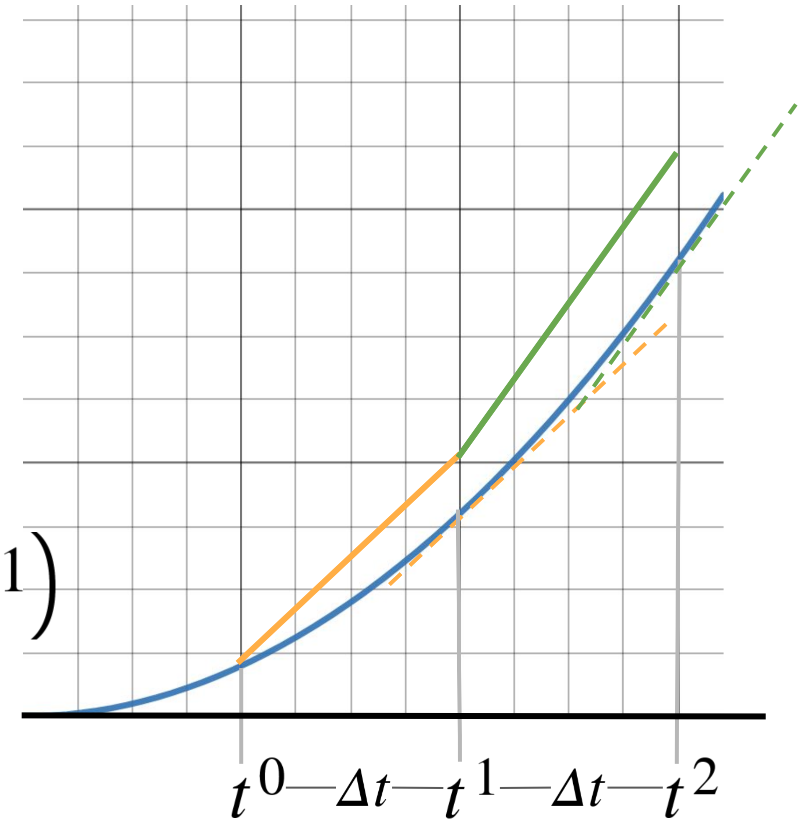
Numerical differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

Backwards Euler Method

$$y^{n+1} = y^n + \Delta t f\left(t^{n+1}, y^{n+1}\right)$$

$$y\left(t^0\right) = c$$



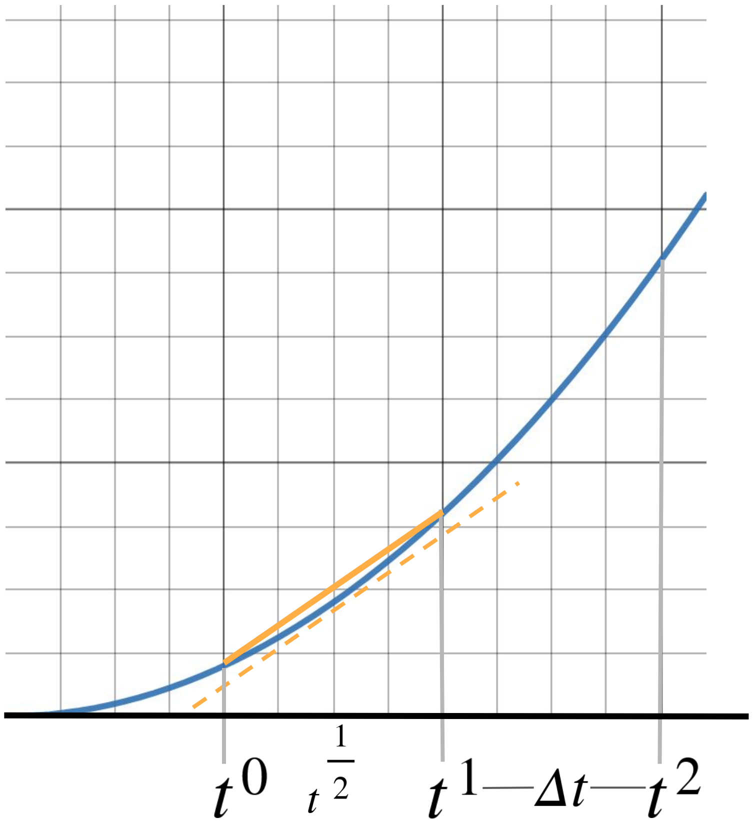
Numerical differential equations

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y)$$

Midpoint Method

$$y^{n+1} = y^n + \Delta t f\left(t^{n+\frac{1}{2}}, y^{n+\frac{1}{2}}\right)$$

$$y(t^0) = c$$



Numerical differential equations

To org-mode for some examples

‘computational’ fluid dynamics

$$\boldsymbol{F} = \boldsymbol{m}\boldsymbol{a}$$

‘computational’ fluid dynamics

$$\boldsymbol{F} = m\boldsymbol{a}$$

$$\rho = \frac{m}{V}$$

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = \boldsymbol{a}$$

‘computational’ fluid dynamics

$$\rho \frac{D\boldsymbol{u}}{Dt} = \boldsymbol{F}$$

‘computational’ fluid dynamics

$$\rho \frac{D\boldsymbol{u}}{Dt} = \boxed{\boldsymbol{F}}$$

‘computational’ fluid dynamics

$$\boldsymbol{F} = -\nabla P + \mu \nabla^2 \boldsymbol{u} + \rho f$$

Pressure **gradient**

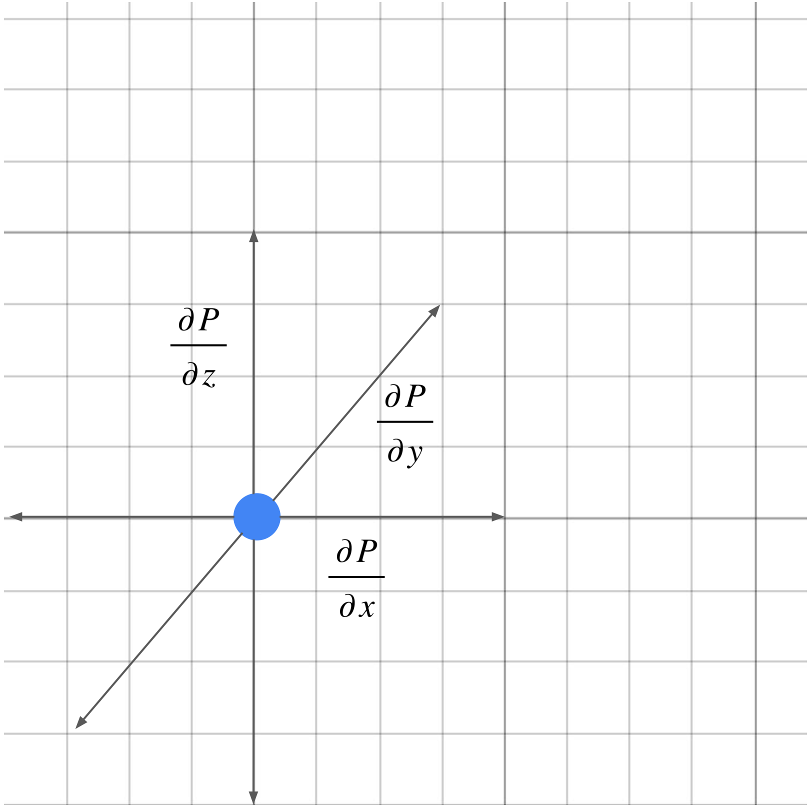
Dissipative force
due to *viscosity*

External **forces**:
***Gravity*;**
Surface tension;
Magnetic;
etc.

‘computational’ fluid dynamics

$$\nabla P = \left(\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z} \right)$$

$$P\big|_{\Omega} = c$$



‘computational’ fluid dynamics

$$\frac{D\boldsymbol{u}}{Dt} = \frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u}$$

‘computational’ fluid dynamics

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\left((\boldsymbol{u} \cdot \nabla) \boldsymbol{u} + \frac{1}{\rho} \nabla P \right) + \frac{\mu}{\rho} \nabla^2 \boldsymbol{u} + \boldsymbol{f}$$

Advection term | **Pressure** term | **Dissipative** term | External **forces**

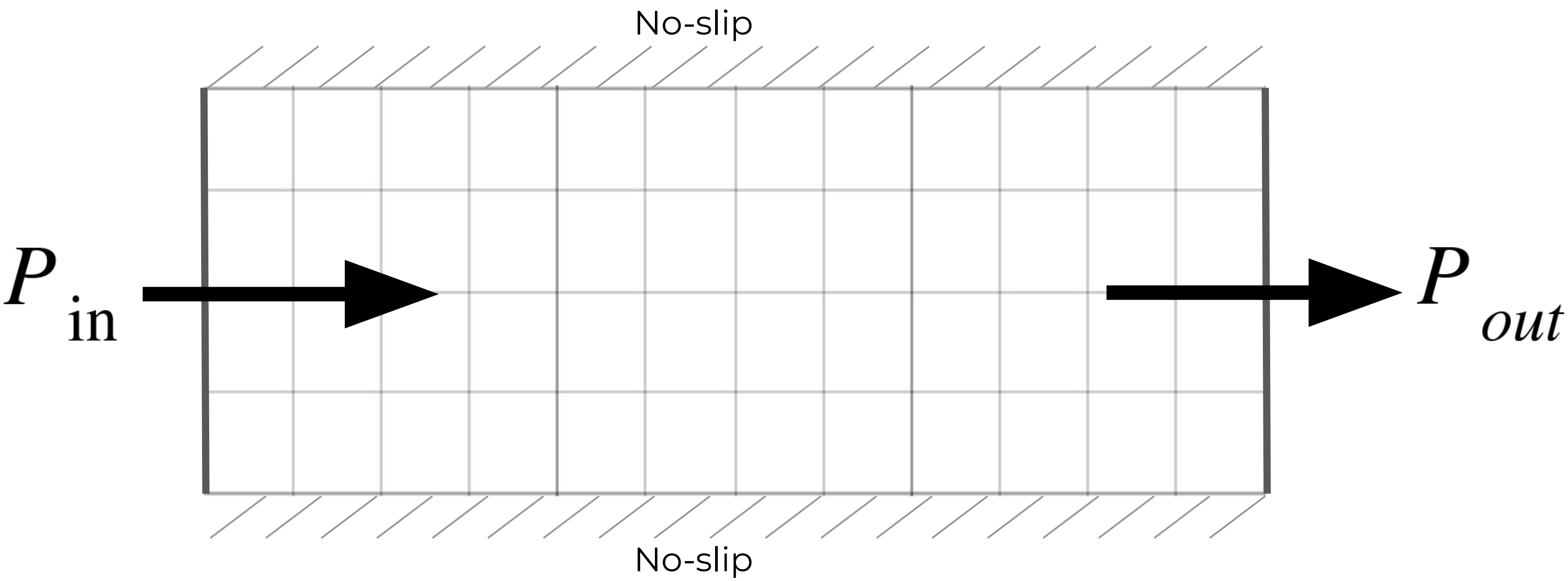
‘computational’ fluid dynamics

$$\frac{\partial \boldsymbol{u}}{\partial t} = -\left((\boldsymbol{u} \cdot \nabla)\,\boldsymbol{u} + \frac{1}{\rho} \nabla P \right) + \frac{\mu}{\rho} \nabla^2 \boldsymbol{u} + \boldsymbol{f}$$

$$\frac{\partial \rho}{\partial t} = -\rho \nabla \cdot \boldsymbol{u}$$

Velocity **divergence**

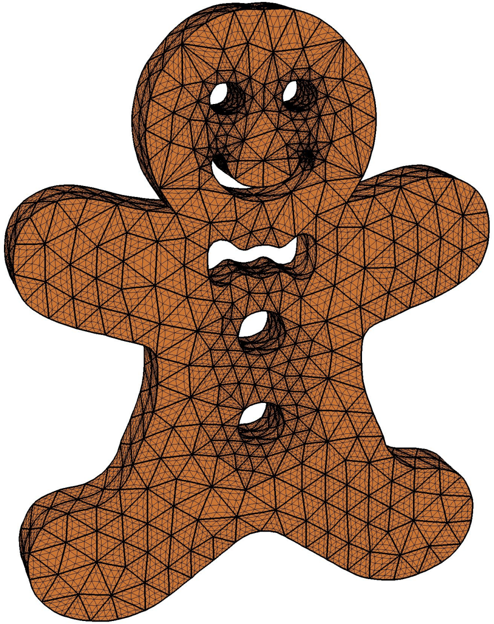
Basics of ‘computational’ fluid dynamics

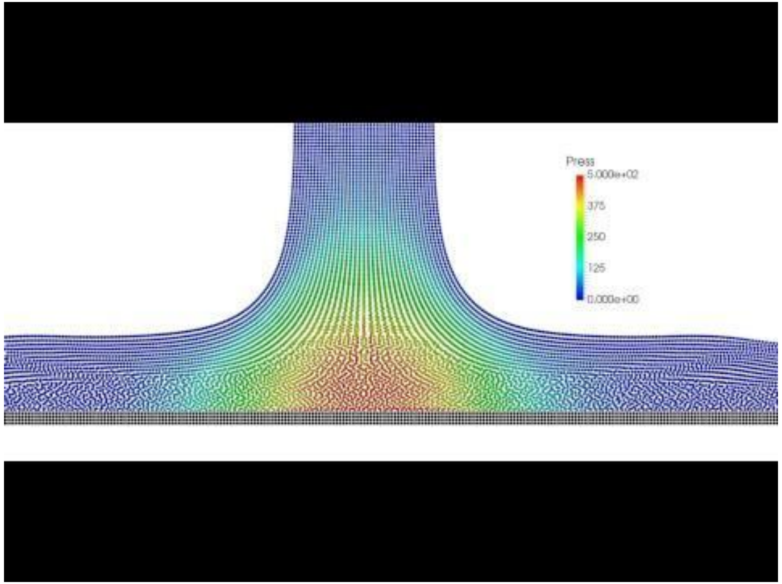


$$P_{in} > P_{out}$$

Computational Fluid Dynamics: part2

- Eulerean and Lagrangin forms
- Basic spatial discretisation methods
- Viscosity
- Pipe-flow (Poiseuille flow)
- Advance spatial discretisation: mesh generation and particle methods




Press
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375
250
125
0.000e+00