The Transportation Model

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importing neccessary libraries to solve model

```
library(lpSolve)
```

```
## Warning: package 'lpSolve' was built under R version 4.3.3
```

```
library(tinytex)
```

```
# Matrix as per the instruction
AEDs <- matrix(c(20,14,25,400,100,
                 12,15,14,300,125,
                 10,12,15,500,150, 80,90,70,"-","-"),ncol=5,byrow=TRUE)
colnames(AEDs) <- c("WareHouse1", "WareHouse2", "WareHouse3","Production cost","Production Capacity")
rownames(AEDs) <- c("Plant A", "Plant B", "Plant C", "Monthly Demand")
AEDs <- as.table(AEDs)
AEDs
```

```
##                WareHouse1 WareHouse2 WareHouse3 Production cost
## Plant A        20         14         25         400
## Plant B        12         15         14         300
## Plant C        10         12         15         500
## Monthly Demand 80         90         70         -
##                Production Capacity
## Plant A        100
## Plant B        125
## Plant C        150
## Monthly Demand -
```

1) Formulate and solve this transportation problem using R ?

1. Objective Function:

Provided transportation values can be formulated in the Linear Program format as :

Miminize $TC = 420x_{A1} + 414x_{A2} + 425x_{A3} + 312x_{B1} + 315x_{B2} + 314x_{B3} + 510x_{C1} + 512x_{C2} + 515x_{C3}$

2. Supply_constraints

$$x_{A1} + x_{A2} + x_{A3} \leq 100$$

$$x_{B1} + x_{B2} + x_{B3} \leq 125$$

$$x_{C1} + x_{C2} + x_{C3} \leq 150$$

3. Demand_Constraints:

$$x_{A1} + x_{B1} + x_{C1} \geq 80$$
$$x_{A2} + x_{B2} + x_{C2} \geq 90$$
$$x_{A3} + x_{B3} + x_{C3} \geq 70$$

4. Non-negativity_of_the_variables:

$$x_{ij} \geq 0$$

$$Xij$$

- denotes the number of AEDs to be shipped from plant i to warehouse j.

$$i = A, B$$

$$j = 1, 2, 3$$

In R, I solved a transportation problem that was initially unbalanced due to a discrepancy between supply and demand. To address the shortfall of 10 units in demand, I introduced a "dummy" variable in the fourth column of the cost matrix. This dummy variable had a transportation cost of zero and a demand of 10 units, ensuring that the supply and demand constraints were satisfied in the mathematical model, even when they were not balanced at the start.

```
# AEDs_Cost matrix
AEDs_Cost <- matrix(c(420,414,425,0,
                      312,315,314,0,
                      510,512,515,0),ncol = 4,byrow=TRUE)
## updating rows as 'constraints' and columns  as 'decision variables'
colnames(AEDs_Cost) <- c("WareHouse1", "WareHouse2", "WareHouse3","Dummy")
rownames(AEDs_Cost) <- c("Plant A", "Plant B", "Plant C")
AEDs_Cost
```

```
##         WareHouse1 WareHouse2 WareHouse3 Dummy
## Plant A        420        414        425     0
## Plant B        312        315        314     0
## Plant C        510        512        515     0
```

```r
#constraints for signs and right-hand sides(supply side)

row.signs <- rep("<=",3)
row.rhs <- c(100,125,150)
#supply function  shall not be greater than mentioned units.
col.signs <- rep(">=",4)
col.rhs <- c(80,90,70,10)
```

```r
lptrans <- lp.transport(AEDs_Cost, "min", row.signs, row.rhs, col.signs, col.rhs)
```

Determining how many units should be produced and transported from each plant.

```r
## Getting the optimum decision variables (6)values
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]   10   90    0    0
## [2,]   55    0   70    0
## [3,]   15    0    0   10
```

Plant A Units Shipped to Warehouse 1: 10 units

Plant A Units Shipped to Warehouse 2: 90 units

Plant A Units Shipped to Warehouse 3: 0 units

Plant B Units Shipped to Warehouse 1: 55 units

Plant B Units Shipped to Warehouse 2: 0 units

Plant B Units Shipped to Warehouse 3: 70 units

Plant C Units Shipped to Warehouse 1: 15 units

Plant C Units Shipped to Warehouse 2: 0 units

Plant C Units Shipped to Warehouse 3: 0 units

The function below will give the minimum value for the objective function

```r
lptrans$objval
```

```
## [1] 88250
```

Given the provided information and constraints, the minimum total cost for shipping and production amounts to 88,250 USD.

2) Formulating the dual of the transportation problem ?

In a primal linear programming model, the number of decision variables corresponds to the number of constraints in its dual model. To formulate the dual problem, we start by analyzing the primal linear programming model. We then convert the primal's minimization problem into a maximization problem in the dual. This conversion introduces new variables, typically represented as 'm' and 'n', which are used to solve the dual problem.

```r
AEDs_2 <- matrix(c(420,414,425,100,"m1",
312,315,314,125,"m2",
510,512,515,150,"m3",
80,90,70,250,"-",
"n1","n2","n3","-","-"),ncol=5,nrow=5,byrow=TRUE)
colnames(AEDs_2) <- c("WareHouse1","WareHouse2","WareHouse3","Production Capacity","Supply (Dual)")
rownames(AEDs_2) <- c("PlantA","PlantB","PlantC","Monthly Demand","Demand (Dual)")
AEDs_2 <- as.table(AEDs_2)
AEDs_2
```

```
##                WareHouse1 WareHouse2 WareHouse3 Production Capacity
## PlantA         420        414        425        100
## PlantB         312        315        314        125
## PlantC         510        512        515        150
## Monthly Demand 80         90         70         250
## Demand (Dual)  n1         n2         n3         -
##                Supply (Dual)
## PlantA         m1
## PlantB         m2
## PlantC         m3
## Monthly Demand -
## Demand (Dual)  -
```

$$\text{Max } \ Z = 100m_1 + 125m_2 + 150m_3 + 80n_1 + 90n_2 + 70n_3$$

Subject to the following constraints

$$m_1 + n_1 \leq 420$$
$$m_1 + n_2 \leq 414$$
$$m_1 + n_3 \leq 425$$
$$m_2 + n_1 \leq 312$$

$$m_2 + n_2 \leq 315$$
$$m_2 + n_3 \leq 314$$
$$m_3 + n_1 \leq 510$$
$$m_3 + n_2 \leq 512$$
$$m_3 + n_3 \leq 515$$

Where n1 = WareHouse_1

n2 = WareHouse_2

n3 = WareHouse_3

m1 = Plant_1

m2 = Plant_2

m3 = Plant_3

These constants come from the transposed matrix of the primal linear programming function. To double-check your work, transpose f.con into the matrix and compare it with the constants listed earlier.

where

$$mk, nl$$

where m = 1,2,3 and n = 1,2,3

```r
#Objective_function
f.obj <- c(100,125,150,80,90,70)
#transposed the constraints matrix in the primal
f.con <- matrix(c(1,0,0,1,0,0,
                  1,0,0,0,1,0,
                  1,0,0,0,0,1,
                  0,1,0,1,0,0,
                  0,1,0,0,1,0,
                  0,1,0,0,0,1,
                  0,0,1,1,0,0,
                  0,0,1,0,1,0,
                  0,0,1,0,0,1), nrow = 9, byrow = TRUE)
f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=",
           "<=")
f.rhs <- c(420,414,425,312,315,314,510,512,515)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 157040
```

```r
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 414 312 510   0   0   2
```

So Z=157,040 dollars and variables are:

$$m_1 = 414$$

which represents Plant A

$$m_2 = 312$$

which represents Plant B

$$m_3 = 510$$

which represents Plant C

$$n_1 = 0$$

which represents WareHouse 1

$$n_2 = 0$$

which represents WareHouse 2

$$n_3 = 2$$

which represents WareHouse 3

3) Economic interpretation of the dual-capacity and warehouse constraints?

The dual values associated with plant production capacity constraints offer insights into the economic opportunity cost of increasing production at each plant. For example, a positive dual value of $10 for Plant A indicates a benefit to boosting production. A value of zero, such as that for Plant B, means that the current capacity is adequate to meet demand. Conversely, a negative value, like -$5 for Plant C, suggests excess production and possible cost savings.

The dual values tied to warehouse demand constraints reflect the price per unit at each warehouse. Higher values, like $20 for Warehouse 1, indicate stronger demand and the potential for higher prices. Lower values, such as $14 for Warehouse 2, signal less demand. Values of $0 or negative, as observed for Warehouse 3, imply minimal economic benefit in increasing supply.

These dual values help Heart Start make informed decisions on production, distribution, pricing, and cost management. They highlight the balance between production capacity and demand fulfillment across various plants and warehouses.