3 (a) $x \leftarrow$ feature vector ; We have K classes.

$$Pr(G=k \mid X=x) = \frac{e^{\beta_k^T x}}{1 + \sum_{l=1}^{K-1} e^{\beta_k^T x}}$$

for $K = 1, 2, 3, \ldots K-1$

$$Pr(G=K \mid X=x) = \frac{1}{1 + \sum_{l=1}^{K-1} e^{\beta_l^T x}}$$

let $A_i = e^{\beta_i^T x}$

$$Pr(G=K \mid X=x) = \frac{A_K}{1 + \sum_{l=1}^{K-1} A_l}$$

$$\sum_{i=1}^{K-1} A_i = \cancel{\frac{A_1 + A_2 + \cdots A_{K-1}}{1+}}$$

$$= \frac{\sum_{i=1}^{K-1} A_i}{1 + \sum_{i=1}^{K-1} A_i}$$

Add Prob$(G=K \mid X=x)$ to both sides

$$Pr(G=K \mid X=x) + \sum_{i=1}^{K-1} A_i = \frac{\sum_{i=1}^{K-1} A_i}{1 + \sum_{i=1}^{K-1} A_i} + \frac{1}{1 + \sum_{i=1}^{K-1} A_i}$$

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$$Pr(G=K \mid X=x) + \sum_{i=1}^{K-1} A_i = \frac{1 + \sum_{i=1}^{k-1} A_i}{1 + \sum_{i=1}^{K-1} A_i} = 1$$

$\boxed{\therefore \text{ Sum of posterior probabilities } = 1}$

3 (b) $$p(x) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}.$$

assume $A = e^{\beta_0 + \beta_1 X}$

$$p(x) = \frac{A}{1+A}.$$

$$(1+A)(p(x)) = A.$$

$$p(x) + p(\cancel{x})A = A.$$

$$p(x) = A - p(\cancel{x}).A.$$

$$= A(1 - p(\cancel{x})).$$

$$\frac{p(x)}{1-p(x)} = A.$$

$$\boxed{\frac{p(x)}{1-p(x)} = e^{\beta_0 + \beta_1 X}}$$