Test	**Hypothesis**	**Conclusion**	**Examples**
Geometric Series	$a_n = ar^n$	$\sum_{n=0}^{\infty} a_n = \frac{a}{1-r}$ for $\lvert r\rvert < 1$ and diverges otherwise	$\sum_{n=0}^{\infty} 5\left(\frac{1}{2}\right)^n = \frac{5}{1-\frac{1}{2}} = 10$. $\sum_{n=0}^{\infty} 2^n$ diverges. $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ converges.
p-series	$a_n = \frac{1}{n^p}$	$\sum_{n=1}^{\infty} a_n$ converges for $p > 1$ and diverges otherwise	$\sum_{n=0}^{\infty} \frac{1}{\sqrt{n^3}}$ converges. $\sum_{n=0}^{\infty} \frac{1}{\sqrt[3]{n^2}}$ diverges.
Telescoping series	There is significant cancellation in s_n	Compute $\lim_{n\to\infty} s_n$ directly	For $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$, $s_n = 1 - \frac{1}{n+1}$ and so $\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \lim_{n\to\infty} s_n = 1$. $\sum_{n=1}^{\infty} \log\left(1+\frac{1}{n}\right)$ diverges.
Test for Divergence	$\lim_{n\to\infty} a_n \neq 0$	$\sum_{n=1}^{\infty} a_n$ diverges	Let $a_n = \frac{n^2+1}{2n^2+9}$. Since $a_n \to \frac{1}{2} \neq 0$, $\sum_{n=1}^{\infty} a_n$ diverges. $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ diverges.
Integral Test	f continuous, positive, decreasing on $[1,\infty)$, $a_n = f(n)$	If $\int_1^{\infty} f(x)\,dx$ (converges, diverges), then $\sum_{n=1}^{\infty} a_n$ (converges, diverges)	$\sum_{n=1}^{\infty} e^{-n}$ converges since $\int_1^{\infty} e^{-x}$ converges. $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges since $\int_1^{\infty} \frac{1}{x}$ diverges. $\sum_{n=1}^{\infty} \frac{\log n}{n}$ diverges. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.
Comparison Test	$0 < a_n \leq b_n$, $\sum_{n=1}^{\infty} b_n$ converges	$\sum_{n=1}^{\infty} a_n$ converges	$\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges since $\frac{1}{n^2+1} < \frac{1}{n^2}$ and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges. $\sum_{n=1}^{\infty} \frac{5}{2n^2+4n+3}$ converges. $\sum_{n=1}^{\infty} \frac{2+\cos n}{\sqrt{n^3}}$ converges.
	$a_n \geq b_n > 0$, $\sum_{n=1}^{\infty} b_n$ diverges	$\sum_{n=1}^{\infty} a_n$ diverges	$\sum_{n=1}^{\infty} \frac{2}{n}$ diverges since $\frac{1}{n} < \frac{2}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges. $\sum_{n=1}^{\infty} \frac{\log n + 3}{n}$ diverges.
Limit Comparison Test	$a_n \geq 0$, $b_n \geq 0$, $\lim_{n\to\infty} \frac{a_n}{b_n} = c \in (0,\infty)$ (Test inconclusive if limit is zero or ∞)	Both $\sum_{n=1}^{\infty} a_n$, $\sum_{n=1}^{\infty} b_n$, (converge , diverge)	$\sum_{n=1}^{\infty} \frac{1}{2^n-1}$ converges since $\lim_{n\to\infty} \frac{\frac{1}{2^n-1}}{\frac{1}{2^n}} = 1$ and $\sum_{n=1}^{\infty} \frac{1}{2^n}$ converges. $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$ diverges.
Alternating Series Test	$a_n > 0$, $a_n \geq a_{n+1}$, $\lim_{n\to\infty} a_n = 0$	$\sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges	$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$ converges since $\frac{1}{n} > 0$, $\frac{1}{n} > \frac{1}{n+1}$, $\lim_{n\to\infty} \frac{1}{n} = 0$. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{\sqrt{n}}$ converges. Both converge conditionally.
Ratio Test	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L < 1$	$\sum_{n=1}^{\infty} a_n$ is absolutely convergent	$\sum_{n=1}^{\infty} \frac{2^n}{n!}$ converges since $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 0$. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^3}{3^n}$ is absolutely convergent.
(Inconclusive if $L = 1$)	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = L > 1$ or $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = \infty$	$\sum_{n=1}^{\infty} a_n$ is divergent	$\sum_{n=1}^{\infty} \frac{3^n}{n^3}$ diverges since $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 9$. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{2^k}{k}$ diverges.
Root Test	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L < 1$	$\sum_{n=1}^{\infty} a_n$ is absolutely convergent	$\sum_{n=1}^{\infty} \left(\frac{2n+6}{5n-3}\right)^n$ converges since $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = \frac{2}{5}$.
(Inconclusive if $L = 1$)	$\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L > 1$ or $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = L = \infty$	$\sum_{n=1}^{\infty} a_n$ is divergent	$\sum_{n=1}^{\infty} \frac{2^n}{\sqrt{3^n}}$ diverges since $\lim_{n\to\infty} \sqrt[n]{\lvert a_n\rvert} = \frac{2}{\sqrt{3}}$.