Quantum Approximate Optimization Algorithm (QAOA) for the Modern Energy Systems

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***Abstract*— Quantum Computing (QC) has been around for a long time now. It is a ground fact that the era of the quantum supremacy as the technology is growing at ever faster rate not very far. Currently Technology is in Noisy Intermediate Scale- Quantum Era. QAOA is a quantum algorithm for combinatorial optimization problems to find the extremum, that starts with an ansatz. QAOA is essentially an algorithm to approximate the solution of a combinatorial optimization problem. In given paper basic understanding of QAOA has been proposed, later three different use cases of how QAOA plays important role in Energy System optimization is delineated using IBM QISKIT platform. For All the use case IBM QASM Simulator has been used to simulate the quantum environment.**

I. Introduction

Use of Quantum Computers (QC) in solving NP-Hard and NP-Complete problems is one of the potential area where QC can outperform conventional computers. Currently, Noisy Intermediate Scale Quantum (NISQ) computers are developed by many firms and a large amount of research going on in Quantum informatics theory. The two main classifications of quantum computational optimization: (i) Quantum Annealing (QA) physical heuristic and (ii) Quantum Approximate Optimization Algorithm (QAOA) on the gate-model quantum computer. This paper is dedicated to QAOA algorithms on the gate-model quantum computer due to its robustness proven over the time and how gate model quantum computers are constantly performing better in all the aspects in last few years post development.[1]

The QAOA is one of the leading heuristic algorithms in quantum computing, due to its simplicity and robustness. The QAOA is already implemented as an algorithm applied to a simplified real-world aircraft assignment problem known as the tail assignment problem. The QAOA is also successfully applied to other (NP-Hard) linear algebra problems. As compared to other quantum algorithms QAOA has an advantage due to its lower depth characteristics. For this reason, QAOA is more robust against noise and decoherence. Furthermore, the QAOA is robust against certain systematic errors.[2]

Unit commitment is one of the most important optimization problems in large-scale electrical power industry the economical operations of the power system. The Unit Commitment problem is generally formulated as Mix-Integer Non-linear problems i.e. MIQP and it is very difficult to solve due to the nonlinear cost function and the combinatorial nature of the set of feasible solutions. It has been proven that UC is NP-complete, so it is impossible to develop an algorithm with polynomial computation time to solve it. Various techniques to solve the UC problem have been proposed based on mixed-integer programming, heuristic units, evolutionary programming, constraint logic programming, interior point method, and neural network to solve advanced UC models, especially when high renewable energy sources are integrated into the power systems are of critical importance. With the increase in the number of Energy resources, it is ever hard to develop an algorithm to solve the UC problem.[3]

Fault detection in large power line is one area where there is a lot of research currently focused on in the industry. The economical cost for black out is gigantic and many severe industries like hospital suffer from the power loss. More importantly power loss and fluctuations cost lives of valuable power devices.

Thus, detecting fault in large lines are one of the serious topic. There are many existing methods which uses real time Neural Networks to diagnosis faults in the power lines. QAOA due to its quickness in solving large matrix and Unconstrained problems plays important part in the systems.[4]

With Increasing amount of Distributed Energy Resources (DERs) deploying in the power system and grids trend of Peer-to-peer energy transfer is increasing. The most important aspect of P2P market is auction mechanism and price clearance of the time and energy block. Mostly Knapsack problems is formulated around the scenario and assets are divided by solving the Knapsack problem. Mainly MIP or MINLP problems are used to solve such problems using classical solvers like GuRoBI or CPLEX. With use of QAOA knapsack problem soled more effectively and quickly by using QISKIT runtime.[5]

II. Case Study-I: Unit Commitment problem to tackle renewable uncertainty using QAOA.

Unit commitment is one of the important problems to solve in the energy system. With more DERs adding to the energy grid uncertainty is increasing exponentially and to solve this issue better forecasting methods should be coupled with conventional energy source optimization. Main problem when it comes to tackle uncertainty is Switching on cost of the large power plants and its trade of with generation cost caused by raw material as well as man force. For the case study purpose six different energy resources has been considered for the generation with their generation and operation costs.

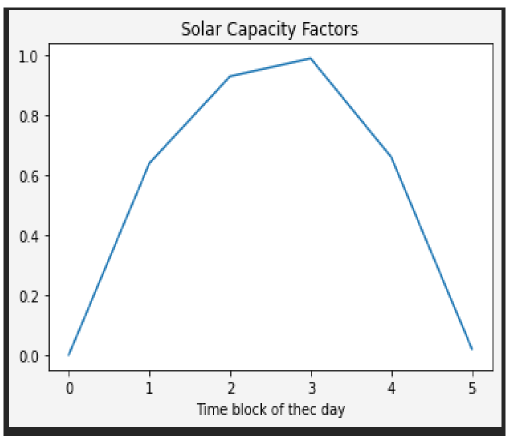

Figure 1 Solar Generation Output

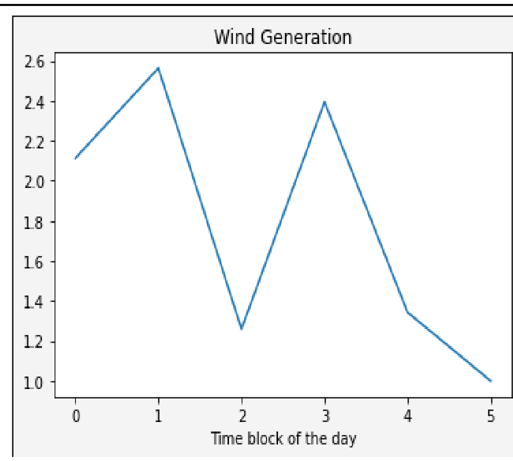

Figure 2 Wind Generation capacity

Energy Resources	Capacity(MW)	Operating Cost(1/1000)	Switch On Cost(1/1000)
Nuclear	6	23	50
Coal	6	38	8
Gas	5	33	1
Hydro	3	13	2
Solar	8	10	0
Wind	6	10	0

Table 1 Operation and Switch on Cost

The given unit commitment problem generally formulated as Mix Integer Quadratic Program optimization problem given by

$$\min \sum f_i$$

$$\text{s.t. } f_i = A_i y_j + B\text{i}P_i + C_i P_i^2$$

$$y_i \in \{0,1\}$$

$$P_{\min j} y_i < P_i < P_{\max j} y_i$$

Mostly Unconstrained optimization problems are mapped in current quantum computers so to accommodate given problem MIQP converted into the Quadratic Unconstrained Binary Optimization (QUBO) problem. QUBO problem converted into Ising Hamiltonian, which are solved by quantum anstraz to produce binary string solution with minimal ground state energy.

Binary Variable	6
Total Variable	36

Table 2 Variables for the problem

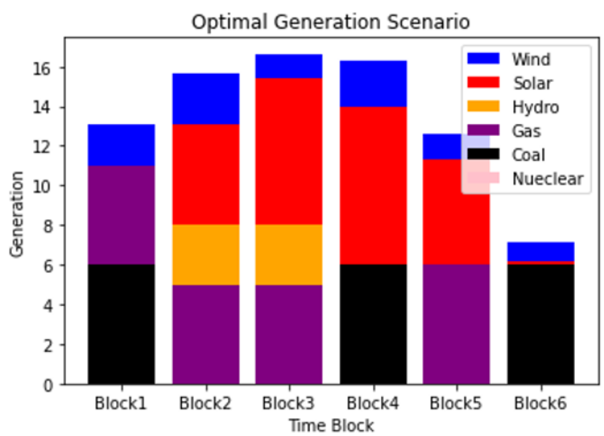

Figure 3 Optimal Scenario Output

Energy Load data	https://energy.acm.org/resources/
Data divided in 6 blocks	1 block= 4Hrs

Table 1 Data Information

The given solution suggests that constant use of Gas plant due to low operating cost and Coal plant as base unit since running cost as well as fuel cost is balanced. The given problem is also solved using intel i7 16 GB RAM processor using classical computing which gave maxed up RAM usage waring and failed to compute the problem using NumPy Eigenvector Solver provided by IBM.

III. Fault detection in Power Line Using QAOA variations

Fault diagnosis in the power line and power system is one of the most important part of the important. This not only save the damage prone power system equipment like transformers and switch yard equipment but also save economical cost caused by blackout in the area. For the case study A power system has been simulated in MATLAB and faults has been introduced to the system for dataset generation

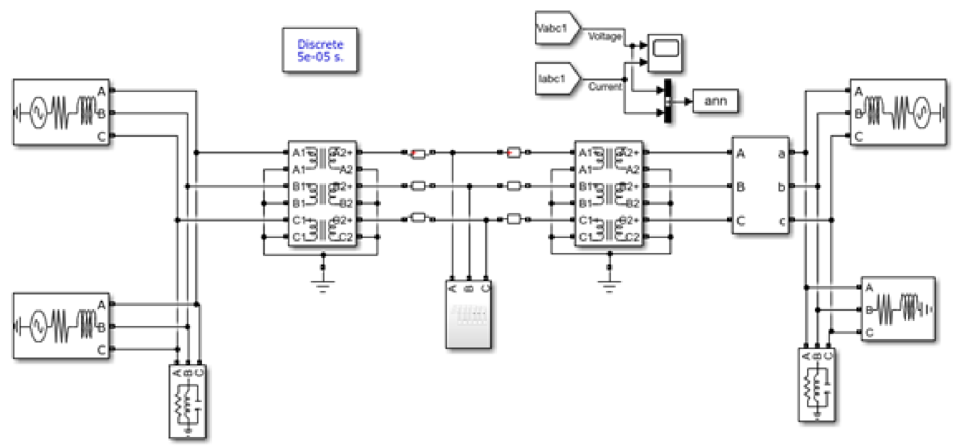

Figure 4 for Power system Design in MATLAB

For the case study purposes we are considering 10 rows of the data due to limitation in QRAM as well size of Matrix. With the current state of Noisy Intermediate Scale Quantum (NISQ) technology, the current approach to using quantum optimization algorithms to perform unsupervised clustering is difficult to scale to larger data sets. The current implementation of this problem with ten rows of data from the Motor Trends Car data set creates a 10x10 matrix. Each additional row expands this matrix by n+1, and the matrix by n*n. This means 10 rows would be have 100 values compared to the 20 values in a 20x20 matrix. This rapid increase in problem size means that we had to use a small data set to keep the size of the weight matrix required. to perform the graph optimization problem to a size compatible with the modest quantum hardware available.

There are three different methods used to compare the algorithm (1) Normal QAOA (2) Warm start QAOA (3) QAOA on IBM Qiskit API runtime. It is noted that Warm start QAOA performed the fastest in all of three methods with accurate solution. The energy of ground state was equal to the energy state when problem solved by classical solvers.

IIIA. Warm start QAOA

QAOA has lacked theoretical guarantees on its performance as well as its ability to outperform classical algorithms. Warm start QAOA has been shown to have a higher probability of sampling the optimal solution. For more details on the math underlying Warm-start QAOA there is paper by [7][8]. The QAOA method uses Max-Cut optimization instead of K-Mean to cluster the data.

IIIB. Method of Warm Start QAOA

1. Transform the problem data into a QUBO quadratic formulation
2. Relax the QUBO formulation and its binary variables into a semi-definite convex problem
3. solve Semi-definite problem, using a classical optimizer so it becomes a multiple continuous variables in [0,1] range
4. Convert problem to an Ising Hamiltonian

5. Pass the relaxed semi-definite continuous variables to form the mixer operator for the Warm-started QAOA

For the warm start QAOA GuRoBi solver is used to find optimal starting point as well as SPSA solver to optimize the parameters while calculations.

	Normal QAOA	Warm Start QAOA	IBM Runtime QAOA	Classical NumPy
Energy	5.6593	-1365.7178	88.1463	-1353.0691
Function Objective	3526.239	4250.3475	3558.1558	4250.3475
Process Time	0.4823s	1.0527	0.8164	65.0972s

Table 3 QAOA Result Comparison

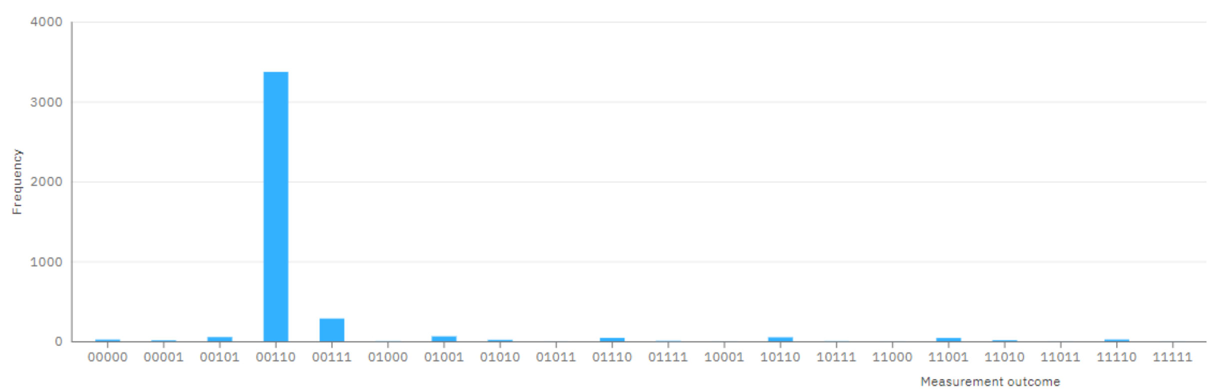

Figure 5 Histogram of Binary String for optimal solution

IV. Peer to Peer Energy Price clearing mechanism Based on QAOA Knapsack Problem

Distributed generation also creates need for effective peer-to-peer energy market where multiple sellers and buyers transact energy within. This paper suggests a trading market platform designed for making peer-to-peer (P2P) energy transactions. The problem is formulated in terms of Knapsack 0-1 Problem and QAOA Knapsack problem is implemented to bring maximum financial benefit to prosumers. Case study is presented for a microgrid of 1 seller and 7 buyers and simulation results show that during every auction seller gets the highest economic surplus, while consumers get locally produced energy at cheaper than grid price. For the given study battery constrains also have been considered for the transactions. [5][6]

Different 7 electric load has been considered and modelled using System Advisory model(SAM) using historical data available publicly on https://energy.acm.org/resources. This case has been specifically considered as Single central seller and multiple independent buyer. For the case all buyers are trading energy using Battery Energy Storage System(BESS) has been used and constrain for BESS has been implemented to conserve the life spam of the batteries.

The knapsack problem has been formulated around the seller input and buyers bid according to

$$\max\left[\sum_{i=1}^{k} w_i * p_i\right]$$

$$\sum_{i=1}^{k} w_i < E$$

$$20 < SOC_{t-1} < 80$$

Where,

w_i = Amount $Of\ energy$

$p_i = Price\ Of\ energy$

$E = \ Total\ Energy\ for\ sale$

SOC_{t-1}= $State\ of\ charge\ at\ t-1\ iteration$

Based on the given Knapsack problem Quadratic Constrain Quadratic optimization problem has been formulated using IBM qiskit. This knapsack problem solved at each time block to get optimal solution of energy distribution. Here is sample of QCQO problem at particular time frame:

```
Maximize
 obj: 32 x_0 + 8 x_1 + 10 x_2 + 60 x_3 + 117 x_4 + 88 x_5 + 2 x_6
Subject To
 c0: 4 x_0 + 2 x_1 + 2 x_2 + 5 x_3 + 9 x_4 + 8 x_5 + x_6 <= 23
 c1: x_3 = 0

Bounds
 0 <= x_0 <= 1
 0 <= x_1 <= 1
 0 <= x_2 <= 1
 0 <= x_3 <= 1
 0 <= x_4 <= 1
 0 <= x_5 <= 1
 0 <= x_6 <= 1

Binaries
 x_0 x_1 x_2 x_3 x_4 x_5 x_6
End

result: [1. 0. 1. 0. 1. 1. 0.]
```

Figure 6 Sample Quadratic Constrain Program

Hour	Seller Quantity	Buyer 1 Quantity	Buyer 1 Bid	X1	Buyer 2 Quantity	Buyer 2 Bid	X2	Buyer 3 Quantity	Buyer 3 Bid	X3	Buyer 4 Quantity	Buyer 4 Bid	X4	Buyer 5 Quantity	Buyer 5 Bid	X5	Buyer 6 Quantity	Buyer 6 Bid	X6	Buyer 7 Quantity	Buyer 7 Bid	X7	Profit
10	12	5	8	1	3	4	0	3	5	0	6	12	1	8	13	1	7	11	0	1	2	1	218
11	23	4	8	1	2	4	0	2	5	1	5	12	0	9	13	1	8	11	1	1	2	0	247
12	16	4	8	1	2	4	1	1	4	1	3	11	1	6	15	0	5	14	1	1	6	1	153
1	24	2	8	0	3	8	0	4	9	1	5	12	1	8	14	1	7	13	1	3	9	0	299
2	19	3	7	1	5	10	1	4	6	0	6	9	1	1	7	1	3	10	1	2	8	0	162
3	29	5	8	1	3	4	0	5	11	1	5	11	1	9	12	1	8	11	1	2	12	0	346

Table 3 Transection Summary for 6-time block

V. Conclusion and future work

QAOA is one of the powerful quantum algorithms out there currently. While using for Mix Integer Quadratic programing problems, Classical optimizer GuRoBi performed better in the smaller unit commitment problems. But when number of variable increases GuRoBi is no longer performing at par with QAOA. For the future work a broder Unit commitment would be considered and solved by forming MINLP problems with the help of classical optimizer to compare efficiency and error with QAOA methods. When normal QAOA method is used for the Fault detection of the power system it misidentified fault in earlier scenarios, although The results of research show that Warm-start QAOA produces more consistent results than other algorithms on quantum simulators. This work also showed that non-convex quadratic programs can be relaxed successfully using the GuRoBi optimizer and used to make QAOA consistently produce consistently better performance. For the energy market mechanism No classical solver could optimize the process of mechanism while QAOA method did solved the knapsack problem quickly. For the future work MIP problem as a knapsack problem modelled and solved by GuRoBi to compare the robustness of both the algorithms.

VI. References

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